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Hidden Cost of Sanctions in a Dynamic Principal-Agent Model: Reactance to Controls and Restoration of Freedom*

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Abstract

This study examines the effect of the principal's control over the agent's behavior in a dynamic principal-agent model with hidden information. We show the condition that the agent who has a similar preference for actions as the principal dares to choose the unpreferred action when the principal imposes a sanction on such an action. This also makes the principal worse off even when imposing sanctions is materially costless. When the principal incurs a cost on sanctions, they cease implementing them after observing the unpreferred action taken by the agent. Our results of the hidden cost of control correspond to the insight from the psychological reactance theory: when an agent's freedom is threatened, they resist it to restore the freedom.

Keywords: Dynamic principal-agent model, Hidden cost of controls, Psychological reactance, Ratchet effects, Sanction.
JEL Classification: D82, D86, D91, M52.

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1. Introduction

The incongruence in preferences between the principal (she) and the agent (he) as well as private information held by the agent deteriorates efficiency in agency models. To mitigate inefficiency, the principal implements controls as well as rewards for the agent's actions to direct him to act according to her preferred behavior. However, controls often do not work well and, consequently, exacerbate the agency problem. This malfunction of controls is referred to as the "hidden cost of controls" (e.g., Falk and Kosfeld, 2006) : the principal's unexpected cost caused by the agent's reaction to controls. However, for the principal, whether the hidden costs are eventually outweighed by the gain of controls is still a controversial issue (e.g., Schnedler and Vadovic, 2011). This implies that it is also important to understand how the principal reacts, in turn, to the agent's reactions. Beyond economics literature, the *psychological reactance theory* (Brehm, 1966; Brehm and Brehm, 1981) states that when the agent's freedom is threatened, he resists it to restore the freedom. This phenomenon can be considered a hidden cost of controls. To confirm whether to restore freedom, the principal's reaction to the agent's reactance must be studied. The contributions of this study are to show not only how the agent reacts to the controls of the principal and how the reaction affects the principal's welfare, but also how the principal responds to the agent's reaction in our dynamic agency model.

In our model, the principal and agent interact for two periods: In each period, the agent chooses an action, either x or y. Although the principal prefers the agent to take action x, the agent's preferred action depends on his type: one type of agent prefers x to y, whereas another type of agent prefers y to x. For example, x can be a mission-oriented action that is the right direction for the organization to which both the principal and agent belong, whereas y is a self-interested action that benefits only the agent. Whether the agent cares about such a mission depends on his preference. In this sense, we can interpret that an agent who prefers x to y is a motivated agent, whereas the one who prefers y to x is a selfish agent.[1] The following two

[1] We borrow the terms "motivated" and "selfish" agent from Besley and Ghatak (2006), although their definitions are not the same as those used in our study.

components are the crux of our model. First, to make the agent choose the principal's preferred action x, the principal can use financial incentives and sanctions as controls. The former is a monetary reward for the agent taking x, while the latter is a non-monetary sanction for taking y. A sanction is supposed to be a control device, such as reprimands, moralization, stigmatization, or threatening with demotions, which the agent experiences. Second, the principal faces two kinds of uncertainty regarding the agent's preferences. The first uncertainty is the agent's preferences for actions. We suppose that while the agent's material payoff for taking x is normalized to be zero regardless of his type, the payoff for taking y is either positive or negative, depending on his type: it is negative (positive) for the motivated (selfish) agent. The second uncertainty is the agent's tolerance for sanctions. Although sanctions make both types of agents worse off, how much the agent suffers from them depends on the agents' types of tolerance. In our model, the principal offers a contract at the beginning of each period. This signifies that the principal can offer a second-period contract after observing the first-period action taken by the agent.

We first show an equilibrium in which the motivated agent with a tolerance takes the unpreferred action y, which we refer to as *reactance*. In addition, reactance leads to the result that the availability of sanctions decreases the principal's payoff when the probability of the motivated agent with a tolerance is sufficiently high. This result can be interpreted as the hidden cost of controls for the principal: the principal unintentionally reduces her welfare when she imposes sanctions to lead the agent to act according to her preferred behavior, but this disincentivizes the agent. Although reactance occurs even when sanctions are unavailable, we specify the condition that it is more likely to happen when sanctions are available than when they are not. These results hold when imposing sanctions is materially costless for the principal.

To observe and explain the reasons for our results, we first confirm that sanctions have two effects on the agent's decisions: (1) sanctions directly reduce the agent's material benefit of opting for the principal's unpreferred behavior; (2) if the principal imposes a sanction on her unpreferred behavior, she can reduce the reward for the preferred behavior to make the selfish agent take it; thus, the information rent for the motivated agent is reduced. Because this reduction of reward weakens the incentives for the motivated agent to imitate a selfish agent, the availability of sanctions makes the principal better off. Although this observation is true even in a static setting, the principal's problem is more complicated in our dynamic model.

Consider the second period. After observing the agent's first-period behavior, the principal updates her belief about the agent's preferences for actions and tolerance. Considering this belief formation process with the above observation in a static setting, we find that a mixed strategy equilibrium occurs from the interaction between the conditional probability of the selfish agent with a tolerance after observing the unpreferred action y in the first period and the incentives to take y for the motivated agent with a tolerance who anticipates the second-period reward after taking y in the first period. When the conditional probability is high, the principal should raise the second-period reward to prevent the selfish agent from taking y. However, such a high reward causes the motivated agent to imitate a selfish one. In this case, the probability decreases, and the principal should lower the reward. Consequently, a low reward makes the motivated agent take x, and the probability increases. This emergence of a mixed equilibrium is true even when sanctions are unavailable. However, when the principal can impose sanctions, she can reduce the second-period reward to prevent the selfish agent from taking y, which enables her to offer the contract with a positive reward with certainty. If the disutility from being sanctioned is sufficiently small, the reduction in reward is also small, which increases the expected value of information rent. This incentivizes the motivated agent with a tolerance to take y, and reactance is more likely to occur when sanctions are available.

We next show that even with a small material cost of imposing sanctions, there is a possibility that although the principal imposes a sanction in the first period, she refrains from this in the second period, after observing reactance in the first period. To observe this, we assume that the intolerant agent never takes unpreferred action when a sanction is imposed. Then, under the condition that the agent took the unpreferred action, the agent must be tolerant. As in the costless sanctioning case, there is an equilibrium in which the principal mixes a contract comprising a sufficiently high reward with that comprising no reward. If the principal offers a contract with no reward for the preferred action, then imposing a sanction does not affect the tolerant agent's behavior. Although the selfish agent takes the unpreferred action, the motivated agent takes the preferred action. As imposing sanctions is costly and the principal is aware that the agent is tolerant, she decides not to impose sanctions. Note that this phenomenon occurs only when reactance behavior is observed. If this behavior does not occur–as taking the unpreferred action implies that the agent is selfish–the principal never offers a contract with no reward: the principal prefers to offer a contract with a sufficiently high reward to prevent the selfish agent from taking the unpreferred action.

We can summarize our results as corresponding with those presented in the psychological reactance theory, which theory explains the reactance behavior and restoration of freedom as follows: When the agent's freedom of choice is threatened or eliminated, he dares to take an unpreferred action for the principal, who restricts the freedom to restore the freedom even if the agent also does not prefer to take such an action. If we interpret the principal's imposing sanctions as a thread to freedom for the agent, after observing the reactance behavior by the motivated agent, the principal may decide not to impose sanctions, which can be interpreted as the agent's freedom being restored. In this sense, our results support the insight from psychological reactance theory.

The rest of the paper is organized as follows. Section 2 introduces our model. In Section 3, we examine the benchmark case where sanctions are unavailable. In Section 4, we study the equilibrium strategies of both the principal and the agent when sanctions are available. Section 5 demonstrates the effects of the availability of sanctions. In Section 6, we analyze the case where sanctions are materially costly for the principal. Section 7 provides the related literature and Section 8 concludes the paper.

2. Model

In our model, a principal and an agent interact for two periods, $t \in \{1, 2\}$. Both players are risk-neutral and do not discount the future. Furthermore, each player's payoff in period t depends on the agent's choice, which is denoted by $d_t \in \{x, y\}$. We suppose that the agent's choices are observable by the principal. Although the payoff for both players is normalized to 0 when $d_t = x$, the principal's payoff is -1 and that of the agent's is $u \in \{\underline{u}, \overline{u}\}$ when $d_t = y$. This means that the principal prefers x to y, but the agent's preferred choice depends on his type. We assume that $\underline{u} < 0 < \overline{u} < 1$ and $\overline{u} + \underline{u} > 0$. The first assumption implies that (i) without the principal's intervention, an agent with $u = \overline{u}$ prefers to take x, but one with $u = \underline{u}$ prefers to take y, and (ii) x is efficient for any type realization. From the principal's perspective, an agent with $u = \underline{u}$ is considered a *motivated agent* because the preferred actions are aligned. In contrast, an agent with $\overline{u}$ is perceived as a *selfish agent* because the preferences are misaligned. The second assumption implies that the degree of preferring a motivated agent x is weaker than that of preferring a selfish agent y. The prior probability is $\Pr(u = \overline{u}) = \beta$, and the true value of u is the agent's private information.

In each period, the principal has two means that cause the agent to take x. One is to give a monetary reward $m_t \geqslant 0$ to the agent for taking x.[2] The other is to impose a non-monetary sanction on the agent taking y. The principal's choice of imposing sanctions at t is binary: $s_t \in \{0, 1\}$; $s_t = 0$ signifies imposing no sanction and $s_t = 1$ means the opposite.[3] The principal's cost of imposing a sanction is denoted by $c \geqslant 0$. The agent is averse to the imposition of a sanction. When the agent is imposed with a sanction, he incurs a cost $a \in \{\underline{a}, \overline{a}\}$ where $0 < \underline{a} < \overline{a}$. We refer to the agent that has $a = \underline{a}$ ($a = \overline{a}$) as an agent with (without) a

[2]We assume that a monetary reward is non-negative.
[3]Supplementary material I considers a case in which $s_t \in [0, 1]$.

tolerance. The prior is $\Pr(a = \underline{a}) = \gamma$, and the true value of a as well as u is the agent's private information. We assume that u and a have no correlation. In each period, the principal offers a contract (s_t, m_t) before the agent's decision. Note that the second period contract (s_2, m_2) can depend on first-period behavior. In this case, we state $(s_2(d_1), m_2(d_1))$.

In summary, in each period t, the principal's instantaneous payoff is $-I(d_t = x)m_t - I(d_t = y)(1 + cs_t)$ and that of the agent is $I(d_t = x)m_t + I(d_t = y)(u - s_t a)$, where $I(E) = 1$ if E is true, otherwise, $I(E) = 0$. The agent's type can be specified using the private information regarding his payoff from taking y and cost of the imposed a sanction, (u, a). We focus on perfect Bayesian Nash equilibria satisfying the dominance criterion (PBE). The timing is summarized as follows:

$t = 1$
1. The principal offers $(s_1, m_1) \in \{1, 0\} \times \mathbb{R}_+$.
2. The agent chooses $d_1 \in \{x, y\}$.
3. (s_1, m_1) is implemented depending on d_1.

$t = 2$
1. After observing d_1, the principal offers $(s_2(d_1), m_2(d_1)) \in \{1, 0\} \times \mathbb{R}_+$.
2. The agent chooses $d_2 \in \{x, y\}$.
3. $(s_2(d_1), m_2(d_1))$ is implemented depending on d_2.

3. The Benchmark Case: No Sanctions

As a benchmark, this section examines the case in which sanctions are unavailable. We can interpret this case as the case when the principal makes a long-term commitment of $s_1 = s_2 = 0$. First, consider period 2. Let $P_{d_1} = \Pr(u = \overline{u} \mid d_1)$ be the updated probability of the selfish agent after the principal observes d_1. To make the selfish agent take x, the principal must pay $m_2 = \overline{u}$. Further, because the motivated agent also takes x when $m_2 = \overline{u}(> 0 > \underline{u})$, the principal's payoff is $-\overline{u}$. If the principal offers $m_2 < \overline{u}$, $m_2 = 0$ is optimal because only the motivated agent takes x. In this case, the principal's expected payoff is $-P_{d_1}$. As a result, $m_2 = \overline{u}$ is optimal if $P_{d_1} \geqslant \overline{u}$; otherwise, $m_2 = 0$ is optimal. Next, based on this second-period behavior, we examine the first period. Let $B_1 := \frac{\overline{u}+\underline{u}}{\frac{1}{\overline{u}}+\underline{u}}$, and $B_2 := \frac{\overline{u}-\underline{u}}{1-\underline{u}}$. Note that $B_1 < \overline{u} < B_2$. We have the following results:

Proposition 1. *The equilibrium behavior in the first period and the principal can be characterized as follows:*

(i) $\beta > B_2$*:* $m_1 = \overline{u}$ *is optimal for the principal. In this case, any type of agent takes* x*.*

(ii) $\beta \in (B_1, B_2)$*:* $m_1 = \overline{u} + \underline{u}$ *is optimal for the principal. In this case, the selfish agent takes* y *and the motivated agent takes* x*.*

(iii) $\beta < B_1$*:* $m_1 = 0$ *is optimal for the principal. In this case, the selfish agent takes* y *and the motivated agent takes a mix of* x *and* y *(takes* y *with probability* $\frac{\beta}{1-\beta}\left(\frac{1-\overline{u}}{\overline{u}}\right)$*).*

When β is high–that is, the prior belief of the selfish agent is high–the cost caused by the selfish agent taking y is high. To avoid this cost, the principal must offer a high m_1 such that the selfish agent takes $d_1 = x$ (Proposition 1 (i)). However, the principal bears the additional cost of providing such a high m_1 because she needs to pay it not only to the selfish agent but also to the motivated agent who would take x even if $m_1 = 0$. As β decreases, this negative effect of a high m_1 becomes serious. Subsequently, the principal reduces m_1, although it still needs to be positive. Reducing m_1 weakens the incentive for both types of agents to take $d_1 = x$. Further, increasing $m_2(y)$ provides an incentive for the selfish agent to take $d_2 = x$, even though it increases the cost incurred by the selfish agent taking $d_1 = y$. Through this contract, the principal can separate the selfish from the motivated agent (Proposition 1 (ii)). As β becomes much lower, the principal can reduce m_1 further, but this gives an incentive for the motivated agent to take $d_1 = y$ under a positive $m_2(y)$. In this case, however, as β is adequately low, after observing y is taken, the agent is extremely likely to be motivated, which makes the principal offer $m_2(y) = 0$. This, in turn, incentivizes the motivated agent to take x, which also motivates the principal to offer $m_2(y) = \overline{u}$ for forcing the selfish agent to take x. This cycle makes the principal take the mixed strategy between $m_2(y) = 0$ and $\overline{u}$, and the motivated agent takes the mixed strategy $d_1 = x$ and y (Proposition 1 (iii)). In summary, when β is sufficiently small, the motivated agent, who has no material benefit of taking y, may prefer to take y in an equilibrium. We refer to this behavior as a *reactance* by the motivated agent.

4. The Availability of Sanctions

In this section, we analyze how the principal takes advantage of her availability to impose sanctions for taking y, and how this opportunity influences the agent's behavior. As an additional sanction can reduce future payoff and present cost of taking y, the benefit of the agent in taking y decreases at first glance. However, the availability of sanctions may affect the possibility of reactance. By considering these effects, the principal determines whether to impose a sanction in each period.

For the sake of simplicity, we make the following assumptions:

Assumption 1. *(i)* $v_1 := \overline{u} - \underline{a} > 0 > v_2 := \underline{u} - \underline{a} > v_3 := \overline{u} - \overline{a} > v_4 := \underline{u} - \overline{a}$, *(ii)* $v_1 + v_2 > 0 > \overline{u} + v_3$, *and (iii)* $(1 + c)\gamma < 1$.

Assumption 1 holds when $\underline{a}$ is close to 0, $\overline{a}$ is sufficiently large, and c is adequately small.

