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- Aggregated TFP and Price Level -

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Abstract

We aggregate an economy consisting of two commodities, two factors, and two producers into an economy with one commodity, two factors and one producer. Our aggregation method has three characteristic features. One is that an aggregated TFP and price level are defined respectively by individual TFPs and prices of commodities. Another is that our aggregation method includes an aggregation of production functions that has been considered intractable. We resolve that difficulty by specifically devoting attention to equilibrium. The other is that the total values of an original and an aggregated economy are identical.

Key words: aggregation, macro production function, price level, TFP

JEL classifications: E23, D24, B41, O41

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1 Introduction

Economists exploit a simple model such as one-sector model or two-sector model to obtain a general view of economic reality. Economists quite commonly describe a macro-economy using one macro production function or an aggregate production function. A significant feature of macro production function is its usefulness in capturing production technologies at macroeconomic levels. Since Solow (1956) proposed a macro production function with total factor productivity (TFP), many macroeconomists have used TFP to explain GDP fluctuations (see Solow (1957) and Hayashi and Prescott (2002)). Although aggregate production functions are very popularly used, it remains unclear whether one can assume the existence of such a macro production function that is consistent with the individual producers' production functions. This study scrutinizes the consistency of a macro production function and underlying individual production functions with devoting special attention to equilibria.

The real economy includes various production sectors with many numbers of firms in each sector. Although it might be readily apparent and safe to say that different firms have different production technologies and that different sectors have different production technologies, many economists use aggregate production functions when discussing a sector-level or macro economy. Behind the scenes we adopt a tacit understanding, i.e., we assume an aggregation of an economy including many producers and many commodities to that with one producer and one commodity. Now, we ask how we construct an aggregated economy.

We limit our interest in the aggregation of economy with Cobb–Douglas production functions for the following three reasons. One is that our aim is to give a successful example for a starting point of aggregation. Another is that we have no answer explaining how we aggregate individual producers' TFPs to a macroeconomic TFP. As described herein, we provide a formula describing the aggregate TFP in relation with individual TFPs. The other reason is a technical difficulty. Our objective necessarily includes difficulties of aggregation of production functions, which has been discussed in the Cambridge Controversies (Cohen and Harcourt (2003)). As many researchers have known from the controversies, the difficulty of describing the aggregate production by using only producer-associated parameters has led to negative results. Among the few exceptions, Houthakker (1955–1956) applied activity analysis and demonstrated that when individual production technologies follow a Pareto distribution, an aggregate technology takes a Cobb–Douglas form.[1] That is, the controversy has been wrapped up by Fisher (1982) and Felipe and Fisher (2003) with a pessimistic perspective. Most studies exploring the controversies assume identical producers that have homogeneous production technologies, which makes life easier to derive a aggregate production function. However, assuming that individual firms have heterogeneous technologies can pose a challenge.

In the latest discussion by Baquaee and Farhi (2019), they provide a successful way to construct an aggregate production function. They introduce an "aggregator" function and find the aggregate production function as an solution of a net production maximization problem with all the resource constraints. However, their discussion have not checked the following two consistencies. One is this. Is the price of commodity obtained aggregation consistent with those in the individual commodities? Suppose that the above question is answered, and that a hypothetical "aggregated producer" (hereafter AP) exists. Does he behaves as a rational producer? We still need to know if the total value of AP's product is consistent with the aggregated total value of individual producers' products. This is the other consistency problem. We intend to clarify these two consistencies.

This study targets aggregation of a (2,2,2)-economy to a (1,2,1)-economy, where an (ℓ, m, n)-economy is an economy including ℓ kinds of commodities, m factors and n number of producers with Cobb–Douglas functions, which might be mutually different. Attaining this goal, we embark on constructing one aggregated production function, an aggregate TFP as a function of individual TFPs and a relation between the price level and prices of commodities.

The main result of this paper is Theorem 2 in Section three, which is summarized as presented below.

(a) The aggregated share of capital in factor income is a weighted sum of those of individual shares.

