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A note on the equilibrium of a monopoly providing a pure network good and the stand-alone effect: A reconsideration of the coordination problem

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Abstract

In this note, we reconsider the coordination problem in the case of a monopoly providing a pure network good, such as telecommunications: a problem previously examined by Rohlfs (1974). As in Lambertini and Orsini (2004), we find that the coordination problem relating to critical mass is not associated with the presence of network effects but is more the property of consumer expectations. Assuming a pure network good and passive expectations, we demonstrate this from the perspective of Rohlfs (1974; 2001), i.e., the coordination problem is associated with critical mass and the role of a stand-alone effect.

Keywords: pure network good, network effect, stand-alone value, critical mass, coordination problem, start-up problem, passive expectations, monopoly.

JEL classification: D42, D62, L12.

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1. Introduction

The purpose of this note is to reconsider the coordination problem (or in other words, the start-up problem) in the case of a monopoly providing a pure network good such as telecommunications and Internet businesses, in which the critical mass of the expected network size is very important for the network industry to be sustainable. Since seminal work by Rohlfs (1974), which focuses on telecommunications industry, many subsequent studies have examined the start-up problem relating to the issue of critical mass in markets with network effects.[1] Most recently, Lambertini and Orsini (2004, p. 124) reconsider Rohlfs (1974) as follows.

> A recurrent theme in the literature on network goods is the start-up problem, i.e., how to attract a significant number of customers so as to offer an appealing good or service to additional consumers. Intuitively, joining the network is more valuable to the generic consumer the larger is the size of the network. This may give rise to a coordination problem, since the market performance of a service/product depends upon the achievement of a *critical mass* of adopters/consumers. **The most widely used illustration of this issue dates back to Rohlfs (1974), assuming that the utility associated with consumption is fully determined by the network effect. This can be the case, e.g., of telecommunication networks, which is the example used by Rohlfs himself.** Thereafter, the coordination problem related to the issue of the critical mass has been generally associated to the presence of network effects. **Yet, this is not true in general, since there exist many goods which exhibit network externalities but carry also an intrinsic utility justifying by itself consumption.** These considerations suggest that the issue of a critical mass is crucial only for a subset of all the goods yielding network effects (emphasis added).

As examples, Lambertini and Orsini (2004, footnote 1, p. 124) take the case of personal computers, compact disc players, televisions, etc. As first emphasized in their quote, Rohlfs (1974) focuses on the case in which "… the utility associated with consumption is fully determined by the network effect", such as telecommunication devices (e.g., telephones and telefax machines). Following the definitions of Amir and Lazzati (2011, p. 2395), we understand that this corresponds

[1] For example, see Shy (1998, pp. 256–259) concisely summarizes Rohlfs' (1974) model.

to the case of a pure network good, which characterizes that the intrinsic utility from consumption depends on the expected network size.

Furthermore, we should also consider that consumer expectations play an important role in markets with network effects. Following the definitions in Hurkens and López (2014, p. 1007), we address responsive and passive expectations. Responsive expectations are where firms first compete in prices (or quantities). Consumers then form expectations about network sizes and make optimal purchasing decisions, given the prices and their expectations. Alternatively, passive expectations are where consumers first form expectations about network sizes and firms then compete in prices (or quantities), so consumers make optimal purchasing decisions, given their expectations. These decisions lead to actual market shares and network sizes.

Thus, in equilibrium, realized and expected network sizes are the same (see Katz and Shapiro, 1985). That is, Lambertini and Orsini (2004) assume the case of responsive expectations. Rohlfs (1974) also implicitly assumes this case. As demonstrated by Lambertini and Orsini (2004, p. 127), a stable equilibrium exists, and thus, the coordination problem relating to critical mass does not arise given the presence of network effects. This is true when consumer expectations are responsive. However, in this note, we demonstrate that the coordination problem relating to critical mass is associated with expected network sizes when consumers have passive expectations.

