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Pollution, Human Capital, and Growth Cycles

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Abstract

To investigate the growth effect of pollution, we apply an optimal growth framework in which human and physical capital accumulation are two growth engines. Pollution is emitted from the stock of physical capital and has a negative impact on the formation of human capital. In this simple growth model, sustainable endogenous growth never occurs and a unique steady state emerges because of the negative impact of pollution. The model shows that (i) if the extent of the external effect of pollution is relatively small, the steady state is stable and the economy starting in the neighborhood of the steady state converges to it, (ii) if the extent of the external effect is relatively large, the steady state is unstable and the economy diverges away from it, and (iii) a Hopf bifurcation occurs at a certain intermediate extent of the external effect. The numerical analysis illustrates the global dynamic behavior in which the economy exhibits a closed orbit as sufficient time passes if the steady state is unstable.

Keywords: pollution, human capital, Hopf bifurcation, limit cycle, endogenous business cycles

JEL Classification Numbers: O41; O44; E32

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1 Introduction

Many researchers have empirically demonstrated that pollution negatively impacts human health (e.g., Bell et al. 2004; Chay and Greenstone 2003; Currie and Neidell 2005; Dockery et al. 1993; Pope et al. 2002; Graff Zivin and Neidell 2012, 2013). If pollution harms human health, it probably impedes the formation of human capital, thereby reducing economic growth, because illness has direct and indirect negative impacts on mental and physical functioning that interfere with learning and working performance. In this paper, we investigate how pollution that impedes the formation of human capital affects the dynamic behavior of an economy by applying an optimal growth framework.

In our model, there are two growth engines: physical capital accumulation and human capital accumulation, as in the model of Lucas (1988). Although without pollution, the economy would experience sustainable endogenous growth, the economy never exhibits endogenous growth because of the negative external effect of pollution. In our model, the stock of physical capital emits pollution if sufficient physical capital accumulates, and this negatively affects the formation of human capital. More concretely, whereas one unit of investment in physical capital production produces one unit of physical capital, one unit of investment in human capital production produces less than one unit of human capital, being impeded by pollution. As physical capital accumulates further, the negative external effect of pollution on the formation of human capital is strengthened. Therefore, the economy is prevented from experiencing endogenous growth.

The outcome that the economy cannot experience endogenous growth is intuitive. The representative agent does not intend to invest much in human capital production because pollution disturbs the formation of human capital. Instead, she invests more in physical capital production, thereby increasing the supply of physical capital. As a result, the shadow price of physical capital (and general goods) declines in each period relative to the case without pollution. Our model proves that the value of consumption (i.e., the shadow price of general goods times consumption) is constant in each period. Then, if the shadow price becomes low, the current consumption becomes large. As such, the allocative inefficiency coming from the investment decision causes overconsumption in each period relative to the case without pollution, and thus, the amount of general goods produced decreases. Therefore, both human capital and physical capital accumulate to a lesser extent, and endogenous growth never occurs.

The investigation of the local dynamics shows that (i) if the extent of the external effect of pollution is relatively small, the steady state is stable and the economy starting in the neighborhood of the steady state converges to it, (ii) if the extent of the external effect is relatively large, the steady state is unstable and the economy diverges away from it, and (iii) a Hopf bifurcation occurs at a certain intermediate extent of the external effect and a limit cycle emerges around the steady state. The Hopf bifurcation in the optimal growth model was first obtained by Benhabib and Nishimura (1979); we apply it to the current growth model. Additionally, the numerical analysis illustrates the global behavior of the dynamical system in which the economy exhibits a closed orbit as sufficient time passes if the steady state is unstable.

Our study belongs to the literature on growth and the environment. Many researchers have investigated growth and the environment over the past forty years by applying an optimal growth framework with infinitely lived agents.[1] Among others, Forster (1973), Tahvonen and Kuuluvainen (1993), and van der Ploeg and Withagen (1991) study the growth effect of pollution with Ramsey-type growth models when pollution affects an instantaneous utility function or a neoclassical production function. Bovenberg and Smulders (1995) and Xepapadeas (1997) also investigate the growth effect of pollution by employing endogenous growth models, á la Romer (1986) and á la Lucas (1988), respectively. Unlike our study, these studies do not obtain endogenous business cycles in equilibrium. In contrast, Wirl (2004) and Bosi and Desmarchelier (2018a) show that a limit cycle emerges in equilibrium by extending the model of Ayong Le Kama (2001), in which pollution negatively affects an environmental resource that has a positive effect on an instantaneous utility function. Furthermore, Bosi and Desmarchelier (2018b), (2018c), and (2019) develop Ramsey-type growth models

[1] See Xepapadeas (2005) for a comprehensive survey of the literature.

in which pollution affects the disease transmission mechanism (which indirectly impacts the aggregate labor supply) and/or consumption demand, and derive a limit cycle. In our model, a limit cycle is also derived, but unlike these models, pollution has a negative effect on the formation of human capital.