(b) The price level is a geometric mean of prices of individual commodities.

[1] Consistent with Houthakker (1955–1956), Jones (2005) explained that when individual producers' production technologies take a Leontief form and the coefficients assigned to capital and labor follow a Pareto distribution, the production function converges to Cobb–Douglas form. Because of their contributions, we know how we to construct Cobb–Douglas production.

(c) The aggregate TFP is determined by each TFP and shares of capital in two producers as well as consumer expenditure rates to two commodities.

We concentrate ourselves to aggregate an equilibrium in a (2,2,2)-economy to that in an aggregated (1,2,1)-economy. The characteristic feature of this paper is in that we emphasize the equilibrium. In other words, to establish results (a), (b) and (c), we make two equilibria of an original and aggregated economy to be mutually consistent.

2 Model

Consider an economy with two producers, i.e., two production sectors. In this section, we define *a general equilibrium in production.*

2.1 Production function of individual sector

Sectors are in a competitive market and are indexed by $i = 1, 2$. Also, i-th sector levels of production, capital, and labor are denoted respectively as Y_i, K_i, and L_i. We assume homogeneous capital and labor. Each sector i has a technology described by a production function of

$$Y_i = F_i(K_i, L_i) = A_i {K_i}^{\theta_i} {L_i}^{1-\theta_i}, \ i = 1, 2 \tag{1}$$

where constant A_i and θ_i denote sector i's TFP and the capital share rate ($0 < \theta_i < 1$) respectively, $i = 1, 2$. We define sector i's capital–labor ratio as $k_i = K_i/L_i$ and the production per capita as $y_i = Y_i/L_i$, which implies $y_i = f_i(k_i) = A_i {k_i}^{\theta_i}$. Let p_i denote the price of sector i's product, r rental ratio, and w wage. We have the marginal condition for profit maximization as

$$r = p_i \frac{\partial F_i}{\partial K_i}(K_i, L_i), \quad w = p_i \frac{\partial F_i}{\partial L_i}(K_i, L_i), \ i = 1, 2.$$

We can rewrite these conditions with the wage–rental ratio $\omega = w/r$ as

$$\omega = \frac{w}{r} = \frac{f_i(k_i) - k_i {f_i}'(k_i)}{{f_i}'(k_i)} = \frac{1 - \theta_i}{\theta_i} k_i, \ i = 1, 2.$$

That is, the marginal condition determines the capital–labor ratio $k_i, i = 1, 2$ for given ω. We can then define the capital–labor ratio as a function of ω,

$$k_i(\omega) = \frac{\theta_i}{1-\theta_i}\omega, \ i = 1, 2, \tag{2}$$

and substitute it back to the marginal condition. In doing so, we have

$$\frac{p_i}{r}A_i = \frac{1}{\theta_i}k_i(\omega)^{1-\theta_i} = \left(\frac{1}{\theta_i}\right)^{\theta_i}\left(\frac{\omega}{1-\theta_i}\right)^{1-\theta_i}, \ i = 1, 2. \tag{3}$$

From the equation above, we can find that the ratio of commodity price–rental ratio $p_{ri} = p_i/r$ is determined by ω. Furthermore, it is apparent that all the endogenous variables in each sector are determined by ω through profit maximization because $y_i(\omega) = f_i(k_i(\omega))$.

2.2 General equilibrium in production

Given positive levels of aggregate capital and aggregate labor K, L, we denote the rate of expenditure to commodity i in the total income as α_i, $i = 1, 2$, which we designate as expenditure coefficients. We assume that expenditure coefficients α_i, $i = 1, 2$ are constants satisfying $\alpha_1 + \alpha_2 = 1$, and $\alpha_i > 0, i = 1, 2$.