The next aspect we emphasize in Lambertini and Orsini's (2004) quote is "… many goods which exhibit network externalities but carry also an intrinsic utility justifying by itself consumption". Lambertini and Orsini (2004) use this to define that a parameter s, being an intrinsic utility (or satisfaction) from consumption, is independent of the network size. They then introduce the parameter into consumer surplus. Using the definitions of Amir and Lazzati (2011, p. 2395), their model is then the case of a network good with a strictly positive stand-alone value.

With respect to the stand-alone effect, Rohlfs (2001, p. 197) describes the start-up problem as follows.

> *Suppliers must have extraordinarily good products or services to reach a critical mass*. All the successful products and services that we examined constituted major technological advances and met important customer needs. At the other extreme, Picturephone was a hard product to sell–even apart from the Bell System's failure to solve the start-up problem. In between are several products that constituted important technological advances but could not reach a critical mass. These include analog fax machines, digital compact cassette players, minidisc players, digital videodisc players, and commercial e-mail (before the rapid growth of the Internet).
> *A valuable stand-alone application is extremely helpful in solving the start-up problem*. Such an application can generate a large initial user set before suppliers have to do anything to solve the start-up problem. If the stand-alone application is sufficiently valuable, the start-up problem solves itself. That is precisely what happened to VCRs, because the stand-alone application of time-shifting of television programs was so valuable. **The start-up problem is much more difficult for pure bandwagon products, which have no such stand-alone application** (emphasis added).

We thus note that the start-up problem is the same as the coordination problem and pure bandwagon products are the same as pure network goods. As explained in the emphasized section of the quote, we can see that the role of a stand-alone value is very important in solving the start-up problem for a pure network good. By introducing a stand-alone value into the utility function of a pure network good, we demonstrate here that there exists a stable equilibrium under passive expectations and that the start-up problem can then be solved.

2. The Model

2.1 An inverse demand function of a pure network good with no stand-alone value

Assuming the presence of a direct network effect as already observed in the information and communications technology industry, such as telecommunications and Internet businesses, we consider a unit-linear market where there is a continuum of consumers, indexed $\theta \in [0,1]$. To simplify, consumers are uniformly distributed with a density of one in the market, and the utility function (willingness to pay) of consumer θ is given by: $u(\theta) = N(S^e)\theta$, where $N(S^e)$ denotes a network effect function of expected network sizes, $S^e \in [0,1]$.[2] Given the price, a consumer purchases at most either one unit of the product or none. Hence, the surplus of consumer θ is expressed as: $v(\theta) = \max\{u(\theta) - P, 0\}$. The index of the marginal consumer with the same surplus from purchasing either one unit of the product or none is $\hat{\theta} = \dfrac{P}{N(S^e)}$.[3] The quantity demanded of the network product is given by: $X = 1 - \hat{\theta}$, $X \in [0,1]$. We obtain the following inverse demand function:

$$P(X, S^e) = N(S^e)(1 - X). \tag{1}$$

Given (1), if consumers expect that any consumers do not purchase the products, i.e., $S^e = 0$, it holds that $N(0) = 0$. In particular, we have $P(X, S^e = 0) = 0$. This implies that the product is a pure network good (see Amir and Lazzati, 2011, p. 2395).

We assume that a marginal cost of production is constant. For example, the marginal costs of production, such as running costs, in network industries are either negligible or zero, i.e., $C(X) = cX$, $N(S^e) > c \geq 0$, for $S^e \in (0,1]$.[4] Thus, the profit function of monopoly is expressed

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[2] We assume $N'(S^e) \equiv \dfrac{\partial N(S^e)}{\partial S^e} > 0$ for $\theta \in [0,1]$. Furthermore, it holds that $\dfrac{\partial u(\theta)}{\partial N(S^e)} = \theta > 0$. That is, the larger the value of θ is, the higher the marginal utility of the network effects becomes.

[3] S^e implies the expected number of consumers acquiring the product.

