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## Menu-pricing and Quality Decisions of a Platform Monopolist

**Tetsuya Shinkai**

(School of Economics, Kwansai Gakuin University)

**Naoshi Doi**

(Otaru University of Commerce—Economics)

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SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho  
Nishinomiya 662-8501, Japan

# Menu-pricing and Quality Decisions of a Platform Monopolist \*

Tetsuya Shinkai<sup>†</sup>

School of Economics, Kwansei Gakuin University  
1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan

Naoshi Doi

Otaru University of Commerce– Economics,  
3-5-21 Midori, Otaru, Hokkaido 047-8501, Japan

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## Abstract

This article examines menu-pricing and quality decisions of a platform monopolist for two types of sellers and buyers on a two-sided market. Under the GPD (general Pareto distribution) valuation of buyers for transaction services, we show that unique optimal services fees exist for sellers and buyers. The two types of services (premium and spot) are offered to both sellers and buyers. An optimal premium membership fee and the quality service level are considered for the premium type of buyers in a platform optimization problem. Assuming that the unit cost of the product is fixed, we show that the optimal membership fee/the level of quality service for premium-type buyers decreases/increases as the service cost for premium-type sellers increases. However if delivery fees charged by transport companies for spot-type sellers increase, the optimal membership fee/level of quality service increases/decreases. However, if the demand for services of the platform for both types of buyers increases, both the optimal membership fee and quality level of services increase.

Keywords: Platform monopoly; Menu-pricing; Quality decisions; Two-sided market.  
JEL Classification Codes: D21, D43, L13, L15

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<sup>†</sup>Corresponding author. School of Economics, Kwansei Gakuin University, 1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan. E-mail: shinkai@kwansei.ac.jp. Phone: +81-798-54-6967. Fax: +81-798-51-0944.

# 1 Introduction

In recent years, the COVID-19 pandemic has affected digital services delivered by digital platforms in the mobility and hospitality sectors because of lockdown, stay-at-home policies, and social distancing provisions by governments in many countries. The same policies have boosted home delivery, digital communication, and the consumption of some digital goods. Amazon.com is one of the digital platforms that has been affected by these policies. Originally started as a business distribution system that used the internet to sell books and other merchandise. Amazon has developed a massive market around the world that handles everything from consumers' internet orders for a variety of goods to collection of payments, inventory adjustment, and delivery. The sudden jump in demand for internet trade services due to these policies against the pandemic has pushed up the delivery services cost of Amazon for its customers in both buyers' and sellers' side markets. Based on July 26, 2022 Nikkei newspaper electric version in Nikkei.com<sup>1</sup>, Amazon raised its "Prime" membership fees in major European countries and the US. The rate of increase is substantial, ranging from 20% to 43%. This rise of fees may be attributable to the increases in logistics costs. In Japan, however, the fee is left unchanged, though logistics costs are rising there too.

Our research question is 'Is the rise of the "Prime" membership fee to pass on to consumers (buyer) the higher logistics cost? ' If this is true, why does Amazon not rise the "Prime" membership fee only in Japan? To answer this question, we consider the pricing and quality decision of the platform monopolist for two types of sellers and buyers in a two-sided internet market by extending Wan and Wright's (2017) model to a model

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<sup>1</sup>Nikkei newspaper electric version article on July 26(2022), (in Japanese) <https://www.nikkei.com/article/DGXZQOGN264TF0W2A720C2000000/>

including discrimination of (premium, spot) services for both sellers and buyers side markets and an endogenous level of quality for premium buyers. By deriving monopoly equilibrium choices and conducting comparative statics of the choices on service (delivery) cost, we explore how an increase in service (delivery) cost affects the annual membership fee for premium buyers through the level of quality for premium buyers.

There are works in literature on two-sided (multi-sided) market and the economics of platforms, e.g. Rochet and Tirole (2003,2006), Caillaud and Jullien (2003), Armstrong (2006), and Bellflamme and Peitz (2018, 2019a, 2019b, 2021).

Rochet and Tirole (2006) present a rough definition of two-sided (more generally, multi-sided) markets ‘as markets in which one or several platforms enable interactions between end-users and try to get two (or multiple) sides “on board” by appropriately charging each side.’ They state that the theory of two-sided markets is related to the theories of network externalities and of (market or regulated) multiproduct pricing. Thus, the multiproduct pricing literature does not consider externalities in the consumption of assorted products or services.

This study is in the flow of the latter multiproduct pricing literature. Accordingly, our model we construct in this study does not include network externality but includes decision of the quality of services that the platform offers to one types of buyers. In our model, a platform monopolist offers menu-pricing for both buyer and seller side markets, two differentiated (premium, spot) services as Amazon does.

Contributions of this study are: (1) We propose the model on pricing and quality decision of platform monopolist for two types of sellers and buyers in a two-sided market. This model, is possibly the first one that explicitly considers menu-pricing in a two-sided service market. (2) In the model, we show that the platform can conduct second-degree price discrimination for the two-sided markets on its shopping site.(3)Provided that the

unit cost of the product is fixed, we show that the optimal membership fee decreases as the service cost for the premium type sellers rises. (4) We present as (3), an example of the result of Doi and Shinkai (2023), in that a cost increase may decrease price (i.e., a negative pass-through) and may increase quality when quality and prices are endogenously chosen in a monopoly. (5) We present the actual behavior of Amazon Japan in that it tries to reduce the service cost for premium-type sellers and not to decrease the membership fee for premium buyers in the future.

## 2 The Model

This section introduces the model. Subsection 2.1 provides the overview. Subsections 2.2, 2.3, and 2.4 explains in detail the modeling of buyers, sellers, and a platform, respectively.

### 2.1 Overview of the model

We consider a platform that facilitates transactions between sellers and buyers through intermediation on the website. Figure 1 shows the framework of the model. All sellers provide homogenous items with the same marginal cost. Buyers are heterogeneous with respect to their valuation of the item. Modeling regarding sellers and buyers is based on the model of Wang and Wright (2017), while we extend it to incorporate two types of transaction services the platform provides.

The platform provides two types of transaction services: “premium” and “spot.” The premium transaction provides a high-quality delivery of an item, such as a fast delivery and delivery at the designated time. The premium transaction can be used only by the “member” buyers who have paid a membership fee.<sup>2</sup> The spot transaction with a

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<sup>2</sup>In the real economy, the platform differentiates between the buying customers (consumers) who

standard-quality delivery can be used by any buyer with no payment to the platform.

Buyers are divided by their membership status. An “existing member” (hereafter EM) has already paid the membership fee and holds the membership status. EMs can use both the premium and spot transactions without payment to the platform. A “non-member” (hereafter NM) does not hold the membership status. While NMs can use spot transaction freely, they should become new members by paying the membership fee to use the premium service. EMs are heterogeneous in their valuation of an item, as are NMs.

The platform decides on the membership fee ( $A$ ), the quality of the premium service ( $X$ ), and charges levied on sellers ( $T^P$  for a premium transaction and  $T^S$  for a spot transaction).<sup>3</sup> The service quality of the spot transaction is standardized to one. We focus on a one-shot profit maximization of the platform and conduct comparative statics for the equilibrium.

## 2.2 Buyer side

This subsection explains the buyer side. Because we consider a platform problem to set not only the charges per transaction ( $T^P$  and  $T^S$ ) but also the membership fee ( $A$ ), we assume that there are two groups of buyers with different membership statuses: EMs (existing members) and NMs (nonmembers). Behind this assumption, we implicitly as-

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purchase various goods on the platform. For example, if a consumer is an Amazon Prime member (annual membership fee 4900 yen), the “Amazon Prime Eligible Item” is displayed on the site. The item is then shipped to the purchaser by the next day (through one-day delivery). However, if the buyer is not an Amazon Prime member, the “Amazon Prime Eligible Item” is not displayed. The buyer can view and order only the items that can be purchased as a spot transaction. The items are not shipped through one-day delivery.

