Fertility, Income Growth and Capital Accumulation*

Masaya Yasuoka**

The analyses described in this paper use an endogenous fertility model with human capital accumulation in a closed economy, with subsequent examination of how fertility and education investment for children change. Results of theoretical analysis indicate that the child allowance raises fertility and reduces educational investment. However, the subsidy for education investment reduces fertility and raises education investment. Being different from the case of a small open economy, the effects on education investment can be magnified or diminished because of physical capital accumulation. An increase in the contribution rate of pension benefits reduces income growth. These results are attributable to physical capital accumulation. Moreover, this paper presents derivation of the first best allocations to support discussion of how policies should be provided.

JEL：J11, J13, H22

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1. Introduction

This paper sets an endogenous fertility model with human capital accumulation in a closed economy. Because of the closed economy, the child care policy effects on income growth and other indicators differ

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** Correspondence to: School of Economics, Kwansei Gakuin University, 1-155, Uegahara Ichiban-Cho, Nishinomiya, Hyogo 662-8501, Japan Tel.: +81-798-54-6993, E-mail: yasuoka@kwansei.ac.jp

from those that occur in a small open economy. Given a change of the physical capital stock per capita, this paper presents a derivation showing that the effects of a subsidy for education investment can differ compared to those of a case of a small open economy.

Many related papers have described examinations of the effects of child care policies on fertility and education investment for children. Van Groezen, Leers, and Meijdam (2003) and Yasuoka and Goto (2011) derive that a child allowance raises fertility in a small open economy. In a closed economy, a child allowance cannot always raise fertility, as demonstrated by van Groezen and Meijdam (2008), Fanti and Gori (2009), and Yasuoka and Goto (2015) because of the effect by which the child allowance reduces household income. Miyazaki (2013) examines how the pension contribution rate affects fertility.

Subsidies for educational investment are examined mainly using human capital growth models. Glomm and Ravikumar (1992) set public education investment such that the education cost is fully covered by the government expenditure. They examine the income growth rate and inequality. Zhang (1997) and Yasuoka and Miyake (2014) set an endogenous fertility model with human capital growth and derive the effects of a child allowance and education subsidies on fertility and the human capital growth rate, i.e. income growth, in a small open economy. De la Croix and Doepke (2003) examine a closed economy model that incorporates quality and quantity of children. Nevertheless, child care policy effects are not examined sufficiently.

The arguments examined in this paper insist on the importance of physical capital accumulation as the effect of policy. Generally, a pay-as-you-go pension reduces household saving and decreases physical capital accumulation. Therefore, if the contribution rate decreases, physical capital accumulation is stimulated and the pension benefit can be pulled up because the income per capita rises. This result is demonstrated by Fanti and Gori (2010). Fertility and education investment can be affected by physical capital accumulation because the household income changes. The effect on fertility is examined by Wigger (1999).[1)] Fenge and Meier (2005), Meier and Wrede (2010), and Yasuoka (2018) examine the pension incentive policy for fertility and education investment.

Some studies assess the manner in which monetary policy affects fertility. Fanti (2012) and Chang, Chen, and Chang (2013) set an endogenous fertility model with the monetary stock and derive that an increase in the money stock affects fertility. As reported by Yakita (2006), monetary policy affects physical capital accumulation because of changes in the inflation rate. Then, the household income changes, which can affect fertility.

This paper presents examination of the effects of a child allowance and a subsidy for education investment. Compared with a small open economy, the negative effect of child allowances on education investment can be magnified in a closed economy. However, the effects of a subsidy for education investment can be diminished. Moreover, the contribution rate for pension benefits affects income growth not only via physical capital but also via human capital accumulation.

The remainder of this paper is presented follows. Section 2 presents the model. Section 3 explains derivation of the equilibrium. Section 4 examines how the child allowance, education subsidy and pension policy affect fertility, the human capital growth rate (income growth rate), and the physical capital stock per unit of effective labor. Section

1) As shown by Nishimura and Zhang (1992), if children are considered as an investment by which children give a gift to their parents during the old period, then the increase in pension benefit can be lessened.

5 presents discussion of the results obtained from these analyses and presents a comparison with the case of a small open economy. In section 6, we derive the first best allocations of how the policies are executed. The final section concludes this paper.

2. Model

As described in this paper, the model includes agents of three types: households, firms, and a government.