As in the previous section, we start by considering the second period. Given $m_2(d_1)$ and s_2, the agent takes x if only if $m_2(d_1) \geqslant u - s_2 a$. First, suppose that $s_2 = 1$. In this case, even without a reward, only the selfish agent with a tolerance takes y. Subsequently, the probability of this agent (i.e., $(u, a) = (\overline{u}, \underline{a})$) matters for the decision on $m_2(d_1)$. Let $P^*_{d_1} = \Pr((u, a) = (\overline{u}, \underline{a}) \mid d_1)$ be the principal's updated probability of the selfish agent with a tolerance after observing d_1. As similar to the analysis in the benchmark case, if $(1 + c)P^*_{d_1} \geqslant v_1$, $m_2(d_1) = v_1$ is optimal; otherwise, $m_2(d_1) = 0$ is optimal. Second, suppose that $s_2 = 0$. If $P_{d_1} = \Pr(u = \overline{u} \mid d_1) \geqslant \overline{u}$, $m_2(d_1) = \overline{u}$ is optimal; otherwise, $m_2(d_1) = 0$ is optimal. We have the following lemma:

Lemma 1. *The best responses of each player in the second period can be summarized as follows:*

The principal's behavior:

(i) If $P_{d_1} = \min\{(1+c)P^*_{d_1}, P_{d_1}, v_1\}$, $(s_2(d_1), m_2(d_1)) = (0,0)$ *is optimal.*

(ii) If $(1+c)P^*_{d_1} = \min\{(1+c)P^*_{d_1}, P_{d_1}, v_1\}$, $(s_2(d_1), m_2(d_1)) = (1,0)$ *is optimal.*

(iii) If $v_1 = \min\{(1+c)P^*_{d_1}, P_{d_1}, v_1\}$, $(s_2(d_1), m_2(d_1)) = (1, v_1)$ *is optimal.*

The agent's behavior:

If $s_2 = 1$*, the selfish agent with a tolerance takes* y *if and only if* $m_2(d_1) < v_1$*; the other types take* x *for any* $m_2(d_1) \geqslant 0$*. If* $s_2 = 0$*, the selfish agent takes* y *if and only if* $m_2(d_1) < \overline{u}$*; the other types take* x *for any* $m_2(d_1) \geqslant 0$*.*

Note that the principal has the following four possible options: (i) The principal neither imposes a sanction against y nor rewards for x: As only the selfish agent takes y, the principal's expected payoff is $-P_{d_1}$. (ii) The principal imposes a sanction against y but gives no reward for x: As only the selfish agent with a tolerance takes y, the principal's expected payoff is $-(1+c)P^*_{d_1}$. (iii) The principal imposes a sanction against y and rewards for x: In this case, all types of agents take x. Then, the principal's expected payoff is $-v_1$. (iv) The principal imposes no sanction against y but rewards for x. In this case, the principal must pay $-\overline{u}$ to prevent the selfish agent from taking y, and her expected payoff is $-\overline{u}$. Among these, (iv) is never better than (iii) because $\overline{u} > v_1 = \overline{u} - \underline{a}$. Lemma 1 states that either (i), (ii), or (iii) is optimal.

Next, we consider the first period. As in the previous section, we examine whether there is an equilibrium in which the motivated agent takes y. Consider the case where $s_1 = 1$. For each reward plan, the equilibrium behavior is summarized in the following lemma:

Lemma 2. *Suppose that* $s_1 = 1$*. The first-period behavior can be characterized as follows:*

(i) The agent without a tolerance takes x*.*

(ii) The selfish agent with a tolerance takes x *if only if* $m_1 \geqslant v_1$*.*

(iii) As for the motivated agent with a tolerance:

a) If $\beta > v_1$, the agent takes x if only if $m_1 \geqslant v_1 + v_2$.

b) If $\beta < v_1$, the agent takes x if $m_1 \geqslant v_1 + v_2$. If $m_1 < v_1 + v_2$, the agent takes y with probability $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$ and takes x with the complementary probability. In this case, in the second period, the principal sets $(s_2(y), m_2(y)) = (1, v_1)$ with probability $\frac{m_1-v_2}{v_1}$ and $(s_2(y), m_2(y)) = (0, 0)$ with the complementary probability.

By Assumption 1, the agent without a tolerance never takes y. This is because, for this agent, the cost of being sanctioned by taking y is much higher than the benefit of earning an information rent by taking it and imitating a selfish agent with a tolerance. Next, we focus on the behavior of the agent with a tolerance. When m_1 is sufficiently large ($m_1 \geqslant v_1$), both types of agents are eager to take x. As m_1 decreases ($v_1 > m_1$), the selfish agent with a tolerance decides to take y, whereas the motivated agent with a tolerance still chooses to take x. The motivated agent with a tolerance also has an incentive to take y as m_1 is decreasing further ($v_1 + v_2 > m_1$).

In our dynamic model, rewards in the second period influence behavior in the first period. First, suppose β is adequately high such that $\beta > v_1$. Even if the motivated agent with a tolerance takes y, the principal's updated belief of the selfish agent with a tolerance (P_y^*) is sufficiently large because of a high β. Then, the principal is willing to offer $m_2(y) = v_1$. However, this induces the motivated agent with a tolerance to take $d_1 = y$ to earn the information rent. When m_1 is small ($m_1 < v_1 + v_2$), this temptation has a more serious effect on the principal's welfare. Second, suppose that β is adequately small such that $\beta < v_1$. Even if the motivated agent with a tolerance takes y, the principal's updated belief of the selfish agent with a tolerance (P_y^*) is sufficiently low because of a small β. Then, the principal is willing to offer $m_2(y) = 0$. This leads the motivated agent with a tolerance to take $d_1 = x$. However, the updated belief above becomes equal to 1, following which the principal should set $m_2(y) = v_1$. This, in turn, leads to a mixed strategy equilibrium as in Proposition 1 (iii).

To observe the role of imposing sanctions on y in the first period, we consider the equilibrium behavior in this period when $s_1 = 0$ and we have the following lemma:

Lemma 3. *Suppose that $s_1 = 0$. The first-period behavior can be characterized as follows:*

(i) The selfish agent takes x if only if $m_1 \geqslant \overline{u}$.

(ii) For the motivated agent:

a) If $v_1 > (1 + c)\gamma$ or $m_1 > v_1 + \underline{u}$, the agent takes x.

b) If $v_1 < (1 + c)\gamma$ and $m_1 < v_1 + \underline{u}$, the agent takes y with probability $\frac{\beta}{1-\beta}(\frac{(1+c)\gamma - v_1}{v_1})$ and x with the complementary probability. In this case, in the second period, the principal sets $(s_2(y), m_2(y)) = (1, v_1)$ with probability $\frac{m_1 - \underline{u}}{v_1}$ and $(s_2(y), m_2(y)) = (1, 0)$ with the complementary probability.

The crucial difference between $s_1 = 1$ and $s_1 = 0$ regarding the equilibrium behavior of the agent appears in the separation: When $s_1 = 1$, separation occurs between tolerant and intolerant agents, whereas when $s_1 = 0$, it occurs between selfish and motivated agents, as in the case when sanctions are unavailable.

The following lemma summarizes the unique equilibrium payoff of the principal, given that m_1 and s_1, which is denoted by $\pi(m_1, s_1)$:

Lemma 4. *For the given m_1 and s_1, the unique equilibrium payoff of the principal can be* *summarized as follows:*

$$\pi(m_1, 1) = \begin{cases} -m_1 - \min\{(1+c)\gamma\beta, v_1\} & \text{if} \quad m_1 > v_1 \\ -\gamma\beta(1+c+v_1) - (1-\gamma\beta)m_1 & \text{if} \quad m_1 \in (v_1+v_2, v_1) \\ -\gamma(1+c+v_1) - (1-\gamma)m_1 & \text{if} \quad m_1 \in [0, v_1+v_2) \text{ and } v_1 < \beta \\ -\frac{\gamma\beta}{v_1}(1+c+v_1) - \left(1 - \frac{\gamma\beta}{v_1}\right)m_1 & \text{if} \quad m_1 \in [0, v_1+v_2) \text{ and } v_1 > \beta. \end{cases}$$

$$\pi(m_1, 0) = \begin{cases} -m_1 - \min\{(1+c)\gamma\beta, v_1\} & \text{if} \quad m_1 > \overline{u} \\ -\beta(1+\min\{v_1, (1+c)\gamma\}) - (1-\beta)m_1 & \text{if} \quad m_1 \in (v_1+\underline{u}, \overline{u}) \\ -\beta(1+(1+c)\gamma) - (1-\beta)m_1 & \text{if} \quad m_1 \in [0, v_1+\underline{u}) \text{ and } v_1 > (1+c)\gamma \\ -\frac{(1+c)\gamma\beta}{v_1}(1+v_1) - \left(1 - \frac{(1+c)\gamma\beta}{v_1}\right)m_1 & \text{if} \quad m_1 \in [0, v_1+\underline{u}) \text{ and } v_1 < (1+c)\gamma. \end{cases}$$

The first observation using Lemma 4 is that when the sanctioning is costless (i.e., $c = 0$), $s_1 = s_2 = 1$ is always optimal.[4]

Proposition 2. *Here, suppose that $c = 0$; subsequently, $s_2 = 1$ in any Perfect Bayesian Equilibrium. For the principal, $s_1 = 1$ is always optimal.*

This proposition implies that when sanctioning is costless, the principal never refrains from sanctioning for taking y. By imposing sanctions, the agent's benefit of taking y decreases. This seems to decrease the probability of the unpreferred action being taken directly. Moreover, the principal can save the payment on x to make the agent take y. Therefore, as sanctioning is costless, there is no reason to forgo the option of sanctioning.

However, Proposition 2 does not imply that the probability of taking the unpreferred action becomes lower when the principal imposes sanctions for the agent taking y. In the next section, we show that allowing the sanctioning option can increase the probability of taking the unpreferred action, and we further explore the role of forgoing the sanctioning option on the probability of taking y and the principal's payoff.

[4]Our main results hold even when $c > 0$. We will study the case of costly sanctions ($c > 0$) in section 6.

5. Hidden Cost of Sanctions

To study how the availability of sanctions influences the agent's behavior and the principal's welfare, we compare the equilibrium reactance probabilities and the principal's payoffs between the cases where sanctions are available and unavailable.[5] Even though imposing sanctions does not incur any monetary cost, the principal may suffer from the agent's reactance when she imposes sanctions on the agent's behavior. We can interpret this negative effect of imposing sanctions as a hidden cost of sanctions for the principal. The following lemma characterizes when the motivated agent takes y with a positive probability.

Lemma 5. *(i) Suppose that $\beta > v_1$. The motivated agent with a tolerance takes y with certainty in the unique equilibrium if and only if*

$$\frac{v_1 + v_2}{1 + v_1 - \beta(1 - v_2)} > \gamma.$$

(ii) Suppose that $\beta < v_1$. The motivated agent with a tolerance takes y with probability $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$ in the unique equilibrium if and only if

$$\frac{1}{\beta}\frac{v_1 + v_2}{\frac{1}{v_1} + v_2} > \gamma.$$

From Proposition 1 and Lemma 5, Figure 1 summarizes when the motivated agent takes y depending on β and γ in the cases when sanctions are available and unavailable. Let R_s and R_{ns} be the probability that the motivated agent takes y in the first period when sanctions are available and unavailable, respectively.

First, when sanctions are unavailable, Proposition 1 shows that R_{ns} is positive for small β. Proposition 1 (iii) shows that if $\beta < B_1$, the probability that the motivated agent takes y is increasing in β. Since this makes the principal worse off, she would reward for taking x and the motivated agent to take x. Therefore, if β is high, R_{ns} becomes 0. Next, when sanctions

[5] In this section, we still assume that $c = 0$.

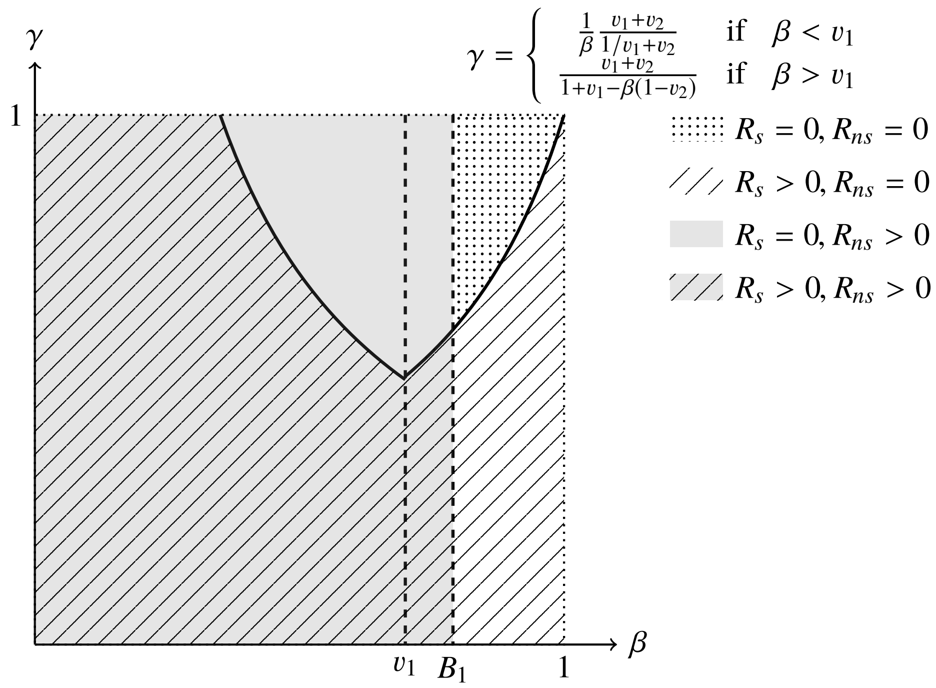

Figure 1: The region where the probability of reactance is positive

are available, R_s is positive for small γ. Note that when γ is small, the agent's type is more likely to be intolerant to sanctions. This implies that sanctions work as well as incentives to choose x for the agent with a tolerance, and the principal need not give a reward for taking x additionally. However, the motivated agent with a tolerance still has incentives to take y, and R_s becomes positive for small γ.

Next, we focus on the region of the β-γ plane where $R_s > 0$ and $R_{ns} > 0$ and compare these probabilities.

Proposition 3. *Consider the region of the β-γ plane where $R_s > 0$ and $R_{ns} > 0$.*

(i) If $\beta > v_1$, $R_s > R_{ns}$ if only if $\gamma > \frac{\beta}{1-\beta}\frac{1-\bar{u}}{\bar{u}}$.

(ii) If $\beta < v_1$, $R_s > R_{ns}$ if only if $\gamma > \frac{1-\bar{u}}{1-v_1}\frac{v_1}{\bar{u}}$.

In the region surrounded by the dashed line of Figure 2, $R_s > 0$ and $R_{ns} > 0$. Proposition 3 indicates that in this region, the availability of sanctions is more likely to cause the agent's reactance ($R_s > R_{ns}$) when γ is sufficiently high

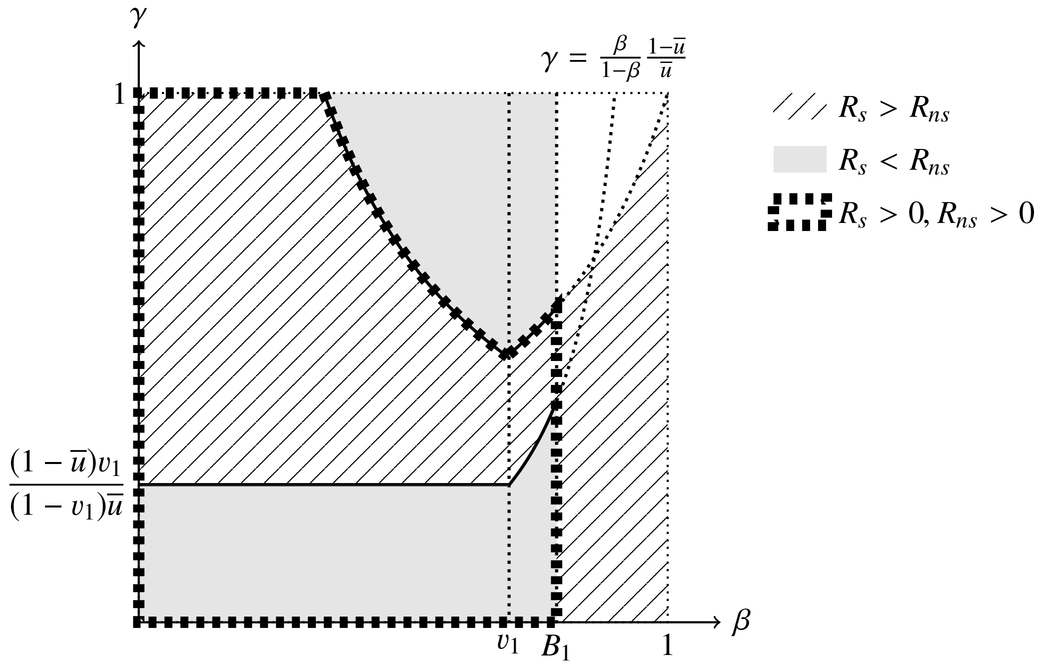

Figure 2: The probability of reactance

This is because, at first, the availability of sanctions increases the probability of reactance by the motivated agent with a tolerance–which occurs owing to the indifferent condition for the mixed strategy equilibrium. Accordingly, given the condition where y is taken in the first period, the second-period expected payoff under zero reward ($m_2(y) = 0$) must increase if that under a positive reward increases. Note that when sanctioning is available, $m_2(y) = v_1$ is sufficient for discouraging the selfish agent with a tolerance from taking y while it requires $m_2(y) = \overline{u} > v_1$ when sanctioning is unavailable. Next, the availability of sanctioning increases the second period expected payoff under $m_2(y) > 0$. This incentivizes the principal to offer $m_2(y) = v_1$, which, in turn, motivates the agent to take y. As the agent without a tolerance never takes y, it only increases the probability of the motivated agent with a tolerance taking y. As γ is the proportion of agents with a tolerance, the total probability of reactance is higher when sanctioning is available with high γ.