<sup>3</sup>According to the charge system of Amazon, we assume that transaction charges are levied on sellers. The results described in the subsequent sections does not change when charges are assumed to be levied on buyers.

sume a dynamic situation depicted in Figure 2.<sup>4</sup> In each time period, NMs enter the market and decide whether to buy one unit of the good. The choice set is purchasing it by using the premium transaction, purchasing it by using the spot transaction, and not buying it on the platform (including purchasing it through another channel). Note that if NMs choose the premium transaction, they should become a new member by paying the membership fee. Membership is effective during two periods, that is, the period when an NM pays the membership fee and the subsequent period. This means that the NMs who choose the premium service by paying the fee in period  $t - 1$  become EMs in period  $t$ . The EMs can use the premium transaction with no payment to the platform. In addition, the platform provide the transaction service for not only the good focused on here ( $c$ ) but also another good ( $c'$ ). The EMs in the market of good  $c$  in period  $t$  include buyers who has become a member for purchasing the good  $c'$  by using the premium transaction in period  $t - 1$ .

The utility of buyers is modeled following Wang and Wright (2017). Because Wang and Wright (2017) consider only one type of transaction service a platform provides, we extend the model to incorporate the two types of transaction services into the model. The utility from not buying is standardized to zero.

The net utility that a buyer purchases an item with the spot transaction is assumed to be as follows:

$$u^S = c(1 + b) - p^S, \quad (1)$$

where  $p^S$  is the price that an buyer pays to an seller for an item with the spot transaction. Following Wang and Wright (2017), the valuation of an item is assumed to be proportionate to the marginal cost ( $c$ ).<sup>5</sup> Buyers' heterogeneity in the valuation of an

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<sup>4</sup>On this idea of time flow of purchase decision of buyers, we owe to the comment of Hiroaki Ino.

<sup>5</sup>As discussed in Wang and Wright (2017), the assumption that buyers' values for a good can be

item is represented by  $b$ . Its cumulative distribution function is denoted by  $F(b)$  with the support  $[0, \bar{b}]$ . Only the buyers know their own value  $b$ , and  $F$  is known to the public.

The valuation of getting an item is higher when the premium transaction is used than when the spot transaction is used, because the former is associated with a higher-quality delivery than the latter. Specifically, the net utility for the premium transaction is assumed to be as follows:

$$u^P = c(1 + b(1 + X)) - p^P, \quad (2)$$

where  $p^P$  is the price in the case of the premium transaction and  $X$  is the quality of the premium transaction (the quality of the spot is standardized to zero). This functional form indicates that the value of a higher quality is not uniform across buyers: it is evaluated more by a buyer who feels a larger value for an item (i.e., a larger  $b$ ) than by a buyer with a smaller value. Because the quality of the delivery of the premium transaction is linked to how fast a buyer can obtain the item transacted, the quality is supposed to be important for buyers who highly appreciate the item.

Note that  $u^S$  and  $u^P$  are common across EMs and NMs. The only difference between EM and NM buyers is whether a buyer has paid the membership fee or not. In turn, we consider the decision problems of EMs and NMs.

EMs can use the premium transaction without a payment of the membership fee because they have already paid it previously (Figure 2). An EM chooses one option that gives the highest utility among three options: the premium transaction ( $u^P$ ), the spot transaction ( $u^S$ ), and not to buy (0).

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scaled by  $c$  is plausible, and supported by previous empirical studies, for example, Einav et al. (2015). By quasiexperimentally observing a large number of auctions for various goods across the internet, they found that the distribution of buyer valuation for a good is proportional to the transaction price.



Figure 3 shows how the choices of EMs are divided into three according to their evaluation for an item ( $b$ ). The solid line represents the utility from the spot transaction ( $u^S$ ). The dashed line is that from the premium transaction ( $u^P$ ). While both increase in  $b$ , the slope is larger for  $u^P$  than for  $u^S$ . This is because for the premium transaction, the utility from the service quality, as well as that from an item itself, increases as  $b$  increases. Figure 3 shows that the option chosen by an EM is changed according to  $b$ . A buyer chooses not to buy if  $b$  is smaller than  $\tilde{b}$ , where  $\tilde{b}$  is the value of  $b$  such that  $u^S = 0$  and  $\tilde{b} = \frac{p^S - c}{c}$ . A buyer uses the spot transaction if  $b$  is in the medium range of  $[\tilde{b}, \hat{b}^E]$ , where  $\hat{b}^E$  is the value of  $b$  such that  $u^S = u^P$  and  $\hat{b}^E = \frac{p^P - p^S}{cX}$ . If a buyer has  $b$  larger than  $\hat{b}^E$ , the premium transaction is chosen.

We turn to the choice problem of NMs. While NMs can use the spot transaction freely, they should pay the membership fee to the platform if they use the premium service. If they pay the fee, they become a member until the next period (Figure 2). NMs are therefore assumed to decide whether they become a member or not based on the total utility of the present and next periods. To simplify the model, the discount rate is assumed to be zero. We assume that NMs believe that the prices and quantity in the next period are the same as in the present period. This assumption means that the option chosen in the next period becomes the same as in the present period. NMs therefore compare the utilities obtained from paying the membership fee and purchasing an item with the premium service in the present and next periods ( $2u^P - A$ ), purchasing it with the spot service in both periods ( $2u^S$ ), and not buying it in both periods (0).

Figure 4 depicts how the option chosen by an NM is changed according to  $b$ . The solid line represents the total utility of the two periods for the spot transaction ( $2u^S$ ). The dashed line is the total utility net the payment of the membership fee ( $2u^P - A$ ). As in Figure 3, the slope of the dashed line is larger than that of the solid line. Figure 4 shows

that the chosen option depends on  $b$ . An NM purchases no item if  $b$  is less than  $\tilde{b}$ , where  $\tilde{b}$  is the value of  $b$  such that  $2u^S = 0$  and  $\tilde{b} = \frac{p^S - c}{c}$ . This threshold is the same as in Figure 3, which is defined such that  $u^S = 0$ . An NM chooses the spot transaction if  $b$  is in the range of  $[\tilde{b}, \hat{b}^N]$ , where  $\hat{b}^N = \frac{p^P - p^S + \frac{A}{2}}{cX}$  is the value of  $b$  such that  $2u^S = 2u^P - A$ . The premium transaction is chosen by NMs with  $b$  larger than  $\hat{b}^N$ .

In this study, we concentrate our analysis on the situation where both EMs and NMs are divided into three groups: choosing the premium transaction, choosing the spot transaction, and not buying. The situation can be depicted in Figures c and d.<sup>6</sup> We therefore assume that parameters are in the range such as that in equilibrium,  $p^S$ ,  $p^P$ ,  $A$ , and  $X$  satisfy 1)  $0 < \tilde{b} < \hat{b}^E$  and 2)  $\hat{b}^N < \bar{b}$ .<sup>7</sup> Note that  $\hat{b}^E < \hat{b}^N$  by definition. Intuitively, NMs are unlikely to choose the premium service compared to EMs due to the membership fee.

For each status of buyers (i.e., EM and NM), there is a continuum of potential buyers with mass normalized to one. The distribution of the evaluations for an item ( $b$ ) is assumed to be same for each status.<sup>8</sup> Following Wang and Wright (2017), we assume that the distribution of  $1 + b$  is the generalized Pareto distribution. Accordingly, the distribution of  $b$ ,  $F(b)$ , is as follows:

$$F(b) = 1 - (1 + \lambda(\sigma - 1)b)^{\frac{1}{1-\sigma}} \quad (3)$$

where  $\sigma < 2$  is the shape parameter and  $\lambda > 0$  is the scale parameter. While Wang and

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<sup>6</sup>In the real world, Amazon Prime members can purchase not only the Amazon Prime Eligible Items but also other items.

<sup>7</sup>We can confirm that these assumptions are satisfied in the equilibrium derived in the next.

<sup>8</sup>An interpretation of this assumption is that the EMs who has become a member to purchase another item ( $c'$ ) in the previous period (see Figure 2) tend to have lower  $b$ . Another interpretation is that although NMs naively believe that their valuation of an item is never changed, their valuation is newly drawn in the subsequent period.