2.1 Household

Households exist in three periods: childhood, adulthood, and the old period. In childhood, individuals receive education investment from their parents. In adulthood, the individuals decide their own number of children n_t, education investment for the children e_t, consumption during adulthood c_{1t}, and saving s_t for consumption during the old period c_{2t+1}. Here, t denotes the period. For these analyses, we use a three-period overlapping generations model: In any t period, children, younger people, and older people all co-exist. The budget constraint in adulthood is

$$s_t = w_t h_t - c_{1t} - (z_t - q_t)n_t - (1 - x_t)e_t n_t - T_t. \tag{1}$$

In that equation, z_t denotes the child care cost for a child. With child allowance q_t, the net child care cost is given as $z_t - q_t$. In the equation, x_t represents the subsidy rate for education investment. w_t and h_t respectively denote the wage rate per unit of effective unit of labor and human capital stock. T_t stands for the lump sum tax to provide resources for child care policies.

In the old period, the budget constraint is given as

$$(1 + r_{t+1})s_t + p_{t+1} = c_{2t+1}. \tag{2}$$

As shown there, r_{t+1} denotes the real interest rate. In the old period, individuals obtain pension benefit p_{t+1}.

Considering (1) and (2), the lifetime budget constraint can be reduced.

$$w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}} = c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} + (z_t - q_t)n_t + (1-x_t)e_t n_t. \tag{3}$$

The human capital accumulation of children is formed by the input of education investment and parental human capital stock. It is assumed as

$$h_{t+1} = He_t^{\varepsilon} h_t^{1-\varepsilon}, 0 < H, 0 < \varepsilon < 1. \tag{4}$$

Parents care for the number of children, human capital of children, and consumption. The utility function is assumed as presented below.

$$u_t = \alpha \ln n_t h_{t+1} + \beta \ln c_{1t} + (1-\alpha-\beta)\ln c_{2t+1},$$
$$0 < \alpha < 1, 0 < \beta < 1, \alpha+\beta < 1 \tag{5}$$

The optimal allocations to maximize utility (5) subject to the lifetime budget constraint (3) and human capital accumulation (4) are expressed as presented below.

$$n_t = \frac{\alpha(1-\varepsilon)}{z_t - q_t}\left(w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{6}$$

$$e_t = \frac{\varepsilon(z_t - q_t)}{(1-\varepsilon)(1-x_t)}, \tag{7}$$

$$c_{1t} = \beta\left(w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{8}$$

$$c_{2t+1} = (1-\alpha-\beta)(1+r_{t+1})\left(w_t h_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{9}$$

2.2 Firms

Final goods are produced by inputting physical capital stock K_t. The effective labor $L_t = N_t h_t$. N_t denotes the population size of younger people in period t. Production function Y_t is assumed as

$$Y_t = AK_t^{\theta}L_t^{1-\theta}, 0 < A, 0 < \theta < 1. \tag{10}$$

Assuming a perfectly competitive market, the wage rate and the interest rate are given by marginal productivity as

$$w_t = A(1-\theta)k_t^{\theta}, \tag{11}$$

$$1 + r_t = A\theta k_t^{\theta-1}, \tag{12}$$

where $k_t = \frac{K_t}{L_t}$. For these analyses, we assume that the physical capital stock is fully depreciated in a single period.

2.3 Government

Government child care policies provide a child allowance and education subsidy. In addition to these policies, a pension benefit is provided for older people. We respectively consider child care cost $z_t = \bar{z}w_t h_t$, a child allowance $q_t = \bar{q}w_t h_t$, education subsidy $x_t = x$, and pension benefit $p_{t+1} = \tau n_t w_{t+1} h_{t+1}$. In addition, $\bar{q}$, x and τ are, respectively, constant over time. With a balanced budget, the government budget constraint is given as[2)]

$$T_t = \bar{q}w_t h_t n_t + xe_t n_t + \tau w_t h_t n_{t-1}. \tag{13}$$

3. Equilibrium

This section presents derivation of the equilibrium. This model includes the assumption of the child care cost as $z_t = \bar{z}w_t h_t$. In

2) To examine the effects of policies one by one, no difference exists between the separate government budgets and not separate one. This paper adopts lump-sum taxation, but a proportional income tax can be considered. However, even if one considers a proportional income tax, the effect is the same as that of a case of lump-sum tax because the households supply labor inelastically.

addition, $\bar{z}$ is constant over time.[3] Given h_t and K_t, the growth rate of human capital $1+g$ is given as[4]

$$1+g_t=\frac{h_{t+1}}{h_t}=H\left(\frac{\varepsilon(\bar{z}-\bar{q})w_t}{(1-\varepsilon)(1-x)}\right)^{\varepsilon}. \tag{14}$$

The capital market equilibrium condition is given as $K_{t+1}=N_t s_t$. Then, the following equation can be derived:

$$K_{t+1}=N_t\left((1-\alpha-\beta)(w_t h_t-T_t)-\frac{(\alpha+\beta)p_{t+1}}{(1+r_{t+1})}\right), \tag{15}$$