[4] This is a necessary condition for a positive output equilibrium to exist.

as:

$$\Pi\left(X,S^e\right)=\left\{P\left(X,S^e\right)-c\right\}X=\left\{N\left(S^e\right)\left(1-X\right)-c\right\}X. \tag{2}$$

2.2 The monopoly equilibrium under a fulfilled expectation

Given the expected network sizes under passive expectations, the monopolist decides the output to maximize the profit. Using equations (1) and (2), the first-order condition (FOC) of profit maximization is given by:

$$\frac{\partial\Pi}{\partial X}=P\left(X,S^e\right)-c-N(S^e)X=N(S^e)\left(1-2X\right)-c=0, \tag{3}$$

where $X\in[0,1]$ and $S^e\in\left[0,1\right]$. The second-order condition (SOC) is given by: $\frac{\partial^2\Pi}{\partial X^2}=-2N(S^e)\leq 0$. It also holds that $\left.\frac{\partial\Pi}{\partial X}\right|_{X=0}=N(S^e)-c>0$ for $S^e\in\left(0,1\right]$ [5] and $\left.\frac{\partial\Pi}{\partial X}\right|_{X=1}=-N(S^e)-c<0$ for $S^e\in[0,1]$.

Using equation (3), we have the fulfilled expectation equilibria which are satisfied with the following conditions:

$$X=\frac{1}{2}\left\{1-\frac{c}{N(S^e)}\right\} \text{ and } S^e=X, \text{where } X\in[0,1] \text{ and } S^e\in[0,1]. \tag{4}$$

For example, we assume that $N(S^e)=\beta S^e$, $\beta=0.1$, and $c=0.01$. We can use this to plot Figure 1, which illustrates that there are two stable equilibria and one unstable equilibrium, i.e., $X^*=0$, $\tilde{X}\doteq 0.138$, and $X^{**}\doteq 0.361$. In particular, the unstable equilibrium, $\tilde{X}$, corresponds

[5] When $S^e=0$, $\left.\frac{\partial\Pi}{\partial X}\right|_{X=0}=N(0)-c=-c<0.$

to the critical mass. With respect to consumer expectations, if $\tilde{X} > S^e$, the pure network good market cannot be sustainable, i.e., $X^* = 0$. This is a coordination problem or in other words, a start-up problem. Conversely, if $\tilde{X} < S^e$, the market is sustainable, i.e., $X^{**} \doteq 0.361$.

Remark 1. Zero marginal costs, i.e., $c = 0$.

We assume that production (running) costs are zero. This is because we observe low and even negligible marginal running costs in network industries, e.g., telecommunication and Internet businesses. Given equation (3), assuming zero marginal cost, the FOC is given by: $\frac{\partial \Pi}{\partial X} = N(S^e)\left(1-2X\right) = 0$. We have the fulfilled expectation equilibrium which is satisfied with the following conditions: $N(S^e)\left(1-2X\right) = 0$ and $S^e = X$, where $X \in [0,1]$ and $S^e \in [0,1]$. We can then draw Figure 2. In this case, $\tilde{X} = 0$ is an unstable equilibrium, i.e., the critical mass, and $X^{**} = \frac{1}{2}$ is a stable equilibrium. Thus, if $S^e > 0$, the market is sustainable.

Remark 2. The case of responsive expectations

Under responsive expectations, consumers believe that the expected network size is equal to the announced output level of the monopolist, i.e., $S^e = X$. In other words, the monopolist can commit to the announced output level before its decision about actual output production. In this case, the inverse demand function is rewritten as $P\left(X, X\right) = N\left(X\right)\left(1-X\right)$. For example, we assume the following linear network effect function: $N(X) = \beta X$, where $1 > \beta > 0$. In this case, the inverse demand function is concave in X. Given this function, we obtain the same results as Shy (1998, pp. 256–259) and Lambertini and Orsini (2004, p. 127).