Wright (2017) proceed their analysis with the general distribution, our extended model has difficulty analytically deriving the equilibrium unless some restrictions are placed on the distribution. We therefore focus on the case with  $\sigma = 0$ , which makes  $F(b) = \lambda b$  and means uniform distribution of  $b$ . The support of the distribution is  $[0, \frac{1}{\lambda}]$ , that is,  $\bar{b} = \frac{1}{\lambda}$ .

We can derive the demand quantities as follows. For the EMs, demand for the spot and premium transactions is

$$q^{E,S} = F(\hat{b}^E) - F(\tilde{b}) = \frac{\lambda}{cX} (p^P - (1 + X)p^S) + \lambda \quad (4)$$

and

$$q^{E,P} = 1 - F(\hat{b}^E) = \frac{\lambda}{cX} (-p^P + p^S) + 1, \quad (5)$$

respectively. For the NMs, demand for the spot and premium transactions is

$$q^{N,S} = F(\hat{b}^N) - F(\tilde{b}) = \frac{\lambda}{cX} \left( p^P + \frac{A}{2} - (1 + X)p^S \right) + \lambda \quad (6)$$

and

$$q^{N,P} = 1 - F(\hat{b}^N) = \frac{\lambda}{cX} \left( -p^P - \frac{A}{2} + p^S \right) + 1, \quad (7)$$

respectively.

## 2.3 Seller side

This subsection describes the seller side. Following Wang and Wright (2017), we assume that homogeneous sellers, which have a common per-unit cost ( $c$ ), engage in Bertrand competition. Although we extend the model to incorporate the two types of transaction services, sellers compete in price in each transaction market.

We also extend the model of Wang and Wright (2017) to explicitly model the costs of delivery of an item. A distinguishing feature of the platform service for a transaction between a buyer and a seller, like the services provided by Amazon, is that it necessarily entails a physical delivery. An aim of this study is to provide comparative statics regarding the costs of delivery. We assume that while sellers in the spot transactions deliver an item by using an external carrier (e.g., FedEx), sellers in the premium transaction leave deliveries to the platform. The platform is assumed to involve delivery for the premium transaction to guarantee the high-quality delivery service.<sup>9</sup>

Specifically, the costs for sellers are as follows. For a premium transaction, the seller bears the marginal costs ( $c$ ) and the transaction charge paid to the platform ( $T^P$ ). Because the delivery is handled by the platform for the premium transaction, the delivery costs are levied not on sellers but on the platform. Note, however, that the delivery costs may be passed through to  $T^P$ . For a spot transaction, the seller bears the delivery costs ( $f$ ) in addition to the marginal costs ( $c$ ) and the transaction charge ( $T^S$ ).

Due to the assumption of Bertrand competition, the price that a buyer pays to a seller in the equilibrium should be

$$p^P = c + T^P \tag{8}$$

for the premium transaction and

$$p^S = c + T^S + f \tag{9}$$

for the spot transaction.

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<sup>9</sup>Indeed, in the real economy, most transactions of the Amazon Prime Eligible Item are supplied by the sellers using the Fulfillment-by-Amazon service for sellers, in which sellers collectively leave Amazon to inventory control and delivery. In contrast, spot transactions are shipped by external carriers in most cases.

## 2.4 Platform

The decision variables of the platform for maximizing its profit are the charges per transaction ( $T^P$  and  $T^S$ ), the membership fee ( $A$ ), and the quality of the premium transaction service ( $X$ ). While a dynamic scenario as depicted in Figure 2 is implicitly assumed in our model, we focus on a one-shot problem in the dynamic situation. A complete analysis of the dynamic problem remains for future research.

The platform bears costs per transaction. For the spot transaction, costs per transaction is denoted as  $k^S$  and is likely to be almost zero. For the premium transaction, costs are assumed to be  $k^P + \gamma X^2$  and depend on the service quality ( $X$ ). When the quality is zero (i.e., the same quality as the spot transaction), the costs per premium transaction is  $k^P$ . This may be larger than  $k^S$  because  $k^P$  includes the delivery costs, which the platform bears for the premium transaction. The delivery costs may increase according to the quality of the premium transaction. We assume a quadratic function to represent this effect ( $\gamma X^2$ ).

The profit maximization problem of the platform is as follows<sup>10</sup>:

$$\max_{T^S, T^P, A, X} \pi = (T^S - k^S)(q^{E,S} + q^{N,S}) + (T^P - k^P - \gamma X^2)(q^{E,P} + q^{N,P}) + Aq^{N,P}. \quad (10)$$

The variables chosen by the platform is expressed by capital letters. The first term of the profit ( $\pi$ ) represents profits from the spot transaction. The second term is those from the premium transaction. The last term stands for the revenues of the membership fee. The solution of this problem is derived in the next section.

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<sup>10</sup>The target profit maximized by a platform is assumed to yield in a steady state period  $t$ . It is caused by actions in the surrounded dotted red line box in the time flow in Figure 1.

### 3 Derivation of a platform monopoly equilibrium

Substituting (??), (??) and (??) into the r.h.s. of  $\pi$ , (10) yields:

$$\begin{aligned}
\max_{T^S, T^P, A, X} \pi &= \lambda (T^S - k_S) \cdot \frac{4(T^P - (1 + X)(T^S + f_T)) + A}{2Xc} + (T^P - k_P - \gamma X^2) \\
&\times \left( 2 - \frac{\lambda}{2Xc} ((4(T^P - T^S - f_T) + A)) \right) \\
&+ A \left( 1 - \lambda \cdot \frac{2(T^P - T^S - f_T) + A}{2Xc} \right). \tag{11}
\end{aligned}$$

The first order conditions of this maximization problem are

$$\begin{aligned}
\frac{\partial \pi}{\partial T^S} &\equiv \pi_S = \frac{\lambda}{2Xc} (8T^P - 4k_P + 3A - 4\gamma X^2 + (1 + X)(-8T^S - 4f_T + 4k_S)) = 0 \\
&\Leftrightarrow 8T^P - 4k_P + 3A - 4\gamma X^2 + (1 + X)(-8T^S - 4f_T + 4k_S) = 0, \tag{12}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi}{\partial T^P} &= \frac{\lambda(4\gamma X^2 - 3A - 8T^P + 8T^S + 4f_T + 4k_P - 4k_S) + 4cX}{2Xc} = 0 \\
&\Leftrightarrow \lambda(4\gamma X^2 - 3A - 8T^P + 8T^S + 4f_T + 4k_P - 4k_S) + 4cX = 0, \tag{13}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi}{\partial A} &= \frac{\lambda(\gamma X^2 - 2A - 3T^P + 3T^S + 2f_T + k_P - k_S) + 2cX}{2Xc} = 0 \\
&\Leftrightarrow \lambda(\gamma X^2 - 2A - 3T^P + 3T^S + 2f_T + k_P - k_S) + 2cX = 0, \tag{14}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \pi}{\partial X} &= \frac{1}{2cX^2} \left[ \left( \begin{array}{l} 4(T^P)^2 - 8T^S T^P + (4\gamma X^2 + 3A - 4f_T - 4k_P + 4k_S)(T^P - T^S) \\ +4(T^S)^2 + A^2 + (\gamma X^2 - k_P + k_S - 2f_T)A - 4f_T(\gamma X^2 - k_P + k_S) \end{array} \right) \lambda \right. \\
-8c\gamma X^3] &= 0 \Leftrightarrow \\
&\left\{ \begin{array}{l} \gamma(A + 4(T^P - T^S - f_T))X^2 + 4(T^P - T^S)^2 - (4(k_P - k_S + f_T) - 3A)(T^P - T^S) \\ + A^2 - (k_P - k_S + 2f_T)A + 4(k_P - k_S)f_T \end{array} \right\} \\
-8c\gamma X^3 &= 0 \tag{15}
\end{aligned}$$

Solving (14) with respect to  $A$ , we obtain

$$A = \frac{1}{2\lambda} (2cX + \lambda(\gamma X^2 + 3(T^S - T^P) + k_P - k_S + 2f_T)) \tag{16}$$

