On the Equivalence between Ad Valorem and Specific Taxation under Imperfect Competition

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It is widely recognized that under imperfect competition ad valorem and specific commodity taxes are non-equivalent in terms of welfare. This paper provides a counter-example by developing a general equilibrium model of Cournot/Bertrand oligopoly and monopolistic competition. We show that if the specific tax rate exceeds its ad valorem equivalent, welfare improves under the shift from specific to ad valorem taxes that leaves the government revenue unchanged. In other words, the implication under perfect competition survives all the market structures above.

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1 Introduction

Which is more efficient between ad valorem and specific (unit) taxes? Under distortion-free perfect competition, both taxes are equivalent in the sense that they lead to the same welfare.[1)] A similar result holds for a shift from specific to ad valorem taxation that leaves the total tax revenue unchanged. To be precise, the above tax reform raises welfare if and only

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1) See, for example, Stiglitz and Rosengard (2015, p. 546).

if the initial level of specific tax rate is higher than its ad valorem equivalent. However, it is almost impossible to find such a perfectly competitive industry, and most industries are characterized by imperfect competition.

This paper examines the welfare effect of an increase in ad valorem tax and a decrease in specific tax that keeps the government revenue constant in three market structures; Cournot, Bertrand and monopolistic competition. There is a large literature that compares the two taxes under imperfect competition, but this paper differs from it in the following respects. First, unlike the previous works, we use the same model to achieve the above purpose. As we review below, the existing literature typically assumes a partial equilibrium oligopoly model with a quasi-linear utility function and a general equilibrium monopolistic cometition. In contrast, we develop a unified model by assuming a continuum of industries, some of which are subject to ad valorem taxes and the other of which are subject to specific taxes. Our modeling strategy greatly draws on Neary (2003, 2016). Second, we pay special attention to the general equilibrium effect of taxation. Third and most importantly, we show the equivalence between the two tax forms in Cournot, Bertrand and monopolistic competition. In other words, the implication under perfect competition is still useful under imperfect competition as well.

While this paper is related to all sub-fields in public economics and industrial organization, we mention only two strands of literature that are most closely related. The first related literature concerns the comparison between ad valorem and specific taxation under imperfect competition. Assuming a homogeneous Cournot model with/without free entry, Delipalla and Keen (1992) show that the revenue-neutral tax shift from specific to ad valorem taxes enhances welfare. This result is challenged by Anderson et al. (2001) by allowing for product differentiation and Bertrand competition. They find that the specific tax is more efficient than the ad valorem tax under Bertrand competition.[2)]

The second strand of related literature owes to Neary (2003, 2016). Assuming a continuum of oligopolistic industries and that firms are large in their market but small in the economy as a whole, Neary (2003) examines the effect of competition policy, and Neary (2016) examines the determinant and welfare effect of international trade.[3)] Since Neary's (2003, 2016) approach is not only tractable but also overcomes the problems inherent in general equilibrium oligopoly theory, it is increasingly applied mainly to international trade.[4)] In contrast, we incorporate Cournot, Bertrand and monopolistic competition into Neary's model, and explore the welfare implication of the revenue-neutral tax reform.

Our main result is that in Cournot, Bertrand and monopolistic competition the revenue-neutral tax reform improves welfare if and only if the specific tax exceeds the corresponding ad valorem equivalent. In other words, the result under perfect competition survives the above three types of imperfect competition. We recognize that this finding rests on a number of simplifying assumptions, it makes a certain contribution to literature in the sense that the insight under perfect competition is useful even under imperfect competition.

This paper is organized as follows. Section 2 introduces the model and assumptions that are commonly employed in different market structures. Sections 3, 4 and 5 examine the welfare effect of the revenue-neutral shift from ad valorem to specific taxation in Cournot, Bertrand and monopolistic competition, respectively. Section 5 concludes.

2) Keen (1998) is a survey on the comparison between ad valorem and specific taxation.

3) Colacicco (2015) provides a comprehensive survey on Neary's (2003, 2016) model and its application to international trade.

4) As far as I searched the Google Scholar, I found no paper that applies Neary's model to public economics (as of January 7, 2021).