The remainder of the paper is organized as follows. The next section presents the model, the growth engine of which is human and physical capital. In section 3, we derive an equilibrium in which we obtain differential equations with respect to human and physical capital and investigate the dynamical system locally. In section 4, we conduct a numerical exercise to observe the global behavior of the dynamical system and section 5 concludes our study.

2 Model

An economy goes from time $t = 0$ to $t = +\infty$ in continuous time and is inhabited by identical infinitely lived agents. Their population is constant and normalized to one. A representative agent produces general goods with a Cobb-Douglas production technology $y = Ah^{\alpha}k^{1-\alpha}$ $(0 < \alpha < 1)$, the inputs of which are human capital h and physical capital k, where A is the technology level. Since the general goods are used for investment or consumption in each period, the flow budget constraint is given by

$$Ah^{\alpha}k^{1-\alpha} = c + i^h + i^k, \tag{1}$$

where c is consumption and i^h and i^k are investments in the formation of human and physical capital, respectively. The accumulation of human and physical capital follows the equations below:

$$\dot{h} = \bar{k}^{-\sigma} i^h - \delta h \tag{2}$$

$$\dot{k} = i^k - \delta k, \tag{3}$$

where the depreciation rates of human and physical capital are the same, which is given by δ. In Eq. (2), $\bar{k}^{-\sigma}$ ($\sigma > 0$) is an external effect that the accumulation of physical capital has on the formation of human capital. If k is less than one, investment in human capital production is enhanced by the external effect, and if k is greater than one, it is disturbed by the effect. One may imagine that if sufficient physical capital accumulates, it begins to emit pollution that negatively affects the production of human capital. In what follows, we simply call σ the extent of the external effect. Because our interest is in the situation in which physical capital accumulates sufficiently that it emits pollution, we focus our following analysis on such a case unless otherwise stated. As σ increases, the extent of the negative external effect becomes large.

2.1 Utility maximization

The representative agent maximizes her lifetime utility $\int_0^\infty e^{-\rho t} \ln(c) dt$ subject to Eqs. (1)-(3); namely, she solves the following maximization problem:

$$\max \int_0^\infty e^{-\rho t} \ln(c) dt$$

subject to

$$\begin{aligned} y &= Ah^\alpha k^{1-\alpha} = c + i^h + i^k \\ \dot{h} &= \bar{k}^{-\sigma} i^h - \delta h \\ \dot{k} &= i^k - \delta k, \end{aligned}$$

where $\rho > 0$ is the subjective discount rate. The current-value Hamiltonian is set as follows:

$$H := \ln(c) + \lambda(Ah^\alpha k^{1-\alpha} - c - i^h - i^k) + \lambda^h(\bar{k}^{-\sigma} i^h - \delta h) + \lambda^k(i^k - \delta k),$$

where λ, λ^h, and λ^k are the shadow prices of general goods, human capital, and physical capital, respectively. The first-order conditions are given by

$$\frac{\partial H}{\partial c} = \frac{1}{c} - \lambda = 0 \tag{4}$$
$$\frac{\partial H}{\partial i^h} = -\lambda + \lambda^h \bar{k}^{-\sigma} = 0 \tag{5}$$
$$\frac{\partial H}{\partial i^k} = -\lambda + \lambda^k = 0 \tag{6}$$
$$\frac{\partial H}{\partial h} = -\dot{\lambda}^h + \rho\lambda^h \iff \lambda\alpha\frac{y}{h} - \delta\lambda^h = -\dot{\lambda}^h + \rho\lambda^h \tag{7}$$
$$\frac{\partial H}{\partial k} = -\dot{\lambda}^k + \rho\lambda^k \iff \lambda(1-\alpha)\frac{y}{k} - \delta\lambda^k = -\dot{\lambda}^k + \rho\lambda^k, \tag{8}$$

where one should note that in equilibrium, it holds that $\bar{k} = k$. The transversality conditions are given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda^h h = \lim_{t\to\infty} e^{-\rho t}\lambda^k k = 0. \tag{9}$$

3 Equilibrium

In this section, we define a dynamic competitive equilibrium and characterize the dynamical system.

Proposition 1. *The shadow prices of human and physical capital satisfy the following differential equations in equilibrium:*

$$\dot{\lambda}^h = \left(\rho + \delta - \alpha\frac{k^{-\sigma}y}{h}\right)\lambda^h \tag{10}$$

and

$$\dot{\lambda}^k = \left(\rho + \delta - (1-\alpha)\frac{y}{k}\right)\lambda^k. \tag{11}$$

Proof. From Eq. (5), we have $\lambda = \lambda^h k^{-\sigma}$. Substituting this into Eq. (7) yields Eq. (10). Similarly, from Eq. (6), we have $\lambda = \lambda^k$. Substituting this into Eq. (8) yields Eq. (11). □

Proposition 2. *The laws of motion of human and physical capital satisfy the following* *differential equations in equilibrium:*

$$\dot{h} = \left((\sigma+\alpha-1)h + \alpha k^{1-\sigma}\right)\left(\frac{Ah^{\alpha-1}k^{1-\alpha-\sigma}}{\sigma}\right) - \delta(h+k^{1-\sigma}) - \frac{1}{\lambda^k k^\sigma} \tag{12}$$

and

$$\dot{k} = \left((1-\alpha)h - \alpha k^{1-\sigma}\right)\left(\frac{Ah^{\alpha-1}k^{1-\alpha}}{\sigma}\right). \tag{13}$$

Proof. See the Appendix.