Substituting (16) into (12), (13) and (15), rearranging them, we obtain

$$5\lambda\gamma X^2 + (8\lambda(2T^S - (k_S - f_T)) - 6c)X + \lambda(5(k_P - k_S) + 2f_T - 7(T^P - T^S)) = 0, \tag{17}$$

$$5\lambda\gamma X^2 + 2cX + \lambda(2f_T + 5(k_P - k_S) - 7(T^P - T^S)) = 0, \tag{18}$$

and

$$\begin{aligned}
& 3\lambda^2\gamma^2X^4 - 24\lambda c\gamma X^3 - (4c^2 + \lambda^2\gamma(16f_T - 17(T^P - T^S)) - 2\lambda^3\gamma(k_P - k_S + 2f_T))X^2 \\
& + \lambda^3 \left( \begin{aligned} & (2f_T + k_P - k_S - 3(T^P - T^S))^2 - 2(2f_T + k_P - k_S)(2f_T + k_P - k_S - 3(T^P - T^S)) \\ & + 16f_T(k_P - k_S) - 2(T^P - T^S)^2 - 2(T^P - T^S)(2f_T + 5(k_P - k_S)) \end{aligned} \right) \\
= & 0. \tag{19}
\end{aligned}$$

From (18), we have

$$-7\lambda(T^P - T^S) = -5\lambda\gamma X^2 - 2cX - \lambda(5(k_P - k_S) + 2f_T). \tag{20}$$

Substituting (20) into (17) yields

$$8(\lambda(2T^S - (k_S - f_T)) - c)X = 0.$$

Thus, we obtain

Solving (20) with respect to  $T^P - T^S$ , we have

$$T^P - T^S = \frac{1}{7\lambda} (\lambda(5(k_P - k_S) + 2f_T + 5\gamma X^2) + 2cX). \tag{21}$$

From (??) and (21), we obtain

$$T^P = \frac{1}{14\lambda} (10\lambda\gamma X^2 + 4cX + 7c + \lambda(10k_P - 3k_S - 3f_T)) \tag{22}$$

and

$$T^{S*} = \frac{1}{2} \left( \frac{c}{\lambda} + k_S - f_T \right). \tag{23}$$



Substituting (21) into (19) and rearranging it, we obtain

$$\frac{8}{7\lambda} (-\lambda\gamma X^2 + cX - \lambda(k_P - k_S - f_T)) (-3\lambda\gamma X^2 + cX + \lambda(k_P - k_S - f_T)) = 0. \quad (24)$$

Next, we present assumptions for existence of the optimal service level for buyers of the platform.

**Assumption 2**

$$k_P - k_S - f_T > 0.$$

**Assumption 3**

$$c^2 > 4\gamma\lambda^2 (k_P - k_S - f_T).$$

**Lemma 1** *Under the assumptions 1, 2 and 3, there exists a unique optimal service level  $X^*$  for buyers on the platform. This satisfies the first order condition, (24) and second order condition of the reduced optimization problem (11),*

$$X^* = \frac{1}{6\gamma\lambda} \left( c + \sqrt{c^2 + 12\gamma\lambda^2 (k_P - k_S - f_T)} \right). \quad (25)$$

For the proof of the lemma, see Appendix 1.

Substituting (25) into (21), we obtain  $T^{P^*} - T^{S^*}$ . By substituting it and (25) into (16), we obtain  $A^*$ . From the expressions  $T^{P^*} - T^{S^*}$  and (??), we can also derive  $T^{P^*}$ . Hence, we present the following proposition without proof:

**Proposition 1** *Suppose that the demand function for a particular product in the market sold on the internet shopping site of the platform monopolist is given by  $X_c(p_d) =$*

$1 - F(\frac{p_d}{c}) = 1 - \lambda(\frac{p_d}{c} - 1)$ , where  $p_d$  is the demand price,  $\lambda > 0$ . Under assumptions 1, 2 and 3, there exists an optimal annual membership fee of  $A^*$  to be paid by premium buyers, optimal per-transaction services fees  $T^{P^*}$  and  $T^{S^*}$  to be paid by type  $P$  and  $S$  sellers for providing optimal service level  $X^*$  to buyers on the platform, as given in Lemma 1. These optimal choices of the platform are

$$A^* = \frac{1}{63\gamma\lambda^2} \left( 4c\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} - 48\gamma\lambda^2(k_P - k_S - f_T) + 4c^2 \right), \quad (26)$$

$$T^{P^*} = \frac{1}{126\gamma\lambda^2} \left( 11c\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} + 57\gamma\lambda^2 \left( \frac{40}{19}k_P - k_S - f_T \right) + 63\gamma\lambda c + 11c^2 \right), \quad (27)$$

$$X^* = \frac{1}{6\gamma\lambda} \left( c + \sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} \right),$$

and

$$T^{S^*} = \frac{1}{2} \left( \frac{c}{\lambda} + k_S - f_T \right).$$

In the proposition, we see that the optimal service fee per transaction intermediation for type  $S$  seller on the platform,  $T^{S^*}$  depends on the service costs  $k_S$  and  $f_T$ . The shipping fee that a type  $S$ 's seller has to pay to the transportation agents is not reflected in the service costs  $k_P$  and  $\gamma$ . However,  $X^*$ ,  $A^*$ ,  $T^{P^*}$  depend on all of its services costs,  $k_S$ ,  $f_T$ ,  $k_P$  and  $\gamma$ . This illustrates the real pricing behavior of a platform owing to the real economy, such as Amazon, which not only supplies the intermediary services

of transactions between sellers and buyers through an internet platform both but also substitutes transportation and inventory service to sellers.

Through tedious and troublesome calculation, we can see that optimal solutions,  $X^*$ ,  $A^*$ ,  $T^{P*}$  and  $T^{S*}$  satisfy the second order condition.

In the next section, we investigate how the unit changes in services costs for both types of sellers and buyers on the platform and the quality costs of the premium services for premium buyers affect the optimal service fees for both types of sellers and buyers and the demand price of the transaction.

## 4 Comparative Statics with respect to cost parameters

In this section, we explore how the optimal service fees for both types of sellers,  $T^{P*}$ ,  $T^{S*}$ ,  $X^*$  and  $A^*$  change when  $k_P$ ,  $f_T$ ,  $\gamma$ , and  $\lambda$  change.

We assume that the quality of the service for type  $P$  sellers is higher than that for type  $S$  sellers on a realistic basis. In a real economy, Amazon offers its  $P$  type sellers (i.e., for the FBA system users), inventory storage, packing, and shipping services, and commission sales services. However, to type  $S$  sellers it only offers the commission sale services. Accordingly, our assumption that  $k_P > k_S > 0$ , is plausible.

**Lemma 2**  $\frac{\partial T^P}{\partial X} > 0$ , if  $c, \gamma, \lambda > 0$  and  $X \geq 0$ , then  $\frac{\partial A}{\partial X} \geq 0 \Leftrightarrow \frac{c}{2\lambda\gamma} \geq X \geq 0$ ,  $\frac{\partial X^*}{\partial k_P} > 0$ , and  $\frac{\partial X^*}{\partial f_T} < 0$ .

From Lemma 2, we obtain the following proposition on the comparative statistics results of optimal service fees  $T^{P*}$ ,  $A^*$ , and  $X^*$  with respect to  $f_T$ , and  $k_P$ . For the proof, see appendix.