Considering $k_t=\frac{K_t}{N_t h_t}$, the dynamics of k_t is given as shown below.

$$n_t(1+g_t)k_{t+1}=(1-\alpha-\beta)w_t-\frac{(\alpha+\beta)p_{t+1}}{(1+r_{t+1})h_t} \tag{16}$$

Without policies, the dynamics of k_t is shown as presented below.

$$k_{t+1}=\frac{(1-\alpha-\beta)A^{1-\varepsilon}(1-\theta)^{1-\varepsilon}(1-\varepsilon)^{1-\varepsilon}\bar{z}}{\alpha\varepsilon^{\varepsilon}}k_t^{(1-\varepsilon)\theta} \tag{17}$$

Considering (6)-(9), (11)-(15), and (17), one obtains c_{t+1}, n_t, e_t, w_t, r_{t+1}, g_t, k_{t+1} for given k_t.

The balanced growth path can be given as $k_{t+1}=k_t=k$. Then, the growth rate of human capital g_t is constant rate g. Without policy parameters, i.e., $\bar{q}=0$, $x=0$ and $\tau=0$, the growth rate of human capital in the balanced growth path is given as

$$1+g=H\left(\frac{\varepsilon\bar{z}w}{1-\varepsilon}\right)^{\varepsilon}. \tag{18}$$

where

$$w=A(1-\theta)k^{\theta}. \tag{19}$$

The interest rate in the balanced growth path is

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3) This paper relies on the assumption that the child care cost is proportional to the wage rate. This is a consistent assumption because child care services are provided by nursing labor. The wage rate is a cost of providing childcare. Yasuoka and Miyake (2010) consider the labor market of child care services explicitly and derive that child care service costs depend on the wage rate.

4) In this model, the growth rate of human capital coincides with the income growth rate.

$$1 + r = A\theta k^{\theta - 1}. \tag{20}$$

The fertility is given as

$$n = \frac{\alpha(1-\varepsilon)}{\bar{z}}. \tag{21}$$

Substituting (18)-(21) into (17), the capital stock per unit of effective unit of labor can be derived such that the following equation holds:[5)]

$$k = \left(\frac{\bar{z}^{1-\varepsilon} A^{1-\varepsilon} (1-\theta)^{1-\varepsilon} (1-\alpha-\beta)}{\alpha H (1-\varepsilon)^{1-\varepsilon} \varepsilon^{\varepsilon}} \right)^{\frac{1}{1-\theta(1-\varepsilon)}}. \tag{22}$$

Considering (18)-(22), we obtain w, r g, k, n in the balanced growth path.

4. Policy Effects

This section presents an examination of how policies such as an increase in child allowance, education subsidy, and pension benefit affect the income growth rate and fertility.

4.1 Child Allowance

This subsection presents an examination of the child allowance effect. Then, the government budget constraint (13) changes to the following equation as

$$T_t = \bar{q} w_t h_t n_t. \tag{23}$$

Considering (6), (14), and (23), we obtain the following fertility and income growth rate. An increase in child allowance level $\bar{q}$ raises fertility.

$$n = \frac{\alpha(1-\varepsilon)}{\bar{z} - (1 - \alpha(1-\varepsilon))\bar{q}}. \tag{24}$$

$$1 + g_t = \frac{h_{t+1}}{h_t} = H \left(\frac{\varepsilon(\bar{z} - \bar{q}) w_t}{1-\varepsilon} \right)^{\varepsilon}. \tag{25}$$

5) See Appendix for a detailed proof of the local stability condition.

Then, an increase in child allowance $\bar{q}$ raises fertility. We obtain $\frac{dn}{d\bar{q}} > 0$.

The physical capital stock is given such that the following equation holds:

$$n(1+g)k = (1-\alpha-\beta)(1-\bar{q}n)w. \tag{26}$$

Regarding the variables used therein, g, w, r, n are given respectively as (11), (12), (24), and (25). From total differentiation of (11), (24), (25), and (26) with respect to k, g, w, r, n, $\bar{q}$ at the approximation of $\bar{q}=0$, one can obtain $\frac{dk}{d\bar{q}}$ as[6)]

$$\frac{dk}{d\bar{q}} = -\frac{w(1-\alpha-\beta)}{\alpha(1+g)(1-\theta(1-\varepsilon))} < 0 \tag{27}$$

We can obtain $\frac{dk}{d\bar{q}} < 0$. The child allowance reduces the capital stock per unit of effective labor.