Finally, we compare the principal's payoffs between the two cases. Let π_s and π_{ns} be the principal's optimal payoff when a sanction is available and unavailable, respectively. The

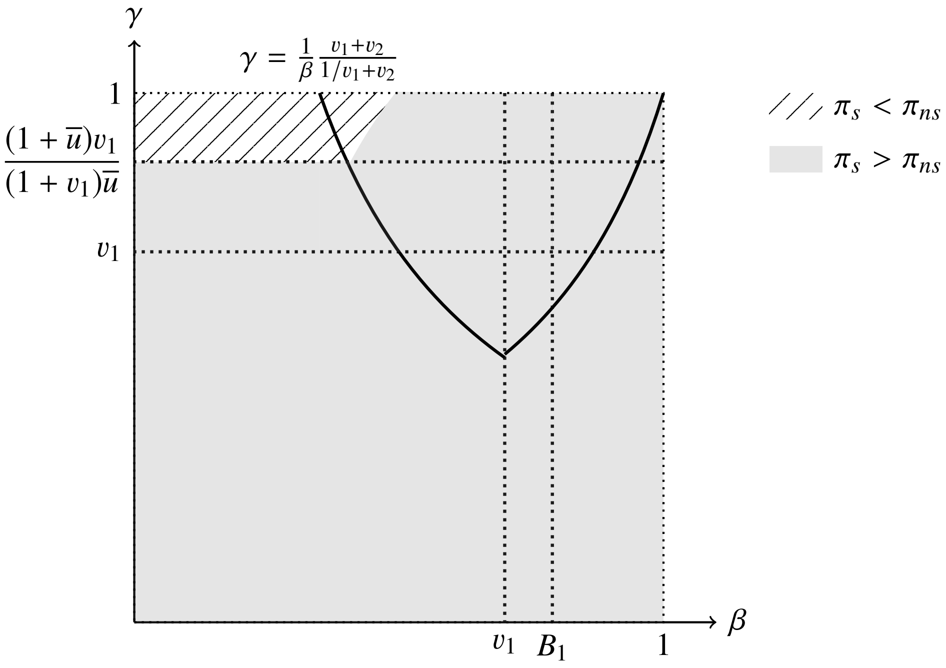

Figure 3: Comparison of the principal's payoffs

following proposition shows that the hidden cost of sanctions can be extremely severe, such that the availability of sanctions reduces the principal's payoff.

Proposition 4. *If* $\beta < \min\{v_1, B_1, \frac{1}{\gamma}\frac{v_1+v_2}{1/v_1+v_2}\}$ *and* $\gamma > \frac{1+\overline{u}}{\overline{u}}\frac{v_1}{1+v_1}$, $\pi_{ns} > \pi_s$.

The intuition is similar to why the probability of reactance is higher when sanctioning is available ($R_s > R_{ns}$). When β is sufficiently small, the principal can be better off by saving rewards even if it caused the agent's reactance. As shown in Proposition 3, if γ is sufficiently large, the probability of reactance is higher when sanctioning is available. This clearly showed that the loss of the principal's payoff is greater when sanctioning is available. Consequently, the principal can improve her welfare by forgoing her option of sanctions if β is small and γ is large.[6] Additionally, note that the threshold of γ, $\frac{1+\overline{u}}{\overline{u}}\frac{v_1}{1+v_1}$, is decreasing in $\underline{a}$. Therefore, if β is sufficiently small, as $\underline{a}$ gets larger, $\pi_{ns} > \pi_s$ is more likely to happen.

[6] In most of the other regions, however, $\pi_s > \pi_{ns}$, as illustrated in Figure 3. See the proof for Proposition 4 in Appendix A for the full classification.

6. Costly Sanctions: Psychological Reactance and Restoration of Freedom

As briefly mentioned in the Introduction, the psychological reactance theory (PRT) defines psychological reactance as the motivational state that is hypothesized to occur when freedom is threatened or eliminated.[7] Freedoms are the beliefs that individuals hold about how they may act. As individuals perceive specific freedoms, anything that makes exercising freedom more difficult represents a threat to freedom. PRT also contends that individuals will be motivated to reestablish that freedom when their perceived freedom is threatened or eliminated.

After studying how and when the agent's reactance to controls occurs when the principal can impose sanctions without material cost, we now examine how an agent restores his freedom and how reactance for the restriction of freedom occurs when the principal must incur a material cost to impose sanctions. We establish the following proposition:

Proposition 5 (Restoration of freedom)**.** *Suppose that* $v_1 > \beta$ *and* $c > 0$*. If* γ *is sufficiently small, the principal offers* $(s_1, m_1) = (1, 0)$ *and* $(s_2(x), m_2(x)) = (1, 0)$*. When* y *is taken at the first period, the principal mixes* $(s_2(y), m_2(y)) = (0, 0)$ *and* $(s_2(y), m_2(y)) = (1, v_1)$*.*

Proposition 5 says that the principal imposes sanctions in the first period. After observing the agent's reactance, the principal gives up imposing sanctions with a positive probability in the second period. This result can be interpreted as follows: When the principal aims to restrict the agent's behavior by imposing sanctions for her unpreferred actions, the agent dares to choose such actions as reactance. Subsequently, the principal no longer implements controls after observing the agent's reactance, allowing the agent to restore the freedom of choice of his behavior.

The intuitive explanation is as follows. As shown in Lemma 2, with a certain condition, after observing the unpreferred action taken, the principal mixes a zero reward and a positive

[7] The following description of PRT is based on and quoted partially from Brehm (1966) and Brehm and Brehm (1981).

reward. When she decides to provide a zero reward, imposing sanctions does not affect the agent's second-period action. This is because if the agent takes the unpreferred action when a sanction is imposed in the first period, he must be tolerant of being sanctioned. Under a zero reward, the selfish agent would take the unpreferred action, independent of whether a sanction is imposed while the motivated agent has no incentive to take the unpreferred action. Therefore, imposing sanctions does not curtail the unpreferred action. As imposing sanctions is costly, even if the cost is adequately small, no sanction is imposed.

Furthermore, note that the principal never mixes contracts without reactance. This is because, without reactance, the agent who takes the unpreferred action is the selfish agent, in which case there is no reason to offer a zero reward. Subsequently, the principal imposes sanctions to save her payment. Therefore, even if sanctioning is costly, when it is small enough, the principal imposes sanctions after the unpreferred action is taken. In other words, without reactance, the principal never fails to impose sanctions. This implies that, in the equilibrium, there would be a positive correlation between reactance and restoring freedom. In this sense, Proposition 5 is consistent with the insights from PRT.

Finally, uncertainty in tolerance is crucial to Proposition 5. If the agent with a tolerance is absent (i.e., $\gamma = 0$), the agent never takes y, and thus, reactance is never observed in the equilibrium path. If the agent without a tolerance is absent (i.e, $\gamma = 1$),[8] $s_1 = 1$ is not offered when $(s_2(y), m_2(y)) = (0, 0)$ is offered with a probability. To observe this, note that a mixed strategy equilibrium occurs if $m_1 = 0$. If the first period reward is zero, sanctioning in the first period is ineffective as it does not directly reduce the unpreferred action—as the agent is known to be tolerant—and sanctioning has no role in reducing m_1. Therefore, when sanctioning is costly, not imposing sanctions is optimal at period 1. In other words, sanctioning never (strictly) decreases over time.

[8] This case is excluded by Assumption 1 (iii).

7. Related Literature

Through an experiment conducted in principal-agent setting, Falk and Kosfeld (2006) study a hidden cost of punishment and show the negative effect of introducing it. Compared to the case that allows only a bonus contract, they show that the amount of effort decreases when the principal can impose a fine in addition to providing a bonus. Schnedler and Vadovic (2011) also study a hidden cost of control in an experiment and show that the negative effect disappears if control is legitimate. We theoretically study a similar hidden cost arising from the principal's option that may reduce the agent's payoff in a dynamic agency model: We show the condition when the principal gives up imposing sanctions as well as when the principal suffers from such a cost.

Akin to the literature based on dynamic agency models where a hidden cost of controls exists, our study is closely related to Schnedler and Vanberg (2014) and Buehler and Eschenbaum (2020). While Schnedler and Vanberg (2014) show that the availability of payment discourages the unpreferred actions, Buehler and Eschenbaum (2020) show that punishment can escalate after an unpreferred action is taken.[9] Our study is related to the literature of ratchet effects, which are based on dynamic agency models where the principal's option may influence the agent's actions (e.g., Freixas, Guesnerie, and Tirole, 1985; Laffont and Tirole, 1988). The main finding in the literature is that the agent restricts his output to prevent future performance requirement from ratcheting up.[10] In contrast to extant literature, our dynamic agency model has two features: (i) contracts that constitute both rewards as monetary incentives and sanctions as a control device and (ii) two kinds of agent private information. We show how the availability

[9]As Tirole (2016) discusses, in an ordinal dynamic contract model, this logic does not always work. If the principal wishesto reward (punish) her (un)preferred behavior more after learning that the agent is misaligned, she also does so in the first stage. Buehler and Eschenbaum (2020) discuss that even in such a case, if the principal cannot offer a long-term contract (i.e., committing to a future contract in the first stage), there is a case of increasing punishment after learning the agent's misaligned preferences. This study also assumes that the principal cannot offer long-term contracts.

[10]Ex-ante private information of the agent is assumed in most of the previous literature as well as in our model.Tan (2020) shows that freedom of work improves the incentive for innovation without assuming ex-ante private information.

of imposing sanctions causes the agent's reactance–and how his reactance influences the principal's payoff–as well as the type of contract the principal offers after facing the reactance.

Moreover, the informed principal models can explain the reactance behavior of the agent: As the principal has private information, rewarding signals the principal's information, such as the difficulty of the task (e.g., Bénabou and Tirole, 2003; Sliwka, 2007), or the distribution of unpreferred agents in environments where strategic complementarity matters, such as public goods provision (e.g., van der Weele, 2012). Then, the agent(s) learn the low benefit of their effort from the amount of reward, which demotivates their effort exertion. In our model, the agent instead of the principal has private information.

Finally, in social psychology, the *psychological reactance theory* is one of the influential hypotheses for reactance behavior (Brehm, 1966; Brehm and Brehm, 1981). In this theory, the reactance behavior is explained as follows. If a person's freedom to choose is threatened or eliminated, even a preferred person engages in unpreferred behavior to restore the freedom. Our result provides an economic rationalization of this behavior. Imposing sanctions can be interpreted as a restriction of freedom. As shown above, because of reactance behavior, the principal decides not to impose sanctions. Thus, the agent has restored his freedom to choose.[11] This result contrasts the work of Buehler and Eschenbaum (2020), who show that punishment can escalate after that unfavorable action is taken.

8. Discussion and Conclusion

This study evaluates the effect of the availability of sanctions in a dynamic principal-agent model where the agent has two kinds of private information: preference for actions and tolerance of sanctions. Our main results show that sanctioning can exacerbate ratchet effects, which,

[11] Kumashiro and Miyagawa (2017) provide another economic explanation of psychological reactance. They consider an advisor-advisee relationship and show an equilibrium, wherein the advisee does the opposite of the adviser's recommendation. In contrast to our model, doing the opposite is not punished. In their model, the source of reactance is that obeying the recommendation damages the advisee's self-esteem.

in turn, increases unpreferred actions by the motivated agent with a tolerance. Consequently, the sanctioning option reduces the principal's payoff even if sanctioning is costless, in some cases. When sanctions are costly, even with a negligible cost, the principal can decrease the sanction level after the unpreferred action is taken.

Before closing our study, we discuss several potential extensions of the current model to investigate the robustness of our results.[12]

First, we assume that the decision on sanctioning is binary although the provision of rewards is continuous. We can extend our model to include an intermediate sanctioning level, and show that our main results are robust if the available sanction level is bounded. For this point, boundedness of the available amount is one crucial difference between rewards and sanctions in our model. Consequently, even if the cost of sanctioning is negligible, sanctions cannot perfectly substitute rewards. Nevertheless, sanctioning has a role in reducing the necessary reward to prevent the unpreferred action from being taken. In this sense, sanctions complement the role of rewards; as Proposition 5 shows, rewards and sanctions are simultaneously offered with a probability, but both are not offered with the complementary probability.

Second, as an alternative situation, we can consider that the principal provides a (non-monetary) prize for a preferred action rather than a sanction for an unpreferred action. Briefly, there is not a remarkable difference in the equilibrium behavior when a prize is introduced instead of a sanction, namely, a prize also exacerbates reactance behavior. However, we show that reactance behavior can be less likely under reward-prize contracts than that under reward-sanction contracts.

Third, we note that in the basic model, if $c = 0$, the principal imposes sanctions (Proposition 2). This result depends on the symmetry in disutility of the principal from the unpreferred action. The principal indisputably incurs more disutility if a motivated agent takes the unpreferred action; as sanctioning increases reactance, the principal decides to not impose sanctions in the first period.

[12]We provide formal discussion and proof for each extension in Supplementary Material.

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Appendix:

A. Proofs

A.1. Omitted proofs in section 3

Proof of Proposition 1. Suppose that the principal commits $s_1 = s_2 = 0$. In the second period, the selfish agent takes x if only if $m_2 \geqslant \overline{u}$ and the motivated agent never takes y.

Consider the first period. Note that the payoff of the selfish agent is $u = \overline{u}$ regardless of $m_2(d_1) \in \{0, \overline{u}\}$ because he can get $\overline{u}$ by taking x when $m_2(d_1) = \overline{u}$ and by taking y when $m_2(d_1) = 0$. This means that the first-period decision by the selfish agent does not depend on the second-period contract: the selfish agent takes x if only if $m_1 \geqslant \overline{u}$.

When $\overline{u} > m_1$, we consider two cases: (1) $\overline{u} > m_1 > \overline{u} + \underline{u}$ and (2) $m_1 < \overline{u} + \underline{u}$. First, when $\overline{u} > m_1 > \overline{u} + \underline{u}$, the motivated agent takes x because the second period payment is at most $\overline{u}$. Next, when $m_1 < \overline{u} + \underline{u}$, the selfish agent takes y because $\overline{u} + \underline{u} < \overline{u}$. Now, consider the strategy that the motivated agent takes y with probability α. Then, the belief satisfying the dominance criterion is

$$P_y = \frac{\beta}{\beta + (1-\beta)\alpha} \text{ and } P_x = 0.$$

This implies that $m_2(x) = 0$. We have two cases: (a) $\beta > \overline{u}$ and (b) $\beta < \overline{u}$.

When $\beta > \overline{u}$, $P_y \geqslant \beta > \overline{u}$. This implies that independent of the value of α, $m_2(y) = \overline{u}$ is optimal. Then, the motivated agent takes y. When $\beta < \overline{u}$, if the motivated agent takes y (i.e., $\alpha = 1$), $P_y = \beta < \overline{u}$. Then, $m_2(y) = 0$ is optimal for the principal. In this case, the motivated agent has no incentive to take y. In contrast, if the motivated agent takes x (i.e., $\alpha = 0$), $P_y = 1 > \overline{u}$, in which case, $m_2(y) = \overline{u}$. The motivated agent has an incentive to take y, which leads to a mixed strategy equilibrium.

Next, we investigate the mixed strategy equilibrium. Suppose that the principal sets $m_2(y) =$

$\overline{u}$ with probability ρ and $m_2 = 0$ with complementary probability. Then, the motivated agent's indifferent condition is

$$\underline{u} + \rho\overline{u} = m_1 \iff \rho = \frac{m_1 - \underline{u}}{\overline{u}}$$

Also, by the principal's indifferent condition,

$$\frac{\beta}{\beta + (1-\beta)\alpha} = \overline{u} \iff \alpha = \frac{\beta}{1-\beta}\left(\frac{1-\overline{u}}{\overline{u}}\right).$$

In summary, we can calculate the principal's expected payoff as follows.

$$\pi(m_1) = \begin{cases} -m_1 - \min\{\overline{u}, \beta\} & \text{if} \quad m_1 > \overline{u} \\ -\beta(1+\overline{u}) - (1-\beta)m_1 & \text{if} \quad m_1 \in (\overline{u} + \underline{u}, \overline{u}) \\ -(1+\overline{u}) & \text{if} \quad m_1 < \overline{u} + \underline{u} \text{ and } \beta > \overline{u} \\ -\frac{\beta}{\overline{u}}(1+\overline{u}) - (1-\frac{\beta}{\overline{u}})m_1 & \text{if} \quad m_1 < \overline{u} + \underline{u} \text{ and } \beta < \overline{u} \end{cases}$$

Now, we compare the payoffs.

In the case where $\beta < \overline{u}$, the principal's equilibrium profit is $\min\{-\overline{u} - \beta, -\beta(1+\overline{u}) - (1-\beta)(\overline{u} + \underline{u}), -\frac{\beta}{\overline{u}}(1+\overline{u})\}$. Note that

$$\beta(1+\overline{u}) + (1-\beta)(\overline{u} + \underline{u}) = \overline{u} + \beta + (1-\beta)\underline{u} < \overline{u} + \beta$$

Thus, the principal's equilibrium profit is $\min\{-\beta(1+\overline{u}) - (1-\beta)(\overline{u} + \underline{u}), -\frac{\beta}{\overline{u}}(1+\overline{u})\}$. While $m_1 = \overline{u} + \underline{u}$ must hold to achieve $-\beta(1+\overline{u}) - (1-\beta)(\overline{u} + \underline{u})$, $m_1 = 0$ must achieve $-\frac{\beta}{\overline{u}}(1+\overline{u})$. Comparing these profits,

1. $\beta < \frac{\overline{u}+\underline{u}}{1/\overline{u}+\underline{u}}$: $m_1 = 0$ is optimal for the principal. In this case, the selfish agent takes y and the motivated agent takes y with probability $\alpha = \frac{\beta}{1-\beta}\left(\frac{1-\overline{u}}{\overline{u}}\right)$.