**Proposition 2** *Suppose that  $c$  is fixed and both assumptions 2 and 3 hold, then the following statements also hold.*

(i) *If the cost of the services of the platform for type  $P$  sellers,  $k_P$ , increases, then the optimal quality level,  $X^*$  for type  $P$  buyers and the service fee for premium sellers,  $T^{P*}$ , also increase; however, the optimal annual membership fee for type  $P$  buyers,  $A^*$ , decreases.*

(ii) *When the shipping fee charged by the transportation agents (firms) per unit of the item for each type  $S$  seller,  $f_T$ , increases, the optimal quality level,  $X^*$ , for type  $P$  buyers and the service fee for premium/ spot sellers,  $T^{P*} / T^{S*}$ , also decrease; however, the optimal annual membership fee for type  $P$  buyers,  $A^*$ , increases.*

(iii) *If the quality marginal cost of the premium services paid by premium buyers,  $\gamma$ , increases, the optimal quality level  $X^*$  for type  $P$  buyers, optimal per-transaction services fee  $T^{P*} / T^{S*}$  for type  $P / S$  sellers, and optimal annual membership fee for type  $P$  buyers,  $A^*$ , all decrease.*

(iv) *If the scale parameter of the probability distribution  $\lambda$  decreases (or the upper bound of the distribution of the valuation of consumers  $\bar{b}(1 + X)$ , equivalently increases), then  $X^*$ ,  $T^{P*}$ ,  $T^{S*}$  and  $A^*$ , all increase.*

The intuition of the proposition is as follows. In (16), (23) and (22), when  $k_P$  changes, we see that its effect on  $A$  can be divided into *two reciprocal effects*, namely, *the quality reaction effect* of  $T^P$  through  $X$  and *demand reaction effect*. Formally, we can express these changes as  $\frac{\partial A^*}{\partial k_P} = \frac{\partial A}{\partial X^*} \cdot \frac{\partial X^*}{\partial k_P} + \frac{\partial A}{\partial k_P}$ . The quality and demand reaction effects stand for the first and second terms in the right hand side of the formula, respectively. The former effect is shown to be positive but the latter is negative. As shown in the proof of Proposition 2, the latter effect surpasses the former so that  $\frac{\partial A^*}{\partial k_P} < 0$ . Therefore, as

$k_P$ , the cost of the discriminate services for premium (type  $P$ ) sellers, including packing and shipping services by the platform, increases,  $A$  the service price (annual membership fee) for the new premium buyer members, also increases, but it may decrease  $X^{NP}$ , the demand for the new premium buyers of the item. To avoid shrinking in the entry of new premium members, the platform may increase  $X^*$ , the premium service level provided to premium buyers. This, in turn, increase  $A$  and  $T^{P*}$ , the service fee for the premium sellers (*the quality reaction effect*). Note that the premium service for the premium buyers is *no more than the service for the premium sellers*. Both  $X^{EP*}$  and  $X^{P*}$  decrease when  $X^*$  increases because the increase in  $T^{P*}$  implies that  $p_d^{P*} = p_c^{P*}$  (??). To recover the demand for item  $c$ , the platform has to reduce  $A^*$ , the price of new premium buyers (the annual membership fee). (*the demand reaction effect*) The effects for  $X^*$ ,  $A^*$ , and  $T^{P*}$  are *the opposite* when the per unit shipping fee charged by the transportation agents (firms) for each item sold by type  $S$  sellers,  $f_T$  increases. This is because the service demands for type  $S$  and  $P$  sellers on the platform are substitutes for each other.

Statement (i) of the proposition is an example of the result of Doi and Shinkai (2023), because the result shows that a cost increase may decrease price (i.e., a negative pass-through) and increase quality when quality and prices are endogenously chosen in a monopoly.

Statement (i) also explains why only Amazon Japan has not raised the annual membership fee for premium buyers even though its delivery (transportation) service cost for premium buyers has rapidly increased. The reason is the short supply of delivery services over demand of them, which occurred during the COVID-19 pandemic that took place in the recent years.

In Japan, The default option in doorstep delivery services of small parcels of transport companies has been “attended delivery,” in which the delivery is completed by a signature

when the recipient receives a parcel. Therefore, if the recipient is absent, the transport company has to conduct redelivery. Redeliveries increase the delivery service costs of the transport companies. As the delivery service cost per item for premium buyers of Amazon Japan has tremendously increased because of the COVID-19 pandemic, Amazon has decreased its level of premium services  $X^*$  provided to premium buyers. It has also reduced the  $k_P$  by changing its default delivery preference services for premium buyers to “unattended delivery,” where the item is delivered to the specified location regardless of whether the premium buyer is at home or absent. Therefore, delivery can be completed without requiring the recipient’s signature, as a result, there is no need to conduct redelivery. Specifically, except for fresh or perishable foods or frozen foods, Amazon Japan has replaced its outsourcing company of premium delivery services from Yamato Transport or Japan Post to AZ-COM Maruwa Inc.. It manages the Third-party logistics (3PL) services system” as a platform system tying small personal transport companies and linking them to Amazon Japan.

To make this system function well, Amazon needs to change its default delivery preference services for premium buyers from “attended delivery” to “unattended delivery,” so that it can remove the cost-up factor, redelivery and requiring the signature of recipient for small transport companies. By doing so, Amazon Japan can reduce  $k_P$  and small transport companies can stably provide plenty of daily opportunities to the workers delivering services to Amazon’s premium buyers<sup>11</sup>. In statement (i) in proposition 2, if  $k_P$  decreases, then  $X^*$  and  $T^{P*}$  in consequent  $p_d^{P*}$  also decrease. Hence premium sellers’

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<sup>11</sup>By the TBS TV program, “Gacchiri Monday,” broadcasted on February 19th, 2023, AZ-COM Maruwa Inc. has been intermediating on small parcel delivery services of premium buyers between Amazon Japan and small transport companies using 3PL services systems through construction of a platform systems in recent years. Masaru Wasami, President and CEO, in an interview, said that AZ-COM Maruwa Inc. has increased its group sales, which are estimated to reach 171.5 billion yen in March 2023! <https://note.com/gacchiri/n/nf49db8c80369>

demand for of the item increases. This provides sufficient room for  $A^*$  to increase in the future, subsequently boosting the platform's profit.

The intuitions of the statements (ii), (iii) and (iv) are all straightforward, which is why we do not mention them.

## 5 Conclusion

In this study, we analyze price discrimination of a platform monopolist that serves both sellers and buyers with their intermediary services by menu-pricing and quality control. For this purpose, we extended Wang and Wright's (2017) model to include the quality level  $X$  ( $\geq 0$ ) chosen by the platform and one-day delivery services without delivery fees offered to premium buyers, for goods indexed by  $c$ . We assume that the distribution of buyers' valuation of the transaction is presented by the generalized Pareto distribution. The platform serves both sides of the market, premium sellers (exhibitors) and premium buyers, through its internet shopping site. It offers intermediation services for transactions among both sides, inventory storage and packing and shipping services of level  $X$  to premium sellers based on a pay-as-you-go system and premium services of level  $X$  to premium buyers including one-day delivery services without a delivery fee.

Under certain assumptions, we derive the optimal annual membership fee of premium buyers, optimal per-transaction services fees for type  $P$  and  $S$  sellers and the optimal service level for buyers on the platform. We also investigate how the unit changes in services costs for both types of sellers and buyers on the platform and the quality costs paid by the premium buyers affect the optimal service fees for both types of sellers and buyers. We also examine the demand price of the transaction, on the platform.

Consequently, we find a counterintuitive result that *an increase in the cost of the services of platform for premium type sellers increases the optimal quality level for premium type buyers and the service fees for premium sellers: however, it also decreases of the optimal service fee (annual membership fee) for the premium buyers.*

Given the limitation of space, we do not derive the first best (socially optimal) equilibrium and conduct the comparison using our monopoly equilibrium in this study. Our model does not consider network externalities either within-group or cross-group which are important in research on the platform economy. These extension of our study is left for future research.

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## 7 Appendix

### Proof of Lemma 1

From the quartic equation of  $X$ , (24) we have

$$-\lambda\gamma X^2 + cX - \lambda(k_P - k_S - f_T) = 0 \quad (28)$$

or

$$-3\lambda\gamma X^2 + cX + \lambda(k_P - k_S - f_T) = 0. \quad (29)$$

For the quadratic equation (28) of  $X$ , from assumption 2, we have the determinant

$$D_1^X = c^2 - 4\gamma\lambda^2(k_P - k_S - f_T) > 0, \quad (30)$$

so that it has two real solutions

$$X_1 = \frac{c + \sqrt{c^2 - 4\gamma\lambda^2(k_P - k_S - f_T)}}{2\gamma\lambda} > 0$$

and

$$X_2 = \frac{c - \sqrt{c^2 - 4\gamma\lambda^2(k_P - k_S - f_T)}}{2\gamma\lambda} > 0.$$

For the quadratic equation (??) of  $X$ , from assumption 1, we have the determinant

$$D_2^X = c^2 + 12\gamma\lambda^2(k_P - k_S - f_T) > c^2 - 4\gamma\lambda^2(k_P - k_S - f_T) > 0,$$

so that it also has two real solutions.