By total differentiation of (25) with respect to g, k, $\bar{q}$ at the approximation of $\bar{q}=0$, one can obtain $\frac{dg}{d\bar{q}}$ as

$$\frac{dg}{d\bar{q}} = -\frac{\varepsilon(1+g)}{\bar{z}(1-\theta(1-\varepsilon))} < 0. \tag{28}$$

Then, the following proposition can be established.

Proposition 1

The child allowance raises fertility. The human capital growth rate and the capital stock per unit of effective labor are decreased.

This result is the same as that reported by Zhang (1997). However, Zhang (1997) derives the results for a small open economy. Even extending the model for a closed economy, we obtain the same result: the child allowance reduces the human capital growth rate.

Child allowance effects on fertility are reported in many related

6) See the Appendix for total differentiation.

papers such as those by van Groezen, Leers, and Meijdam (2008) and by Yasuoka and Goto (2011). Considering a closed economy, the negative effect of the child allowance on the human capital growth rate is magnified because the wage rate, regarded as the child care cost of having children, decreases. As a result of $\frac{dg}{d\bar{q}} < 0$, substitution of the quantity and quality of children occurs.

4.2 Education Subsidy

This subsection presents derivation of how the education subsidy affects the capital stock per unit of effective labor, the human capital growth rate, and fertility. The government budget constraint (13) is given as

$$T_t = xe_t n_t = \frac{x\varepsilon \bar{z} w h_t}{(1-\varepsilon)(1-x)} n. \tag{29}$$

Then, fertility and income growth rate are given as

$$n = \frac{\alpha(1-\varepsilon)}{\bar{z}} \left(1 - \frac{x\varepsilon\bar{z}}{(1-\varepsilon)(1-x)} n\right). \tag{30}$$

$$1 + g = \frac{h_{t+1}}{h_t} = H \left(\frac{\varepsilon \bar{z} w}{(1-\varepsilon)(1-x)}\right)^{\varepsilon}. \tag{31}$$

The physical capital stock is given as

$$n(1+g)k = (1-\alpha-\beta)\left(1 - \frac{x\varepsilon\bar{z}n}{(1-\varepsilon)(1-x)}\right) w. \tag{32}$$

By total differentiation of (30) with respect to n and x at the approximation of $x = 0$, we can obtain $\frac{dn}{dx} < 0$ as

$$\frac{dn}{dx} = -\alpha\varepsilon n < 0. \tag{33}$$

By total differentiation of (32) with respect to g, k, x, we obtain[7]

$$\frac{dk}{dx} = -\frac{w\varepsilon(1-\alpha-\beta)}{n(1+g)(1-\theta(1-\varepsilon))} < 0. \tag{34}$$

Subsidy for education investment reduces the capital stock per effective labor.

7) See the Appendix for total differentiation.

By total differentiation of (31) with respect to g, k, x at the approximation of $x=0$, one can obtain $\frac{dg}{dx}$ as

$$\frac{dg}{dx}=\frac{\varepsilon(1+g)(1-\theta)(1-\varepsilon)}{1-\theta(1-\varepsilon)}>0. \tag{35}$$

Then, the following proposition can be established.

Proposition 2

The subsidy for education investment decreases fertility and the physical capital stock per unit of effective labor. The human capital growth rate rises.

As demonstrated by Zhang (1997) and Yasuoka and Miyake (2014), a subsidy for education investment reduces fertility and raises the income growth rate in a small open economy. This result is obtainable in the closed economy.

4.3 Pension Policy

The final subsection presents examination of pension effects on fertility, the human capital growth rate, and the physical capital stock per capita. From (13), the government budget constraint is given as

$$T_t=\tau w_t h_t n_{t-1}. \tag{36}$$

The pension benefit is given as $p_t=\tau n w_t h_t$ in the balanced growth path. Consequently, fertility is

$$n=\frac{\alpha(1-\varepsilon)}{\bar{z}}\left(1-\tau+\frac{\tau n(1+g)}{1+r}\right). \tag{37}$$

In addition, $\frac{dn}{d\tau}$ can be derived as

$$\frac{dn}{d\tau}=n\left(\frac{n(1+g)}{1+r}-1\right). \tag{38}$$

With $\frac{n(1+g)}{1+r}-1>0$, an increase in τ raises the household lifetime income by virtue of an increase in the pension benefit: fertility increases.[8)] The physical capital stock per effective labor is given as

$$n(1+g)k=(1-\alpha-\beta)(1-\tau)w-\frac{\tau(\alpha+\beta)n(1+g)w}{1+r}. \tag{39}$$

By total differentiation of (31) with respect to k and τ, we can obtain as[9)]

$$\frac{dk}{d\tau}=-\frac{(1-\alpha-\beta)\left(\dfrac{n(1+g)}{1+r}+\dfrac{(\alpha+\beta)(1-\theta)}{\theta}\right)}{(1+g)n(\varepsilon\theta+1-\theta)}<0. \tag{40}$$

Therefore, an increase in contribution rate reduces the physical capital stock per unit of effective labor. As shown by (18), we can obtain $\frac{dg}{d\tau}<0$ because k decreases. Then, the following proposition can be established.

