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Asset Bubbles, Unemployment, and Financial Market Frictions*

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Abstract

A tractable model with infinitely lived agents is constructed for the examination of bubbles and unemployment. It is demonstrated that the presence of bubbles stimulates capital accumulation and reduces unemployment. The presence of bubbles also changes the effects of government policies that target unemployment and welfare conditions in the labor market. The main findings are as follows: (i) the presence of bubbles is more beneficial to an economy with severe credit constraints; (ii) the presence of bubbles mitigates the negative effects of taxation and unemployment benefits on unemployment and welfare; and (iii) these mitigation effects decrease as credit constraints are relaxed.

Keywords: Asset bubbles, Unemployment, Labor-market matching frictions, Financial frictions.

JEL Classification Numbers: J64, O41, O42.

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1 Introduction

Upward deviations of asset prices from their trend values are often followed by a sharp drop in prices and then a recession. As a result, several researchers have developed dynamic general equilibrium models that characterize these phenomena as the emergence and subsequent collapse of asset price bubbles. These studies focus on the positive effect of bubbles on investment and output and propose several mechanisms that might drive this effect.[1] It should be noted here that these large swings in asset prices are often accompanied by corresponding changes in employment and investment. In fact, Phelps (1999), Fitoussi et al. (2000), and Pan (2020) statistically confirm that high asset prices reduce unemployment. Motivated by these observations, we construct a model with infinitely lived agents that incorporates unemployment. Then, we use this model to investigate how asset bubbles affect unemployment, capital accumulation, and welfare.

In our model, there are three types of economic agents: entrepreneurs, workers, and firms. We also include a representative financial intermediary, which is a veil in our model. Entrepreneurs are potential capital goods producers. In each period, they receive idiosyncratic productivity shocks. Entrepreneurs who draw higher productivity shocks borrow from the financial intermediary and undertake an investment project for capital goods production. Those who draw lower productivity shocks deposit their savings with the financial intermediary without engaging in capital goods production. In other words, depositors (effective lenders) and borrowers appear endogenously in each period. Moreover, entrepreneurs are assumed to face credit constraints in that

[1] According to this growing stream of literature, financial market imperfections and productivity differences across agents are key factors that produce situations in which asset bubbles enhance capital accumulation. For example, Farhi and Tirole (2012), Martin and Ventura (2012), Carvalho et al. (2012), and Kunieda (2014) apply overlapping generations (OLG) models. Furthermore, despite the assumption of infinitely lived agents, in the dynamic general equilibrium models of Kocherlakota (2009), Kiyotaki and Moore (2012), Aoki and Nikolov (2015), Hirano et al. (2015), Kunieda and Shibata (2016), and Hirano and Yanagawa (2017), the presence of bubbles promotes economic growth through a mechanism similar to that in Mitsui and Watanabe (1989), which is the earliest study to show the capital-enhancing effect of bubbles.

they can borrow only up to a certain proportion of their net worth. Therefore, the demand for borrowing is smaller than that in the case of no credit constraints. As a result, in the presence of such constraints, the equilibrium interest rate can be lower than the growth rate of the economy, which makes it possible for an asset bubble to exist.[2]

Workers and firms are subject to labor-market matching frictions. We employ the same matching function as that in Diamond (1982), Mortensen and Pissarides (1999), and Pissarides (2000). A worker who successfully matches with a firm inelastically supplies one unit of labor to the firm and earns a wage income in each period. However, workers may fail to match with a firm owing to labor-market matching frictions. These workers are unemployed and receive unemployment benefits from the government. Both employed and unemployed workers consume all their earnings in each period; that is, they are hand-to-mouth consumers because they cannot borrow in the financial market, and their subjective discount factor is so small that the borrowing constraints are always binding. Firms are endowed with identical constant-returns-to-scale technology. Each firm must hire a worker to operate its business. However, the aforementioned frictions can result in firms failing to hire a worker. Such firms are not engaged in production activities.

Several studies are related to ours. The seminal paper by Miao et al. (2016) investigates the relationship between unemployment and stock market bubbles in an economy with both labor- and financial-market frictions. They derive the implications of labor-market policies such as unemployment benefits and hiring subsidies for macroeconomic variables.[3] Although we share numerous research interests with Miao et al., our research departs from theirs in several respects. First, in their model,

[2]A necessary condition for asset bubbles to appear in growth models is that the equilibrium interest rate is lower than the economic growth rate. See, for example, King and Ferguson (1993).

[3]Vuillemey and Wasmer (2020) study the effect of nonfundamental shocks on unemployment by applying a standard search-and-matching model. However, they derive bubbles from the model without rationality. In contrast, rational bubbles are derived in Miao et al. (2016) and our paper.

bubbles play the key role in increasing a firm's fundamental value through the relaxation of the collateral constraint such that the firm increases its production and employment being stimulated by extrinsic uncertainty. In contrast, in our model, the presence of intrinsically useless assets á la Tirole (1985) can correct allocative inefficiency regarding production resources, and as a result, capital accumulation is promoted so that firms are incentivized to increase employment. Second, whereas Miao et al. do not examine how the extent of financial frictions changes the effects of labor-market policies on unemployment (and other macroeconomic variables), we demonstrate that the effects of labor-market policies on unemployment (and other macroeconomic variables) are crucially dependent upon the extent of these frictions. Moreover, we provide a welfare analysis of the policies. Third, whereas in the model of Miao et al., agents are risk-neutral and thus consumption dynamics does not arise, in our model, agents are risk-averse, so consumption and asset dynamics can be obtained. Furthermore, whereas Miao et al. adopt a Leontief production technology in which there is no substitution between capital and labor, we employ a general neoclassical production technology that exhibits factor substitution. Kocherlakota (2011) investigates the effect of asset bubbles on unemployment, assuming away capital accumulation. Hashimoto and Im (2016, 2019) and Hashimoto et al. (2020) introduce labor-market frictions into OLG models to study the effect of bubbles on both capital accumulation and unemployment. In the models of Hashimoto and Im (2016, 2019), because the financial market is perfect, bubbles have only crowding-out effects on capital accumulation.[4] In the model of Hashimoto et al. (2020), although the financial market is imperfect and the presence of bubbles promotes capital accumulation, how the extent of financial constraints affects unemployment when bubbles are present cannot be investigated because agents cannot borrow at all in the financial market,

[4]Many previous studies have examined the crowding-out effect of asset bubbles on capital accumulation using OLG models (e.g., Tirole, 1985; Weil, 1987; Grossman and Yanagawa, 1993; King and Ferguson, 1993; Futagami and Shibata, 1999, 2000; Kunieda, 2008; Mino, 2008; Matsuoka and Shibata, 2012).

unlike in the current paper. Introducing downward wage rigidity, Hanson and Phan (2017) and Biswas et al. (2020) show that collapses of bubbles may cause a large and prolonged recession with involuntary unemployment. In contrast with their models, we introduce search matching frictions into a dynamic general equilibrium model and analyze the effects of labor-market environments on macroeconomic variables.