Thus, we have

$$X_3 = \frac{c + \sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)}}{6\gamma\lambda} > 0$$

and

$$X_4 = \frac{c - \sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)}}{6\gamma\lambda} < 0.$$

We can easily show that

$$X_1 > X_3 > X_2 > 0 > X_4.$$

The quartic equation of  $X$ , (24) can be rewritten by

$$3\gamma^2\lambda^2X^4 - 4c\gamma\lambda X^3 + 2(3\gamma\lambda^2(k_P - k_S - f_T) + c^2 - \gamma\lambda^2(k_P - k_S - f_T))X^2 - 2\lambda^2(k_P - k_S - f_T)^2 = 0.$$

Let the l.h.s of the above by  $g(X)$ , i.e. ,

$$\begin{aligned} g(X) &= 3\gamma^2\lambda^2X^4 - 32c\gamma\lambda X^3 + 2(3\gamma\lambda^2(k_P - k_S - f_T) + c^2(k_P - k_S - f_T))X^2 \\ &- 2\lambda^2(k_P - k_S - f_T)^2 = 0. \end{aligned} \quad (31)$$

$$\begin{aligned} g''(X) &= 4c^2 - 8\lambda^2\gamma(f_T - k_P + k_S) + 36X^2\lambda^2\gamma^2 - 192cX\lambda\gamma \\ g''(X_3) &= f''\left(\frac{c + \sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}}{6\gamma\lambda}\right) = 4c^2 - 32c\left(c + \sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}\right) + \\ &\left(c + \sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}\right)^2 + 8\lambda^2\gamma(k_P - k_S - f_T) = 20\lambda^2\gamma k_P - 26c^2 - 20\lambda^2\gamma f_T - \\ &30c\sqrt{c^2 - 12\lambda^2\gamma f_T + 12\lambda^2\gamma k_P - 12\lambda^2\gamma k_S - 20\lambda^2\gamma k_S} \\ &= -26c^2 - 30c\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)} + 20\lambda^2\gamma(k_P - k_S - f_T) \\ &= -2(13c^2 + 15c\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)} - 10\lambda^2\gamma(k_P - k_S - f_T)) \\ &< -2(13 \cdot 4\lambda^2\gamma(k_P - k_S - f_T) + 15c\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)} - 10\lambda^2\gamma(k_P - k_S - f_T)) \\ &= -6(14\lambda^2\gamma(k_P - k_S - f_T) + 15c\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}) < 0, \text{ where the inequality} \\ &\text{holds from (30).} \end{aligned}$$

We can easily show that  $g''(X_1) > 0$  and  $g''(X_2) > 0$ . Hence, only  $X^* = X_3 > 0$  satisfies the second order condition out of four real solutions  $X_1, X_2, X_3, X_4$  of the quartic equation (31).

The parameters have the following constrains:  $\lambda > 0, 0 \leq \sigma < 2, \sigma \neq 1, c, k_L > 0$ . ■

## Proof of Lemma 2

From (??) and (21), we have

$$T^P = \frac{5}{7}\gamma X^2 + \frac{2}{7}\frac{c}{\lambda}X + \left(\frac{5}{7}k_P - \frac{3}{14}f_T - \frac{3}{14}k_S + \frac{1}{2}\frac{c}{\lambda}\right)$$

$\frac{\partial T^P}{\partial X} = \frac{2}{7\lambda}(c + 5X\lambda\gamma) > 0$ , since  $c, \gamma, \lambda > 0$  and  $X \geq 0$ . From (??) and (21), we obtain

$$A = \frac{4}{7\lambda}(cX - \lambda(\gamma X^2 + (k_P - k_S - f_T))). \text{ Hence we see that } \frac{\partial A}{\partial X} = \frac{4}{7\lambda}(c - 2X\lambda\gamma) \stackrel{\geq}{\leq}$$

$$0 \Leftrightarrow \frac{c}{2\lambda\gamma} \stackrel{\geq}{\leq} X \geq 0.$$

From (25) and assumption 2, we have

$$\frac{\partial X^*}{\partial k_P} = \frac{\lambda}{\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)}} > 0, \text{ and } \frac{\partial X^*}{\partial f_T} = -\frac{\lambda}{\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)}} < 0. \blacksquare$$

## Proof of Proposition 2

(i) From (25), we can show that

$$\frac{c}{2\lambda\gamma} - X^* = \frac{1}{6\lambda\gamma} \left( 2c - \sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} \right) > 0,$$

$$\text{since } (2c)^2 - (\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)})^2$$

$$= \left( 2c - \sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} \right) \left( 2c + \sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} \right) > 0$$

and  $2c + \sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} > 0$ . Therefore, from Lemma 2,  $\frac{\partial A}{\partial X^*} > 0$ .

$$\begin{aligned} \frac{\partial A^*}{\partial k_P} &= \frac{\partial A}{\partial X^*} \cdot \frac{\partial X^*}{\partial k_P} + \frac{\partial A}{\partial k_P} = \frac{4}{7} \frac{c - 2X^*\lambda\gamma}{\lambda} \cdot \frac{\lambda}{\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)}} + \left(-\frac{4}{7}\right) \\ &= -\frac{1}{21} \frac{8(2\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)} - c)}{\sqrt{c^2 + 12\gamma\lambda^2(k_P - k_S - f_T)}} < 0. \end{aligned}$$

(ii) From (25), (27), and (23), we obtain

$$\frac{\partial X^*}{\partial f_T} = -\frac{\lambda}{\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}} < 0, \quad \frac{\partial T^{P^*}}{\partial f_T} = -\frac{1}{42} \frac{22c + 19\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}}{\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}} < 0,$$

and  $\frac{\partial T^{S^*}}{\partial f_T} = \frac{\partial}{\partial f_T} \left( \frac{1}{2} \left( \frac{c}{\lambda} + k_S - f_T \right) \right) = -\frac{1}{2} < 0$ . From (16),

$$\begin{aligned} \text{(25) and assumption 2, we have } \frac{\partial A^*}{\partial f_T} &= \left( \frac{\partial A}{\partial X} + \frac{\partial A}{\partial(T^S - T^P)} \cdot \left( \frac{\partial(T^S - T^P)}{\partial X} \right) \right) \cdot \frac{\partial X^*}{\partial f_T} + \frac{\partial A}{\partial f_T} \\ &= \left( \frac{1}{\lambda}(c + X^*\lambda\gamma) + \frac{3}{2} \cdot \left(-\frac{1}{7\lambda}(2c + 10X^*\lambda\gamma)\right) \right) \cdot \left( -\frac{\lambda}{\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}} \right) + \frac{4}{7} \\ &= -\frac{8}{21} \frac{c - 2\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}}{\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}} = \frac{8}{21} \frac{-c + 2\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}}{\sqrt{c^2 + 12\lambda^2\gamma(k_P - k_S - f_T)}} > 0. \end{aligned}$$