2 Model

This section constructs a base model that is common to all market structures we will consider. Suppose a continuum of goods on a unit interval $[0,1]$, and a representative consumer solves the following utility maximization problem.

$$\max \quad \int_0^1 \ln X(z)dz \tag{1}$$

$$\text{s.t.} \quad \int_0^1 \sum_{i=1}^{n(z)} p_i(z)x_i(z)dz = I \tag{2}$$

$$\text{where} \;\; X(z) \equiv \left[\sum_{i=1}^{n(z)} x_i(z)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $n(z)$ is the number of differentiated goods in industry z, $p_i(z)$ is the price of variety i, $x_i(z)$ is the consumption of variety i, $X(z)$ is the quantity index of industry z, I is national income, and $\sigma > 1$ is the elasticity of substitution among varieties.[5)] Solving this problem, the demand and inverse demand functions of variety i in industry z are respectively obtained as

$$x_i(z) = \frac{p_i(z)^{-\sigma}I}{\sum_{i=1}^{n(z)} p_i(z)^{1-\sigma}}, \quad p_i(z) = \frac{x_i(z)^{-\frac{1}{\sigma}}I}{\sum_{i=1}^{n(z)} x_i(z)^{\frac{\sigma-1}{\sigma}}}. \tag{4}$$

If we define the price index of industry z as

$$P(z) \equiv \left[\sum_{i=1}^{n(z)} p_i(z)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{5}$$

a convenient relationship holds between $P(z)$ and $X(z)$ such that $P(z)X(z) = I$. Substituting this relationship into the direct utility function (1), the indirect utility function W becomes

$$W = \int_0^1 \ln\left[\frac{I}{P(z)}\right]dz = \ln I - \int_0^1 P(z)dz. \tag{6}$$

5) It is popular to assume a continuous quantity index $[\int_0^{n(z)} x_i(z)^{(\sigma-1)/\sigma}di]^{\sigma/(\sigma-1)}$ under monopolistic competition, but we use the discrete version as in Dixit and Stiglitz (1977) and Krugman (1980).

This indirect utility function serves as a welfare measure in the subsequent sections.

Having described the consumer behavior, let us turn to the firm behavior. We assume that labor is a numeraire so that the wage rate is normalized to unity. Each firm produces a (horizontally) differentiated good with marginal labor requirement $c > 0$ and fixed labor requirement $f \geq 0$. In the existing literature that compares ad valorem and specific taxes, it is customary to assume that both taxes are imposed on firms. In contrast, we split the whole economy into a set of industries in which the ad valorem tax is levied and a set of industries in which the specific tax is levied. Denoting the threshold that separates these sets of industries by $\widetilde{z} \in (0,1)$, the firm profit in each set of industries is defined by

$$\frac{p_i(z)x_i(z)}{1+t} - cx_i(z) - f \qquad \text{for} \quad z \in [0, \widetilde{z}] \tag{7}$$

$$p_i(z)x_i(z) - cx_i(z) - \tau x_i(z) - f \quad \text{for} \quad z \in [\widetilde{z}, 1], \tag{8}$$

where t is the ad valorem tax rate and τ is the specific tax rate. In the following sections, we utilize this model to accomodate a Cournot oligopoly, a Bertrand oligopoly and monopolistic competition to investigate the welfare effect of the revenue-neutral shift from specific to ad valorem taxation.

3 Cournot Oligopoly

The present and next sections address the effects of taxation in Cournot and Bertrand competition, respectively. Just for notational simplicity, we assume that the fixed labor requirement is zero. Under Cournot oligopoly, the first-order condition for profit maximization yields the following per-firm output, industry-wide output and price:

$$x_1 = \frac{(\sigma-1)(n-1)I}{(1+t)\sigma n^2 c}, \quad nx_1 = \frac{(\sigma-1)(n-1)I}{(1+t)\sigma nc}, \quad p_1 = \frac{(1+t)\sigma nc}{(\sigma-1)(n-1)}, \tag{9}$$

for the industries on which the ad valorem tax is levied. Here, subscript 1 represents the industry that is subject to ad valorem taxation. Similarly, the counterparts of the industries the specific tax is imposed are obtained as

$$x_2 = \frac{(\sigma-1)(n-1)I}{\sigma n^2(c+\tau)}, \quad nx_2 = \frac{(\sigma-1)(n-1)I}{\sigma n(c+\tau)}, \quad p_2 = \frac{\sigma n(c+\tau)}{(\sigma-1)(n-1)}, \tag{10}$$

where subscript 2 represents the industry that is subject to specific taxation.