The remainder of this paper is organized as follows. In section 2, we develop our model. Section 3 derives the existence condition for a bubbly steady state and examines the stability of the bubbly and bubbleless equilibria. Section 4 compares unemployment, capital accumulation, and aggregate consumption in the bubbly steady state with their counterparts in the bubbleless steady state. By means of numerical simulations, section 5 investigates how changes in the degree of financial frictions affect unemployment, capital accumulation, and welfare in both the bubbly and bubbleless steady states. Section 5 also investigates the effects of government policies on unemployment and welfare. Section 6 concludes the paper.

2 The model

In this section, we develop a dynamic general equilibrium model with infinitely lived agents. The basic structure of our model follows that of Kunieda and Shibata (2016), but it differs from their model in some respects. First, in their model there are only entrepreneurs and a financial intermediary, whereas our economy also includes workers and firms. Second, we introduce labor market frictions into our model, whereas the labor market in their model is perfect. The economy is measured in discrete time, ranging from period 0 to ∞.

2.1 Entrepreneurs

Entrepreneurs are infinitely lived and are uniformly distributed on $[0, 1]$. Entrepreneur $j \in \Omega$ is endowed with a linear investment technology such that $k_t(j) = A\Phi_{t-1}(j)i_{t-1}(j)$, where Ω is the set of all entrepreneurs, $k_t(j)$ represents the capital goods produced in period t, $\Phi_{t-1}(j)$ is an individual-specific productivity shock in period $t-1$, $i_{t-1}(j)$ is the investment undertaken in period $t-1$, and A is a positive constant. Entrepreneurs who invest one unit of funds in a project in period $t-1$ produce $A\Phi_{t-1}(j)$ units of capital goods, which are sold to firms at price ρ_t in period t. Capital goods depreciate in one period. Note that $\Phi_{t-1}(j)$ is a random variable realized in period $t-1$. and each entrepreneur has information about $\Phi_{t-1}(j)$ before i_{t-1} is undertaken. Although $\Phi(j)$ is idiosyncratic, there is no insurance market for the productivity shocks and, thus, the realization of low productivity cannot be insured. It is assumed that the idiosyncratic productivity shocks $\Phi_0(j), \Phi_1(j), ...$ are independently and identically distributed (i.i.d.) across both time and entrepreneurs. Specifically, we assume that $\Phi(j)$ has support over $[0, \eta]$ and its cumulative distribution function is $G(\Phi(j))$, which is continuous and differentiable on the support.

Entrepreneur j solves the following maximization problem:

$$\max E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \ln c_s(j) \right],$$

subject to

$$i_s(j) + b_s(j) = (\rho_s A \Phi_{s-1} i_{s-1}(j) + r_s b_{s-1}(j))(1-\tau) - c_s(j) \quad \text{for } s \geq 1, \tag{1}$$

$$b_s(j) \geq -\lambda a_s(j), \tag{2}$$

$$i_s(j) \geq 0, \tag{3}$$

where $\beta \in (0,1)$ is the entrepreneur's subjective discount factor, $c_s(j)$ is her consumption in period s, $b_s(j)$ is a deposit if positive and a debt if negative, ρ_s is the capital price, r_s is the gross interest rate, and τ is a tax on the entrepreneur's income, which is constant over time. In inequality (2), $a_s(j) := (\rho_s A \Phi_{s-1} i_{s-1}(j) +$

$r_s b_{s-1}(j))(1-\tau) - c_s(j)$ for $s \geq 1$ is her saving (or the net worth remaining after she consumes in period s). In period $s = 0$, the flow budget constraint is given by $i_0(j) + b_0(j) = e_0(1-\tau) - c_0(j)$, where e_0 is the initial endowment of the entrepreneur at birth, which is common to all entrepreneurs. Inequality (2) is the credit constraint faced by entrepreneur j.[5] $\lambda \in (0, \infty)$ measures the extent of the credit constraint. Inequality (2) implies that an entrepreneur can borrow in the financial market only up to λ times her net worth. It follows from Eq. (1) that $a_s(j) = i_s(j) + b_s(j)$. Then, the credit constraint is rewritten as

$$b_s(j) \geq -\mu i_s(j), \tag{4}$$

where $\mu := \lambda/(1+\lambda) \in (0,1)$ also measures the extent of the credit constraint. Finally, inequality (3) is the nonnegativity constraint on investment.

2.2 Optimal behavior

Let us define

$$\phi_t := r_{t+1}/A\rho_{t+1}. \tag{5}$$

Then, it is optimal for entrepreneurs who are more productive (i.e., $\Phi_t(j) > \phi_t$) to produce capital goods by borrowing up to the limit of their credit constraints. However, it is optimal for entrepreneurs who are less productive (i.e., $\Phi_t(j) \leq \phi_t$) to deposit their net worth with the financial intermediary in order to obtain the gross interest rate, r_{t+1}, without engaging in capital goods production. Henceforth, the former are called capital producers (borrowers) and the latter are called lenders. The cutoff, ϕ_t, divides entrepreneurs into capital producers and lenders. Hence, after observing the idiosyncratic productivity shock, $\Phi_t(j)$, entrepreneur j with net worth

[5] This formulation of credit constraints is standard in the literature (e.g., Aghion et al., 1999; Aghion and Banerjee, 2005; Aghion et al., 2005; Antrás and Caballero, 2009).