Proposition 3

In the case of $\frac{n(1+g)}{1+r}-1>0$, fertility can be increased by an increase in the contribution rate of pension benefit. However, the physical capital stock per unit of effective labor and the human capital growth rate decrease.

One might state from intuition that the condition to increase fertility and the income growth rate depends on $\frac{n(1+g)}{1+r}-1>0$. Fanti and Gori (2010), Miyazaki (2013), and others examine how the contribution rate of a pay-as-you-go pension affects lifetime income. The negative effect of an increase in pension benefits on the physical capital stock

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8) This result can be obtained generally. The condition of $\frac{n(1+g)}{1+r}-1>0$ dictates that the pension benefit is greater than the interest rate for saving; it also dictates that the lifetime income increases. By virtue of the increase in lifetime income, fertility increases.

9) See the Appendix for total differentiation.

is generally considered in the field of economics for a pay-as-you-go pension. In a closed economy, an increase in the contribution rate reduces the income growth rate because the child care cost z_t decreases and the cost of education investment is high.

These propositions are derived in a closed economy model. Different from a small open economy model, the physical capital stock has an important effect on policy. In the next section, we compare a closed economy model with a small open economy model.

5. Comparison of Small Open Economy

In a small open economy, the interest rate (12) is constant over time because the interest rate is given by the world interest rate. Therefore, the physical capital stock per unit of effective labor is fixed by the world interest rate. The wage rate is constant over time, too. Therefore, in a small open economy, no policy effect exists via physical capital accumulation.

In a small open economy, as shown by (27) and (28), we obtain $\frac{dn}{d\bar{q}} > 0$ and $\frac{dg}{d\bar{q}} < 0$ as the effect of the child allowance. The signs of $\frac{dn}{d\bar{q}}$ and $\frac{dn}{dx}$ are the same as those of the case of a closed economy. However, in a closed economy, we obtain $\frac{dk}{d\bar{q}} < 0$. This effect brings about a decrease in the income growth rate because the wage rate decreases. A decrease in the wage rate raises the relative cost of education investment. Therefore, the negative effect of a child allowance on income growth in a closed economy is greater than in the case of a small open economy.

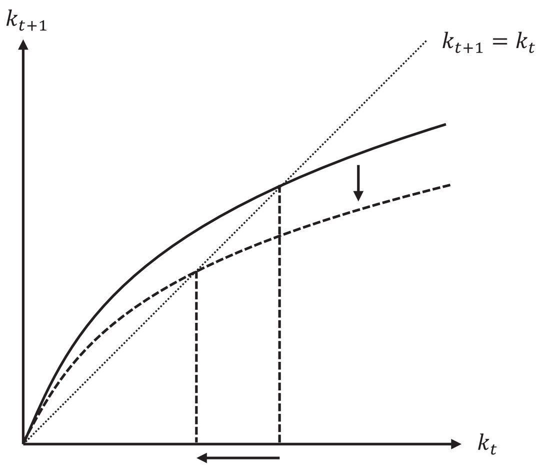
The effect of a subsidy for education investment in a closed economy should be confirmed. As shown by (33) and (35), we obtain $\frac{dn}{dx} < 0$ and $\frac{dg}{dx} > 0$. These signs are the same as those obtained in related reports from the literature by Zhang (1997) and by Yasuoka and Miyake

(2014), who examined a small open economy. However, we obtain the effects on the physical capital stock. Because of $\frac{dk}{dx} < 0$, the positive effect on income growth diminishes.

An increase in pension benefit policy reduces the physical capital stock. This effect reduces the income growth rate because the cost of education investment is high. This result can not be obtained for a small open economy. By considering a closed economy, the policy implications can be rich.

Using a closed economy model with physical capital accumulation, one can verify the effects of the policy on the transitional path. As shown by Fig. 1, both the child allowance and a subsidy for education investment reduce the physical capital stock per effective labor k in the balanced growth path. k_t moves to the new balanced growth path equilibrium. Then k_t continues decreasing. Thereafter, income growth g continues decreasing even if g decreases instantaneously because of

Fig. 1: Dynamics of k_t



an increase in the child allowance. Finally, the instantaneous negative effect on g is magnified.

In the case of a subsidy for education, k decreases depending on the preference parameter. With a decrease in k in the balanced growth path, g decreases. Therefore, even if the subsidy raises g instantaneously, the positive effect on g is weakened because k_t continues decreasing. As described above, the closed economy model brings about the transitional path for the effects of the policy.