2. $\beta > \frac{\overline{u}+\underline{u}}{1/\overline{u}+\underline{u}}$: $m_1 = \overline{u} + \underline{u}$ is optimal for the principal. In this case, the selfish agent takes y

and the motivated agent takes x.

In the case where $\beta > \overline{u}$, the principal's equilibrium profit is $\min\{-2\overline{u}, -\beta(1+\overline{u}) - (1-\beta)(\overline{u}+\underline{u}), -(1+\overline{u})\}$. Note that

$$\beta(1+\overline{u}) + (1-\beta)(\overline{u}+\underline{u}) < 1+\overline{u}$$

Thus, the principal's profit is $\min\{-2\overline{u}, -\beta(1+\overline{u}) - (1-\beta)(\overline{u}+\underline{u})\}$. While $m_1 = \overline{u}+\underline{u}$ must hold to achieve $-\beta(1+\overline{u}) - (1-\beta)(\overline{u}+\underline{u})$, $m_1 = \overline{u}$ must achieve $-2\overline{u}$. Comparing these profits, we obtained the following results.

1. $\beta < \frac{\overline{u}-\underline{u}}{1-\underline{u}}$: $m_1 = \overline{u}+\underline{u}$ is optimal for the principal. In this case, the selfish agent takes y and the motivated agent takes x.

2. $\beta > \frac{\overline{u}-\underline{u}}{1-\underline{u}}$: $m_1 = \overline{u}$ is optimal for the principal. In this case, any type of agent takes x.

We can now show that $B_1 = \frac{\overline{u}+\underline{u}}{1/\overline{u}+\underline{u}} < \overline{u} < \frac{\overline{u}-\underline{u}}{1-\underline{u}} = B_2$, which concludes the proof. □

In the proof of Proposition 1, we specify the equilibrium payoff of the principal. We summarize this in the following lemma as it can be used in other proofs.

Lemma A.1. *Without sanctions, the equilibrium payoff of the principal is the following.*

(i) $\beta > B_2$*: The principal's payoff is* $-2\overline{u}$.

(ii) $\beta \in (B_1, B_2)$*: The principal's payoff is* $-\beta(1+\overline{u}) - (1-\beta)(\overline{u}+\underline{u}) = -(\overline{u}+\beta+(1-\beta)\underline{u})$.

(iii) $\beta < B_1$*: The principal's payoff is* $-\frac{\beta}{\underline{u}}(1+\overline{u})$.

A.2. Omitted proofs in section 4

Proof of Lemma 1. The agent's optimal behavior is summarized as follows:

$$d_2 = \begin{cases} x & \text{if} \quad m_2 \geqslant u - s_2 a, \\ y & \text{if} \quad m_2 \leqslant u - s_2 a. \end{cases}$$

Therefore, the motivated agent never takes y. Next, consider the selfish agent. Suppose that $s_2 = 1$. Then, the selfish but intolerant agent never takes y. In this case, the principal's optimal payment is either $m_2 = v_1$ or $m_2 = 0$. To observe this, let $m_2 > v_1$. In this case, as each type of agent takes x, reducing the amount of payment is a profitable deviation for the principal. If $m_2 \in (0, m_2)$, as the selfish agent with a tolerance takes y, and the other type takes x, reducing the amount of payment is also a profitable deviation for the principal. Next, we show that there is no equilibrium in which the selfish agent with a tolerance takes y with a positive probability when $m_2 = v_1$. If this is the case, the principal offers $m_2 = v_1 + \varepsilon$, in which case, the selfish agent with a tolerance definitely takes x. If $\varepsilon > 0$ is adequately small, this deviation is profitable. This shows that the principal's optimal payment is either $m_2 = v_1$ or $m_2 = 0$. If $m_2 = v_1$, the principal's payoff is $-v_1$; and if $m_2 = 0$, the payoff is $(1 + c)P^*_{d_1}$.

Suppose that $s_2 = 0$. In this case, same as above, we can show that the principal's optimal payment is either $m_2 = \overline{u}$ or $m_2 = 0$, in which case, if $m_2 = \overline{u}$, the principal's payoff is $-\overline{u}$. If $m_2 = 0$, the principal's payoff is P_{d_1}.

As $\overline{u} > v_1 = \overline{u} - \underline{a}$, $(s_1, m_1) = (0, \overline{u})$ is never optimal, which completes the proof. □

Proof of Lemma 2. First note that by Lemma 1, $(s_2, m_2) \in \{(1, 0), (0, 0), (1, v_1)\}$. Then, the second-period payoff is either $u - a$ (taking y when $s_2 = 1$), u (taking y when $s_2 = 0$), 0 (taking x when $m_2 = 0$), or v_1 (taking x when $m_2 = v_1$).

Consider the intolerant agent. By taking x, the payoff of the intolerant agent is at least $m_1 \geqslant 0$. In contrast, by taking y, the payoff of the agent is at most $\underbrace{u - \overline{a}}_{\text{1st p. util.}} + \underbrace{\max\{u, v_1\}}_{\text{2nd p. util.}} < v_3 + \overline{u} < 0$.

Therefore, for the intolerant agent, taking x is optimal.

We then consider the behavior of the agent with a tolerance.

Case 1. Suppose that $m_1 > v_1 + v_2$. Consider the motivated agent with a tolerance. Such an agent never takes y in the second period as it would incur high costs, and the game ends in the second period. Then, as $m_2 \leqslant v_1$, the expected payoff of taking y in the first period is at most $v_2 + v_1 < v_1 < m_1$. Therefore, he takes x in the first period.

Next, if $d_1 = y$ at the equilibrium, this implies that $u = \overline{u}$, and $P_y = 1$. This signifies that $(s_2, m_2) = (0, 0)$ is never optimal if $d_1 = y$.

Consider the selfish agent with a tolerance. Note that if $s_2 = 1$, the selfish agent with a tolerance is indifferent between $m_2 = v_1$ and $m_2 = 0$. This is because he takes x and receives $m_2 = v_1$ when $m_2 = v_1$, whereas he takes y and gains v_1 when $m_2 = 0$. Then, the expected payoff of taking y in the first stage is $v_1 + v_1 = 2v_1$.

In contrast, if he takes x in the first stage, $P_x^* = \gamma\beta$ and $P_x = \beta$. As we assume $(1 + c)\gamma < 1$, $(1 + c)P_x^* < P_x$. Next, by Lemma 1, $s_2 = 1$. Then, the expected payoff of taking y in the first stage is $m_1 + v_1$. Therefore, the selfish agent with a tolerance takes x if only if $m_1 + v_1 \geqslant 2v_1 \iff m_1 \geqslant v_1$.

Case 2. Consider the case that $m_1 < v_1 + v_2$.

Consider the selfish agent with a tolerance. As shown in Case 1, if $s_2 = 1$, the second period payoff of the agent is v_1. If $s_2 = 0$, such an agent takes y, following which the second-period payoff is $\overline{u}$. Thus, the second period payoff of the agent is at least v_1. Therefore, if the selfish agent with a tolerance takes y, his total payoff is at least $v_1 + v_1 = 2v_1$. In contrast, the total payoff of taking x is at most $m_1 + \overline{u}$. Note that $m_1 + \overline{u} < v_1 + v_2 + \overline{u} = v_1 + v_1 + \underline{u} < 2v_1$. Then, the selfish agent with a tolerance takes y. This and the dominance criterion imply that $P_x^* = 0 < P_x$ and $(s_2(x), m_2(x)) = (1, 0)$.

Next, we focus on the equilibrium in which the motivated agent with a tolerance takes

y with probability $\alpha \in [0, 1]$. According to Bayes rule, we have

$$P_y^* = P_y = \frac{\beta}{\beta + \alpha(1 - \beta)}.$$

Then, if $c > 0$, as per Lemma 1, $(s_2(y), m_2(y)) = (1, 0)$ is never taken. If $c = 0$, in the second period, $(s_2(y), m_2(y)) = (1, 0)$ and $(0, 0)$ are indifferent for any α. Therefore, without loss of generality, we only focus on $(s_2(y), m_2(y)) = (0, 0)$ and $(1, v_1)$.

We now have two cases.

a) $\beta > v_1$: Note that $P_y^* = P_y = \frac{\beta}{\beta+\alpha(1-\beta)} \geqslant \beta > v_1$. Then, using Lemma 1, $m_2(y) = v_1$; thus, the motivated agent with a tolerance prefers y to x if only if $v_1 + v_2 \geqslant m_1$, which is satisfied by assumption.

b) $\beta < v_1$: If $\alpha = 1$, $P_y^* = P_y = \beta < v_1$. Then, $m_2(y) = 0$, which implies no benefit of taking y for the motivated agent; thus, $\alpha = 0$ is the best response. In contrast, if $\alpha = 0$, $P_y^* = P_y = 1 > v_1$. Then, the motivated agent with a tolerance prefers to take y, which implies $\alpha > 0$. Hence, we must consider a mixed strategy equilibrium.

   Consider a strategy that the principal mixes $(s_2(y), m_2(y)) = (1, v_1)$ with probability q and $(s_2(y), m_2(y)) = (0, 0)$ with probability $1 - q$. Then, the agent with $(u, a) = (\underline{u}, \underline{a})$ is indifferent between taking x and y if

$$m_1 = v_2 + qv_1 \iff q = \frac{m_1 - v_2}{v_1}.$$

   Meanwhile, by Lemma 1, the principal prefers to mix $(s_2, m_2(y))$ if only if

$$P_y = \frac{\beta}{\beta + \alpha(1 - \beta)} = v_1 \iff \alpha = \frac{\beta}{1 - \beta}\frac{1 - v_1}{v_1}.$$

In summary, as $v_1 \geqslant v_1 - \underline{a} \geqslant v_1 + \underline{u} - \underline{a} = v_1 + v_2$, the agent with type $(\overline{u}, \underline{a})$ takes x if only if

$$m_1 \geqslant \begin{cases} v_1 & \text{if} \quad \beta > \min\{(1+c)\gamma\beta, v_1\}, \\ v_1 - \underline{a} & \text{if} \quad \beta < \min\{(1+c)\gamma\beta, v_1\}. \end{cases}$$

Furthermore, the motivated agent with a tolerance and the principal behave as the statement of the proposition describes. □

Proof of Lemma 3. Note that $\overline{u} > v_1 = \overline{u} - \underline{a} > v_1 + \underline{u}$ as $\underline{u} < 0$.

(1) $m_1 \geqslant \overline{u}$.

Then, as $m_2 \leqslant v_1$, the motivated agent never takes y.

Consider the selfish agent. Suppose that some types of agents take y at the equilibrium. Then, $P_y = 1$ and $P_x \leqslant \beta$. Using Lemma 1, $s_2(y) = 1$. Suppose that $m_2(y) = 0$; then, for the selfish agent without a tolerance, the expected payoff of taking y is $\overline{u} \leqslant m_1$. This implies that the selfish agent without a tolerance never takes y and $P^*_y = 1$. Next, $m_2(y) = v_1$, which is a contradiction. Therefore, $(s_2(y), m_2(y)) = (1, v_1)$. Then, the expected payoff of taking y is $\overline{u} + v_1$.

Suppose that $s_2(x) = 0$; the expected payoff of taking x is $m_1 + \overline{u}$. Hence, taking x is optimal. Suppose that $s_2(x) = 1$; for the selfish agent with a tolerance, the expected payoff of taking x is $m_1 + v_1$. Hence, taking x is optimal. Thus, if some type takes y at the equilibrium, the agent is selfish without a tolerance. Then, $P^*_y = 0 < P_y = 1$, and therefore $(s_2(y), m_2(y)) = (1, 0)$, which is a contradiction.

Consequently, there is no incentive to take x for any type. Now, we check that taking x for each type is an equilibrium behavior. To observe this, set off-path beliefs as $P^*_y = P^*_x$ and $P_y = P_x$. Then, the second-period behavior of the principal is the same, taking x for each type.

(2) Suppose that $m_1 \in (v_1 + \underline{u}, \overline{u})$. Then, it is apparent that the motivated agent does not take y since the payoff of taking y is at most $\underline{u} + v_1 < m_1$. This implies that $P_y = 1$. Thus, $(s_2(y), m_2(y)) \neq (0, 0)$

Note that if the selfish agent takes y, $P_y^* = \gamma$, $P_y = 1$, and $P_x = P_x^* = 0$. Then, if $(s_2(x), m_2(x)) = (1, 0)$, and $(s_2(y), m_2(y)) \in \{(1, v_1), (1, 0)\}$, for $u = \overline{u}$, taking y is optimal, and this is the focused behavior.

We now show that there is no equilibrium in which the selfish agent takes x. To this end, we have the following four cases.

Case (2-1): Consider the case that $(s_2(x), m_2(x)) = (s_2(y), m_2(y))$. Here, in period 1, taking y is better than taking x if only if $\overline{u} \geqslant m_1$. Thus, given that m_1, the selfish agent takes y.

Case (2-2): Consider the case that $(s_2(y), m_2(y)) = (1, v_1)$ and $(s_2(x), m_2(x)) = (1, 0)$. Then, taking y is more likely to be preferable for $u = \overline{u}$. The selfish agent takes y.

Case (2-3): Consider the case that $(s_2(y), m_2(y)) = (1, 0)$ and $(s_2(x), m_2(x)) = (1, v_1)$. Then, after taking y, in the second period, the selfish agent with a tolerance takes y. In contrast, after taking x, in the second period, this agent takes x. Then, for the selfish agent with a tolerance, taking y is better than taking x if only if $\overline{u} + v_1 \geqslant m_1 + v_1$. Thus, given that m_1, the selfish agent with a tolerance takes y. This implies that $P_x^* = 0 < v^*$. Using Lemma 1, $m_2(x) = 0$, which is a contradiction.

Case (2-4): Consider the case that $(s_2(y), m_2(y)) \in \{(1, 0), (1, v_1)\}$ and $(s_2(x), m_2(x)) = (0, 0)$. Consider the selfish agent with a tolerance. Then, for him, the payoff of taking y is $\overline{u} + v_1$ while that of x is $m_1 + \overline{u}$. Therefore, the selfish agent with a tolerance takes y if only if $m_1 \leqslant v_1$.

Consider $m_1 < v_1$. As the selfish agent with a tolerance takes y, consider the case that the selfish agent without a tolerance takes x. Then, $P_x > 0$ and $P_x^* = 0$. Using Lemma 1,

$(s_2(x), m_2(x)) = (1, 0)$, which is a contradiction.

Consider $m_1 \geqslant v_1$. We examine the selfish agent without a tolerance. For the agent, the payoff of taking y is at most $\overline{u} + v_1$, while that of taking x is $m_1 + \overline{u}$. Then, he takes x. Here, $P_x = \beta$ and $P_x^* = \gamma\beta$. As we assume that $(1 + c)\gamma < 1$, $P_x^* < P_x$. Using Lemma 1, $(s_2(x), m_2(x)) = (1, 0)$, which is a contradiction.

(3) Suppose that $m_1 < v_1 + \underline{u}$.

Consider the selfish agent with a tolerance. For him, the payoff of taking y is $\overline{u} + v_1$, while that of x is $m_1 + \overline{u}$. Therefore, the selfish agent with a tolerance takes y if only if $m_1 \leqslant v_1$. As $v_1 + \underline{u} < v_1$, the selfish agent with a tolerance takes y. Then, $P_x^* = 0$. Suppose that there is an equilibrium in which the selfish agent without a tolerance takes x. Then, as $P_x > 0 = P_x^*(1 + c)$, $(s_2(x), m_2(x)) = (1, 0)$. In this case, for the selfish agent without a tolerance, the payoff of taking y is at least $\overline{u}$, while that of taking x is m_1. As $\overline{u} > m_1$, taking y is optimal, which is a contradiction. Therefore, the selfish agent takes y, which implies that $P_x = P_x^* = 0$ and $P_y^* \leqslant \gamma$.

(3a) Assume that $v_1 > (1 + c)\gamma$.

Note that $P_x = P_x^* = 0$ implies that $(s_2(x), m_2(x)) \in \{(0, 0), (1, 0)\}$. As the motivated agent takes x in the second period, he is indifferent between these second period contracts. We now show that the motivated agent takes x. To this end, suppose that, by contradiction, there is an equilibrium in which the motivated agent takes y. Note that the payoff of taking x is m_1. Thus, if taking y is optimal for the motivated agent, $(s_2(y), m_2(y)) = (1, v_1)$ with a probability. In contrast, as $P_y^* \leqslant \gamma$ and $(1 + c)\gamma < v_1$, using Lemma 1, $(s_2(y), m_2(y)) \neq (1, v_1)$, which is a contradiction. Therefore, for the motivated agent, taking x is the unique equilibrium behavior.