$a_t(j)$ in period t plans to invest, borrow, or lend, as follows:

$$i_t(j) = \begin{cases} 0 & \text{if } \Phi_t(j) \leq \phi_t \\ \frac{a_t(j)}{1-\mu} & \text{if } \Phi_t(j) > \phi_t, \end{cases} \tag{6}$$

and

$$b_t(j) = \begin{cases} a_t(j) & \text{if } \Phi_t(j) \leq \phi_t \\ -\frac{\mu}{1-\mu} a_t(j) & \text{if } \Phi_t(j) > \phi_t. \end{cases} \tag{7}$$

By defining $R_s(j) := \max\{r_s, \frac{\rho_s A \Phi_{s-1}(j) - r_s \mu}{1-\mu}\}$, and from Eqs. (6) and (7), the flow budget constraint of entrepreneur j in period s is expressed as

$$a_s(j) = (1-\tau) R_s(j) a_{s-1}(j) - c_s(j). \tag{8}$$

Entrepreneur j solves the intertemporal maximization problem subject to Eq. (8). The Euler equation for all $t \geq 0$ is given by

$$\frac{1}{c_t(j)} = \beta(1-\tau) E_t \left[R_{t+1}(j) \frac{1}{c_{t+1}(j)} \right]. \tag{9}$$

Because the lifetime utility function is log-linear, it follows from Eqs. (8) and (9) and the transversality condition $\lim_{s\to\infty} \beta^s E_t \left[a_{t+s}(j)/c_{t+s}(j) \right] = 0$ that

$$a_{t+1}(j) = \beta(1-\tau) R_{t+1}(j) a_t(j). \tag{10}$$

2.3 Financial intermediary

As in Grandmont (1983) and Rochon and Polemarchakis (2006), we assume a representative financial intermediary. The financial sector is competitive and, thus, the representative financial intermediary does not earn a profit. The financial intermediary accepts deposits from lenders and lends funds to capital producers. The financial intermediary purchases an intrinsically useless asset using the excess total savings. Therefore, the intermediary's balance sheet is given by

$$L_t + B_t = D_t, \tag{11}$$

where L_t and D_t are aggregate loans and deposits, respectively. The nominal supply of the intrinsically useless asset is constant, and denoted by M. It follows that $p_t M = B_t$, where p_t is the price of the asset. Because this asset is freely disposable, B_t cannot be negative. When B_t is strictly greater than zero, a bubble on the asset occurs because a bubble is defined as the difference between the fundamental and market values of an asset. Because there is no opportunity for the financial intermediary to earn a profit, it follows that $p_t/p_{t-1} = r_t$ in equilibrium. As such, a dynamic equation with respect to B_t is obtained as

$$B_t = r_t B_{t-1}. \tag{12}$$

2.4 Final goods sector

To produce final goods, a firm hires a worker. However, workers and firms face search-matching frictions in the labor market. A firm that matches with a worker purchases capital goods as input. We denote such a firm as firm h. Firm h produces final goods, $Y_t(h)$, in period t. Its production technology is represented by $Y_t(h) = F(K_t(h), N_t(h))$, where $N_t(h)$ and $K_t(h)$ are the labor and capital employed by the firm, respectively. As previously assumed, capital depreciates entirely in one period. The production function is at least twice continuously differentiable, concave, homogeneous of degree one, and increasing with respect to both $K_t(h)$ and $N_t(h)$. It is assumed that $F(0, N_t(h)) = 0$ and $F(K_t(h), 0) = 0$. We define $f(k_t(h)) := F(K_t(h)/N_t(h), 1)$, where $k_t(h) := K_t(h)/N_t(h)$ is the capital–labor ratio of firm h and $f(k_t(h))$ satisfies $f(0) = 0$. Because a firm hires only one worker, it holds that $N_t(h) = 1$ and the capital–labor ratio is equal to the capital per firm. Because $f(k_t(h))$ is increasing and concave, it follows that $f'(k_t(h)) > 0 > f''(k_t(h))$.

Under perfect competition in the capital market, the marginal productivity of capital is equal to its price:

$$\rho_t = f'(k_t(h)).$$

Because every firm faces a common capital price, they employ the same amount of capital. Thus, we can drop the index h in the above equation, yielding

$$\rho_t = f'(k_t). \tag{13}$$

The remainder of the output allotted between a firm and its worker is given by

$$\pi_t := f(k_t) - f'(k_t)k_t. \tag{14}$$

For simplicity, we impose Assumption 1 in the following analysis.

Assumption 1 $[f'(k_t)k_t]' > 0$.

Assumption 1 holds when $F(K_t(h), N_t(h))$ is of the Cobb-Douglas class.

2.5 Workers

Each worker lives forever and is endowed with one unit of labor at the beginning of each period. The population of workers is equal to N. If a worker matches with a firm, she is hired by the firm and earns a wage income, w_t. Otherwise, she becomes a jobless person and receives the unemployment benefit γ_t from the government. The government taxes income at rate τ_t^w and the tax revenue covers the unemployment benefit. Thus, after-tax income is denoted by $\omega_t^\iota(1-\tau_t^w)$, where ι represents a worker's employment status and ω_t^ι is a worker's income: $\iota = e$ and $\omega_t^e = w_t$ if employed, and

$\iota = u$ and $\omega_t^u = \gamma_t$ if unemployed. The workers are hand-to-mouth consumers; that is, they consume their current income entirely. Thus, their consumption is

$$c_t^{w,\iota} = \omega_t^\iota (1 - \tau_t^w) \tag{15}$$

for all $t \geq 0$, where $c_t^{w,\iota}$ is a worker's consumption. A worker's expected lifetime utility is given by

$$\begin{aligned} U_t^{w,\iota} &= c_t^{w,\iota} + E_t \sum_{s=1}^{\infty} \left[\tilde{\beta}^s c_{t+s}^{w,\iota} \right] \\ &= \omega_t^\iota (1 - \tau_t^w) + E_t \sum_{s=1}^{\infty} \left[\tilde{\beta}^s c_{t+s}^{w,\iota} \right], \end{aligned} \tag{16}$$

where $\tilde{\beta} \in (0, \beta)$ is the worker's subjective discount factor.[6] We assume that the probability of a worker matching with a firm from period $t+1$ onward is independent of her current status ι.

2.6 Government

The government runs a balanced budget to provide unemployment benefits for workers:

$$\tau \int_{j \in \Omega} (\rho_t A \Phi_{t-1}(j) i_{t-1}(j) + r_t b_{t-1}(j)) dj + \tau_t^w \left(w_t (1 - u_t) + \gamma_t u_t \right) N = \gamma_t u_t N, \tag{17}$$

where u_t is the unemployment rate. The left-hand side of Eq. (17) is equal to the aggregate tax revenue and the right-hand side represents the total payments for unemployment benefits. The tax rate on entrepreneurs' income, τ, is assumed to be constant over time, and γ_t is assumed to be a constant fraction of wages, γw_t. Thus, the tax rate on workers' income, τ_t^ω, is determined endogenously to satisfy Eq. (17).