Definition 1 *Given levels of capital stocks K and labor stocks L, a pair of prices and an allocation of production $(((p^*_{ri})^2_{i=1}, \omega^*),\ (Y^*_i, K^*_i, L^*_i)^2_{i=1})$ is called a general equilibrium in production if and only if it satisfies the following conditions.*

$$p^*_{ri} = \frac{1}{f_i'(K^*_i/L^*_i)}, \ i = 1, 2, \tag{4}$$

$$\omega^* = \frac{f_i(K^*_i/L^*_i) - f_i'(K^*_i/L^*_i)K^*_i/L^*_i}{f_i'(K^*_i/L^*_i)}, \ i = 1, 2, \tag{5}$$

$$\sum_{i=1}^{2} K^*_i = K \text{ and } \sum_{i=1}^{2} L^*_i = L, \tag{6}$$

$$\alpha_i(K + \omega^* L) = p^*_{ri}F_i(K^*_i, L^*_i) = p^*_{ri}Y^*_i, \ i = 1, 2. \tag{7}$$

(4) and (5) are the marginal conditions for the respective sectors' profit maximization. (6) represents the condition that balances demand and the

supply in factor markets. (7) represents the equilibrium in the commodity markets. This paper assumes an expenditure on the commodity i is proportional to income, although it is presumed to be derived as a result of consumer's optimization problem. Individual producers achieve allocative efficiency in a general equilibrium in production. May (1949) and Felipe and Fisher (2003) require an efficiency condition for the aggregation of production functions too. Gorman (1953) assigns an important role to an efficiency condition when discussing aggregation of the utility function, which can be analogous to our argument.

Theorem 1 *Assume that an individual producer has the production function of* (1)*. Then for any positive amount of initial endowment of production factors* (K, L)*, and for given positive expenditure coefficients* α_1, α_2*, satisfying* $\alpha_1 + \alpha_2 = 1$*, there exists a unique general equilibrium in production.*

[Proof] Equilibria in production factors are defined by

$$\begin{cases} K = k_1(\omega)L_1 + k_2(\omega)L_2 \\ L = L_1 + L_2 \\ K_i = k_i(\omega)L_i, \ i = 1, 2. \end{cases} \tag{8}$$

By setting $k = K/L$ and $\rho_i = L_i/L, i = 1, 2$, we obtains the capital market clearing condition, which is

$$k = k_1(\omega)\rho_1 + k_2(\omega)\rho_2.$$

From the market clearing condition of commodity i, we obtains

$$\alpha_i(\omega L + K) = p_{ri}F_i(k_i(\omega)L_i, L_i) = L_i p_{ri} A_i \left(k_i(\omega)\right)^{\theta_i} = L_i \frac{k_i(\omega)}{\theta_i}.$$

By aggregating the equality above with respect to i and by using the capital market condition, we know the following relation.

$$(\alpha_1\theta_1 + \alpha_2\theta_2)(k + \omega) = k_1(\omega)\rho_1 + k_2(\omega)\rho_2 = k.$$

Solve the equation with respect to ω to obtain

$$\omega^* = \frac{1 - \alpha_1\theta_1 - \alpha_2\theta_2}{\alpha_1\theta_1 + \alpha_2\theta_2} k. \tag{9}$$

Because $K_i^* = L_i k_i(\omega^*) = \alpha_i\theta_i(\omega^* L + K), i = 1, 2$, we have

$$K_i^* = \frac{\alpha_i\theta_i}{\alpha_1\theta_1 + \alpha_2\theta_2} K, i = 1, 2$$
$$L_i^* = \frac{K_i^*}{k_i(\omega^*)} = \frac{\alpha_i(1-\theta_i)}{1 - \alpha_1\theta_1 - \alpha_2\theta_2} L, i = 1, 2.$$

Demand and supply in the labor market balance. Next, we can examine the i-th commodity market equilibrium.

$$\begin{aligned} p_{ri}^* F_i(K_i^*, L_i^*) &= \frac{K_i^*}{\theta_i} = \frac{\alpha_i}{\alpha_1\theta_1 + \alpha_2\theta_2} K \\ &= \alpha_i \left(K + \frac{1 - \alpha_1\theta_1 - \alpha_2\theta_2}{\alpha_1\theta_1 + \alpha_2\theta_2} K \right) \\ &= \alpha_i \left(K + \frac{\omega^*}{k} K \right) = \alpha_i \left(K + \omega^* L \right) \end{aligned}$$