Based on the industry-wide outputs and prices in (9) and (10), we now derive the national income. The definition of national income is

$$\begin{aligned} I &\equiv \text{labor income} + \text{aggregate profits} + \text{tax revenue} \\ &= L + \int_0^{\widetilde{z}} n\pi_1 dz + \int_{\widetilde{z}}^1 n\pi_2 dz + T, \end{aligned}$$

where the first term in the last equation is labor income, the second and the third terms are the aggregate profits in the industries with ad valorem and specific taxation, respectively, and T is the tax revenue. The per-firm profits and tax revenue are defined by

$$\pi_1 = \left(\frac{p_1}{1+t} - c\right)x_1, \quad \pi_2 = (p_2 - c - \tau)x_2, \quad T = \frac{\widetilde{z}ntp_1x_1}{1+t} + (1-\widetilde{z})\,n\tau x_2. \tag{11}$$

Substituting (9) and (10) into (11) and applying the resulting expressions to the above definition of national income, the closed form of I is derived as

$$I = \frac{\sigma nL}{(\sigma-1)(n-1)} \cdot \frac{(1+t)(c+\tau)}{\widetilde{z}(c+\tau) + (1-\widetilde{z})\,(1+t)c}. \tag{12}$$

This completes the full illustration of the Cournot model. Once I is determined in (12), we can obtain the other endogenous variables by substituting (12) into them.

In the rest of this section, we will study the effects of the shift from specific to ad valorem taxation such that the total tax revenue T is unchanged.

For this purpose, let us note that tax revenue T takes the form of[6)]

$$T = \frac{L}{(\sigma-1)(n-1)} \cdot \frac{\tilde{z}\sigma nt(c+\tau) + (1-\tilde{z})(\sigma-1)(n-1)(1+t)\tau}{\tilde{z}(c+\tau) + (1-\tilde{z})(1+t)c}. \quad (13)$$

Totally differentiating this with respect to t and τ, the revenue-neutral tax reform requires $dT = (\partial T/\partial t)dt + (\partial T/\partial \tau)d\tau = 0$, which is rewritten as

$$d\tau = -\frac{\tilde{z}(c+\tau)\{[\tilde{z}(\sigma+n-1) + (\sigma-1)(n-1)]\,\tau + \sigma nc\}}{(1-\tilde{z})(1+t)c\{[\tilde{z}(\sigma+n-1) + (\sigma-1)(n-1)]\,t + (\sigma-1)(n-1)\}} dt. \quad (14)$$

Since the coefficient of dt is negative, the specific tax rate must be lowered ($d\tau < 0$) if the ad valorem tax rate is raised ($dt > 0$) and the government revenue is kept constant. In the analysis below, we focus on this case; the opposite case can be analyzed just by reversing the sign of dt and $s\tau$.

We are now ready to examine the welfare effect of the above tax reform. Recalling that the indirect utility function is

$$W = \ln I - \tilde{z}\ln P_1 - (1-\tilde{z})\ln P_2,$$

where P_1 and P_2 are the price indices in the industries in which ad valorem and specific taxes are levied, respectively, and are given by

$$P_1 = \frac{\sigma n^{\frac{2-\sigma}{1-\sigma}}(1+t)c}{(\sigma-1)(n-1)}, \quad P_2 = \frac{\sigma n^{\frac{2-\sigma}{1-\sigma}}(c+\tau)}{(\sigma-1)(n-1)}.$$

Substituting these price indices and (12) into the indirect utility function above and rearranging terms, welfare is shown to depend on the two tax rates as follows.

$$W = (1-\tilde{z})\ln(1+t) + \tilde{z}\ln(c+\tau) - \ln[\tilde{z}(c+\tau) + (1-\tilde{z})(1+t)c] + const. \quad (15)$$

$$\text{where} \quad const. = \ln\left(n^{\frac{2-\sigma}{1-\sigma}}L\right) - \tilde{z}\ln c.$$

By relating the rule of the revenue-neutral tax reform (14) to (15), we establish:

6) The derivation of (13) is as follows. Subsituting (12) into (9) and (10) yields the outputs and prices as a function of t and τ. Further substitution of them into T in (11) leads to (13).