A dynamic competitive equilibrium is defined as a sequence of the shadow prices $\{\lambda^h, \lambda^k\}$ and human and physical capital stocks $\{h, k\}$ for $t \geq 0$ that satisfy Eqs. (10)-(13) and the transversality conditions (9), given h_0 and k_0. In the equilibrium dynamics, λ^h and λ^k are non-predetermined variables that can jump and h and k are state variables that cannot jump.

3.1 Dynamical system

By using the transversality conditions (9), we can reduce the four-dimensional dynamical system consisting of Eqs. (10)-(13) to a two-dimensional dynamical system with respect to h and k. To do so, we prepare a useful lemma below.

Lemma 1. *It holds that*

$$\lambda^h h + \lambda^k k = \frac{1}{\rho} \tag{14}$$

for all $t \geq 0$ in equilibrium.

Proof. See the Appendix.

Eq. (14) implies that the sum of the current values of human and physical capital is equal to the sum of the present values of consumption flows.[2] This equation holds regardless of the presence of the negative external effect of pollution. Before arranging the dynamical system by using Eq. (14), we observe the relationship among human capital, physical capital, and consumption in Remark 1 below.

[2] Since $\lambda c = 1$, the sum of the present values of consumption flows is computed as $\int_0^\infty e^{-\rho t} \cdot 1 dt = 1/\rho$, which is the right-hand side of Eq. (14).

Remark 1. *It holds that*

$$c = \rho(k^\sigma h + k) \tag{15}$$

for all $t \geq 0$ in equilibrium.

Proof. From Eqs. (4), (5), and (6), it follows that $\lambda c = 1$ and $\lambda = \lambda^k = \lambda^h k^{-\sigma}$ in equilibrium. Substituting these equations into Eq. (14) yields Eq. (15). □

Note from Remark 1 that overconsumption occurs in each period when the accumulation of physical capital emits pollution. To consider this, suppose that $k > 1$. If there were no negative external effects of pollution, consumption would be determined by $c = \rho(h + k)$ given human and physical capital, which implies that a certain proportion of the sum of human and physical capital is optimally consumed in each period. However, now that there is a negative external effect of pollution, the representative agent consumes more than the optimal level because $k^\sigma > 1$.

From Eqs. (5), (6), and (14), we have $1/\lambda^k = \rho(k^\sigma h + k)$. Substituting this equation into Eq. (12) yields the following equation:

$$\dot{h} = \left((\sigma + \alpha - 1)h + \alpha k^{1-\sigma}\right)\left(\frac{Ah^{\alpha-1}k^{1-\alpha-\sigma}}{\sigma}\right) - (\rho + \delta)(h + k^{1-\sigma}). \tag{16}$$

Given $k(0)$ and $h(0)$, the equilibrium path with respect to k and h is given by Eqs. (13) and (16). Because of the transversality conditions, the shadow prices, λ^h and λ^k, jump such that the economy reaches the manifold that includes the set of the general solutions of Eqs. (13) and (16). In what follows, our analysis focuses on the dynamical system with respect to h and k that is given by Eqs. (13) and (16), or equivalently,

$$\begin{cases} \dot{h} = ((\sigma + \alpha - 1)h + \alpha k^{1-\sigma})\left(\frac{Ah^{\alpha-1}k^{1-\alpha-\sigma}}{\sigma}\right) - (\rho + \delta)(h + k^{1-\sigma}) \\ \dot{k} = ((1-\alpha)h - \alpha k^{1-\sigma})\left(\frac{Ah^{\alpha-1}k^{1-\alpha}}{\sigma}\right). \end{cases} \tag{17}$$

3.2 Steady state

From Eq. (17), we obtain the steady-state values of human and physical capital as follows:

$$h^* := \left[\left(\frac{(1-\alpha)A}{\rho+\delta} \right)^{\frac{1-\sigma}{\alpha}} \left(\frac{\alpha}{1-\alpha} \right) \right]^{\frac{1}{\sigma}} \tag{18}$$

and

$$k^* := \left[\left(\frac{(1-\alpha)A}{\rho+\delta} \right)^{\frac{1}{\alpha}} \left(\frac{\alpha}{1-\alpha} \right) \right]^{\frac{1}{\sigma}}. \tag{19}$$

It is straightforward to show that both h^* and k^* increase as σ decreases if $A\alpha^\alpha(1-\alpha)^{1-\alpha} > \rho+\delta$. In particular, Remark 2 below clarifies the case in which σ is close to zero.