6. First Best Solution

In this section, we derive the first best solution. To derive the solution, we set the following social welfare function as

$$W = \sum_{s=t}^{\infty} \rho^{s-t} (\alpha \ln n_s h_{s+1} + \beta \ln c_{1s} + (1-\alpha-\beta) \ln c_{2s+1}). \tag{41}$$

In that equation, $\rho (0 < \rho < 1)$ denotes the discount rate of utility of each generation.[10)]

The resource constraint can be presented as

$$Ak_s^{\theta} - \frac{c_{1s}}{h_s} - \frac{c_{2s}}{h_s n_{s-1}} - \frac{z_s n_s}{h_s} - \frac{e_s n_s}{h_s} - n_s(1+g_s)k_{s+1} = 0. \tag{42}$$

Based on constraints (4) and (42), we derive the first best solution to maximize social welfare.

The first best allocations are derived as

$$1 + g = \frac{A\rho\theta(1 - \bar{z}(1-\theta)n)k^{\theta}}{nk}, \tag{43}$$

10) Social welfare consists of the utility of each generation from zero period to infinity. At the zero period, younger and older people co-exist. However, older people finish consumption in the young period before the zero period, then only the utility of older people at the zero period is included in the social welfare function. This social welfare function does not include the weight of population. This setting is the same as that used by van Groezen, Leers and Meijdam (2003).

$$\frac{c_{2t}}{c_{1t}} = \frac{(1-\alpha-\beta)n}{\rho\beta}, \tag{44}$$

$$\frac{c_{1t}}{h_t} = \frac{\rho\beta y}{\rho+1-\alpha-\beta}, \tag{45}$$

$$\frac{e_t}{h_t} = \frac{\varepsilon}{n}\left(\rho\beta y\left(\frac{1}{\rho+1-\alpha-\beta}+\frac{1}{\alpha}\right)-\frac{\beta(1+\rho)n(1+g)k}{\alpha}\right), \tag{46}$$

$$\frac{n(1+g)k}{y}\left(1-\frac{\beta\varepsilon(1+\rho)}{\alpha}\right) = \frac{1-(1-\alpha-\beta+(1-\varepsilon)\rho\beta)}{\rho+1-\alpha-\beta}+\frac{\varepsilon\rho\beta}{\alpha}-\bar{z}(1-\theta)n, \tag{47}$$

where $y = Ak^{\theta}$.

From (43)-(47), one can obtain the first best allocations at the balanced growth path, k, n, $\frac{c_{1t}}{h_t}$, $\frac{c_{2t}}{c_{1t}}$, $\frac{e_t}{h_t}$. The policy implications can be obtained from these allocations. With low ρ, i.e., if the government does not consider the utility of the future generation, then income growth rate g is expected to be small, as shown by (43). Because of $1+g = H\left(\frac{e_t}{h_t}\right)^{\varepsilon}$, $\frac{e_t}{h_t}$ should be small. Therefore, a subsidy for education investment must not be provided.

7. Conclusions

This paper sets an endogenous fertility model with human capital accumulation in the closed economy. Because of the closed economy, the effects of a child care policy on income growth and other outcomes differ from those of the case of small open economy. Because of a change of physical capital stock per unit of effective labor, this paper presents derivation by which the effect of the policy for education investment can be magnified or diminished because substitution between the quality and quantity of children reduces the cost of children. A change of the contribution rate for pension benefits affects the income growth rate not only via physical capital but also via human capital accumulation. Moreover, this paper presents derivation of the first best allocations to discuss the policies.

In a small open economy, policy effects can be observed simply. Considering a closed economy, the dynamics of physical capital accumulation make the results of policy effects complicated. In addition, a transitional path of the effects of policies exists. However, these results suggest many policy implications.

References

Chang, W., Chen Y., & Chang J. 2013. "Growth and welfare effects of monetary policy with endogenous fertility." Journal of Macroeconomics, vol. 35, pages 117-130.

De la Croix D. & Doepke, M. 2003. "Inequality and growth: Why differential fertility matters," American Economic Review, vol. 93(4), pages 1091-1113.

Fanti, L. 2012. "Fertility and money in an OLG model," Discussion Papers 2012/145, Dipartimento di Economia e Management (DEM), University of Pisa, Pisa, Italy.

Fanti, L. & Gori, L. 2009. "Population and neoclassical economic growth: A new child policy perspective," Economics Letters, vol. 104(1), pages 27-30.

Fanti, L. & Gori, L. 2010. "Increasing PAYG pension benefits and reducing contribution rates," Economics Letters, Elsevier, vol. 107(2), pages 81-84.