Proposition 1. *The revenue-neutral shift from specific to ad valorem taxation raises welfare if and only if the specific tax rate (τ) is higher than the corresponding ad valorem equivalent (tc), i.e.*

$$sign\{dW\} = sign\{(\tau - tc)dt\}. \tag{16}$$

Proof. Totally differentiating (15) with respect to t and τ and substituting the policy rule (14) yield

$$\begin{aligned} dW &= \frac{\partial W}{\partial t} dt + \frac{\partial W}{\partial \tau} d\tau \\ &= \left(\frac{\partial W}{\partial t} + \frac{\partial W}{\partial \tau} \cdot \frac{d\tau}{dt} \right) dt, \end{aligned}$$

where the terms in the parentheses are

$$\begin{aligned} &\underbrace{\frac{\widetilde{z}\,(1-\widetilde{z})\,(\tau - tc)}{(1+t)\,[\widetilde{z}(c+\tau) + (1-\widetilde{z})\,(1+t)c]}}_{\partial W/\partial t} \\ &+ \underbrace{\frac{\widetilde{z}\,(1-\widetilde{z})\,(tc - \tau)}{(c+\tau)\,[\widetilde{z}(c+\tau) + (1-\widetilde{z})\,(1+t)c]}}_{\partial W/\partial \tau} \\ &\times \underbrace{\frac{-\widetilde{z}(c+\tau)\,\{[\widetilde{z}(\sigma+n-1) + (\sigma-1)(n-1)]\,\tau + \sigma n c\}}{(1-\widetilde{z})\,(1+t)c\,\{[\widetilde{z}(\sigma+n-1) + (\sigma-1)(n-1)]\,t + (\sigma-1)(n-1)\}}}_{d\tau/dt} \\ = &\frac{\widetilde{z}(\tau - tc)A}{(1+t)c\,[\widetilde{z}(c+\tau) + (1-\widetilde{z})\,(1+t)c]\,B}, \end{aligned}$$

where A and B are given by

$$\begin{aligned} A &= \widetilde{z}\,\{[\widetilde{z}(\sigma+n-1) + (\sigma-1)(n-1)]\,\tau + \sigma n c\} \\ &\quad + (1-\widetilde{z})\,c\,\{[\widetilde{z}(\sigma+n-1) + (\sigma-1)(n-1)]\,t + (\sigma-1)(n-1)\} > 0 \\ B &= [\widetilde{z}(\sigma+n-1) + (\sigma-1)(n-1)]\,t + (\sigma-1)(n-1) > 0. \end{aligned}$$

As a result of these lengthy manipulations, we reach (16). ||

4 Bertrand Oligopoly

This section turns to Bertrand competition in which firms choose the price. The analysis in this section is motivated by Anderson et al. (2001). In a Hotelling model of Bertrand duopoly with asymmetric marginal costs, they show that specific taxation welfare-dominates ad valorem taxation. Then, they claim that 'This argument (weakly) suggests that it is primarily the mode of competition that isesponsible for the result.' (p. 243) In this section, we reexamine this conclusion of Anderson et al. (2001) by checking the validity of Proposition 1 in a Bertrand oligopoly.