(3b) Assume that $v_1 < (1 + c)\gamma$.

We show that there is no pure strategy equilibrium: Suppose that the principal takes a pure strategy in the second period. If the motivated agent takes x, $(1+c)P_y^* = (1+c)\gamma > v_1$ and $P_y = 1 > v_1$. Then, $m_2(y) = v_1$. However, in this case, for type $u = \underline{u}$, the payoff of taking x is m_1, while that of taking y is $\underline{u} + m_2(y) = \underline{u} + v_1$. Therefore, taking y is optimal. On the contrary, if the motivated agent takes y, $P_y^* = \beta\gamma$; thus, $(1+c)P_y^* < v_1$. Next, $m_2(y) = 0$. Consequently, taking x is optimal.

Hence, we need to consider a mixed strategy equilibrium. As per Lemma 1, in the second period, the principal mixes m_2 if only if $(1+c)P_y^* = v_1$ or $P_y = v_1$. Let α be the probability that the motivated agent takes y. Then,

$$P_y = \frac{\beta}{\beta + (1-\beta)\alpha}, \qquad (1+c)P_y^* = \frac{(1+c)\beta\gamma}{\beta + (1-\beta)\alpha}.$$

As we assume that $(1+c)\gamma < 1$, $P_y > (1+c)P_y^*$ for any α. By Lemma 1, in the second period, the principal mixes $(s_2, m_2) = (1, v_1)$ and $(1, 0)$ if only if $(1+c)P_y^* = v_1$. To satisfy this equality, let α be the probability that the motivated agent takes y. Then,

$$(1+c)P_y^* = \frac{(1+c)\beta\gamma}{\beta + (1-\beta)\alpha} = v_1.$$

By solving the equality,

$$\alpha = \frac{\beta}{1-\beta}\left(\frac{(1+c)\gamma}{v_1} - 1\right),$$

which is the focused probability. Since $\beta\gamma < v_1 < (1+c)\gamma$, $\alpha \in (0,1)$.

Consider the agent's behavior. The motivated agent should be indifferent between taking x and y. Let q be the probability that $m_2(y) = v_1$. Note that the motivated does not take y in the second period. Then, the expected payoff of taking x equals that of y if only if $qv_1 + \underline{u} = m_1$. By solving this equality,

$$q = \frac{m_1 - \underline{u}}{v_1},$$

which is the focused probability.

□

Proof of Lemma 4. (a) Suppose that $s_1 = 1$. According to Lemma 2, if $m_1 > v_1$, as the agent with any type takes x, $P_x^* = \gamma\beta$ and $P_x = \beta$. Then, using Lemma 1, the principal's payoff in the second period is $-\min\{(1+c)\gamma\beta, \beta, v_1\}$. As $(1+c)\gamma\beta < \beta$, $-\min\{(1+c)\gamma\beta, \beta, v_1\} = -\min\{(1+c)\gamma\beta, v_1\}$.

If $m_1 \in (v_1 + v_2, v_1)$, as per Lemma 2, the selfish agent with a tolerance takes y, and the others take x. Then, $P_y^* = P_y = 1 > P_x > P_x^* = 0$. In the second period, using Lemma 1, the principal sets $(s_2(y), m_2(y)) = (1, v_1)$, and $(s_2(x), m_2(x)) = (1, 0)$. Then, in period 2, the agent with any type takes x. As $P((u, a) = (\overline{u}, \underline{a})) = \gamma\beta$, the principal's payoff is $-\gamma\beta(1 + c + v_1) - (1 - \gamma\beta)m_1$.

If $m_1 \in [0, v_1 + v_2)$ and $v_1 < \beta$, according to Lemma 2 (1), (2), and (3a), the agent with a tolerance takes y, and the others take x. Then, $P_y^* = P_y = \beta > v_1$ and $P_x > P_x^* = 0$. In the second period, as per Lemma 1, the principal sets $(s_2(y), m_2(y)) = (1, v_1)$, and $(s_2(x), m_2(x)) = (1, 0)$. Then, in period 2, the agent with any type takes x. As $P(a = \underline{a}) = \gamma$, the principal's payoff is $-\gamma(1 + c + v_1) - (1 - \gamma)m_1$.

In contrast, if $m_1 \in [0, v_1 + v_2)$ and $v_1 > \beta$, according to Lemma 2 (1), (2), and (3b), the motivated agent with a tolerance takes y with probability $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$ while the others' behaviors are the same. Then, the principal's payoff is

$$-\left(\beta\gamma + (1-\beta)\gamma\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}\right)(1 + c + v_1) - \left(1 - \beta\gamma - (1-\beta)\gamma\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}\right)m_1$$
$$= -\frac{\gamma\beta}{v_1}(1 + c + v_1) - \left(1 - \frac{\gamma\beta}{v_1}\right)m_1.$$

(b) Suppose that $s_1 = 0$. Using Lemma 3, if $m_1 > \overline{u}$, any type of agent takes x in the first period. Then, the principal pays m_1 with probability 1. As in case (a), according to Lemma 1, the principal's expected payoff in the second period is $-\min\{(1+c)\gamma\beta, v_1\}$.

Suppose that $m_1 \in (v_1 + \underline{u}, \overline{u})$, or $m_1 < v_1 + \underline{u}$ and $v_1 > (1+c)\gamma$. Then, using Lemma 3 (1) and (2a), in the first period, the selfish agent takes y, and the motivated agent takes x. This implies that $P_y = 1 > P_y^* = \gamma$ and $P_x = P_x^* = 0$. Then, the principal pays m_1 with probability $1-\beta$, in which case, the principal sets $(s_2, m_2(x)) = (0,0)$ and no type of agent takes y in the second period. On the contrary, with probability β, the agent takes y, and the principal receives payoff -1, in which case, in the second period, the principal's expected payoff is $-\min\{(1+c)\gamma, v_1\}$.

Next, consider $m_1 < v_1 + \underline{u}$ and $v_1 > (1+c)\gamma$. According to Lemma 3(2b), with probability $\beta + (1-\beta)\left(\frac{\beta}{1-\beta}\left(\frac{(1+c)\gamma}{v_1} - 1\right)\right)$, the agent takes y. Then, in the second period, $(s_2(y), m_2(y)) = (1, v_1)$ with probability $\frac{m_1 - \underline{u}}{v_1}$ and $(s_2(y), m_2(y)) = (1,0)$ with the complementary probability. If $(s_2(y), m_2(y)) = (1,0)$, with probability P_y^*, the agent takes y. Note that as shown in the proof of Lemma 3(2b), $(1+c)P_y^* = v_1$.

On the contrary, in the first period, with probability $(1-\beta)\left(1 - \frac{\beta}{1-\beta}\left(\frac{(1+c)\gamma}{v_1} - 1\right)\right)$, the agent takes x and then, the principal pays m_1. In summary, the principal's payoff is

$$
\begin{aligned}
&-\left(\beta + (1-\beta)\left(\frac{\beta}{1-\beta}\left(\frac{(1+c)\gamma}{v_1} - 1\right)\right)\right)(1+v_1) \\
&\quad -(1-\beta)\left(1 - \frac{\beta}{1-\beta}\left(\frac{(1+c)\gamma}{v_1} - 1\right)\right)m_1 \\
&= -\beta\frac{(1+c)\gamma}{v_1}(1+v_1) - \left(1 - \beta\frac{(1+c)\gamma}{v_1}\right)m_1.
\end{aligned}
$$

□

Proof of Proposition 2. Suppose that $v_1 > \beta$. By Lemma 4, if $c = 0$, the principal's expected payoff is

$$\pi(m_1, 1) = \begin{cases} -m_1 - \gamma\beta & \text{if} \quad m_1 > v_1 \\ -\gamma\beta(1 + v_1) - (1 - \gamma\beta)m_1 & \text{if} \quad m_1 \in (v_1 + v_2, v_1) \\ -\gamma(1 + v_1) - (1 - \gamma)m_1 & \text{if} \quad m_1 \in [0, v_1 + v_2) \text{ and } v_1 < \beta \\ -\frac{\gamma\beta}{v_1}(1 + v_1) - \left(1 - \frac{\gamma\beta}{v_1}\right) m_1 & \text{if} \quad m_1 \in [0, v_1 + v_2) \text{ and } v_1 > \beta. \end{cases}$$

$$\pi(m_1, 0) = \begin{cases} -m_1 - \gamma\beta & \text{if} \quad m_1 > \overline{u} \\ -\beta(1 + \min\{v_1, \gamma\}) - (1 - \beta)m_1 & \text{if} \quad m_1 \in (v_1 + \underline{u}, \overline{u}) \\ -\beta(1 + \gamma) - (1 - \beta)m_1 & \text{if} \quad m_1 \in [0, v_1 + \underline{u}) \text{ and } v_1 > \gamma \\ -\frac{\gamma\beta}{v_1}(1 + v_1) - \left(1 - \frac{\gamma\beta}{v_1}\right) m_1 & \text{if} \quad m_1 \in [0, v_1 + \underline{u}) \text{ and } v_1 < \gamma. \end{cases}$$

Note that $-\gamma\beta(1+v_1) - (1-\gamma\beta)(v_1+v_2) = -v_1 - \gamma\beta - (1-\gamma\beta)v_2$ and $-v_1 - \gamma\beta - (1-\gamma\beta)v_2 > -v_1 - \gamma\beta$ as $v_2 < 0$. Then, if $s = 1$, the principal's maximum payoff is

$$\pi(s_1 = 1) = \begin{cases} \max\{-v_1 - \gamma\beta - (1 - \gamma\beta)v_2, -\gamma(1 + v_1)\} & \text{if} \quad v_1 < \beta \\ \max\{-v_1 - \gamma\beta - (1 - \gamma\beta)v_2, -\frac{\gamma\beta}{v_1}(1 + v_1)\} & \text{if} \quad v_1 > \beta \end{cases}$$

If $s_1 = 0$, the principal's maximum payoff is

$$\pi(s_1 = 0) = \begin{cases} \max\{\overline{u} - \gamma\beta, -\beta(1 + \gamma)\} & \text{if} \quad v_1 > \gamma \\ \max\{-v_1 - \beta - (1 - \beta)\underline{u}, -\frac{\gamma\beta}{v_1}(1 + v_1)\} & \text{if} \quad v_1 < \gamma \end{cases}$$

We have four cases.

(1) $v_1 < \beta$ and $v_1 > \gamma$: Then, we have that $-\gamma(1 + v_1) > -\beta(1 + \gamma)$. By $v_2 < 0$ and $\overline{u} > v_1$, we also have that $-v_1 - \gamma\beta - (1 - \gamma\beta)v_2 > -\overline{u} - \gamma\beta$. This implies that $\pi(s_1 = 1) > \pi(s_1 = 0)$.

(2) $v_1 < \beta$ and $v_1 < \gamma$: Note that as $v_2 = \underline{u} - \underline{a}$, and $\gamma, \beta \in (0, 1)$, one can show that $-v_1 - \gamma\beta - (1 - \gamma\beta)v_2 > -v_1 - \beta - (1 - \beta)\underline{u}$. Moreover, as $\beta > v_1$, $-\gamma(1 + v_1) > -\frac{\beta}{v_1}\gamma(1 + v_1)$. Therefore, $\pi(s_1 = 1) > \pi(s_1 = 0)$.

(3) $v_1 > \beta$ and $v_1 > \gamma$. Then, as $v_1 > \gamma$, $-\frac{\gamma}{v_1}\beta(1 + v_1) > -\beta(1 + \gamma)$. We also note that

$-v_1 - \gamma\beta - (1-\gamma\beta)v_2 > -\overline{u} - \gamma\beta$. This implies that $\pi(s_1 = 1) > \pi(s_1 = 0)$.

(4) $v_1 > \beta$ and $v_1 < \gamma$. As $-v_1 - \gamma\beta - (1-\gamma\beta)v_2 > -v_1 - \beta - (1-\beta)\underline{u}$, $\pi(s_1 = 1) \geqslant \pi(s_1 = 0)$. In contrast, if $v_1 + \gamma\beta + (1-\gamma\beta)v_2 > \frac{\gamma\beta}{v_1}(1+v_1)$, $\pi(s_1 = 1) = \pi(s_1 = 0)$.

Therefore, in any case, $s_1 = 1$ is optimal. Moreover, by case (4), $s_1 = 0$ is optimal if and only if $\gamma > v_1 > \beta$ and $v_1 + \gamma\beta + (1-\gamma\beta)v_2 > \frac{\gamma\beta}{v_1}(1+v_1)$. Moreover, as $P^*_{d_1} = \Pr((u,a) = (\overline{u}, \underline{a}) \mid d_1) \leqslant \Pr(u = \overline{u} \mid d_1) = P_{d_1}$, in the second stage, $s_2 = 1$ is optimal. □

A.3. Omitted Proofs in section 5

Proof of Lemma 5. By Proposition 2, when sanctions are available for the principal, $s_1 = s_2 = 1$ is optimal. In this case, the principal's payoff is

$$\pi(m_1, 1) = \begin{cases} -m_1 - \min\{\gamma\beta, v_1\} & \text{if} \quad m_1 > v_1 \\ -\gamma\beta(1+v_1) - (1-\gamma\beta)m_1 & \text{if} \quad m_1 \in (v_1+v_2, V) \\ -\gamma(1+v_1) - (1-\gamma)m_1 & \text{if} \quad m_1 \in [0, v_1+v_2) \text{ and } v_1 < \beta \\ -\frac{\gamma\beta}{v_1}(1+v_1) - \left(1 - \frac{\gamma\beta}{v_1}\right)m_1 & \text{if} \quad m_1 \in [0, v_1+v_2) \text{ and } v_1 > \beta. \end{cases}$$

We have two cases.

(a) $\beta > v_1$: In this case, the principal's optimal payoff is $-\min\{v_1+\gamma\beta+(1-\gamma\beta)v_2, \gamma(1+v_1)\}$. By Lemma 2, the motivated agent with a tolerance takes y with probability 1 if $m_1 < v_1 + v_2$. To implement this agent's behavior in the equilibrium, the principal sets $m_1 = 0$, in which case, the payoff is $-\gamma(1+v_1)$. Thus, the motivated agent with a tolerance takes y with a positive probability in the equilibrium if and only if $\frac{v_1+v_2}{1+v_1-\beta(1-v_2)} > \gamma$.

(b) $\beta < v_1$: In this case, the principal's optimal payoff is $-\min\{v_1 + \min\{\gamma\beta, v_1\}, v_1 + \gamma\beta + (1-\gamma\beta)v_2, \frac{\gamma\beta}{v_1}(1+v_1)\}$. Note that as $\beta < v_1$, $v_1 + \min\{\gamma\beta, v_1\} = v_1 + \gamma\beta > v_1 + \gamma\beta + (1-\gamma\beta)v_2$. Thus, the principal's optimal payoff is $-\min\{v_1 + \gamma\beta + (1-\gamma\beta)v_2, \frac{\gamma\beta}{v_1}(1+v_1)\}$.

By Lemma 2, the motivated agent with a tolerance takes y with probability $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$ if and only if $m_1 < v_1 + v_2$. To implement this agent's behavior in the equilibrium, the principal sets

$m_1 = 0$, in which case, the payoff is $-\frac{\gamma\beta}{v_1}(1+v_1)$. Thus, the motivated agent with a tolerance takes y with probability $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$ in the unique equilibrium if $\frac{v_1+v_2}{\frac{1}{v_1}+v_2} > \beta\gamma$. □

Proof of Proposition 3. Without the sanctioning option, when the motivated agent takes y, the probability is $\frac{\beta}{1-\beta}\frac{1-\overline{u}}{\overline{u}}$ as shown in the proof of Proposition 1. Thus, $R_{ns} = \frac{\beta}{1-\beta}\frac{1-\overline{u}}{\overline{u}}$.

Using Lemma 5, when sanctioning is available, the motivated agent takes y. The probability is 1 when $\beta > v_1$; otherwise, it is $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$. As the motivated agent without a tolerance never takestakes y, the total probability that the motivated agent takes y is γ when $\beta > v_1$; otherwise, the probability is $\gamma\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$. This completes the proof. □

Proof of Proposition 4. The following lemmata summarize the comparison of π_s and π_{ns}. The proof of the proposition is an immediate corollary of Lemma A.3 (1).

Lemma A.2. *(1) If $\pi_{ns} > \pi_s$, $\pi_{ns} = -\frac{\beta}{\overline{\overline{u}}}(1+\overline{u})$. (2) If $\gamma < v_1$, $\pi_s > \pi_{ns}$. (3) If $\beta > v_1$, $\pi_s > \pi_{ns}$.*

Lemma A.3. *Consider the case that $\beta < \min\{B_1, v_1\}$.*

(1) Suppose that $\gamma\beta < \frac{v_1+v_2}{1/v_1+v_2}$ and $\beta < B_1$. Then, $\pi_{ns} > \pi_s$ if and only if $\gamma > \frac{1+\overline{u}}{\overline{u}}\frac{v_1}{1+v_1}$.

(2) Suppose that $\frac{v_1+v_2}{1/v_1+v_2} < \gamma\beta$. Then, $\pi_{ns} > \pi_s$ if and only if $\frac{\beta}{\overline{\overline{u}}}(1+\overline{u}) < \gamma\beta(1-v_2) + v_1 + v_2$.