Therefore, demand and supply of the i-th commodity market balance as well.
∎

From the proof of Theorem 1, the amount of production in equilibrium of each sector can be obtained as

$$Y_i = A_i \left(\frac{\alpha_i\theta_i}{\alpha_1\theta_1 + \alpha_2\theta_2} K \right)^{\theta_i} \left(\frac{\alpha_i(1-\theta_i)}{\alpha_1(1-\theta_1) + \alpha_2(1-\theta_2)} L \right)^{1-\theta_i}, \; i = 1, 2. \quad (10)$$

3 Aggregated Economy

Definition 2 *When considering an economy with production functions* (1)*, for a given amount of production factors* $(K, L), K > 0, L > 0$ *and given positive expenditure coefficients* α_1 *and* α_2*, let the general equilibrium in production be* $(((p_{ri}^*)_{i=1}^2, \omega^*), \; (Y_i^*, K_i^*, L_i^*)_{i=1}^2)$*. There exist a production function* $F(\tilde{K}, \tilde{L})$ *and a pair of price and allocation of production* $((p_r^*, \omega^*), (Y^*, K^*, L^*))$ *such that conditions* (i), (ii), and (iii) *bellow are satisfied.*

(i) Y^*, K^*, L^* *is a solution of the profit maximization problem of*

$$\max_{Y, \tilde{K}, \tilde{L}} p_r^* Y - \tilde{K} - \omega^* \tilde{L} \quad \textit{subject to } Y = F(\tilde{K}, \tilde{L}).$$

(ii) *Demand and supply are in balance.*

$$K^* = K, \quad L^* = L, \quad \omega^* L + K = p_r^* Y^*.$$

(iii) *Functions F, F_1, and F_2 satisfy the aggregation condition of*

$$p_r^* Y^* = p_r^* F(K, L) = p_{r1}^* F_1(K_1^*, L_1^*) + p_{r2}^* F_2(K_2^*, L_2^*).$$

We say the pair of price and allocation of production $((p_r^*, \omega^*), (Y^*, K^*, L^*))$ a general equilibrium of aggregated economy. We say the function $F(\tilde{K}, \tilde{L})$ is the aggregate production functions $F_i(K_i, L_i)$, $i = 1, 2$ when a general equilibrium of aggregated economy exists.

Note that in Definition 2 we claim two kinds of consistencies between a (2,2,2)-economy and an aggregated (1,2,1)-economy. We claim in (i) that the AP behaves as a maximizer of profit and in (iii) that the total value of AP's product is identical with sum of those of individual producers' products.

The necessary condition for condition (i) in Definition 2 is

$$p_r^* \frac{\partial F}{\partial \tilde{K}} = 1, \; p_r^* \frac{\partial F}{\partial \tilde{L}} = \omega^*.$$

If we can show the aggregate production function as $F(\tilde{K}, \tilde{L}) = A\tilde{K}^\theta \tilde{L}^{1-\theta}$, the expression above is equivalent to

$$p_r^* A = \frac{1}{\theta \tilde{k}^{\theta-1}}, \quad \omega^* = \frac{1-\theta}{\theta}\tilde{k}, \quad \tilde{k} = \frac{\tilde{K}}{\tilde{L}}. \tag{11}$$

Because $F(\tilde{K}, \tilde{L})$ is a concave function, (11) is a sufficient condition for profit maximization.