Since the procedure of solving the model and examining the welfare effect of tax reform is the same as that of the previous section, we will shorten the argument to the necessary extent. The profit of each firm is given by (7) and (8) under the demand functions (4). Solving the first-order conditions for profit maximization yields

$$p_1 = \frac{(\sigma n - \sigma + 1)(1+t)c}{(\sigma - 1)(n-1)}, \quad nx_1 = \frac{(\sigma - 1)(n-1)I}{(\sigma n - \sigma + 1)(1+t)c}, \tag{17}$$

for the goods on which the ad valorem tax is levied. Similarly, the counterparts for the goods on which the specific tax is levied are

$$p_2 = \frac{(\sigma n - \sigma + 1)(c+\tau)}{(\sigma - 1)(n-1)}, \quad nx_2 = \frac{(\sigma - 1)(n-1)I}{(\sigma n - \sigma + 1)(c+\tau)}. \tag{18}$$

Substituion of these prices and industry outputs into (11), and further substitution of the resulting expression into the definition of I lead to

$$I = \frac{(\sigma n - \sigma + 1)L}{(\sigma - 1)(n-1)} \cdot \frac{(1+t)(c+\tau)}{\widetilde{z}(c+\tau) + (1-\widetilde{z})(1+t)c}. \tag{19}$$

This completes the description of our Bertrand model. As in the Cournot model, all the endogenous variables such as prices and outputs are derived by substituting (19) into them.

Having derived the Bertrand equilibrium, let us examine the welfare effect of the revenue-neutral shift from specific to ad valorem taxation. Relating the equilibrium prices and outputs in (17) and (18) to (11), the total tax revenue turns to depend on the two taxes as follows.

$$T = \frac{L}{(\sigma-1)(n-1)} \cdot \frac{\widetilde{z}(\sigma n - \sigma + 1)t(c+\tau) + (1-\widetilde{z})(\sigma-1)(n-1)(1+t)\tau}{\widetilde{z}(c+\tau) + (1-\widetilde{z})(1+t)c}. \tag{20}$$

Totally differentiating (20) with respect to t and τ, the revenue-neutral tax reform $dT = 0$ has the rule

$$d\tau = -\frac{\widetilde{z}(c+\tau)\left\{[\widetilde{z}n + (\sigma-1)(n-1)]\,\tau + (\sigma n - \sigma + 1)c\right\}}{(1-\widetilde{z})(1+t)c\left\{[\widetilde{z}n + (\sigma-1)(n-1)]\,t + (\sigma-1)(n-1)\right\}}dt. \tag{21}$$

Nore here that as in the Cournot model the specific tax must be lowered if the government is to raise the ad valorem tax and the tax revenue is to be unchanged.[7)]

In order to assess the welfare effect of the above tax reform, we now compute welfare as a function of the two tax rates. The indirect utility function is a function of the national income I in (19) and two price indices, which are obtained as

$$P_1 = \frac{\sigma n^{\frac{1}{1-\sigma}}(\sigma n - \sigma + 1)(1+t)c}{(\sigma-1)(n-1)}, \quad P_2 = \frac{\sigma n^{\frac{1}{1-\sigma}}(\sigma n - \sigma + 1)(c+\tau)}{(\sigma-1)(n-1)},$$

in the Bertrand case. Then, we have the closed form of welfare

$$W = (1-\widetilde{z})\ln(1+t) + \widetilde{z}\ln(c+\tau) - \ln\left[\widetilde{z}(c+\tau) + (1-\widetilde{z})(1+t)c\right] + const. \tag{22}$$
$$\text{where} \quad const. = \ln\left(n^{\frac{1}{\sigma-1}}L\right) - \widetilde{z}\ln c,$$

by subtituting I, P_1 and P_2 into the indirect utility function. Relating the tax reform rule (21) to this welfare measure, we obtain the following result.

Proposition 2. *Proposition 1 holds in Bertrand competition as well. That is, the revenue-neutral shift from specific to ad valorem taxation raises welfare if and only if the specific tax rate (τ) is higher than the corresponding*

7) This follows from the fact that the coefficient of dt in (21) is negative.

ad valorem equivalent (tc).