Remark 2. *Suppose that $A\alpha^\alpha(1-\alpha)^{1-\alpha} > \rho+\delta$. Then, it follows that $\lim_{\sigma\to 0} h^* = \infty$ and $\lim_{\sigma\to 0} k^* = \infty$.*

Proof. The claim immediately follows from Eqs. (18) and (19). □

Remark 2 implies that absent the negative external effect of pollution, the economy would exhibit endogenous growth, the engines of which are the accumulation of human and physical capital. If there were no external effects of pollution, i.e., if $\sigma = 0$, the growth rate on the balanced growth path in equilibrium would be equal to $A\alpha^\alpha(1-\alpha)^{1-\alpha}-(\rho+\delta)$. However, the negative external effect that pollution has on human capital formation produces finite steady-state values of human and physical capital. Note that although the extent of the negative external effect is infinitesimal but not zero, the steady state appears. The intuition behind the outcome that the economy does not exhibit endogenous growth is as follows. Suppose that sufficient physical capital accumulates such that k is greater than one; thus, physical capital emits pollution at a certain time. Since the production of human capital is disturbed by pollution, the representative agent refrains from investing in human capital production, but instead, she invests more in physical capital production. Then, the supply of physical capital increases, and the shadow price of physical capital (and general goods) decreases in each period relative to the case without pollution. Because the value of current consumption is one (i.e., $\lambda c = 1$), if the shadow price of general goods becomes low, current consumption becomes large. The allocative inefficiency regarding the investment decision causes overconsumption, as seen in Remark 1, and the lower amount of general goods production in each period relative to the case without pollution. Eventually, both human capital and physical capital accumulate less, and thus, endogenous growth never occurs.

3.3 Dynamics of h and k

Linearizing the dynamical system (17) around the steady state, we obtain the local system as follows:

$$\begin{pmatrix} \dot{h} \\ \dot{k} \end{pmatrix} = J \begin{pmatrix} h - h^* \\ k - k^* \end{pmatrix}, \tag{20}$$

where J is the Jacobian of the system, which is given by

$$J = \begin{pmatrix} -\frac{(1-\alpha)(\rho+\delta)}{\alpha\sigma} & \left(\frac{(1-\alpha)(1-\sigma)-\alpha\sigma^2}{\alpha\sigma}\right)(\rho+\delta)^{\frac{1+\alpha}{\alpha}}(A(1-\alpha))^{-\frac{1}{\alpha}} \\ \frac{1}{\sigma}(\rho+\delta)^{\frac{\alpha-1}{\alpha}}(A(1-\alpha))^{\frac{1}{\alpha}} & -\left(\frac{1-\sigma}{\sigma}\right)(\rho+\delta) \end{pmatrix}. \tag{21}$$

Let $\mathrm{Tr}(J)$ and $\mathrm{Det}(J)$ denote the trace and determinant of J, respectively, which are computed as

$$\mathrm{Tr}(J) = (\rho+\delta)\left(1 - \frac{1}{\alpha\sigma}\right) \tag{22}$$

and

$$\mathrm{Det}(J) = (\rho+\delta)^2. \tag{23}$$

Since $\mathrm{Det}(J) > 0$, the real parts of the eigenvalues have the same sign and whether the sign is positive or negative is determined by the trace. From Eq. (22), if $\alpha\sigma < 1$ (> 1), the sign of the real parts is negative (positive). The following proposition characterizes the steady state and the local dynamic behavior of the economy in terms of the range of parameter values of σ.

Proposition 3. *The following hold.*

- *If $0 < \sigma < 1/\alpha$, the steady state is stable, and (h, k) starting with any initial values of*

$(h(0), k(0))$ in the neighborhood of the steady state converges to the steady state.

- *If $1/\alpha < \sigma$, the steady state is unstable, and (h, k) starting with any initial values of $(h(0), k(0))$ in the neighborhood of the steady state diverges away from the steady state.*

Proof. See the Appendix.

Furthermore, if σ increases from 0, a Hopf bifurcation occurs as summarized in the following remark.

Remark 3. *A Hopf bifurcation occurs at $\sigma = 1/\alpha$, and a limit cycle appears at a certain value of $\sigma \in (1/\alpha - \epsilon, 1/\alpha + \epsilon)$ where $\epsilon > 0$.*

Proof. At $\sigma = 1/\alpha$, $\mathrm{Tr}(J)$ becomes zero with $\mathrm{Det}(J)$ remaining positive. Additionally, the derivative of the real parts of the eigenvalues with respect to σ is positive. Then, a Hopf bifurcation occurs at $\sigma = 1/\alpha$. □

For the criterion to determine whether the Hopf bifurcation is subcritical or supercritical, see Nishimura and Shigoka (2019).