Proof of Lemma A.2. (1) If the principal commits $s_1 = s_2 = 0$, by Lemma A.1, her payoff is $-\min\{\frac{\beta}{\overline{\overline{u}}}(1+\overline{u}), \beta(1+\overline{u}) + (1-\beta)(\overline{u}+\underline{u}), 2\overline{u}\}$. In contrast, without long-term commitment, if $s_1 = 0$, by Lemma 4, the principal's payoff is no less than $-\min\{\overline{u}+v_1, \beta(1+v_1)+(1-\beta)(v_1+\overline{u})\}$. Note that $\overline{u}+v_1 < 2\overline{u}$, and $\beta(1+v_1)+(1-\beta)(v_1+\overline{u}) < \beta(1+\overline{u})+(1-\beta)(\overline{u}+\underline{u})$. This implies that $\pi_{ns} > \pi_s$ only if $\pi_{ns} = -\frac{\beta}{\overline{\overline{u}}}(1+\overline{u})$. By Lemma A.1, $\pi_{ns} = -\frac{\beta}{\overline{\overline{u}}}(1+\overline{u})$ if and only if $\beta < B_1$. This implies that if $\beta > B_1$, $\pi_s > \pi_{ns}$.

(2) Suppose that $\gamma < v_1$. Suppose by contradiction that $\pi_s < \pi_{ns}$. Then, as shown above, $\pi_{ns} = -\frac{\beta}{\overline{\overline{u}}}(1+\overline{u})$. By Lemma 4, as $v_1 > \gamma$, if the principal sets $(s_1, m_1) = (0, 0)$, her expected payoff is $-\beta(1+\gamma)$. Then, as $\gamma < v_1 < \overline{u} < 1$, $\pi_s = -\beta(1+v_1) > -\frac{\beta}{\overline{\overline{u}}}(1+\overline{u}) \geqslant \pi_{ns}$. Thus,

$\pi_s > \pi_{ns}$.

(3) Suppose that $\beta > v_1$. By (1), we focus on the case that $\pi_{ns} = -\frac{\beta}{\overline{u}}(1+\overline{u})$. Now consider the case that $s_1 = 1$, and $m_1 = v_1$, in which case, $\pi(v_1, 1) = -v_1 - \min\{\gamma\beta, v_1\}$.

If $\gamma\beta < v_1$, $\pi(v_1, 1) = -v_1 - \gamma\beta$. Then, $\pi_{ns} > \pi(v_1, 1)$ if and only if $\beta\left(\frac{1+\overline{u}}{\overline{u}} - \gamma\right) < v_1$. As $\beta > v_1$, this inequality implies that $1 < 1/\overline{u} < \gamma$, which is a contradiction.

If $\gamma\beta > v_1$, $\pi(v_1, 1) = 2v_1$. Then, $\pi_{ns} > \pi(v_1, 1)$ if and only if $\beta\frac{1+\overline{u}}{\overline{u}} < 2v_1$. As $\beta > v_1$, this inequality implies that $\frac{1+\overline{u}}{\overline{u}} < 2$, which is contradictory to $\overline{u} < 1$.

Therefore, in each case, $\pi_s \geqslant \pi(v_1, 1) > \pi_{ns}$. □

Proof of Lemma A.3. As we assume that $\beta < B_1$, $\pi_{ns} = -\frac{\beta}{\overline{u}}(1+\overline{u})$ by Lemma A.1. Also by $\beta < v_1$, and using Lemma 4, we can summarize the payoff as follows:

$$\pi_s = \begin{cases} -\frac{\gamma\beta}{v_1}(1+v_1) & \text{if} \quad \gamma\beta < \frac{v_1+v_2}{1/v_1+v_2} \\ -\gamma\beta(1+v_1) - (1-\gamma\beta)(v_1+v_2) & \text{if} \quad \gamma\beta \in \left(\frac{v_1+v_2}{1/v_1+v_2}, \frac{v_1-v_2}{1-v_2}\right) \\ -2v_1 & \text{if} \quad \gamma\beta > \frac{v_1-v_2}{1-v_2}. \end{cases}$$

Note that we can show that $\frac{v_1+v_2}{1/v_1+v_2} < v_1 < \frac{v_1-v_2}{1-v_2}$. Note also that as we assume $\beta < v_1$, $\gamma\beta > \frac{v_1-v_2}{1-v_2}$ implies that $\gamma > \frac{v_1-v_2}{v_1-v_1v_2} > 1$ as $v_2 < 0$ and $v_1 < 1$. Thus, we focus on whether $\gamma\beta > \frac{v_1+v_2}{1/v_1+v_2}$ holds. Then, the statement of the lemma immediately follows by comparing π_s and π_{ns}. □

□

A.4. Omitted proof in section 6

Proof of Proposition 5. Suppose that $v_1 > \beta$. Let $\gamma \to 0$. Then, by Lemma 4,

$$\pi(m_1, 1) = \begin{cases} -m_1 & \text{if} \quad m_1 > v_1, \\ -m_1 & \text{if} \quad m_1 \in (v_1 + v_2, v_1), \\ -m_1 & \text{if} \quad m_1 \in [0, v_1 + v_2), \end{cases}$$

$$\pi(m_1, 0) = \begin{cases} -m_1 & \text{if} \quad m_1 > \overline{u}, \\ -\beta - (1-\beta)m_1 & \text{if} \quad m_1 \in (v_1 + \underline{u}, \overline{u}), \\ -\beta - (1-\beta)m_1 & \text{if} \quad m_1 \in [0, v_1 + \underline{u}). \end{cases}$$

Then, setting $(s_1, m_1) = (1, 0)$ maximizes the principal's expected payoff. Next, using Lemma 2 (3b), the principal mixes $(s_2(y), m_2(y)) = (1, v_1)$ and $(s_2(y), m_2(y)) = (0, 0)$. Moreover, as per Lemma 2 (3b), as $(u, a) = (\overline{u}, \underline{a})$ takes y, $P_x^* = 0$. Also, through Lemma 1, $(s_2(x), m_2(x)) = (1, 0)$. □

Hidden Cost of Sanctions in a Dynamic Principal-Agent Model: Reactance to Controls and Restoration of Freedom

Supplementary Material

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I. Intermediate Sanctioning Level

This section deals with the case that sanctioning can take an arbitrary value in $[0, \bar{s}]$. We will show that with some additional assumptions, when (1) $c = 0$ and β is small enough, or (2) $c > 0$ and γ is sufficiently small, similar results to the basic model hold even in this extension.

Suppose that $\bar{s} \geqslant \overline{u}/\underline{a}$. Then, $s_1 = s_2 = \overline{u}/\underline{a}$ and $m_1 = m_2 = 0$ is optimal because with this contract, each agent takes x, and the principal pays nothing and bears no cost. Therefore, consider the case that $\bar{s} < \overline{u}/\underline{a}$. Further, we assume that $\overline{u}/\overline{a} < \bar{s} < \overline{u}/\underline{a}$. We normalize $\bar{s} = 1$ and assume Assumption 1.

Consider the second period. Note that the agent with $u = \underline{u}$ never takes y and the agent with $u = \overline{u}$ takes y if only if $\overline{u} - a \cdot s_2 > m_2$. Now we have three cases. (1) If all types take x, $\overline{u} - \underline{a}s_2 \leqslant m_2$. As each type of agent never takes x, the size of s_2 does not affect the principal's payoff. Then, $s_2 = 1$ is optimal, in which case, $m_2 = \overline{u} - \underline{a} = v_1$.

(2) If the selfish agent without a tolerance takes x but the selfish agent with a tolerance takes y with such s_2, the principal's continuation payoff is

$$-(P^*_{d_1}(1 + cs_2) + (1 - P^*_{d_1})m_2).$$

and $\overline{u} - \underline{a}s_2 \geqslant m_2 \geqslant \overline{u} - \overline{a}s_2$. In this case, we can say that $s_2 = 0$ is never optimal. If it is the case, $m_2 \geqslant \overline{u}$. Then, as $1 > \overline{u}$, the principal's payoff is at most $-\overline{u}$, which is less than that in case (1). Then, the pair of $s_2 = \overline{u}/\overline{a}$ and $m_2 = 0$ is optimal. Otherwise, $s_2 < \overline{u}/\overline{a}$, in which case, $m_2 = \overline{u} - \overline{a}s_2$ is optimal. Then, the principal's payoff is linear in s_2, and then, a corner solution is optimal. As the corner solution is $s_2 \in \{0, \overline{u}/\overline{a}\}$, we are done.

(3) If the selfish agent without a tolerance takes y, the selfish agent with a tolerance also takes y, in which case, $m_2 = s_2 = 0$ is optimal. In this case, the principal's payoff is P_{d_1}.

Using this result, the optimal contract in the second period is summarized as follows.

Lemma I.1. *The second period behavior in each equilibrium of the continuation game is characterized as follows:* ***the principal's behavior:***

(a) If $(1 + c\frac{\overline{u}}{\overline{a}})P^*_{d_1} = \min\{(1 + c\frac{\overline{u}}{\overline{a}})P^*_{d_1}, P_{d_1}, v_1\}$, $(s_2, m_2) = (\frac{\overline{u}}{\overline{a}}, 0)$ *is optimal.*

(b) If $P_{d_1} = \min\{(1 + c\frac{\overline{u}}{\overline{a}})P^*_{d_1}, P_{d_1}, v_1\}$, $(s_2, m_2) = (0, 0)$ *is optimal.*

(c) If $v_1 = \min\{(1 + c\frac{\overline{u}}{\overline{a}})P^*_{d_1}, P_{d_1}, v_1\}$, $(s_2, m_2) = (1, v_1)$ *is optimal.*

Now consider the first-period behavior. Assume the following off-path belief: if all types take x, $P^*_y = 1$, and if all types take y, $P_x = 0$. Additionally, to simplify the discussion, we assume that $2\overline{u} < \overline{a}$, and $\beta < v_1$.

First, we consider the case that $c = 0$.

Proposition I.1. *Assume that* $s_t \in [0, 1]$ *and Assumption 1. If* $c = 0$ *and* β *is sufficiently small, in the optimal contract,* $(s_1, m_1) = (1, 0)$ *is offered, and the motivated agent with a tolerance takes* y *with probability* $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$.

Combining this proposition with Lemma I.1, the offered contract and agent's behaviors are almost the same as that in the basic model when β is sufficiently small. Therefore, the discussions of Section 5 that hold when β is sufficiently small continue to hold even when s_t can take an intermediate value in $[0, 1]$.

Proof of Proposition I.1. We focus on the agent's behaviors taken in the first period, and provide the maximal payoff of each case. The possible cases are the following: (a) all types of agents take x, (b) only the selfish agent with a tolerance takes y, (c) only the selfish agent takes y, (d) only the agent with a tolerance takes y, (e) the agent other than the motivated agent without a tolerance takes y, and (f) all types take y. We show that when β is small enough, (d) is implemented in the optimal contract.

(a) Suppose that each type takes x. In this case, as $P_y^* = P_y = 1$, $(s_2(y), m_2(y)) = (1, v_1)$. On the other hand, $P_x^* = \gamma\beta$ and $P_x = \beta$. Then, as $\beta < v_1$, $(s_2(x), m_2(x)) = (\frac{\overline{u}}{\overline{a}}, 0)$. When the selfish agent with a tolerance takes x, $\overline{u} - \underline{a}\frac{\overline{u}}{\overline{a}} + m_1 \geqslant \overline{u} - \underline{a}s_1 + v_1$ should be satisfied. Other types take x if the selfish agent with a tolerance take x. Thus, $s_1 = 1$, and $m_1 = v_1 - \underline{a}(1 - \frac{\overline{u}}{\overline{a}})$ in the optimal contract. Then, the principal's payoff is

$$-(\beta\gamma + v_1 - \underline{a}(1 - \frac{\overline{u}}{\overline{a}}))$$

(b) Suppose that $(u, a) = (\overline{u}, \underline{a})$ takes y in the equilibrium. In this case, $P_y^* = P_y = 1$ and $P_x^* = 0$. Thus, $(s_2(y), m_2(y)) = (1, v_1)$, and $(s_2(x), m_2(x)) = (\frac{\overline{u}}{\overline{a}}, 0)$. Then, $m_1 \geqslant \underline{u} - \underline{a}s_1 + v_1$ and $m_1 \geqslant \overline{u} - \overline{a}s_1 + v_1$ are satisfied. Therefore, in the optimal contract $s_1 = 1$, $m_1 = v_1 + v_2$, in which case, the principal's payoff is

$$-\beta\gamma(1 + v_1) - (1 - \beta\gamma)(v_1 + v_2).$$

(c) Suppose that only the selfish agent takes y. In this case, $U_y(\overline{u}, \overline{a}) - U_x(\overline{u}, \overline{a}) - m_1 + \overline{u} - \overline{a}s_1 \geqslant 0 > U_y(\underline{u}, \underline{a}) - U_x(\underline{u}, \underline{a}) - m_1 + \underline{u} - \underline{a}s_1$, where $U_d(u, a)$ is the second period expected payoff for type (u, a) after taking d. Suppose that $s_1 > 0$. In this case, type $(\overline{u}, \underline{a})$ (selfish agent with a tolerance) is more likely to take y than type $(\overline{u}, \overline{a})$ (selfish agent without a tolerance), $P_y = 1$, $P_y^* \geqslant \gamma$, and $P_x^* = 0$. Then, $(s_2(x), m_2(x)) = (\frac{\overline{u}}{\overline{a}}, 0)$, $m_2(y) = 0$, $U_x(u, a) = 0$ and $U_x(u, a) = v_1$ for each $(u, a) \in \{(\overline{u}, \overline{a}), (\underline{u}, \underline{a})\}$. This implies that $s_1 \leqslant \frac{\overline{u} - \underline{u}}{\overline{a} - \underline{a}} \in (0, 1)$.[13] Now,

[13]Note that $\frac{\overline{u} - \underline{u}}{\overline{a} - \underline{a}} < 1$ is implied by the assumption that $\underline{u} - \underline{a} > \overline{u} - \overline{a}$.

the principal's expected payoff is the following.

$$-\beta(\gamma + (1-\gamma)q)(1+v_1) - ((1-\beta) + \beta(1-\gamma)(1-q))m_1),$$

where q is the probability that type $(\overline{u}, \overline{a})$ takes y. Then, $s_1 = \frac{\overline{u}-\underline{u}}{\overline{a}-\underline{a}}$, and $m_1 = v_1 + \overline{u} - \underline{a}s_1 = v_1 + \underline{u} - \underline{a}s_1 > v_1 + v_2$. In this case, the principal's expected payoff is strictly less than that in case (b). Therefore, this case is never optimal.

If $s_1 = 0$, it is possible that the selfish agent with a tolerance takes x with a probability. Now the principal's payoff is

$$-Q(1+Y) - (1-Q)(m_1 + X) = -(QY + (1-Q)X + Q + (1-Q)m_1),$$

where $Q = \beta(\gamma q' + (1-\gamma)q)$ is the probability that the agent takes y, q' is the probability that $(\overline{u}, \underline{a})$ takes y, $X = \min\{P^*_x, v_1\}$, and $Y = \min\{P^*_y, v_1\}$. We have four cases. If $X = Y = v_1$, $m_2(y) = m_2(x) = v_1$. Then, if the selfish agent with a tolerance takes x, $m_1 \geqslant \overline{u}$. Then, as $\beta < v_1$, the payoff is less than that in case (a). Otherwise, the selfish agent takes y with probability 1, in which case, $P^*_x = 0$, which is a contradiction that $X = v_1$.

Also, if $X = P^*_x$ and $Y = P^*_y$, $m_2(x) = m_2(y) = 0$. If $m_1 \geqslant \overline{u}$, as $QY + (1-Q)X = \gamma\beta$, then, the payoff is also less than that in case (a). Then, $m_2 < \overline{u}$, in which case the selfish agent takes y with probability 1. Now the principal's payoff is at most $-\beta(1+\gamma+v_1)$. This value is strictly less than the payoff that is obtained in case (d).

If $X = v_1$ and $Y = P^*_y$, $m_2(y) = 0$ and $m_2(x) = v_1$. As the selfish agent without a tolerance takes y, $\overline{u} - v_1 \geqslant m_1$. Now the selfish agent with a tolerance takes y with probability 1. This implies that $P^*_x = 0$, which is contradictory to $X = v_1$.