Fenge R. & Meier V. 2005. "Pensions and fertility incentives," Canadian Journal of Economics, vol. 38(1), pages 28-48.

Glomm, G. & Ravikumar, B. 1992. "Public versus private investment in human capital endogenous growth and income inequality," Journal of Political Economy, vol. 100(4), pages 818-834.

Groezen B. van & Meijdam, L. 2008. "Growing old and staying young: population policy in an ageing closed economy," Journal of Population Economics, vol. 21(3), pages 573-588.

Groezen, B. van, Leers, T. & Meijdam, L. 2003. "Social security and endogenous fertility: pensions and child allowances as Siamese twins," Journal of Public Economics, vol. 87(2), pages 233-251.

Miyazaki K. 2013. “Pay-as-you-go social security and endogenous fertility in a neoclassical growth model,” Journal of Population Economics, vol. 26(3), pages 1233-1250.

Meier, V. & Wrede, M. 2010. “Pensions, fertility, and education,” Journal of Pension Economics and Finance, vol. 9(01), pages 75-93.

Nishimura, K. & Zhang, J. 1992. “Pay-as-you-go public pensions with endogenous fertility,” Journal of Public Economics, vol. 48(2), pages 239-258.

Wigger, B. U. 1999. “Pay-as-you-go financed public pensions in a model of endogenous growth and fertility,” Journal of Population Economics, vol. 12(4), pages 625-640.

Yakita, A. 2006. “Life expectancy, money, and growth,” Journal of Population Economics, vol. 19(3), pages 579-592.

Yasuoka, M. 2018. “Fertility and education investment incentive with a pay-as-you-go pension,” Eurasian Economic Review, vol. 8(1), pages 37-50.

Yasuoka, M. & Goto, N. 2011. “Pension and child care policies with endogenous fertility,” Economic Modelling, vol. 28(6), pages 2478-2482.

Yasuoka M. & Goto N. 2015. “How is the child allowance to be financed? By income tax or consumption tax?,” International Review of Economics, vol. 62(3), pages 249-269.

Yasuoka, M. & Miyake, A. 2010. “Change in the transition of the fertility rate,” Economics Letters, vol. 106(2), pages 78-80.

Yasuoka, M. & Miyake, A. 2014. “Fertility rate and child care policies in a pension system,” Economic Analysis and Policy, vol. 44(1), pages 122-127.

Zhang, J. 1997. “Fertility, growth, and public investments in children,” Canadian Journal of Economics, vol. 30(4), pages 835-843.

Appendix

Derivation of $\frac{dk}{d\bar{q}}$

From (24), one can obtain

$$dn = \frac{(1-\alpha(1-\varepsilon))n}{\bar{z}} d\bar{q}. \tag{A.1}$$

From (26), one can obtain

$$\frac{dk}{d\bar{q}} = \frac{\frac{n(1+g)}{\bar{z}} \frac{k}{w}(1-\alpha(1-\varepsilon)-\varepsilon) + n(1-\alpha-\beta)}{\frac{n(1+g)(1-\theta(1-\varepsilon))}{w}}. \tag{A.2}$$

Noting $\frac{k}{w} = \frac{1-\alpha-\beta}{n(1+g)}$, one can obtain (27).

Derivation of $\frac{dk}{dx}$

From (32), one can obtain the following:

$$-\frac{\alpha\varepsilon n(1+g)k}{w}dx + \frac{\varepsilon n(1+g)k}{w}dx + \frac{n(1+g)(1-\theta(1-\varepsilon))}{w}dk$$
$$= -\alpha\varepsilon(1-\alpha-\beta)dx. \tag{A.3}$$

Noting $\frac{n(1+g)k}{w} = 1-\alpha-\beta$, one can obtain (34).

Derivation of $\frac{dk}{d\tau}$

From (39), one can obtain the following

$$n(1+g)k\left(\frac{n(1+g)}{1+r}-1\right)d\tau+(\varepsilon\theta n(1+g)+n(1+g))dk-\frac{\theta(1-\alpha-\beta)w}{k}dk$$
$$= -\left((1-\alpha-\beta)w + \frac{(\alpha+\beta)n(1+g)w}{1+r}\right)d\tau. \tag{A.4}$$

Noting $\frac{n(1+g)k}{w} = 1-\alpha-\beta$, one can obtain (40).