[6] If a worker's subjective discount factor is sufficiently small and she cannot borrow in the financial market, she behaves in a hand-to-mouth manner. King and Leape (1998) and Guiso et al. (2003) provide empirical evidence supporting the existence of hand-to-mouth consumers.

2.7 Labor market

We introduce labor-market matching frictions into our model. That is, we assume the existence of search costs and a matching function between firms and workers. A firm that matches with a worker begins operating without any time lags.

2.7.1 Matching mechanism

Each worker is endowed with one unit of labor in each period. Because firms must pay a fixed cost to search for a worker and workers and firms face matching frictions, unemployment occurs in equilibrium. The match between a worker and a firm is assumed to be broken in one period. Because of free entry into the labor market, each firm is indifferent between entry and no-entry into the labor market and the equilibrium entry rate of firms is determined by macroeconomic conditions. In contrast, all workers search in every period because they can enter the labor market without incurring a cost. We denote the entry rate of firms by υ_t. Thus, we refer to υ_t as the number of firms with vacancies in the labor market. The number of matches is a function of the population of workers, N, and the number of firms with vacancies, υ_t, which is given by $m(N, \upsilon_t)$. We assume that $0 \leq m(N, \upsilon_t) \leq \min\{N, \upsilon_t\}$, for $N \in [0, \infty)$ and $\upsilon_t \in [0, \infty)$, $m(0, \upsilon_t) = 0$, and $m(N, 0) = 0$. The matching function, $m(N, \upsilon_t)$, is continuously differentiable, concave, homogeneous of degree one, and increasing with respect to both N and υ_t. The tightness of the labor market is measured by $\theta_t := \upsilon_t/N \in (0, \infty)$, which is the jobs-to-applicants ratio. The probability that a firm with a vacancy matches with a worker is given by $m(N, \upsilon_t)/\upsilon_t = m(1/\theta_t, 1) =: q(\theta_t)$. $q(\theta_t)$ is continuously differentiable in $(0, \infty)$, where $q'(\theta_t) < 0$, for $\theta_t \in (0, \infty)$, $\lim_{\theta_t \to 0} q(\theta_t) = 1$, and $\lim_{\theta_t \to \infty} q(\theta_t) = 0$. The number of employed workers is equal to the number of matches. Thus, it follows that $(1 - u_t)N = m(N, \upsilon_t)$ or, equivalently,

$$1 - u_t = \theta_t q(\theta_t). \tag{18}$$

Eq. (18) derives the so-called the Beveridge curve, which represents a negative relationship between the unemployment rate and the labor market tightness. More specifically, from Eq. (18), we obtain the unemployment rate, u_t, as a decreasing function of θ_t, $u_t = u(\theta_t)$. It holds that $u'(\theta_t) < 0$ because $\partial[\theta_t q(\theta_t)]/\partial\theta_t = \partial m(1, \theta_t)/\partial\theta_t > 0$.

A firm that matches with a worker produces final goods. The value of a firm that produces final goods, J_t^e, and the value of a firm that does not match with a worker, J_t^u, are given by

$$J_t^e = \pi_t - w_t - \zeta + \frac{1}{\rho_{t+1}} \left[q(\theta_{t+1}) J_{t+1}^e + (1 - q(\theta_{t+1})) J_{t+1}^u \right]$$

and

$$J_t^u = -\zeta + \frac{1}{\rho_{t+1}} \left[q(\theta_{t+1}) J_{t+1}^e + (1 - q(\theta_{t+1})) J_{t+1}^u \right],$$

respectively, where ζ is the search cost that the firm incurs when searching for a worker in the labor market.[7] If the actual revenue $\pi_t - w_t$ is less than ζ, no firms operate. We proceed with our investigation assuming the nontrivial case in which $\pi_t - w_t \geq \zeta$, unless otherwise stated.[8] We can write a firm's expected entry value, Q_t, in the labor market as follows:

$$\begin{aligned} Q_t &= q(\theta_t) J_t^e + (1 - q(\theta_t)) J_t^u, \\ &= q(\theta_t)(\pi_t - w_t) - \zeta + \frac{1}{\rho_{t+1}} Q_{t+1}. \end{aligned} \tag{19}$$

The free-entry condition for the labor market leads to zero profit for each firm,

[7] Since the capital depreciation rate is 1, the net rental rate of capital is equal to $\rho_{t+1} - 1$ in period $t + 1$. Therefore, a firm's expected entry value is discounted by ρ_{t+1}. We assume $\rho_{t+1} > 1$ for all $t \geq 0$.

[8] Each firm incurs a search cost because of recruitment activities such as job interviews and evaluations of reference letters. The firm's operating resources cover the search cost, which is an implicit opportunity cost.

that is, $Q_t = 0$ for any t or, equivalently,

$$\pi_t - w_t = \frac{\zeta}{q(\theta_t)}. \tag{20}$$

2.7.2 Nash bargaining

A firm and its worker divide π_t, which is equal to the output minus the capital payments. according to Nash bargaining: that is, the shares for the firm and the worker are obtained from the following maximization problem:

$$\begin{aligned} w_t &= \arg\max_{w_t} (U_t^{w,e} - U_t^{w,u})^{\epsilon} (J_t^e - J_t^u)^{1-\epsilon} \\ &= \arg\max_{w_t} (1 - \tau_t^w)^{\epsilon} (w_t - \gamma_t)^{\epsilon} (\pi_t - w_t)^{1-\epsilon}, \end{aligned}$$

where $\epsilon \in (0, 1)$ is the worker's bargaining power. When bargaining they solve this problem by viewing γ_t as exogenous because they have no information how γ_t is formed. The Nash bargaining solution is given by

$$w_t = (1 - \epsilon)\gamma_t + \epsilon\pi_t. \tag{21}$$

As stated previously, unemployment benefits are paid to unemployed workers in such a way that $\gamma_t = \gamma w_t$, where $\gamma \in [0, 1)$. Substituting $\gamma_t = \gamma w_t$ into Eq. (21), we obtain

$$w_t = \Theta\pi_t, \tag{22}$$

where $\Theta := \epsilon/\{1 - (1 - \epsilon)\gamma\} \in (0, 1)$ is the worker's output share of π_t. Note that a larger outside option, γw_t, and greater Nash bargaining power, ϵ, lead to a greater share for the worker, Θ. Substituting Eqs. (14) and (22) into Eq. (20) yields

$$(1 - \Theta)(f(k_t) - f'(k_t)k_t) = \frac{\zeta}{q(\theta_t)}. \tag{23}$$

This equation determines the equilibrium value of θ_t, the entry rate of firms into the labor market, υ_t, and the unemployment rate, $u_t = u(\theta_t)$.