Theorem 2 *Assume that the two sectors have production functions given as* (1) *and that* $(((p_{ri}^*)_{i=1}^2, \omega^*),\ (Y_i^*, K_i^*, L_i^*)_{i=1}^2)$ *is the general equilibrium in production for given positive amount of production factor (K, L) and positive expenditure coefficients α_1 and α_2. Define each of the parameters θ, A of the production function $F(\tilde{K}, \tilde{L}) = A\tilde{K}^\theta \tilde{L}^{1-\theta}$ and aggregate price-rental ratio p_r^* as follows*

$$\theta = \alpha_1 \theta_1 + \alpha_2 \theta_2, \tag{12}$$

$$A = \prod_{i=1}^{2} \left(\frac{A_i {\theta_i}^{\theta_i} (1-\theta_i)^{1-\theta_i}}{\theta^\theta (1-\theta)^{1-\theta}} \right)^{\alpha_i}, \tag{13}$$

$$p_r^* = {p_{r1}^*}^{\alpha_1} {p_{r2}^*}^{\alpha_2}. \tag{14}$$

Then, $F(K, L)$ is the aggregate production function. There exists a general equilibrium of aggregatted economy $((p_r^*, \omega^*), (Y^*, K^*, L^*))$ and an aggregate production function $F(\tilde{K}, \tilde{L}) = A\tilde{K}^\theta \tilde{L}^{1-\theta}$.

[Proof] From equations (13) and (14), we have

$$p_r^* A = \prod_{i=1}^{2} \left(\frac{p_{ri} A_i {\theta_i}^{\theta_i} (1-\theta_i)^{1-\theta_i}}{\theta^\theta (1-\theta)^{1-\theta}} \right)^{\alpha_i}.$$

By the definition of a general equilibrium in production, $\frac{K_i^*}{L_i^*} = \frac{\theta_i}{1-\theta_i}\omega^*$. Equations (4) and (9) lead us to

$$p_{ri}^* A_i {\theta_i}^{\theta_i} (1-\theta_i)^{1-\theta_i} = \left(\frac{1-\theta}{\theta} k \right)^{1-\theta_i}, i = 1, 2.$$

Therefore, we obtain the following:

$$p_r^* A = \prod_{i=1}^{2} \left(\frac{\left(\frac{1-\theta}{\theta} k\right)^{1-\theta_i}}{\theta^\theta (1-\theta)^{1-\theta}} \right)^{\alpha_i} = \frac{1}{\theta k^{\theta - 1}}.$$

Because $(((p_{ri}^*)_{i=1}^2, \omega^*),\ (Y_i^*, K_i^*, L_i^*)_{i=1}^2)$ is a general equilibrium in production, we can observe the following by (9).

$$\omega^* = \frac{1-\theta}{\theta} k.$$

Therefore, we obtain (11) and (Y^*, K, L) can be characterized as a solution to (i) of Definition 2. Next we show that condition (ii) holds. $K^* = K$ and $L^* = L$ might be readily apparent. By Euler's theorem,

$$p_r^* Y^* = p_r^* F(K, L) = p_r^* \frac{\partial F}{\partial K} K + p_r^* \frac{\partial F}{\partial L} L = K + \omega^* L.$$

This implies the condition (ii) in Definition 2 holds. Finally, (iii) holds because the third condition of (ii) implies the following:

$$\begin{aligned} p_r^* Y^* &= K + \omega^* L = K_1^* + K_2^* + \omega(L_1^* + L_2^*) \\ &= \sum_{i=1}^{2} (K_i^* + \omega^* L_i^*) = \sum_{i=1}^{2} p_{ri}^* F_i(K_i^*, L_i^*). \blacksquare \end{aligned}$$

The highlight of this paper is Theorem 2, which suggests that, in equilibrium, (a) the share for capital θ in aggregate production is given by the weighted average of individual functions' share of capital θ_1, θ_2. (b) The price level is a weighted geometric mean of individual prices. (c) The Aggregated TFP actually consists of individual sectors' TFPs. Recall that the aggregate production function $F(K, L) = AK^{\theta}L^{1-\theta}$ is a function that explains the quantity of product in equilibrium and that it does not necessarily explain the "aggregated technology", meaning that it only gives the corresponding $Y^* = F(K, L)$ for given (K, L), which is an aggregated quantity of product in general equilibrium. In that sense, we can call the function $F(K, L) = AK^{\theta}L^{1-\theta}$ as a "reduced form" of the aggregated production function. It must be emphasized that the aggregated parameters θ and A include the expenditure coefficients $\alpha_i, i = 1, 2$ instead of containing only technology-associated parameters.