Lastly, if $X = P^*_x$ and $Y = v_1$, $m_2(y) = v_1$ and $m_2(x) = 0$. Then, as the motivated agent with a tolerance takes x, $m_1 \geqslant v_1 + v_2$. Also, as $(\overline{u}, \underline{a})$ takes y, $\overline{u} + v_1 \geqslant m_1 + \overline{u}(1 - \frac{\underline{a}}{\overline{a}})$. Therefore, the selfish agent without a tolerance takes y with probability 1. If $\overline{u} + v_1 =$

$m_1 + \overline{u}(1 - \frac{\underline{a}}{\overline{a}}) \iff m_1 = v_1 + \overline{u}\frac{\underline{a}}{\overline{a}}$, $(\overline{u}, \underline{a})$ takes y with probability q'. Now the principal's payoff is

$$-Q(1 + v_1) - (1 - Q)(v_1 + \overline{u}\frac{\underline{a}}{\overline{a}} + P_x^*) = -Q(1 + v_1) - (1 - Q)(v_1 + \overline{u}\frac{\underline{a}}{\overline{a}}) - \beta\gamma(1 - q')$$
$$= -Qv_1 - (1 - Q)(v_1 + \overline{u}\frac{\underline{a}}{\overline{a}}) - \beta.$$

Then, this payoff is strictly less than that in case (a). Therefore, this case is never optimal. Now consider the case that $m_1 < v_1 + \overline{u}\frac{\underline{a}}{\overline{a}}$, in which case, $m_1 = v_1 + v_2$ is optimal, and the principal's payoff is

$$-\beta(1 + v_1) - (1 - \beta)(v_1 + v_2),$$

which is strictly less than that in case (b). In summary, this case is never optimal.

(d) Suppose that only the agent with a tolerance takes y. Then, the following inequality must hold.

$$U_y(\overline{u}, \overline{a}) - U_x(\overline{u}, \overline{a}) - m_1 + \overline{u} - \overline{a}s_1 < 0 \leqslant U_y(\underline{u}, \underline{a}) - U_x(\underline{u}, \underline{a}) - m_1 + \underline{u} - \underline{a}s_1, \quad (1)$$

where $U_d(u, a)$ is the second period expected payoff for type (u, a) after taking d. In this case, it also holds that $P_x^* = 0$, and thus $s_2(x) = 1$ is optimal. Then, $U_x(u, a) = 0$ for each $(u, a) \in \{(\overline{u}, \overline{a}), (\underline{u}, \underline{a})\}$. As the motivated agent with a tolerance takes y, it also holds that $m_2(y) = v_1$ with a probability. As $m_2(y) = v_1$ with a probability, $v_1 \leqslant P_y^*$ should hold. Then, the principal's expected payoff is the following.

$$-\gamma(\beta + (1 - \beta)q)(1 + v_1) - (\gamma(1 - \beta)(1 - q) + (1 - \gamma))m_1,$$

where q is the probability that type $(\underline{u}, \underline{a})$ takes y. In this case, $(s_1, m_1) = (1, 0)$ is optimal, and as $U_y(\overline{u}, \underline{a}) \leqslant \overline{u}$, (1) is satisfied. Now, by the same logic as the case in Lemma 2, we can show that $q = \frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$ is a unique equilibrium. Then, the expected payoff is $-\frac{\beta\gamma}{v_1}(1+v_1)$.

(e) Suppose that the agent with type $(u, a) \neq (\underline{u}, \overline{a})$ takes y. Then, $P_x^* = 0$ and $m_2(y) = v_1$ with a probability. Also, as the selfish agent without a tolerance takes y, the following inequality holds.

$$U_y(\overline{u}, \overline{a}) + \overline{u} - \overline{a}s_1 \geqslant m_1.$$

As $U_y(\overline{u}, \overline{a}) \leqslant \overline{u}$, and $2\overline{u} < \overline{a}$, $s_1 < 1$. Consider the case that $s_1 > 0$. Then, as the motivated agent without a tolerance takes x, $m_1 \geqslant U_y(\underline{u}, \overline{a}) + \underline{u} - \overline{a}s_1$. If $U_y(\underline{u}, \overline{a}) + \underline{u} - \overline{a}s_1 > 0$, $m_1 = U_y(\underline{u}, \overline{a}) + \underline{u} - \overline{a}s_1$. Otherwise, $m_1 = 0$. In this case, as the payoff of taking y for types $(\overline{u}, \overline{a})$ (selfish, intolerant) and $(\underline{u}, \underline{a})$ (motivated, tolerant) is greater than that for $(\underline{u}, \overline{a})$ (motivated, intolerant), if $m_1 > 0$, an increase in s_1 decreases m_1 without changing all types' behaviors. Then, in the optimal contract, $m_1 = 0$. As $m_2(y) = v_1$ with a probability, $v_1 \leqslant P_y^*$ should hold. Then, the principal's payoff is $-Q(1+v_1)$, where Q is the probability that the agent takes y. If $m_2(y) = v_1$ with probability 1, then, the motivated agent with a tolerance takes y for any $s_1 \leqslant 1$. Then, $Q \geqslant \gamma > \gamma\frac{\beta}{v_1}$, and thus, the principal's payoff is less than that in case (d). Therefore, this case is never optimal. This implies that the motivated agent with a tolerance is indifferent between taking x and y. Then, $Rv_1 + \underline{u} - \underline{a}s_1 = 0$, where R is the probability that the principal sets contract $(s_2, m_2) = (1, v_1)$. Thus, $R < 1$. Then, $P_y^* = v_1$ should hold. As $P_y^* = \frac{\gamma\beta}{Q}$, $Q = \frac{\gamma\beta}{v_1}$. Therefore, the expected payoff is at most that in case (d).

(f) Suppose that all types take y. Then, $P_x^* = 0$. As in case (e), because the motivated agent without a tolerance takes y, $Rv_1 + \underline{u} - \overline{a}s_1 \geqslant m_1$. If $s_1 > 0$, $Rv_1 + \underline{u} - \underline{a}s_1 > m_1$, and then, the motivated agent with a tolerance takes y with probability 1, in which case, $Q \geqslant \gamma$. Then, as in (e), the payoff of the principal is less than that in case (d). Therefore, $s_1 = 0$.

This case is the same as that in Lemma 3. Then, the expected payoff is $-\beta(1+\gamma)$ if $v_1 > \gamma$, and $-\frac{\gamma\beta}{v_1}(1+v_1)$ if $v_1 < \gamma$. In each case, the expected payoff is less than that in case (d).

In summary, if β is sufficiently small, case (d) maximizes the principal's payoff. □

Next, consider the case that γ is small enough but $c > 0$.

Proposition I.2. *Assume that $s_t \in [0,1]$ and Assumption 1. If $c \in (0, \overline{a})$, $\beta < v_1$, and γ is sufficiently small, in the optimal contract, $(s_1, m_1) = (\frac{\overline{u}-a}{\overline{a}-\underline{a}}, 0)$ is offered, $(\underline{u}, \underline{a})$ takes y with probability $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$, and $s_2(y) = 0$ with a probability.*

In this case, $s_1 > 0$. However, in the second period, $(s_2(y), m_2(y)) = (0,0)$ is taken with a probability. Therefore, a similar statement to Proposition 5 holds.

Proof of Proposition I.2. As in the proof of Proposition I.1, we consider cases (a)–(f). Note that even when $c > 0$, for cases (a) and (b), as in the proof of Proposition I.2, we can show that $m_1 > 0$ for any γ. In case (c), as seen above, the selfish agent without a tolerance takes y, otherwise, $m_1 > 0$. In such case, the principal's payoff does not converge to 0 as $\gamma \to 0$.

Then, consider cases (d)–(f), in which cases, $m_1 = 0$ is possible when γ is small enough.

(d) Suppose that only the agent with a tolerance takes y. Then, $P_y^* = P_y$. Then, if y is taken, $(s_2(y), m_2(y)) = (\overline{u}/\overline{a}, 0)$ is never taken. As the motivated agent with a tolerance takes y, it also holds that $m_2(y) = v_1$ with a probability. As $m_2(y) = v_1$ with a probability, $v_1 \leqslant P_y$ should hold. Then, the principal's expected payoff is the following.

$$-\gamma(\beta + (1-\beta)q)(1 + cs_1 + v_1) - (\gamma(1-\beta)(1-q) + (1-\gamma))m_1,$$

where q is the probability that the motivated agent with a tolerance takes y. As in the case for $c = 0$, $(s_2(x), m_2(x)) = (\overline{u}/\overline{a}, 0)$, the motivated and tolerant agent's payoff for taking y is $Rv_1 + \underline{u} - \underline{a}s_1$. If $(\underline{u}, \underline{a})$ takes y with probability 1, $P_y = \beta < v_1$, which is a contradiction. Therefore, $(\underline{u}, \underline{a})$ mixes his action. Then, $Rv_1 + \underline{u} - \underline{a}s_1 = m_1$ holds. As the selfish agent without a tolerance takes x, $Rv_1 + \overline{u} - \overline{a}s_1 \leqslant m_1$. By this inequality, we can show that $(s_1, m_1) = (\frac{\overline{u}-\underline{a}}{\overline{a}-\underline{a}}, 0)$ is the optimal contract.[14] Then, by the same logic as the case in Lemma 2, we can show that $q = \frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$ is a unique equilibrium. Then, the expected payoff is $-\frac{\beta\gamma}{v_1}(1 + cs^* + v_1)$. If $\gamma \to 0$, the expected payoff converges to 0, and therefore, cases (a)–(c) are never optimal.

(e) Suppose that the agent with $(u, a) \neq (\underline{u}, \underline{a})$ takes y. Let q be the probability that the selfish agent without a tolerance takes y and q' be that the motivated agent with a tolerance takes y. The cases for $q = 0$ or $q' = 0$ are considered above. Thus, consider the case $q > 0$ and $q' > 0$. The principal's payoff is $Q(1 + cs_1 + v_1) + (1 - Q)m_1$. As $(\underline{u}, \overline{a})$ takes x, $m_1 \geqslant U_y(\underline{u}, \overline{a}) + \underline{u} - \overline{a}s_1$. If $U_y(\underline{u}, \overline{a}) + \underline{u} - \overline{a}s_1 > 0$, $m_1 = U_y(\underline{u}, \overline{a}) + \underline{u} - \overline{a}s_1$. Otherwise, $m_1 = 0$. If $m_1 = U_y(\underline{u}, \overline{a}) + \underline{u} - \overline{a}s_1$, $q = q' = 1$ is optimal, and then, $Q = 1 - (1 - \gamma)(1 - \beta)$. If γ is sufficiently small, the expected payoff is less than that in case (d). Therefore, consider the case that $m_1 = 0$. Further, if $q = 1$, $Q > \beta$, in which case, the expected payoff is less than that in case (d) for sufficiently small γ. Consider $q \in (0, 1)$, in which case, $U_y(\overline{u}, \underline{a}) + \overline{u} - \overline{a}s_1 = 0$. As $U_y(\overline{u}, \underline{a}) \geqslant U_y(\underline{u}, \underline{a}) = Rv_1$, $s_1 \geqslant \frac{Rv_1 + \overline{u}}{\overline{a}}$. Also, since $Rv_1 + \underline{u} - \underline{a}s_1 \geqslant 0$, $R \geqslant \frac{\overline{u}\frac{\underline{a}}{\overline{a}} - \underline{u}}{v_1(1 - \frac{\underline{a}}{\overline{a}})}$. Therefore, $s_1 \geqslant s^*$. Note that $P_y = \frac{\beta\gamma + \beta(1-\gamma)q}{Q}$, $P_y^* = \frac{\beta\gamma}{Q}$, and $Q = \beta\gamma + \beta(1 - \gamma)q + (1 - \beta)\gamma q'$. Then, as $P_y \leqslant v_1$ and $(1 + c\frac{\overline{u}}{\overline{a}})P_y^* \leqslant v_1$, $Q > \frac{\gamma\beta}{v_1}$. Thus, the expected payoff is less than that in case (d). Therefore, this case is never optimal.

(f) Suppose that all types take y. As the motivated agent without a tolerance takes y, the motivated agent with a tolerance also takes y. Then, $Q \geqslant \beta$. If γ is sufficiently small, this case is never optimal.

In summary, case (d) is optimal for sufficiently small γ. □

Note that this result depends on whether the distribution of the agent's type is discontinuous and the players' payoffs are linear in s. Even with a continuous distribution, if the density of types other than $\{(u, a) : u \in \{\overline{u}, \underline{u}\}, a \in \{\overline{a}, \underline{a}\}\}$ is sufficiently small, similar results hold. One

[14]By fixing m_1, the minimum value of s_1 that satisfies $Rv_1 + \overline{u} - \overline{a}s_1 \leqslant m_1$ is $\frac{Rv_1 + \overline{u} - m_1}{\overline{a}}$. As $c/\overline{a} < 1$, $m_1 = 0$ minimizes the total cost. Then, as $Rv_1 + \underline{u} - \underline{a}s_1 = m_1$ holds, $s_1 = \frac{\overline{u}-\underline{a}}{\overline{a}-\underline{a}}$.

of the characteristics of our study is focusing on corner solutions in the optimal contracts. The linearity in the players' payoffs guarantees the optimality of corner solutions.

II. Prize vs. Sanctions

In the basic model, we assumed that s is a kind of sanction: each agent incurs a cost when sanctioned, but the cost is uncertain. However, as another modeling possibility, we can consider giving a (non-monetary) prize. In this case, each type of agent obtains a positive payoff when he obtains the prize, but the payoff is also uncertain. We distinguish prizes from the monetary reward $(m_t)_{t\in\{1,2\}}$. Further, we assume that the principal can give prizes conditioned on each period of the agent's action in addition to monetary rewards. Prizes would include praise, awards, and so on. Compared with a monetary payoff, the payoff of obtaining such prizes would differ person to person. For example, some may never care about being praised, but some may be greatly influenced by it. In this section, therefore, we consider the case that the principal can give a (non-monetary) prize in addition to a monetary reward instead of imposing a sanction.

To this end, we assume that in each period, the principal can offer a contract, in which if the agent takes x, the principal gives $m_t \geqslant 0$ and $s_t \in \{0, 1\}$; otherwise, the principal gives nothing. The agent's payoff is $(m_t + a \cdot s_t) \cdot I(d_t = x) + u \cdot I(d_t = y)$.

Note that even under this setting, the agent's second-period behavior is the same as that in the basic model. In contrast, the principal's payoff slightly changes because the cost of s is paid when the agent takes x.

the principal's behavior:

(a) If $P^*_{d_1} + (1 - P^*_{d_1})c = \min\{P^*_{d_1} + (1 - P^*_{d_1})c, P_{d_1}, v_1 + c, \overline{u}\}$, $(s_2, m_2) = (1, 0)$ is optimal.

(b) If $P_{d_1} = \min\{P^*_{d_1} + (1 - P^*_{d_1})c, P_{d_1}, v_1 + c, \overline{u}\}$, $(s_2, m_2) = (0, 0)$ is optimal.

(c) If $v_1 + c = \min\{P^*_{d_1} + (1 - P^*_{d_1})c, P_{d_1}, v_1 + c, \overline{u}\}$, $(s_2, m_2) = (1, v_1)$ is optimal.

(d) If $\overline{u} = \min\{P^*_{d_1} + (1 - P^*_{d_1})c, P_{d_1}, v_1 + c, \overline{u}\}$, $(s_2, m_2) = (0, \overline{u})$ is optimal.

To simplify, we assume $v_1 + c < \overline{u}$, which implies that $c < 1$. Then, we can exclude case (d).

With the prize, the principal tends to bear more cost than the case of sanctions. This is because the principal pays the prize cost when the agent takes x, whereas in the case of sanctions, she pays the cost when the agent takes y. As the principal prefers the agent to take x, she makes the agent take x, in which case she needs to pay the cost. Due to this difference, if c is sufficiently large, some difference occurs, as shown below.

Proposition II.1. *Assume $c > 0$, $\beta < v_1 + c$, and $s_1 = 1$. Then, for sufficiently small γ, there is an equilibrium in which the agent without a tolerance takes x, the selfish agent with a tolerance takes y, and the motivated agent with a tolerance takes y with probability $\frac{\beta}{1-\beta}\frac{1-(v_1+c)}{(v_1+c)}$.*

In this case, the principal mixes $(s_2(y), m_2(y)) = (1, v_1)$ and $(s_2(y), m_2(y)) = (0, 0)$, and sets

$$(s_2(x), m_2(x)) = \begin{cases} (0,0) & \text{if} \quad \beta < c, \\ (1,0) & \text{if} \quad \beta > c. \end{cases}$$

Moreover, as $\gamma \to 0$, the principal's payoff is $-(c + \min\{\beta, c\})$.

Proof of Proposition II.1. Note that by Lemma 1, the second-period payoff is either u (taking y) or $as_2 + m_2$ (taking x). Consider the agent with type $(u, a) = (\underline{u}, \overline{a})$. By taking x, the payoff of the agent without a tolerance is at least $m_1 + \overline{a} \geqslant \overline{a}$. In contrast, by taking y, the payoff of the agent is at most $u + \max\{u, \overline{a} + v_1$. Note that as $v_1 = \overline{u} - \underline{a}$, $v_1 + \overline{a} > u$. Thus, if the agent takes y, $u + v_1 + \overline{a} > \overline{a}$. As $\underline{u} < 0$, such type of agent never takes y.

Now consider the other types.

Case 1. Suppose that $m_1 > v_1 + \underline{u}$. Consider the motivated agent with a tolerance. The agent takes x in the second period. Then, the second-period payoff is $m_2 + s_2\underline{a} \leqslant v_1 + \underline{a}$. Consider the first-period payoff. If the agent takes y, his payoff is at most $\underline{u} + v_1 + \underline{a}$. In contrast, if he takes x, his payoff is at least $m_1 + \underline{a}$. Therefore, if $m_1 > v_1 + \underline{u}$, the motivated agent with a tolerance never takes y.