4 Numerical analysis

In the previous section, we investigated the local dynamics of the economy. To observe the global dynamic behavior, we perform a numerical analysis in this section.[3] In particular, we shall see that the economy can exhibit cyclical behavior when the steady state is unstable as in the second case of Proposition 3. Throughout the numerical analysis in this section, we set the parameter values as $\alpha = 0.33$, $\rho = 0.05$, $\delta = 0.10$, and $A = 1$. Regarding σ, we examine three cases: $\sigma = 1$, $\sigma = 1/0.3$, and $\sigma = 1/0.33$. We iterate $200,000$ times for the simulation. From Proposition 3, $\sigma = 1$ yields a stable steady state and $\sigma = 1/0.3$ yields an unstable steady state. $\sigma = 1/0.33$ is the value of σ that produces a Hopf bifurcation.

[3]To perform the numerical analysis, we use MATLAB R2020a with ode45.

Case 1: $\sigma = 1$

Fig. 1 shows the case of $\sigma = 1$. In this case, the steady-state values of human and physical capital are given by $h^* = 0.491$ and $k^* = 45.92$, respectively. The initial values of human and physical capital are $h_0 = 0.01$ and $k_0 = 2.4$, respectively. Starting from the initial values, human capital increases first and overshoots, while physical capital remains relatively unchanged. However, because of the negative external effect of pollution, human capital starts to decline, and physical capital significantly increases. Then, the economy converges to the stable steady state. We examined various initial values, but the convergence outcomes were all the same, which implies that the economy is globally stable if the condition of the first part of Proposition 3 holds.

Case 2: $\sigma = 1/0.3$

The outcome of this case is presented in Fig. 2. In this case, the steady-state values are $h^* = 0.0338$ and $k^* = 3.152$, both of which are smaller than those in Case 1. Of course, this is because the extent of the negative external effect of pollution is more severe than in Case 1. Panel **a** in the figure examines the case in which the economy starts away from the steady state. More concretely, the initial values are $h_0 = 0.01$ and $k_0 = 2.4$, which are the same as those in Case 1. Although the dynamic courses of both human and physical capital on the initial dates are similar to those of Case 1, the economy does not converge to the steady state but exhibits a closed orbit that surrounds the steady state, implying that endogenous business cycles occur. In panel **b**, the economy starts from initial values that are closer to the steady state, which are $h_0 = 0.033$ and $k_0 = 3.15$. Since the steady state is unstable and the eigenvalues of J are imaginary numbers with positive real parts, the economy exhibits oscillation with the amplitude widening over time. Eventually, we obtain the same closed orbit as that in the case of panel **a**.

Case 3: $\sigma = 1/0.33$

Since $\alpha = 0.33$, $\sigma = 1/0.33$ is the value of σ that causes the Hopf bifurcation. Although in this case, there exists a limit cycle in the neighborhood of the steady state for a certain value of $\sigma \in (1/\alpha - \epsilon, 1/\alpha + \epsilon)$, we are curious about the global dynamic behavior. The steady-state values of human and physical capital are given by $h^* = 0.03379$ and $k^* = 3.535$. In panel **a** in Fig. 3, the initial values are again $h_0 = 0.01$ and $k_0 = 2.4$. As in Case 2, the economy converges to a closed obit when sufficient time passes, implying that endogenous business cycles occur. Panel **b** illustrates the case in which the initial values are $h_0 = 0.0377$ and $k_0 = 3.53$, which are even closer to the steady state. Although the eigenvalues of J are pure imaginary numbers, the amplitude of oscillation becomes wider. Although the economy converges to a closed orbit as in Case 2, the convergence takes much more time than in Case 2.

5 Conclusion

We have investigated the growth effect of pollution that is emitted from the stock of physical capital and has a negative impact on the formation of human capital by applying a simple optimal growth framework in which there are two growth engines: human and physical capital accumulation. Because of the negative impact, sustainable endogenous growth never occurs, and a unique steady state emerges despite that the extent of the external effect of pollution is infinitesimal. The analysis shows that if the extent of the external effect is relatively small, the steady state is stable, and the economy starting in the neighborhood of the steady state converges to it. If the extent of the external effect is relatively large, the steady state is unstable, and the economy starting in the neighborhood of the steady state diverges away from it. Furthermore, a Hopf bifurcation occurs at a certain intermediate extent of the external effect. We have also performed a numerical analysis to observe the global dynamic behavior, in which the economy exhibits a closed orbit once sufficient time passes if the steady state is unstable.

One can extend our model such that pollution abatement technologies are introduced and pollution impacts not only the formation of human capital but also consumption demand. Investigating what would happen to the dynamic behavior under these extensions is left for future research.