Another interesting property holds for aggregated production function $F(K, L) = AK^{\theta}L^{1-\theta}$, which is stated in the following theorem.

Theorem 3 *Under the same assumption as that in Theorem* 2 *and the parameters* $\omega^*, A, \theta, p_r^*$ *defined in* (9), (12), (13), and (14)*, the following equality holds:*

$$F(K, L) = \prod_{i=1}^{2} \left(\frac{F_i(K_i^*, L_i^*)}{\alpha_i} \right)^{\alpha_i}. \tag{15}$$

[Proof] Because of (10), the equilibrium quantities of products Y_1^* and Y_2^* satisfy

$$\begin{aligned}
\prod_{i=1}^{2} F_i(K_i^*, L_i^*)^{\alpha_i} &= K^{\theta}L^{1-\theta} \prod_{i=1}^{2} \left(\alpha_i A_i \left(\frac{\theta_i}{\theta}\right)^{\theta_i} \left(\frac{1-\theta_i}{1-\theta}\right)^{1-\theta_i} \right)^{\alpha_i} \\
&= \left({\alpha_1}^{\alpha_1} {\alpha_2}^{\alpha_2} \prod_{i=1}^{2} \left(A_i \left(\frac{\theta_i}{\theta}\right)^{\theta_i} \left(\frac{1-\theta_i}{1-\theta}\right)^{1-\theta_i} \right)^{\alpha_i} \right) K^{\theta}L^{1-\theta} \\
&= \left({\alpha_1}^{\alpha_1} {\alpha_2}^{\alpha_2} \prod_{i=1}^{2} \left(\frac{A_i {\theta_i}^{\theta_i} (1-\theta_i)^{1-\theta_i}}{\theta^{\theta_i}(1-\theta)^{1-\theta_i}} \right)^{\alpha_i} \right) K^{\theta}L^{1-\theta} \\
&= (\alpha_1)^{\alpha_1}(\alpha_2)^{\alpha_2} AK^{\theta}L^{1-\theta}.
\end{aligned}$$

This finding implies that (15) is the case.[2] ∎

[2]Note that $\prod_{i=1}^{2} \left(\theta^{\theta_i}(1-\theta)^{1-\theta_i}\right)^{\alpha_i} = \prod_{i=1}^{2} \left(\theta^{\theta}(1-\theta)^{1-\theta}\right)^{\alpha_i}$.

4 Concluding Remarks

As described herein, we aggregated an economy including two products, two factors, and two producers, a (2,2,2)-economy, to an economy with one product, two factors, and one producer, a (1,2,1)-economy. We can emphasize that the equilibria of two economies, the original (2,2,2)-economy and aggregated (1,2,1)-economy are consistent in the sense (i) that the values of total products of two equilibria are intact and that (ii) the wage rental ratio in two equilibria are identical.

We obtain the three distinct features on our aggregated economy. One is the relation (12) stating that the aggregate capital share rate is the mean of two individual capital share rates weighted by expenditure coefficients. Another is equation (13), which asserts that the aggregate TFP are defined by individual TFPs, capital share rates θ_1, θ_2, and expenditure coefficients α_1, α_2. This result implies that changes in expenditure coefficients may vary the aggregated TEP without technological advances. The other is the equation (14), which shows that the price level of aggregated economy is not the algebraic mean but the geometric mean of commodity prices for which weights are expenditure coefficients.

Difficulties that remain unresolved from this study are the following three questions (p1), (p2), and (p3).

(p1) Under what conditions can a (1,2,1)-economy be disaggregated into a (2,2,2)-economy?

(p2) Can we aggregate an (ℓ, m, n)-economy into a $(1, 2, 1)$ economy?

(p3) Are the aggregated capital share rate θ, the aggregated TFF A, and the price level p_r^* unique?

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