2.3 A stand-alone effect: Introduction of a strictly positive stand-alone value and the equilibrium

Using the perspective of Rohlfs (2001) as quoted earlier, we introduce a strictly positive stand-alone value, which is independent of the expected network sizes, into the utility function as follows: $u(\theta) = N(S^e)\theta + a.$ In particular, we derive the following inverse demand function:

$$P(X, S^e) = N(S^e)(1 - X) + a, \tag{5}$$

where $N(1) > a > c \geq 0.$[6] Thus, the FOC is given by:

$$\frac{\partial \Pi}{\partial X} = N(S^e)(1 - 2X) + a - c = 0, \tag{6}$$

where $X \in [0,1]$ and $S^e \in [0,1]$. The second-order condition (SOC) is given by: $\frac{\partial^2 \Pi}{\partial X^2} = -2N(S^e) \leq 0.$ It also holds that $\left.\frac{\partial \Pi}{\partial X}\right|_{X=0} = N(S^e) + a - c > 0$ and $\left.\frac{\partial \Pi}{\partial X}\right|_{X=1} = -N(S^e) + a - c < 0$[7] for $S^e \in [0,1]$.

Using equation (6), we have the fulfilled expectation equilibrium which is satisfied with the following conditions.

$$X = \frac{1}{2}\left\{1 + \frac{a - c}{N(S^e)}\right\} \text{ and } S^e = X, \text{ where } X \in [0,1] \text{ and } S^e \in [0,1]. \tag{7}$$

For example, we assume that $N(S^e) = \beta S^e$, $\beta = 0.1$, and $a - c = 0.01$. We can use this to draw Figure 3, which illustrates that there exists a stable equilibrium, i.e., $X^{\#} \doteq 0.585$. That is, as argued by Rohlfs (2001), a start-up problem can be solved by introducing a strictly positive stand-

[6] If $c \geq a > 0,$ we obtain the same results as in Section 2.1 and Remark 1.

[7] This is a necessary condition for a partial output equilibrium (i.e., less than unity) to exist.

alone value.

Remark 3. Additively separable network effects and a mixed network good

In previous subsections, we consider the case of a pure network good, which is expressed as a function of multiplicatively added network effects, associated with either zero or a strictly positive stand-alone value. Here, we assume the following utility function with additively separable network effects, which expresses that the intrinsic utility is independent of the network effects, as mentioned in Lambertini and Orsini (2004).

$$u\left(\theta\right)=\alpha\theta+N(S^e), \quad \alpha>c\geq 0.^{8} \tag{8}$$

where $\alpha>N'(S^e)$ and $1>N(1)$. In this case, we derive the following inverse demand function:

$$P\left(X,S^e\right)=\alpha\left(1-X\right)+N\left(S^e\right). \tag{9}$$

In particular, following the definitions of Amir and Lazzati (2011), we examine the case of a mixed network good, i.e., $P\left(X,S^e=0\right)=\alpha\left(1-X\right)>0$. To simplify, assume $\alpha=1>c\geq 0$. The profit function is given by: $\Pi\left(X,S^e\right)=\left\{P\left(X,S^e\right)-c\right\}X=\left\{1-X+N\left(S^e\right)-c\right\}X$. This profit function is basically similar to that of Lambertini and Orsini (2004), except for the assumption of consumer expectations.[9] In this case, we have the following FOC.

$$\frac{\partial\Pi}{\partial X}=1-2X+N(S^e)-c=0, \tag{10}$$

where $X\in[0,1]$ and $S^e\in[0,1]$. We have the fulfilled expectation equilibrium which is satisfied with the following conditions.

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[8] If $c\geq\alpha>0$, we obtain the same results as in Section 2.1 and Remark 1.

[9] Under responsive expectations, i.e., $S^e=X$, the profit function is rewritten as: $\Pi\left(X,X\right)=\left\{1-X+N\left(X\right)-c\right\}X$. We then obtain the same result as Lambertini and Orsini (2004).

$$X = \frac{1}{2}\left\{1 - c + N(S^e)\right\} \text{ and } S^e = X, \text{ where } X \in [0,1] \text{ and } S^e \in [0,1]. \qquad (11)$$

For example, we assume that $N(S^e) = \beta S^e$, $\beta = 0.5$, and $c = 0.5$. We can draw Figure 4, which illustrates there is a single stable equilibrium, i.e., $X^{***} = \frac{1}{3}$.