Appendix

Proof of Proposition 2

From Eqs. (5) and (6), it follows that

$$\frac{\dot{\lambda}^k}{\lambda^k} = \frac{\dot{\lambda}^h}{\lambda^h} - \sigma\frac{\dot{k}}{k} \tag{A.1}$$

From Eqs. (10), (11), and (A.1), we obtain Eq. (13). Eqs. (1)-(4), (6) eliminate c, i^h, and i^k and we obtain

$$\frac{1}{\lambda^k} + \dot{k} + \delta k + k^\sigma(\dot{h} + \delta h) = y. \tag{A.2}$$

Substituting Eq. (13) into Eq. (A.2) yields Eq. (12). □

Proof of Lemma 1

Define $x = \lambda^h h + \lambda^k k$. Then, it follows from Eqs. (2), (3), (5), (10), and (11) that

$$\begin{aligned}\dot{x} &= \dot{\lambda}^h h + \lambda^h \dot{h} + \dot{\lambda}^k k + \lambda^k \dot{k} \\ &= \lambda^h(k^{-\sigma} i^h - \delta h) + \lambda^k(i^k - \delta k) + (\rho + \delta)(\lambda^h h + \lambda^k k) - \lambda^h k^{-\sigma}\alpha y - \lambda^k(1-\alpha)y \\ &= \lambda^k(i^h + i^k) + \rho x - \lambda^k y. \qquad \text{(B.2)}\end{aligned}$$

Eqs. (1) and (B.2) eliminate i^h and i^k and we obtain

$$\dot{x} = \rho x - \lambda_k c. \tag{B.3}$$

Applying Eqs. (4) and (6) to Eq. (B.3) yields

$$\dot{x} = \rho x - 1. \tag{B.4}$$

Solving Eq. (B.4), we have

$$e^{-\rho t}x = x(0) + \frac{1}{\rho}(e^{-\rho t} - 1). \tag{B.5}$$

From the transversality conditions, it follows that $\lim_{t\to\infty} e^{-\rho t}x = e^{-\rho t}(\lambda^h h + \lambda^k k) = 0$. Therefore, from Eq. (B.5) we obtain $x(0) = 1/\rho$ and thus $x = 1/\rho$ for all $t \geq 0$. □

Proof of Proposition 3

Since $\mathrm{Det}(J) > 0$, the real parts of the eigenvalues have the same sign. If $0 < \sigma < 1/\alpha$, it follows that $\mathrm{Tr}(J) < 0$, and thus, the real parts of the eigenvalues are negative. Therefore, the steady state is stable. If $1/\alpha < \sigma$, it follows that $\mathrm{Tr}(J) > 0$, and thus, the real parts of the eigenvalues are positive. Therefore, the steady state is unstable. □

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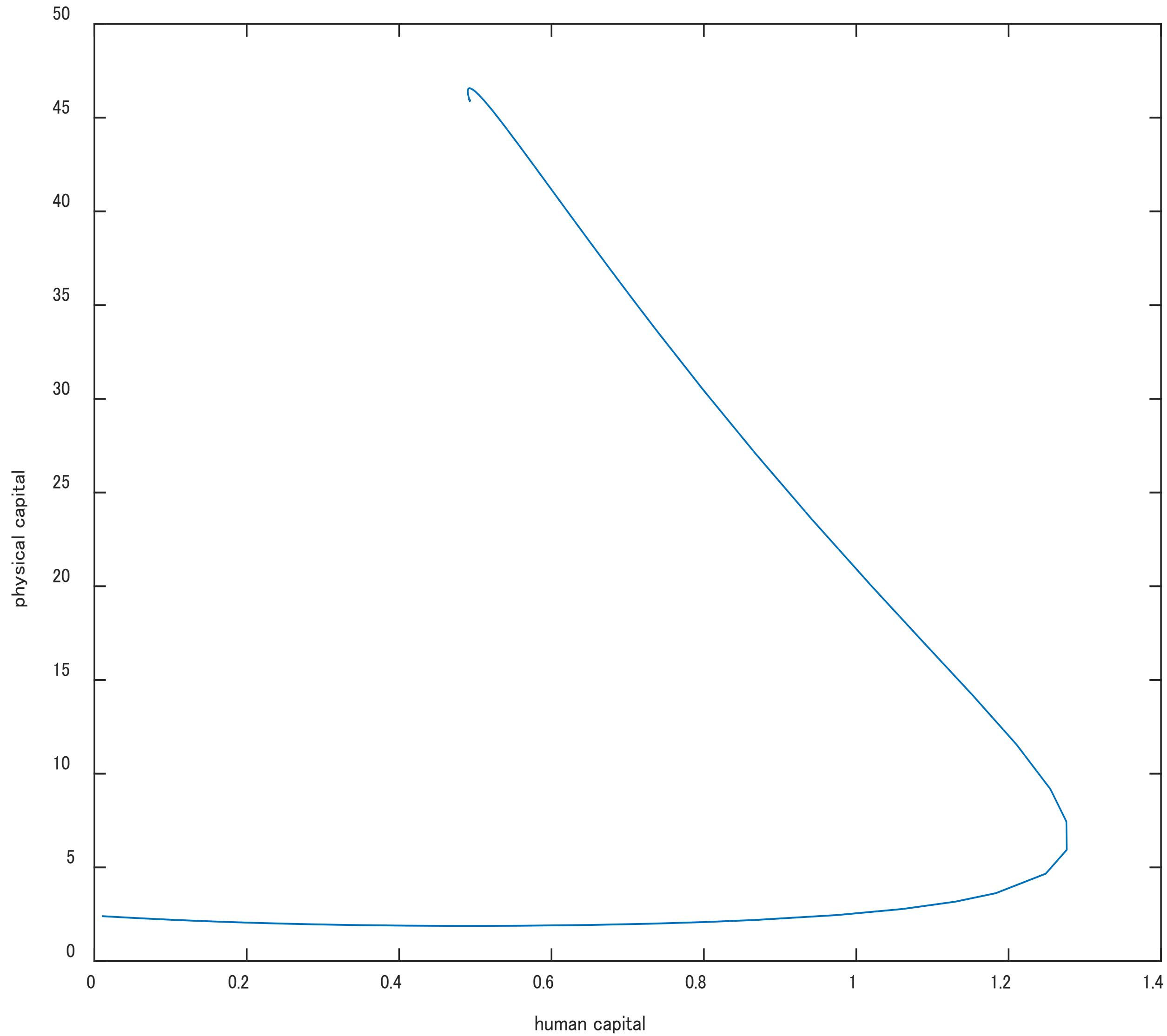
Fig. 1 global dynamics
$\sigma = 1$