Proof. Our proof is the same as the proof of Proposition 1. Totally differentiating (22) with respect to t and τ and substituting (21) into the resulting expression give rise to

$$dW = \frac{\tilde{z}(\tau - tc)C}{(1+t)c\left[\tilde{z}(c+\tau)+(1-\tilde{z})\,(1+t)c\right]\left\{\left[\tilde{z}n+(\sigma-1)(n-1)\right]t+(\sigma-1)(n-1)\right\}}dt,$$

where C is given by

$$\begin{aligned} C &= \tilde{z}\left\{\left[\tilde{z}n + (\sigma-1)(n-1)\right]\tau + (\sigma n - \sigma + 1)c\right\} \\ &\quad + (1-\tilde{z})\,c\left\{\left[\tilde{z}n + (\sigma-1)(n-1)\right]t + (\sigma-1)(n-1)\right\} > 0. \end{aligned}$$

Thus, we have proved the proposition. ||

5 Monopolistic Competition

Thus far, we have addressed the welfare effect of the tax reform in oligopolies in which the number of firms is exogenously fixed. In contrast, this section turns to monopolistic competition. As is well-known, monopolistic competition is different from restricted entry oligopolies in the sense that (i) the number of active firms is endogenously determined by the zero profit condition, and (ii) each firm takes the quantity and price indices as given in choosing output/price. Despite these differences, we will show that Propositions 1 and 2 survive this market structure.

The model is the most primitive version of Dixit and Stiglitz (1977).[8)]

8) Melitz (2003), which has a revolutionary impact on the trade literature, introduces firm heterogeneity. Krugman (1979), Behrens and Murata (2007) and Zhelobodko et al. (2012) abandon the assumption of CES preferences to generate a variable markup, which has been empirically reported. It is beyond the scope of this paper to take into account of these extensions of the Dixit-Stiglitz model since they considerably complicates the analysis. However, it may be possible to utilize the model of Parenti et al. (2017) that compares the solution of (free entry) Cournot, Bertrand and monopolistic competition.

Solving the first-order condition for profit maximization, the zero profit condition, and the market-clearing condition for each variety, the price, per-firm output and number of firms under the ad valorem tax are jointly determined as follows.

$$p_1 = \frac{(1+t)\sigma c}{\sigma - 1}, \quad x_1 = \frac{(\sigma-1)f}{c}, \quad n_1 = \frac{I}{(1+t)\sigma f}. \tag{23}$$

Applying the same procedure to the industries under the specific tax, we have

$$p_2 = \frac{\sigma(c+\tau)}{\sigma - 1}, \quad x_2 = \frac{(\sigma-1)f}{c+\tau}, \quad n_2 = \frac{I}{\sigma f}. \tag{24}$$

Since fee entry derives the firm profit to zero, national income consists of labor income and tax revenue:

$$\begin{aligned} I = L + T &= L + \widetilde{z}\frac{tp_1nx_1}{1+t} + (1-\widetilde{z})\,\tau n x_2 \\ &= L + \left[\frac{\widetilde{z}t}{1+t} + \frac{(1-\widetilde{z})\,(\sigma-1)\tau}{\sigma(c+\tau)}\right] I. \end{aligned}$$

Solving this equation for I yields the closed form of national income:

$$I = \frac{\sigma(1+t)(c+\tau)L}{\sigma\left[\widetilde{z}\tau + (1-\widetilde{z})\,tc + c\right] + (1-\widetilde{z})\,(1+t)\tau}. \tag{25}$$

As is the case of the Cournot and Bertrand models, all the endogenous variables are obtained by substituting (25) into the relevent variables.

Let us now compute the tax revenue as an auxiliary step to examining the welfare effect of the revenue-neutral tax reform. Making use of Eqs. (23)-(25), T becomes

$$T = \frac{\left[\widetilde{z}\sigma t(c+\tau) + (1-\widetilde{z})\,(\sigma-1)(1+t)\tau\right] L}{\sigma\left[\widetilde{z}\tau + (1-\widetilde{z})\,tc + c\right] + (1-\widetilde{z})\,(1+t)\tau}. \tag{26}$$

By totally differentiating (26) with respect to t and τ and invoking the rule of the revenue-neutral tax reform $0 = dT = (\partial T/\partial t)dt + (\partial T/\partial \tau)d\tau$, we get

$$d\tau = -\frac{\widetilde{z}\sigma(c+\tau)^2}{(1-\widetilde{z})\,(\sigma-1)(1+t)^2 c}dt. \tag{27}$$