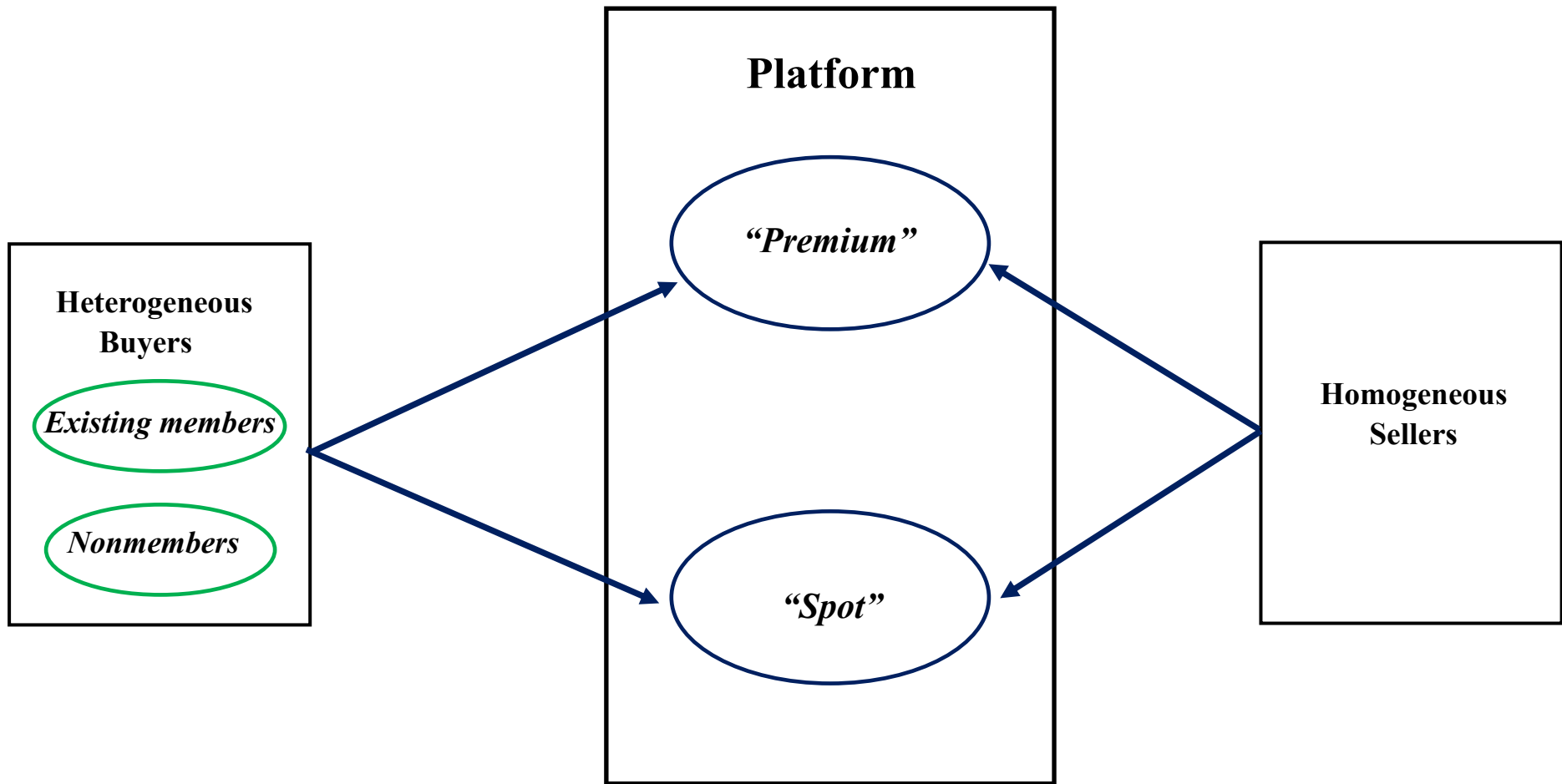
(iii) From (25) (27), (23), (26) and assumption 2, we obtain

$$\frac{\partial X^*}{\partial \gamma} = -\frac{1}{6\lambda\gamma^2\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f_T)}} \left( c\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f_T)} + c^2 + 6\gamma\lambda^2(k_P-k_S-f_T) \right) < 0,$$

$$\frac{\partial T^{P*}}{\partial \gamma} = -\frac{1}{42} \frac{22c+19\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f)}}{\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f)}} < 0, \quad \frac{\partial T^{S*}}{\partial \gamma} = 0, \quad \frac{\partial A^*}{\partial \gamma} = -\frac{4}{63} \frac{c}{\lambda^2\gamma^2\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f_T)}} \left( c\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f_T)} + c^2 + 6\lambda^2\gamma(k_P-k_S-f_T) \right) < 0.$$

(iv) From (25) (27), (23), (26) and assumption 2, we have

$$\frac{\partial X^*}{\partial \lambda} =, \quad \frac{\partial T^{P*}}{\partial \lambda} = -\frac{1}{42} \frac{22c+19\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f)}}{\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f)}} < 0, \quad \frac{\partial T^{S*}}{\partial \lambda} = 0, \quad \frac{\partial A^*}{\partial \lambda} = -\frac{4}{63} \frac{c}{\lambda^2\gamma^2\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f_T)}} \left( c\sqrt{c^2+12\gamma\lambda^2(k_P-k_S-f_T)} + c^2 + 6\lambda^2\gamma(k_P-k_S-f_T) \right) < 0. \blacksquare$$