Define $\bar{k}$ such that $f(\bar{k}) - f'(\bar{k})\bar{k} = \zeta/(1-\Theta)$. Then, it is easy to show from Eq. (23) that for the economy to be feasible, it must hold that $k_t > \bar{k}$ for all $t \geq 0$. Otherwise, no firms can cover the search cost, ζ, and hire a worker. In this case, the economy becomes infeasible and does not produce final goods. In what follows, we focus on the case in which $k_t > \bar{k}$, for all $t \geq 0$.

2.8 Aggregation

We can aggregate variables in the same manner as in Kunieda and Shibata (2016), in which the i.i.d. assumption simplifies the aggregation. The before-tax aggregate income over all entrepreneurs is equal to the total capital income plus the total income from the intrinsically useless asset. Because each entrepreneur's marginal propensity to save is β, the aggregate saving by entrepreneurs is given by

$$\int_{j\in\Omega} a_t(j)dj = \beta(1-\tau)(\rho_t Z_t + r_t B_{t-1}), \tag{24}$$

where $Z_t := \int_{j\in\Omega} A\Phi_{t-1}i_{t-1}(j)dj$ is the total of capital goods produced by high-productivity entrepreneurs. Because the number of firms that match with a worker in period t is $(1-u_t)N$, the capital goods market clearing condition is given as follows:

$$Z_t = k_t(1-u_t)N. \tag{25}$$

In period t, an entrepreneur becomes a lender with probability $G(\phi_t)$ and a capital producer (borrower) with probability $1-G(\phi_t)$. The i.i.d. assumption allows us to apply the law of large numbers to our economy. Thus, the population of lenders is equal to $G(\phi_t)$ in period t and the population of capital producers is equal to $1-G(\phi_t)$. Then, Eqs. (7) and (24) yield $D_t = \beta(1-\tau)(\rho_t Z_t + r_t B_{t-1})G(\phi_t)$ and

$L_t = \beta(1-\tau)(\rho_t Z_t + r_t B_{t-1})\mu[1-G(\phi_t)]/(1-\mu)$. From Eq. (11), the total value of the intrinsically useless asset is obtained as

$$B_t = \beta(1-\tau)(\rho_t Z_t + r_t B_{t-1})\frac{G(\phi_t)-\mu}{1-\mu}. \tag{26}$$

Eq. (26) shows that the intrinsically useless asset has a positive value if $G(\phi_t) > \mu$. In what follows, we focus exclusively on the case in which $G(\phi_t) \geq \mu$, such that B_t is nonnegative. It follows from Eqs. (6) and (24) that the total of capital goods, Z_{t+1}, is given by

$$Z_{t+1} = \frac{\beta(1-\tau)AH(\phi_t)}{1-\mu}(\rho_t Z_t + r_t B_{t-1}), \tag{27}$$

where $H(\phi_t) := \int_{\phi_t}^{\eta} \Phi_t(j)dG(\Phi)$.

2.9 Dynamical system

The dynamical system of our economy is obtained from Eqs. (5), (12), (13), (26), and (27). From these equations, we can derive the dynamical equations of the cutoff, ϕ_t, and the intrinsically useless asset, as follows:

$$\frac{G(\phi_t)-\mu}{1-\mu-\beta(1-\tau)(G(\phi_t)-\mu)} = \frac{\phi_{t-1}(G(\phi_{t-1})-\mu)}{\beta(1-\tau)H(\phi_{t-1})}, \tag{28}$$

and

$$B_t = A\phi_{t-1}f'(k_t)B_{t-1}, \tag{29}$$

respectively.

From Eqs. (18) and (23), we have

$$1-u_t = \frac{\zeta}{(1-\Theta)(f(k_t)-f'(k_t)k_t)}q^{-1}\left(\frac{\zeta}{(1-\Theta)(f(k_t)-f'(k_t)k_t)}\right) := \Psi(k_t). \tag{30}$$

Using Eqs. (12), (13), (26), (27), and (30), we obtain the dynamical equation of capital, as follows:

$$k_{t+1}\Psi(k_{t+1}) = \frac{\beta(1-\tau)H(\phi_t)}{1-\mu-\beta(1-\tau)(G(\phi_t)-\mu)}Af'(k_t)k_t\Psi(k_t). \tag{31}$$

Using Eqs. (12), (13), (25), and (31), Eq. (26) is rewritten as

$$B_t = \frac{\beta(1-\tau)(G(\phi_t)-\mu)}{1-\mu-\beta(1-\tau)(G(\phi_t)-\mu)}f'(k_t)k_t\Psi(k_t)N. \tag{32}$$

3 Steady states and stability

As stated previously, we focus on the case in which $G(\phi_t) \geq \mu$ and restrict the domain of the dynamical equation, Eq. (28), to $[G^{-1}(\mu), \eta)$. Whereas Eq. (28) is solely an autonomous difference equation with respect to ϕ_t, we consider an autonomous dynamical system with respect to ϕ_t and k_t that consists of Eqs. (28) and (31).

Proposition 1 *Consider the dynamical system of Eq. (28) and (31). Then, the following hold.*

(i) There exist two steady states (k^*, ϕ^*) and (k^{**}, ϕ^{**}), where $\phi^* > \phi^{**}$, such that

$$1-\mu-\beta(1-\tau)(G(\phi^*)-\mu) = \beta(1-\tau)H(\phi^*)/\phi^*, \tag{33}$$

$$1 = \phi^* Af'(k^*), \tag{34}$$

$$G(\phi^{**}) = \mu, \tag{35}$$

and

$$1 = \frac{\beta(1-\tau)H(\phi^{**})}{1-\mu}Af'(k^{**}), \tag{36}$$

if and only if $(1-\mu)\phi^{**} < \beta(1-\tau)H(\phi^{**})$. Moreover, the intrinsically useless asset has a positive value in the steady state with ϕ^*. Thus, asset bubbles exist in this steady state.