3. Concluding Remarks

In this note, we reconsidered the coordination problem in the case of a monopoly with a pure network good. As addressed in Lambertini and Orsini (2004), the coordination problem relating to the issue of critical mass has not been associated with the presence of network effects and depends on consumer expectations, i.e., passive expectations. Furthermore, even with passive expectations, in the case of a mixed network good in which the intrinsic utility is independent of the expected network sizes, the problem does not necessarily arise. Assuming a mixed network good and responsive expectations, Lambertini and Orsini (2004) show that the coordination problem relating to critical mass does not arise. However, assuming a pure network good and passive expectations, we demonstrate the perspective of Rohlfs (1974. 2001), i.e., the coordination problem associated with critical mass and the role of a stand-alone effect can be used to solve the problem.

As in Rohlfs (1974. 2001) and Lambertini and Orsini (2004), in this note we assume a one-sided market associated with network effects, which positively affects the utilities of consumers. In other words, we do not consider network effects on the supply side (sellers and suppliers). However, in the digital economy as it exists, we observe the enormous trade of products and services ubiquitously operates through various platforms on Internet systems. For this reason, we should focus on the indirect network effects in two-sided (or even multi-sided) markets with platforms (intermediaries) and reconsider the start-up problem in these markets. That is, the number of suppliers (sellers) affects the utilities of consumers (buyers) whereas the number of consumers affects the revenues of suppliers. In this case, is it possible for a start-up, i.e., “chicken and egg” problem to arise?[10] Although important in the digital economy, we do not address this in this note and defer our attention to future study.

[10] See, for example, Caillaud and Jullien (2003).

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Figure 1. A coordination (s start-up) problem

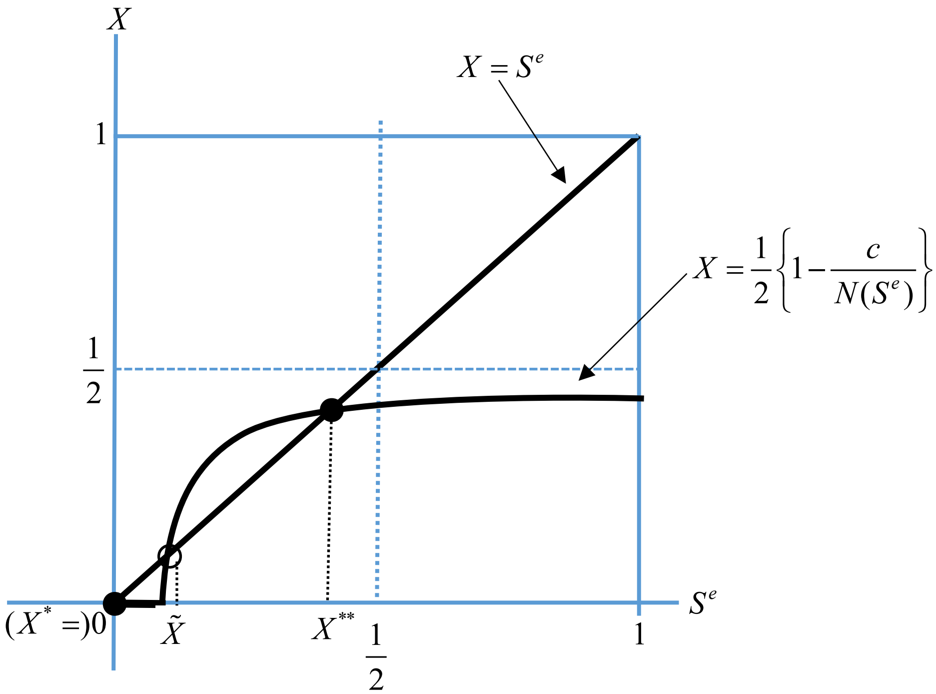

Figure 2.