$h_0 = 0.01, k_0 = 2.4$
$h^* = 0.491, k^* = 45.92$

a

b

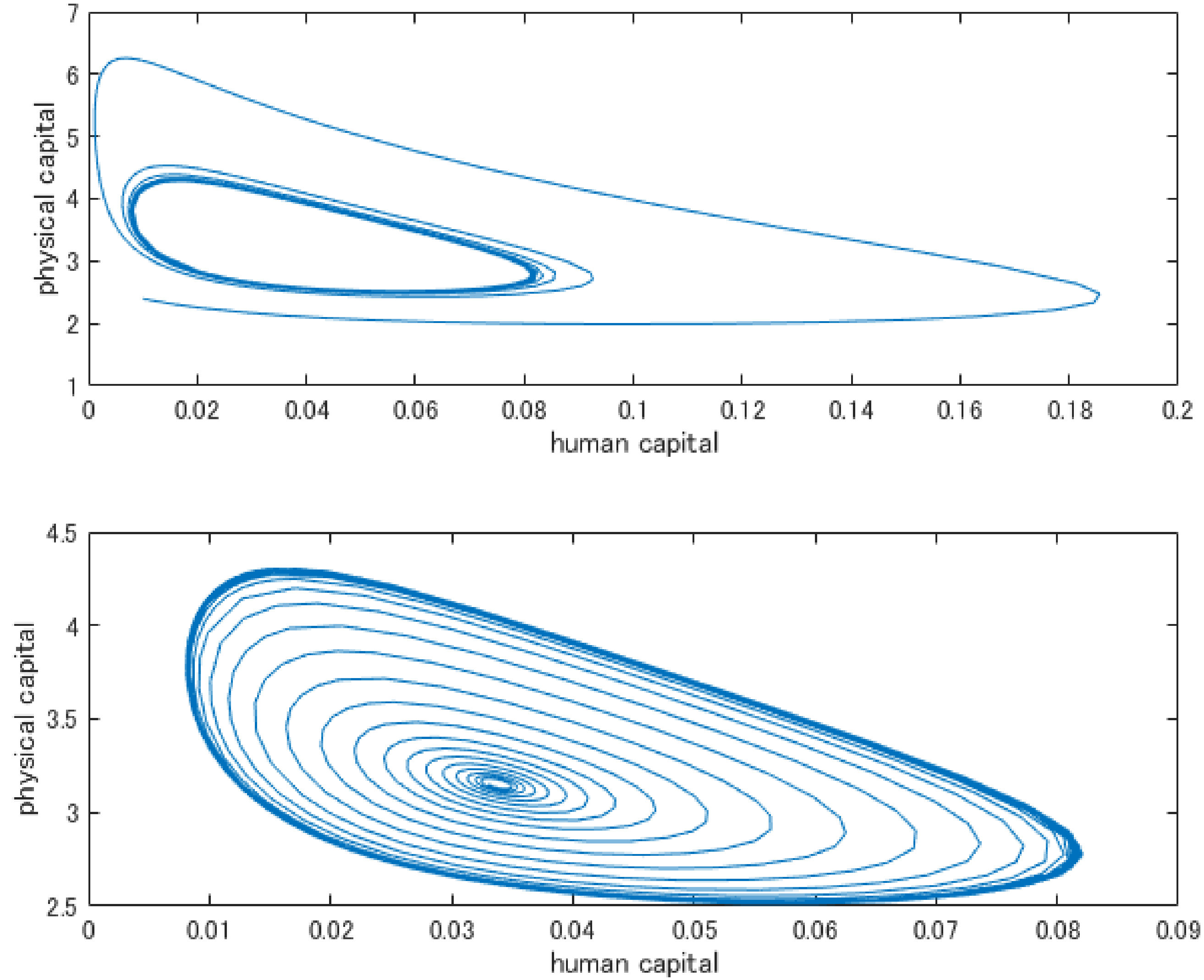
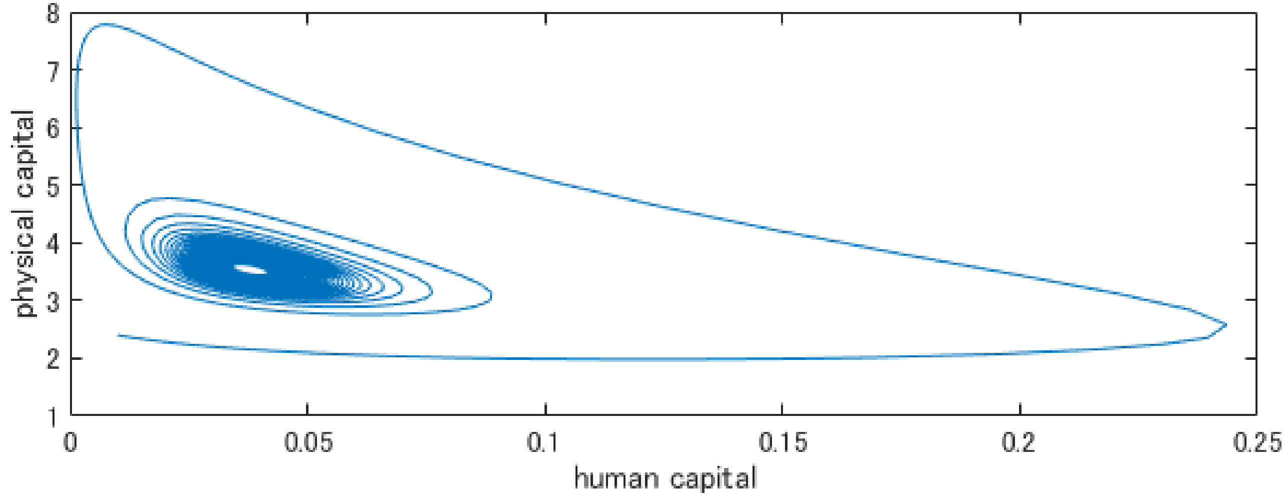