This implies that at the equilibrium, $P_y = 1$, and $P_x \leqslant \beta$. Then, in the second period, by Lemma 1, $s_2(y) = 1$. Consider the selfish agent with a tolerance. His second-period payoff is either $\overline{u}$ (taking y) or $v_1 + \underline{a} = \overline{u}$ (taking x). Therefore, such an agent is indifferent to the second choice. Therefore, he takes x if and only if $m_1 + \underline{a} \geqslant \overline{u} \iff m_1 \geqslant v_1$.

Lastly, consider the selfish agent without a tolerance. His second-period payoff is either $\overline{u}$ (if $(s_2, m_2) = (0, 0)$), $\overline{a}$ (if $(s_2, m_2) = (1, 0)$), or $\overline{a} + v_1$ (if $(s_2, m_2) = (1, v_1)$). Suppose that $m_1 > v_1$. Then, if he takes y, his payoff is at most $\overline{u} + v_1 + \overline{a}$. In contrast, if he takes x, his payoff is at least $m_1 + \overline{a} + \overline{u}$. Therefore, if $m_1 > v_1$, he never takes y.

Consider the case that $m_1 < v_1$. Here, the selfish agent with a tolerance takes y. Then, $P_x^* = 0$. Next, $m_2(x) = 0$. Let α be the probability that the selfish agent without a tolerance takes y. Thus,

$$P_x^* = 0, \quad P_x = \frac{\beta(1-\gamma)(1-\alpha)}{(1-\beta) + \beta(1-\gamma)(1-\alpha)}, \quad P_y^* = \frac{\gamma}{\gamma + (1-\gamma)\alpha}, \quad P_y = 1.$$

Take a small γ that satisfies $\gamma(1-c) < v_1$. This implies that $m_2(y) = 0$. Then, taking y is better than taking x if and only if $\overline{u} + \overline{a} \geqslant m_1 + \overline{a} + \overline{u} \iff 0 \geqslant m_1$. Now taking x is optimal.

Case 2. Suppose that $m_1 < v_1 + \underline{u}$. Then, the selfish agent with a tolerance takes y, which implies that $P_x^* = 0$, and then $m_2(x) = 0$. As in Lemma 2, we consider the equilibrium in which the selfish agent without a tolerance takes x, the motivated agent with a tolerance takes y with probability α, and the principal takes $(s_2(y), m_2(y)) = (1, v_1)$ with probability q, and $(s_2(y), m_2(y)) = (0, 0)$ with the complementary probability $1 - q$.

$$P_x = \frac{\beta(1-\gamma)}{(1-\beta)((1-\gamma) + \gamma(1-\alpha)) + (1-\gamma)\beta)}$$

We have two cases:

a) $\beta < c$. In this case, as $P_x \leqslant \beta$, and $P_x^* = 0$, $P_x < P_x^* + (1 - P_x^*)c = c$. Then, $(s_2(x), m_2(x)) = (0, 0)$.

   For $(\overline{u}, \overline{a})$, taking y is better than taking x if and only if $\overline{u} + q(v_1 + \overline{a}) + (1 - q)(\overline{u}) > m_1 + \overline{a} + \overline{u} \iff (1 - q)v_3 + qv_1 > m_1$.

   Similarly, for $(\underline{u}, \underline{a})$, taking y is better than taking x if and only if $\underline{u} + q(v_1 + \underline{a}) > m_1 + \underline{a} \iff v_2 + q(v_1 + \underline{a}) > m_1$. Note that as $v_2 > v_3$, $v_2 + q(v_1 + \underline{a}) > (1 - q)v_3 + qv_1$. Thus, if $v_2 + q(v_1 + \underline{a}) = m_1$, for $(\overline{u}, \overline{a})$, taking x is optimal. In this case, $q = \frac{m_1 - v_2}{v_1 + \underline{a}}$.

b) $\beta > c > 0$. In this case, for sufficiently small γ, $P_x > c = P_x^* + (1 - P_x^*)c$. Then, $(s_2(x), m_2(x)) = (1, 0)$.

   For the selfish agent without a tolerance, taking y is better than taking x if only if $\overline{u} + q(v_1 + \overline{a}) + (1 - q)(\overline{u}) > m_1 + \overline{a} + \overline{a} \iff (2 - q)(\overline{u} - \overline{a}) + qv_1 > m_1$. As $\overline{u} < \overline{a}$, $(2 - q)(\overline{u} - \overline{a}) + qv_1 < \overline{u} - \overline{a} + v_1 = v_3 + v_1 < 0$. Thus, for the selfish agent without a tolerance, taking x is optimal.

   Similarly, for the motivated agent with a tolerance, taking y is better than taking x if only if $\underline{u} + q(v_1 + \underline{a}) > m_1 + \underline{a} + \underline{a} \iff v_2 + q(v_1 + \underline{a}) - \underline{a} > m_1$. Note that if $m > v_1 + v_2$, for $(\underline{u}, \underline{a})$, taking x is better than taking y.

   In contrast, if $m > v_1 + v_2$: In this case, if $q = \frac{m_1 + \underline{a} - v_2}{v_1 + \underline{a}}$, the motivated agent with a tolerance is indifferent between taking x and y.

Consider the principal's behavior. In each case, as the motivated agent with a tolerance takes y with probability α, $P_y = P_y^* = \frac{\beta}{\beta + (1 - \beta)\alpha}$. Then, for the principal, $(s_2(y), m_2(y)) = (0, 0)$ is better than $(s_2(y), m_2(y)) = (1, 0)$. Now consider α such that

$$P_y = \frac{\beta}{\beta + (1 - \beta)\alpha} = v_1 + c \iff \alpha = \frac{\beta}{1 - \beta} \frac{1 - (v_1 + c)}{(v_1 + c)}$$

As $\beta < v_1 + c$, $\alpha \in (0,1)$.

In summary, if γ is sufficiently small, there exists the following equilibrium.

1. If $m_1 > v_1$, all agents takes x.

2. If $m_1 \in (v_1 + \underline{u} - \underline{a}I(\beta < c), v_1)$, only $(u,a) = (\overline{u}, \underline{a})$ takes y.

3. If $m_1 < v_1 + \underline{u} - \underline{a}I(\beta < c)$: the agent without a tolerance takes x, the selfish agent with a tolerance takes y, and the motivated agent with a tolerance takes y with probability $\alpha_2 = \frac{\beta}{1-\beta}\frac{1-(v_1+c)}{(v_1+c)}$.

   The principal mixes $(s_2(y), m_2(y)) = (1, v_1)$ and $(s_2(y), m_2(y)) = (0,0)$, and

$$(s_2(x), m_2(x)) = \begin{cases} (0,0) & \text{if} \quad \beta < c \\ (1,0) & \text{if} \quad \beta > c \end{cases}$$

The payoff of the principal is as follows:

1. If $m_1 > v_1$,

$$\pi = -(m_1 + c + \min\{\gamma\beta + (1-\beta\gamma)c, \beta, v_1 + c\}).$$

   Then, when $\gamma \to 0$, the maximum value is $\pi = -(v_1 + c + \min\{\beta, c\})$.

2. If $m_1 \in (v_1 + \underline{u} - I(\beta > c)\underline{a}, v_1)$,

$$\pi = -\gamma\beta(1 + v_1 + c) - (1-\gamma\beta)\left(c + m_1 + \min\left\{c, \frac{\beta(1-\gamma)}{\beta(1-\gamma) + (1-\beta)}\right\}\right).$$

   Then, as $\gamma \to 0$, the maximum value is $\pi = -(c + v_1 + \underline{u} - I(\beta > c)\underline{a} + \min\{\beta, c\}) \leqslant -(c + v_1 + v_2 + \min\{\beta, c\})$.

3. If $m_1 < v_1 + \underline{u} - I(\beta > c)\underline{a}$,

$$\pi = -\gamma(\beta + (1-\beta)\alpha_2)(1 + v_1 + c)$$
$$- (1 - \gamma(\beta + (1-\beta)\alpha_2)) \left(m_1 + c + \min\left\{ \frac{\beta(1-\gamma)}{(1-\beta)((1-\gamma) + \gamma(1-\alpha_2)) + (1-\gamma)\beta}, c \right\} \right).$$

Then, as $\gamma \to 0$, the maximum value is $\pi = -(c + \min\{\beta, c\})$.

Then, the third case maximizes the principal's payoff, which completes the proof. □

This proposition shows that if γ is sufficiently small, the probability of reactance is $\frac{\beta}{1-\beta}\frac{1-(v_1+c)}{(v_1+c)}$. This probability is smaller than that in the basic model where a sanction can be imposed instead of a non-monetary prize if $c > 0$. Recall that in the basic model, by Lemma 4 and Proposition 5, if γ and β are sufficiently small, $(s_1, m_1) = (1, 0)$ is optimal, in which case, the probability of reactance is $\frac{\beta}{1-\beta}\frac{1-v_1}{v_1}$. The reactance probability is smaller when a sanction is supplanted by a prize.

The intuition is similar to the previous results. As in the previous cases, the principal and the motivated agent with a tolerance play mixed strategies. If the principal uses non-monetary prizes instead of sanctioning, when she implements $(s_2, m_2) = (1, v_1)$, she incurs a cost of giving the prize. Then, the principal's payoff of offering $(s_2, m_2) = (1, v_1)$ decreases compared with the case that she uses sanctions. For the mixed strategy equilibrium to occur, the payoff of offering $(s_2, m_2) = (0, 0)$ should also decrease, which is increasing in the probability of reactance. Therefore, the probability of reactance decreases. In contrast, the principal's payoff is greater with the sanctioning option, in which case, the payoff is 0 as $\gamma \to 0$. This is because, in such a case, as the probability of the selfish agent is 0, that of taking y is 0. On the one hand, the principal must give a prize with a positive cost with the prize option. On the other hand, the principal must not impose sanctions with the sanctioning option.

We remark that if $\beta < c$, $(s_2(x), m_2(x)) = (0, 0)$, in which case, the selfish agent without a tolerance takes y in the second period, while he takes x in the first period, which never happens in the case of sanctions. This implies that if the principal ceases giving prizes for the preferred action, the agent switches to take the unpreferred behavior.

III. Strategic Forgoing of Sanctions under Asymmetric Damages

We have assumed that the damage for the principal caused by the agent's unpreferred action is constant over the types of agents. However, there may be some cases where the amount of damages depends on the agents' types. For example, consider the case where the damage for the principal from the unpreferred action (y) taken by the agent with $\underline{u}$ is higher than that for the agent with $\overline{u}$. This situation represents the more severe damage the principal has from the unpreferred action taken by this agent, whose preference is more similar to the principal. We assume that it brings payoff $-(1+k)$ where $k > 0$ for the principal if the agent with $u = \underline{u}$ takes y while the payoff for the principal remains -1 if agent with $u = \overline{u}$ takes y. k represents the additional damage for the principal from the unpreferred action taken by the motivated agent compared with the selfish agent. We also assume that the damage is unobservable until the game ends, and then the principal cannot infer the agent's type from her damage.

With a positive k, the reactance behavior of the motivated agent becomes costlier. Imposing sanctions may deteriorate the reactance behavior, increasing the hidden cost of sanctions. Indeed, by Lemmas 2 and 3, in the mixed equilibria, if $c = 0$, the probability of reactance is $\gamma \frac{\beta}{1-\beta} \frac{1-v_1}{v_1}$ if $s_1 = 1$ while it is $\frac{\beta}{1-\beta} \frac{\gamma - v_1}{v_1}$ if $s_1 = 0$. The intuition is as follows. If $s_1 = 1$, only the motivated agent with a tolerance takes unpreferred action, while if $s_1 = 0$, the motivated agent without tolerance can take unpreferred action. Then, with the same probability of reactance, P_y^* becomes larger when $s_1 = 1$. In the mixed strategy equilibrium, the principal is indifferent between a contract with no payment and that with a positive payment. Under the equilibrium contract with a positive payment, independent of s_1, the principal's payoff is $-v_1$. Therefore, the value of P_y^* is independent of s_1. Then, the probability of reactance should be small when $s_1 = 0$. In summary, the probability of reactance increases as s_1 increases. As the reactance is costlier, $s_1 = 0$ can be optimal.

Indeed, the following proposition shows that the increased hidden cost of sanctions deters the principal from imposing sanctions even without the long-term commitment of $s_1 = s_2 = 0$, which is in contrast with the case when $k = 0$ (Proposition 2).

Proposition III.1. *Suppose that $k > 0$ and $c = 0$. Then, if*

$$\gamma > \frac{v_1}{1 + (1 - v_1)k}, \tag{2}$$

for sufficiently small β, $s_1 = 0$ is optimal.

Proof of Proposition III.1. Since the motivated agent has no incentive to take y in the second period, we can apply the analysis in the previous model to that in this extended model. We only need to modify the principal's payoff, which is summarized below.

Lemma III.1. *Assume that $c = 0$ and $\gamma\beta < v_1 < 1$. For each given (m_1, s_1), the unique equilibrium payoff of P, $\pi(m_1, s_1)$ is*

$$\pi(m_1, 1) = \begin{cases} -m_1 - \gamma\beta & \text{if } m_1 > v_1 \\ -\gamma\beta(1 + v_1) - (1 - \gamma\beta)m_1 & \text{if } m_1 \in (v_1 + v_2, v_1) \\ -\gamma(1 + (1 - \beta)k + v_1) - (1 - \gamma)m_1 & \text{if } m_1 \in [0, v_1 + v_2) \text{ and } v_1 < \beta \\ -\frac{\gamma\beta}{v_1}(1 + v_1 + (1 - v_1)k) - \left(1 - \frac{\gamma\beta}{v_1}\right) m_1 & \text{if } m_1 \in [0, v_1 + v_2) \text{ and } v_1 > \beta. \end{cases}$$

$$\pi(m_1, 0) = \begin{cases} -m_1 - \gamma\beta & \text{if } m_1 > \overline{u} \\ -\beta(1 + \min\{\gamma, v_1\}) - (1 - \beta)m_1 & \text{if } m_1 \in (v_1 + \underline{u}, \overline{u}) \\ -\beta(1 + \gamma) - (1 - \beta)m_1 & \text{if } m_1 \in [0, v_1 + \underline{u}) \text{ and } v_1 > \gamma \\ -\frac{\gamma\beta}{v_1}\left(1 + v_1 + \left(1 - \frac{v_1}{\gamma}\right)k\right) - \left(1 - \frac{\gamma\beta}{v_1}\right) m_1 & \text{if } m_1 \in [0, v_1 + \underline{u}) \text{ and } v_1 < \gamma. \end{cases}$$

Let π_s be the principal's payoff with given sanction level s. By the above lemma, π_s is calculated as follows.

$$\pi_1 = \max_{m_1} \pi(m_1, 1) = \begin{cases} \max\{-v_1 - \gamma\beta - (1-\gamma\beta)v_2, -\gamma(1 + (1-\beta)k + v_1)\} & \text{if} \quad v_1 < \beta, \\ \max\left\{-v_1 - \gamma\beta - (1-\gamma\beta)v_2, -\frac{\gamma\beta}{v_1}(1 + (1-v_1)k + v_1)\right\} & \text{if} \quad v_1 > \beta, \end{cases}$$

$$\pi_0 = \max_{m_1} \pi(m_1, 0) = \begin{cases} \max\{-\overline{u} - \gamma\beta, -\beta(1+\gamma)\} & \text{if} \quad v_1 > \gamma, \\ \max\left\{-v_1 - \beta - (1-\beta)\underline{u}, -\frac{\gamma\beta}{v_1}\left(1 + v_1 + \left(1 - \frac{v_1}{\gamma}\right)k\right)\right\} & \text{if} \quad v_1 < \gamma. \end{cases}$$

If β is sufficiently small,

$$\pi_1 = -\frac{\gamma\beta}{v_1}(1 + (1-v_1)k + v_1),$$

$$\pi_0 = \begin{cases} -\beta(1+\gamma) & \text{if} \quad v_1 > \gamma, \\ -\frac{\gamma\beta}{v_1}\left(1 + v_1 + \left(1 - \frac{v_1}{\gamma}\right)k\right) & \text{if} \quad v_1 < \gamma. \end{cases}$$

If $v_1 > \gamma$, $\pi_0 > \pi_1$ if only if inequality (2) holds. If $v_1 < \gamma$, as $\gamma < 1$, $\pi_0 > \pi_1$ holds. In each case, $s_1 = 0$ is optimal. □

Intuitively, as shown in Figure 2, imposing sanctions deteriorates the reactance behavior when β is small and γ is large. Moreover, as only the motivated agent with a tolerance opts for a reactance behavior and causes damage k when $\beta = P(u = \overline{u})$ gets smaller and $\gamma = P(a = \underline{a})$ gets larger, the expected loss from the reactance behavior becomes more severe. Thus, to reduce this possibility of reactance, the principal forgoes her option of imposing sanctions.

When $k = 0$, without the long-term commitment of $s_1 = s_2 = 0$, the benefit of imposing a sanction, namely, curtailing the unpreferred behavior of the selfish agent, exceeds the cost of deteriorating reactance behavior taken by the motivated agent. A positive k increases the latter cost.