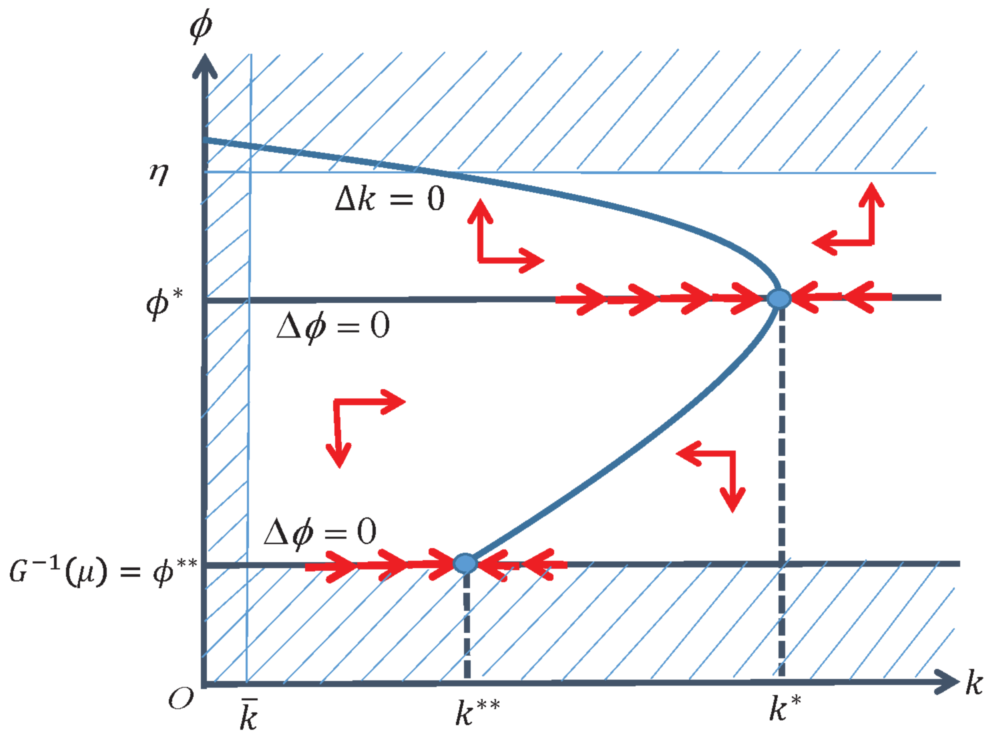

Figure1: Phase diagram. Asset bubbles promote capital accumulation. The shadow area is not a domain of the dynamical system.

(ii) *There exists only one steady state,* (k^{**}, ϕ^{**})*, given by Eqs. (35) and (36), if and only if* $(1-\mu)\phi^{**} \geq \beta(1-\tau)H(\phi^{**})$*. Moreover, the intrinsically useless asset has no value in this steady state.*

Proof: See Appendix A.1.

We call the steady state of (k^*, ϕ^*) a bubbly steady state and the steady state of (k^{**}, ϕ^{**}) a bubbleless steady state.

Proposition 2 *Suppose that the bubbly steady state,* (k^*, ϕ^*)*, exists in our economy (the first case of Proposition 1). Then, the bubbly steady state* (k^*, ϕ^*) *is a saddle point and the bubbleless steady state* (k^{**}, ϕ^{**}) *is totally stable.*

Proof: See Appendix A.2.

Figure 1 depicts the phase diagram of the economy. Because ϕ_0 is a nonpredetermined variable and the bubbly steady state is a saddle point, the bubbly steady state is locally determinate. However, the bubbleless steady state is totally stable. Thus, any sequence of $\{k_{t+1}, \phi_t\}_{t=0}^{\infty}$ with $(k_1, \phi_0) \in (\bar{k}, \infty) \times (0, \phi^*)$ converging to (k^{**}, ϕ^{**}) is an equilibrium. Therefore, the equilibrium is globally indeterminate. Note that any sequence of $\{k_{t+1}, \phi_t\}_{t=0}^{\infty}$ with $(k_1, \phi_0) \in (\bar{k}, \infty) \times (\phi^*, \eta]$ cannot be an equilibrium because ϕ_t becomes greater than η or k_t becomes less than $\bar{k}$ in finite time.

4 Macroeconomic variables in the bubbly and bubbleless steady states

In general equilibrium models, the presence of asset bubbles can change the equilibrium allocation of resources and affect macroeconomic variables. In this section, we analytically investigate the effects of bubbles on capital accumulation, unemployment, and aggregate consumption by comparing the bubbly and bubbleless steady states.

4.1 Capital accumulation

We first investigate the long-run effect of bubbles on capital accumulation. Here, the following lemma is useful.

Lemma 1 *Suppose that the bubbly steady state, (k^*, ϕ^*), exists in the dynamical system consisting of Eqs. (28) and (31) (the first case of Proposition 1). Define the following function:*

$$S(\phi) := \frac{\beta(1-\tau)H(\phi)}{1-\mu-\beta(1-\tau)(G(\phi)-\mu)},$$

the domain of which is $[0, \eta]$. Then, $S(\phi)$ is maximized at $\phi = \phi^*$, and thus $S(\phi^*) > S(\phi^{**})$.

Proof: See Appendix B.

From this lemma, we have Proposition 3.

Proposition 3 *Suppose that the bubbly steady state, (k^*, ϕ^*), exists. Then, more capital accumulates in the bubbly steady state than it does in the bubbleless steady state.*

Proof: From Lemma 1 and Eqs. (34) and (36), we have $S(\phi^*) > S(\phi^{**}) \Longleftrightarrow \phi^* > \beta(1-\tau)H(\phi^{**})/(1-\mu) \Longleftrightarrow f'(k^*) < f'(k^{**}) \Longleftrightarrow k^* > k^{**}$. □

Although the presence of bubbles decreases the aggregate investment, the equilibrium interest rate increases. As a result, less productive entrepreneurs are ruled out of capital production activities and the aggregate productivity in capital production becomes higher. Then, the aggregate capital produced by entrepreneurs and, thus, the capital per worker become greater in the bubbly steady state than those in the bubbleless steady state.