First Best Solution

We set the Lagrange function as shown below.

$$L = \sum_{s=t}^{\infty} \rho^{t-s}(\alpha \ln n_s h_{s+1} + \beta \ln c_{1s} + (1-\alpha-\beta)\ln c_{2s+1})$$
$$+ \sum_{s=0}^{\infty} \lambda_s \left(Ak_s^{\theta} - \frac{c_{1s}}{h_s} - \frac{c_{2s}}{h_s n_{s-1}} - \frac{z_s n_s}{h_s} - \frac{e_s n_s}{h_s} - n_s(1+g_s)k_{s+1}\right) \tag{A.5}$$

The first-order condition is presented below.

$$\frac{\partial L}{\partial n_t} = \frac{\alpha \rho^{t-s}}{n_t} + \lambda_t \left(-(1+g_t)k_{t+1} - \frac{z_t}{h_t} - \frac{e_t}{h_t} \right) + \frac{\lambda_{t+1} c_{2t+1}}{n_t^2 h_{t+1}} = 0, \tag{A.6}$$

$$\frac{\partial L}{\partial k_{t+1}} = -\lambda_t n_t (1+g_t) + \frac{\lambda_{t+1}\theta}{k_{t+1}} (-\bar{z} w_{t+1} n_{t+1} + y_{t+1}) = 0, \tag{A.7}$$

$$\begin{aligned} \frac{\partial L}{\partial e_t} = &\frac{\alpha \varepsilon \rho^{t-s}}{e_t} - \lambda_t \left(\frac{\varepsilon n_t (1+g_t) k_{t+1}}{e_t} + \frac{n_t}{h_t} \right) \\ &+ \frac{\varepsilon \lambda_{t+1}}{e_t} \left(\frac{c_{1t+1}}{h_{t+1}} + \frac{c_{2t+1}}{h_{t+1}} n_t + \frac{z_{t+1} n_{t+1}}{h_{t+1}} + \frac{e_{t+1} n_{t+1}}{h_{t+1}} \right) = 0, \end{aligned} \tag{A.8}$$

$$\frac{\partial L}{\partial c_{1t}} = \frac{\beta \rho^{t-s}}{c_{1t}} - \frac{\lambda_t}{h_t} = 0, \tag{A.9}$$

$$\frac{\partial L}{\partial c_{2t+1}} = \frac{(1-\alpha-\beta)\rho^{t-s}}{c_{2t+1}} - \frac{\lambda_{t+1}}{n_t h_{t+1}} = 0, \tag{A.10}$$

$$\frac{\partial L}{\partial c_{1t+1}} = \frac{\beta \rho^{t+1-s}}{c_{1t}} - \frac{\lambda_{t+1}}{h_{t+1}} = 0, \tag{A.11}$$

where $y_{t+1} = Ak_{t+1}^{\theta}$. From (A.7), (A.9) and (A.11), one can obtain $\frac{c_{1t+1}}{c_{1t}}$ at the balanced growth path as[11)]

$$1 + g = \frac{A\rho\theta(1-\bar{z}(1-\theta)n)k^{\theta}}{nk}. \tag{A.12}$$

From (A.10) and (A.11), one can obtain the following equation:

$$\frac{c_{2t}}{c_{1t}} = \frac{(1-\alpha-\beta)n}{\rho\beta}. \tag{A.13}$$

From (A.6), (A.9), and (A.11), we derive

$$\frac{c_{1t}}{h_t} = \frac{\rho\beta y}{\rho + 1 - \alpha - \beta}. \tag{A.14}$$

From (A.9) and (A.10), one can obtain

$$\frac{c_{2t+1}}{c_{1t}} = \frac{(1-\alpha-\beta)\theta(y-\bar{z}wn)}{\beta k}. \tag{A.15}$$

From (A.8) and (A.9), one can obtain the following equation:

$$\frac{c_{1t}}{h_t} = \frac{\beta}{\alpha} \left((1+\rho)n(1+g)k - \rho A k^{\theta} + \frac{n}{\varepsilon} \frac{e_t}{h_t} \right). \tag{A.16}$$

11) At the balanced growth path, we obtain $c_{1t+1}/c_{1t} = 1 + g$.

From (A.14) and (A.16), one can obtain

$$\frac{e_t}{h_t} = \frac{\varepsilon}{n}\left(\rho\beta y\left(\frac{1}{\rho+1-\alpha-\beta}+\frac{1}{\alpha}\right)-\frac{\beta(1+\rho)n(1+g)k}{\alpha}\right). \tag{A.17}$$

From (42), (A.13), (A.14) and (A.17), one obtains the following:

$$\frac{n(1+g)k}{y}\left(1-\frac{\beta\varepsilon(1+\rho)}{\alpha}\right) = 1-\frac{1-\alpha-\beta+(1-\varepsilon)\rho\beta}{\rho+1-\alpha-\beta} +\frac{\varepsilon\rho\beta}{\alpha}-\bar{z}(1-\theta)n. \tag{A.18}$$