Fig. 2 global dynamics $\sigma = 1/0.3$

a $h_0 = 0.01\ k_0 = 2.4 \quad h^* = 0.0338\ k^* = 3.152$
b $h_0 = 0.033\ k_0 = 3.15 \quad h^* = 0.0338\ k^* = 3.152$

a



b

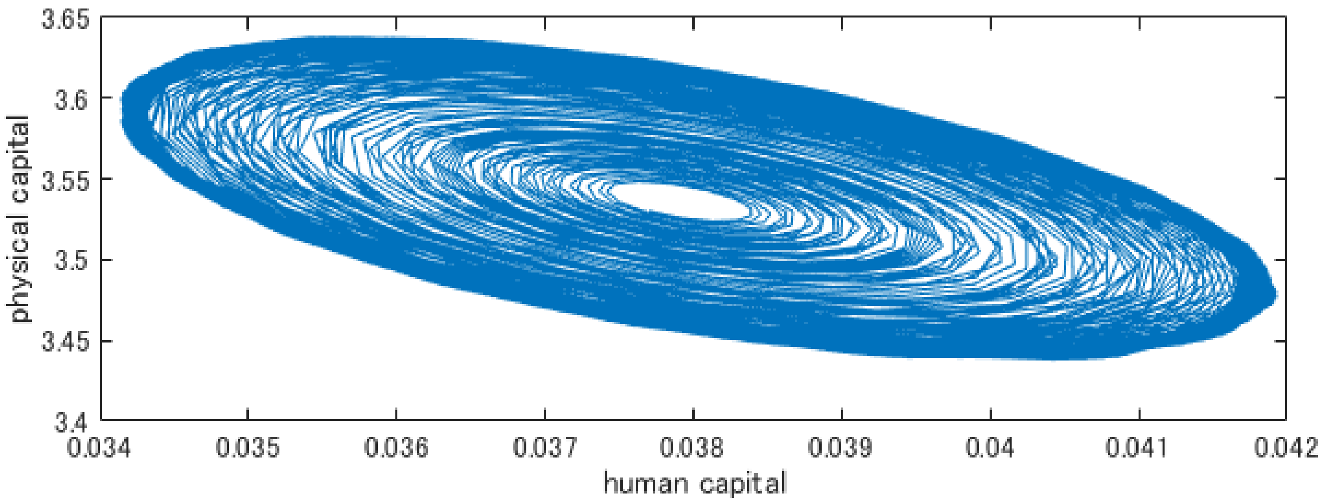


Fig. 3 global dynamics $\sigma = 1/0.33$

a $h_0 = 0.01\ k_0 = 2.4\quad h^* = 0.03779\ k^* = 3.535$
b $h_0 = 0.0377\ k_0 = 3.53\quad h^* = 0.03779\ k^* = 3.535$