4.2 Unemployment

We next consider the effect of bubbles on unemployment. From Eq. (23), we have $\partial\theta/\partial k > 0$, and from Eq. (18), we have $\partial u/\partial\theta < 0$. Therefore, an increase in capital reduces the rate of unemployment. Proposition 4 summarizes these results.

Proposition 4 *Suppose that the bubbly steady state, (k^*, ϕ^*), exists. The unemployment rate in the bubbly stead state, u^*, is lower than that in the bubbleless steady state, u^{**}.*

Intuitively, as more capital accumulates, the profits obtained by firms that have matched with a worker become higher. Thus, more firms enter the labor market. As a result, the unemployment rate decreases (and the tightness of the labor market increases) as shown in Figure 2.

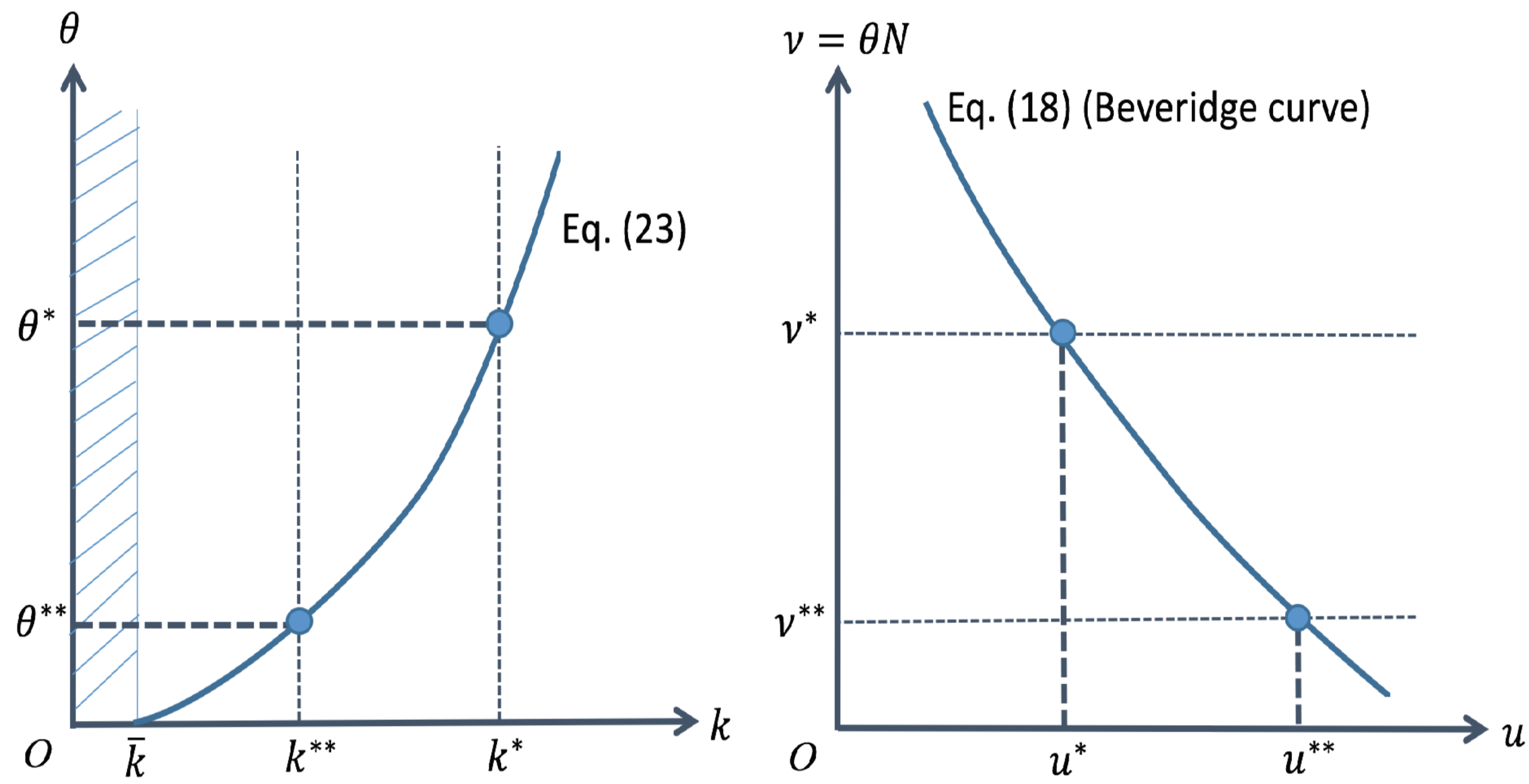

Figure 2: the Job creation condition and the Beveride curve.

4.3 Aggregate consumption

This subsection demonstrates that the aggregate consumption of entrepreneurs and that of workers in the bubbly steady state are both greater than those in the bubbleless steady state. The following lemma derives the aggregate consumption of entrepreneurs and that of workers in the two steady states.

Lemma 2 *Suppose that the bubbly steady state,* (k^*, ϕ^*)*, exists. Then, the following hold:*

(i) The aggregate consumption of entrepreneurs in the bubbly and bubbleless steady states is given by

$$C^{p,*} = (1-\beta)(1-\tau)\Gamma(\phi^*)f'(k^*)k^*\Psi(k^*)N \tag{37}$$

and

$$C^{p,**} = (1-\beta)(1-\tau)\Gamma(\phi^{**})f'(k^{**})k^{**}\Psi(k^{**})N, \tag{38}$$

respectively, where $\Gamma(\phi) = (1-\mu)/[1-\mu-\beta(1-\tau)(G(\phi)-\mu)]$.

(ii) The aggregate consumption of workers in the bubbly and bubbleless steady states is given by

$$C^{w,*} = [\Theta(f(k^*) - f'(k^*)k^*) + \tau\Gamma(\phi^*)f'(k^*)k^*]\,\Psi(k^*)N, \tag{39}$$

and

$$C^{w,**} = [\Theta(f(k^{**}) - f'(k^{**})k^{**}) + \tau\Gamma(\phi^{**})f'(k^{**})k^{**}]\,\Psi(k^{**})N, \tag{40}$$

respectively.

Proof: See Appendix C.

Using this lemma, we can prove the following proposition.