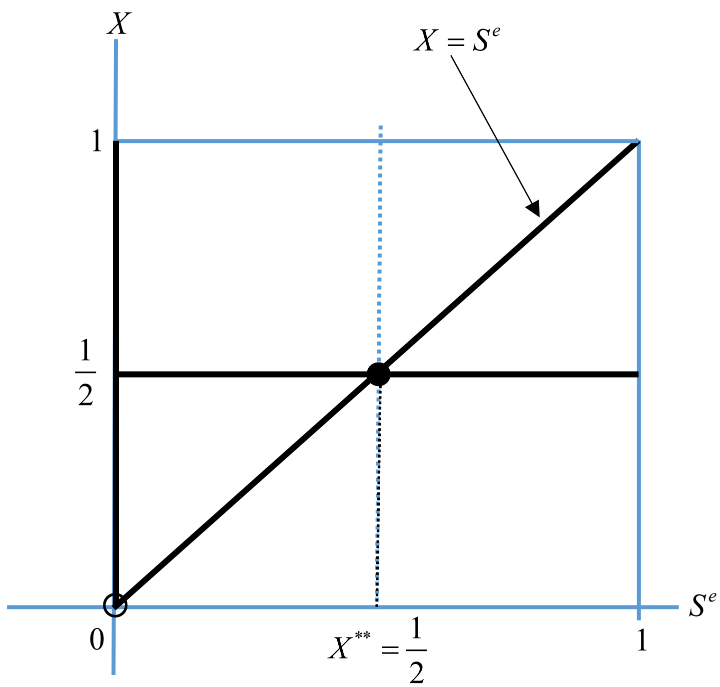

Figure 3. A stand-alone effect

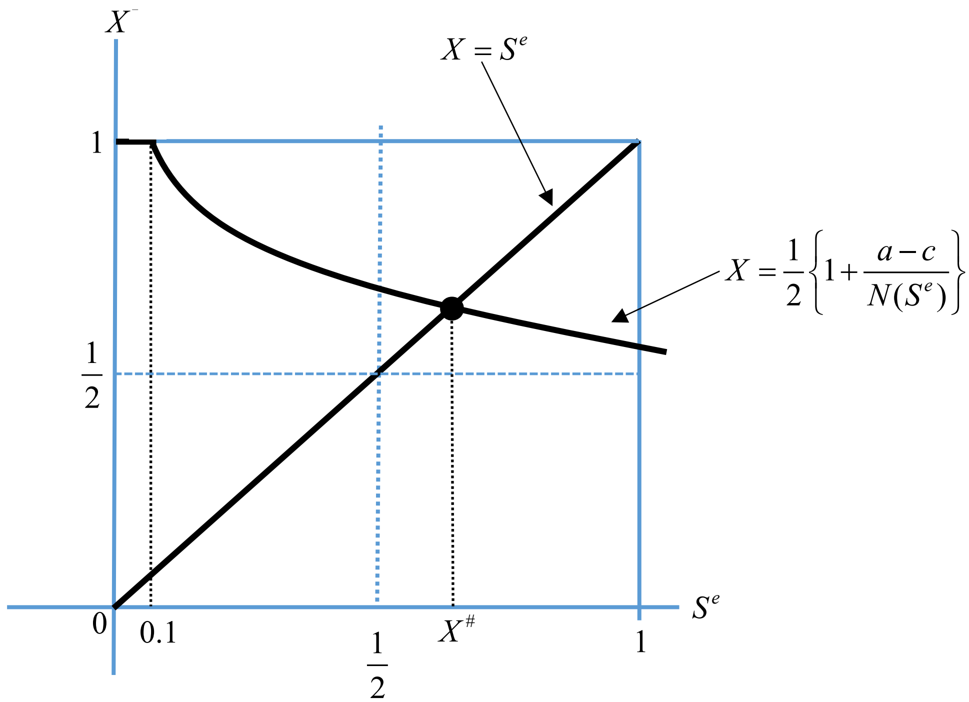

Figure 4.