**Figure 1**

**Framework of the model**

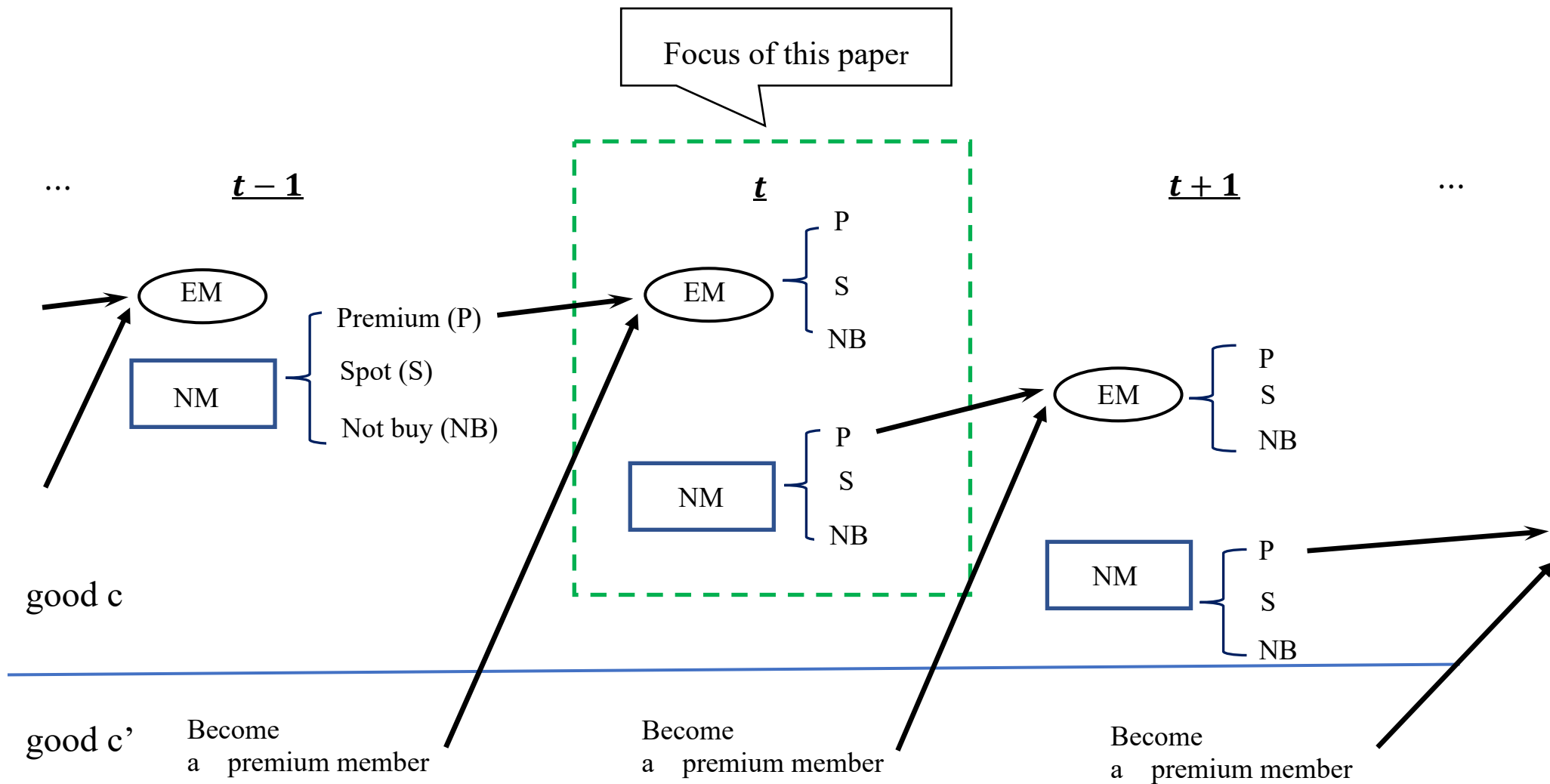


Figure 2

Dynamics behind the model

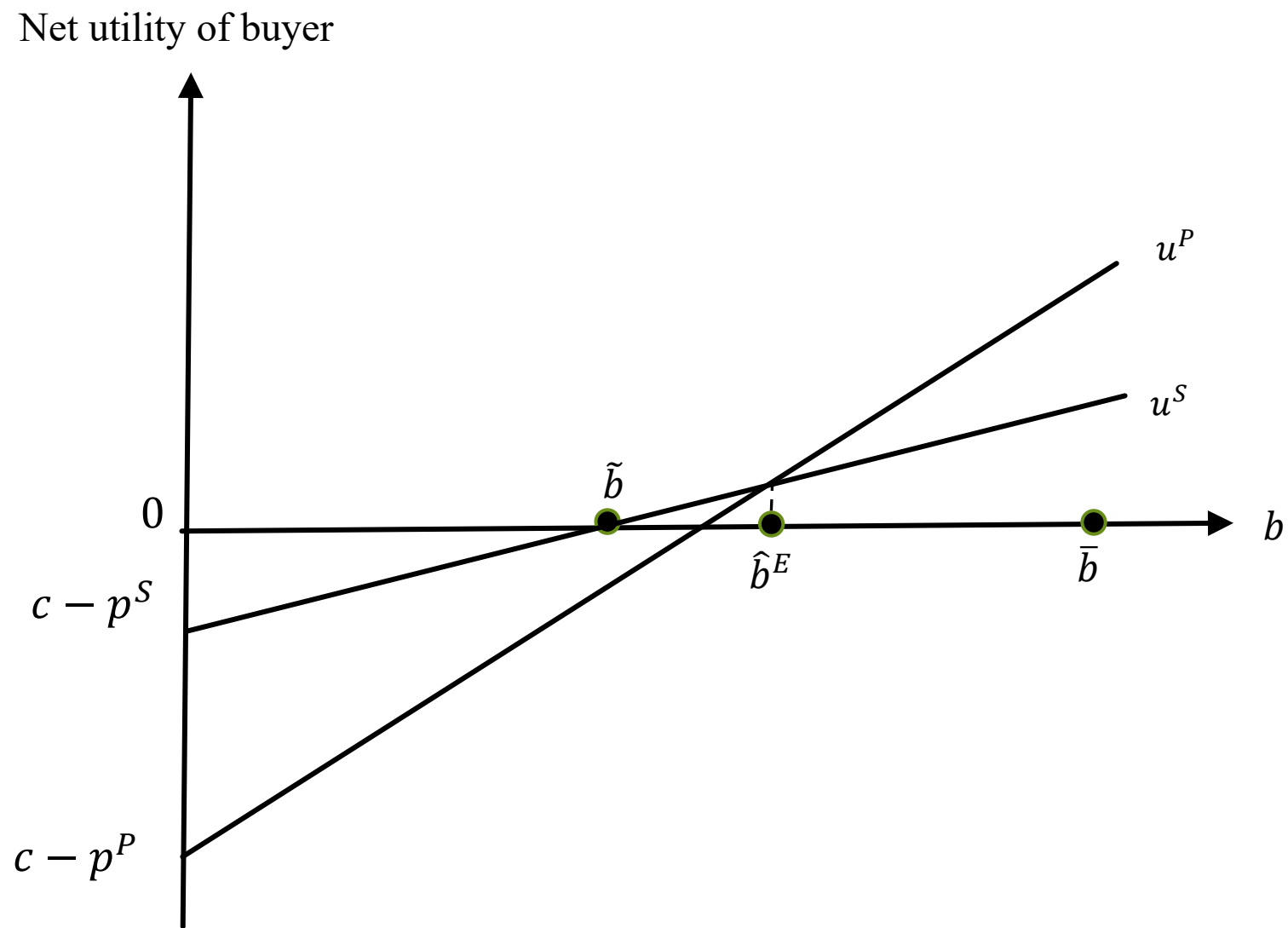


Figure 3

Choice of existing members



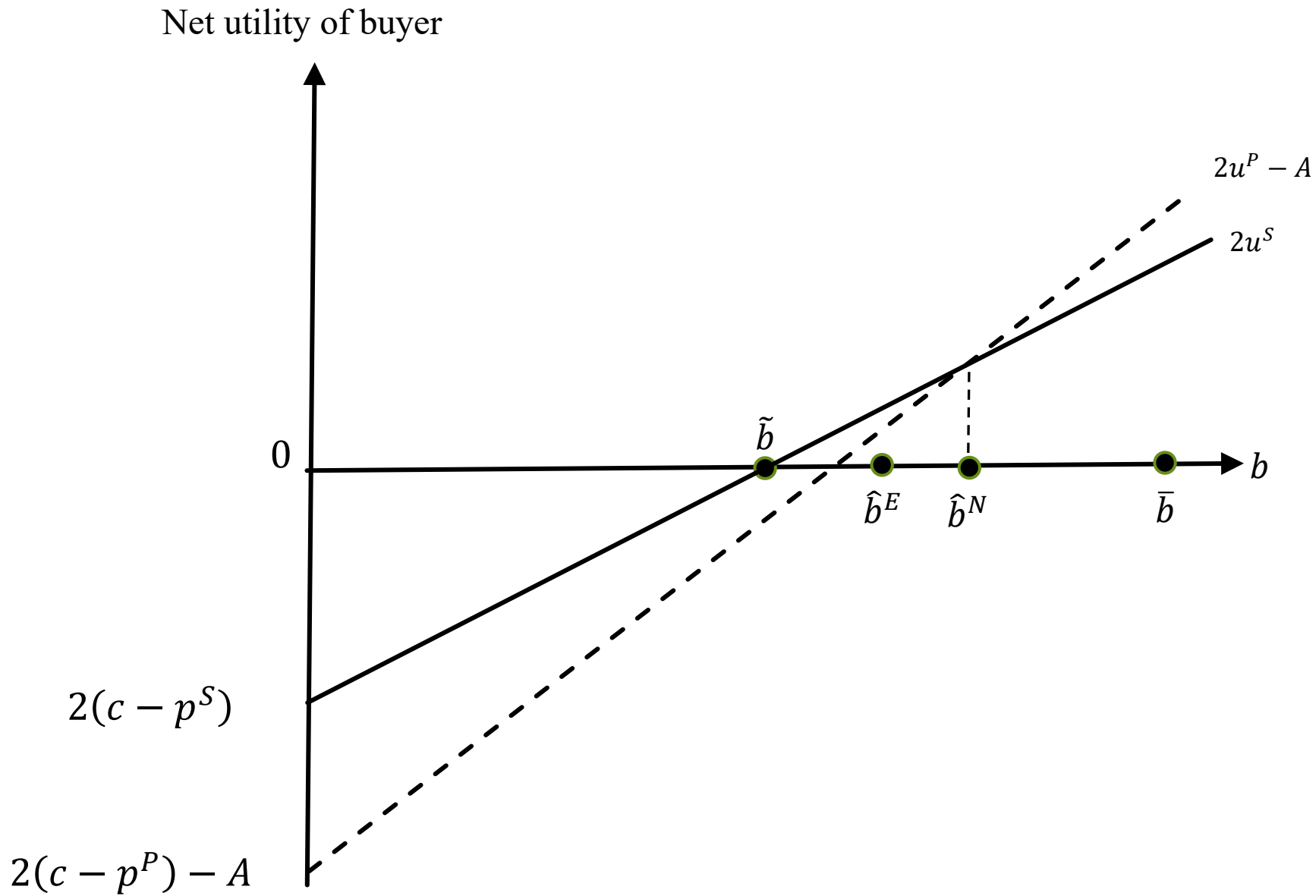


Figure 4

Choice of nonmembers