Having derived the policy reform rule, we will consider its welfare effect. As repeatedly stressed, the biggest difference between monopolistic competition and restricted entry oligopolies lies in whether the number of firms is endogenous or exogenous. Noting this, the closed form of the price indices is obtained as

$$P_1 = \left[\frac{I}{(1+t)\sigma f}\right]^{\frac{1}{1-\sigma}} \frac{(1+t)\sigma c}{\sigma - 1}, \quad P_2 = \left(\frac{I}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{\sigma(c+\tau)}{\sigma - 1}.$$

Substituting these price indices and (25) into the indirect utility function, welfare takes the form

$$\begin{aligned} W &= \ln I - \tilde{z} \ln P_1 - (1-\tilde{z}) \ln P_2 \\ &= \ln I - \tilde{z} \ln \left\{ \left[\frac{I}{(1+t)\sigma f}\right]^{\frac{1}{1-\sigma}} \frac{(1+t)\sigma c}{\sigma - 1} \right\} \\ &\qquad\qquad - (1-\tilde{z}) \ln \left\{ \left(\frac{I}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{\sigma(c+\tau)}{\sigma - 1} \right\} \\ &= \frac{\sigma}{\sigma - 1} \ln I - \frac{\tilde{z}\sigma}{\sigma - 1} \ln(1+t) - (1-\tilde{z}) \ln(c+\tau) + const. \end{aligned} \tag{28}$$

$$\text{where} \quad const. = -\frac{1}{\sigma - 1} \ln(\sigma f) - \ln\left(\frac{\sigma}{\sigma - 1}\right) - \tilde{z} \ln c.$$

Based on the preceding arguments, the welfare effect of the revenue-neutral shift from specific to ad valorem taxation is summarized as follows.

Proposition 3. *Proposition 1 holds in Bertrand competition as well. That is, the revenue-neutral shift from specific to ad valorem taxation raises welfare if and only if the specific tax rate (τ) is higher than the corresponding ad valorem equivalent (tc).*

Proof. Notice first that I in (28) is unchanged under the revenue-neutral tax reform because of the definition of national income, $I = L + T$; if T is unchanged, I is also unchanged. Accordingly, the suggested tax reform affects welfare through the second and third terms in (28). Taking this fact into account and totally differentiating (28) with respect to t and τ, the welfare effect of the tax reform is derived as

$$dW = \left(\frac{\partial W}{\partial t} + \frac{\partial W}{\partial \tau} \cdot \frac{d\tau}{dt} \right) dt = \frac{\widetilde{z}\sigma(\tau - tc)}{(\sigma - 1)(1 + t)^2 c} dt,$$

by utilizing (27). Therefore, the right-hand side of this equation allows us to find that the tax reform raises welfare if and only if $\tau - tc > 0$. ||

6 Concluding Remarks

It is generally believed that perfect competition is so far from the reality that the insight under perfect competition is useless. This paper has provided a counter-example to tackle this claim in a context of comparison between ad valorem and specific taxes. Concretely, we show that under Cournot, Bertrand and monopolistic competition the revenue-neutral shift from specific to ad valorem taxation raises welfare if and only if the specific tax rate is higher than the corresponding ad valorem equivalent. We also note that the general equilibrium model with a continuum of industries plays a key role behind this result.

The primary purpose of this paper is not to pursue the generality of the model, but we must recognize the limitations of our analysis. First, we assume a traditional approach in which firms are homogeneous. However, it is now standard to introduce firm heterogeneity a la Melitz (2003). Second, we have employed a CES -sub-utility function under which the markup is constant. Although this considerably facilitates analysis, recent developments of imperfect competition theory stress the importance of variable markup by abandoning the CES assumption. For example, quadratic preferences are used in Ottaviano et al. (2002) and Melitz and Ottaviano (2008), CARA (constant absolute risk aversion) preferences are used in Behrens and Murata (2007) and more general preferences are used in Zhelobodko et al. (2012). As far as we know, there is little literature in public economics that takes into account firm heterogeneity and/or non-CES preferences. It is important future research agenda to check our result in a more general and realistic setting.

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