Proposition 5 *Suppose that the bubbly steady state,* (k^*, ϕ^*)*, exists. Then, the aggregate consumption of entrepreneurs in the bubbly steady state,* $C^{p,*}$*, is greater than that in the bubbleless steady state,* $C^{p,**}$*.*

Proof: Because $\phi^* > \phi^{**}$ and $\Gamma(\phi)$ is an increasing function with respect to ϕ, it holds that $\Gamma(\phi^*) > \Gamma(\phi^{**})$. From the definition of $\Psi(k)$ and Proposition 4, it follows that $\Psi(k^*) > \Psi(k^{**})$. From Assumption 1, we have $f'(k^*)k^* > f'(k^{**})k^{**}$. Then, from Eqs. (37) and (38), the desired conclusion is obtained. □

In Eqs. (37) and (38), the presence of bubbles has two positive effects on the aggregate consumption of entrepreneurs. The first effect is produced by an increase in the aggregate capital income, that is, $f'(k^*)k^*\Psi(k^*)N > f'(k^{**})k^{**}\Psi(k^{**})N$. The second effect is the wealth effect of the bubbles, which is reflected by $\Gamma(\phi^*) > \Gamma(\phi^{**}) = 1$. The wealth effect increases entrepreneurs' consumption.

Proposition 6 *Suppose that the bubbly steady state,* (k^*, ϕ^*)*, exists. The aggregate consumption of workers in the bubbly steady state,* $C^{w,*}$*, is greater than that in the bubbleless steady state,* $C^{w,**}$*.*

Proof: $\Theta(f(k) - f'(k))$ is an increasing function with respect to k. Thus, $f(k^*) - f'(k^*)k^* > f(k^{**}) - f'(k^{**})k^{**}$. As in Proposition 5, it follows that $\Gamma(\phi^*)f'(k^*)k^*\Psi(k^*)N > \Gamma(\phi^{**})f'(k^{**})k^{**}\Psi(k^{**})N$. Then, from Eqs. (39) and (40), the desired conclusion is obtained. □

In Eqs. (39) and (40), the first term, $\Theta(f(k) - f'(k)k)\Psi(k^{**})N$, is the aggregate wage income in which $\Psi(k)N$ reflects the job-matching effect from Proposition 4. The second term, $\tau\Gamma(\phi)f'(k)k\Psi(k)N$, represents the effect of the redistribution policy from entrepreneurs to workers through the unemployment benefits (see Eq. (17)). These effects mean that workers can receive more in unemployment benefits in the bubbly steady state than they can in the bubbleless steady state.

5 Numerical analysis

In this section, we numerically analyze the effects of changes in the credit constraints, labor market conditions, and redistribution policy on the macroeconomic variables.

5.1 Specification and parameterization

Following Den Haan et al. (2000), we specify the matching function as $m(N, v_t) = N v_t (N^\sigma + v_t^\sigma)^{-1/\sigma}$.[9] The individual-specific productivity, Φ, is uniformly distributed in $[0, 1]$. For the final good production, the Cobb–Douglas production function is assumed, that is, $F(K_t, N) = \bar{A} K_t^\alpha N^{1-\alpha}$, where $\bar{A}$ is the productivity parameter. Then, the production function per worker is given by $f(k_t) = \bar{A} k_t^\alpha$. Under these specifications, we obtain macroeconomic variables such as the capital accumulation, jobs-to-applicants ratio (tightness of the labor market), unemployment rate, and aggregate consumption of entrepreneurs and of workers in the bubbly and bubbleless steady states. We also compute the expected lifetime utility of an entrepreneur and that of a worker in the two steady states. See Appendices D and E for the derivations.

We set $\alpha = 0.33$, $\beta = 0.98$, and $\tilde{\beta} = 0.8$. Note that the workers are more impatient than entrepreneurs and are hand-to-mouth consumers, as explained earlier. We normalize the population of entrepreneurs to one and that of the workers to $N = 100$. Under this parameter setting, we examine the effects of changes in the credit constraints, μ, labor market conditions (including the worker's bargaining power), ϵ, search cost, ζ, unemployment benefit, γ, and tax rate on the entrepreneur's income, τ.

If ϵ and γ are close to one, Θ is also close to one and the economy becomes infeasible because production never occurs. Thus, we must impose a ceiling on ϵ and γ. According to the Organisation for Economic Co-operation and Development (OECD, 2006), unemployment benefits in the 26 OECD countries in 2004 ranged

[9]This matching function satisfies the conditions imposed in Section 2.

between 45% and 83% of net earnings. Thus, we vary γ from 0.65 to 0.8, which is a subset of $[0.45, 0.83]$, in order to examine the effect of γ. Because Den Haan et al. (2000) and Shimer (2005) set $\epsilon = 0.5$ and 0.72, respectively, we vary ϵ from 0.5 to 0.7. As in the case of ϵ and γ, if the search cost, ζ, is high, the economy is infeasible. Therefore, we use a relatively low value of ζ, ranging between 0.12 and 0.14. We set the remaining parameter values as $A = 1$, $\bar{A} = 2.5$, and $\sigma = 4$ to produce plausible values for the tightness of the labor market, θ, and the unemployment rate, u. We fix $\tau = 0.01$, $\epsilon = 0.6$, $\gamma = 0.7$, and $\zeta = 0.12$ when varying one of the parameter values. Table 1 summarizes our parameter settings.

Table 1: Parameterization

$\alpha = 0.33$	$\beta = 0.98$	$\tilde{\beta} = 0.8$	$A = 1$
$\bar{A} = 2.5$	$\sigma = 4$	$\gamma = 0.7$	$\epsilon = 0.6$
$\zeta = 0.12$	$\tau = 0.01$		

5.2 Credit constraints

5.2.1 Effects on macroeconomic variables

Figure 3 shows the effects of μ on the macroeconomic variables in the bubbly and bubbleless steady states. The dotted vertical line in each panel shows $\mu = 0.94$, beyond which bubbles cannot exist. As shown in the panel of bubbles, we can compute the value of the bubbly assets, B, for $\mu > 0.94$. However, it becomes negative, which is ruled out by the free disposability of the asset.

Before discussing the effects of μ, we numerically confirm the outcomes obtained in Propositions 3–6. Figure 3 indicates that the capital per worker in the bubbly steady state, k^*, is always higher than that in the bubbleless steady state, k^{**}. This is because the presence of bubbles always promotes capital accumulation (Proposition 3). Moreover, the jobs-to-applications ratio in the bubbly steady state, θ^*, is higher

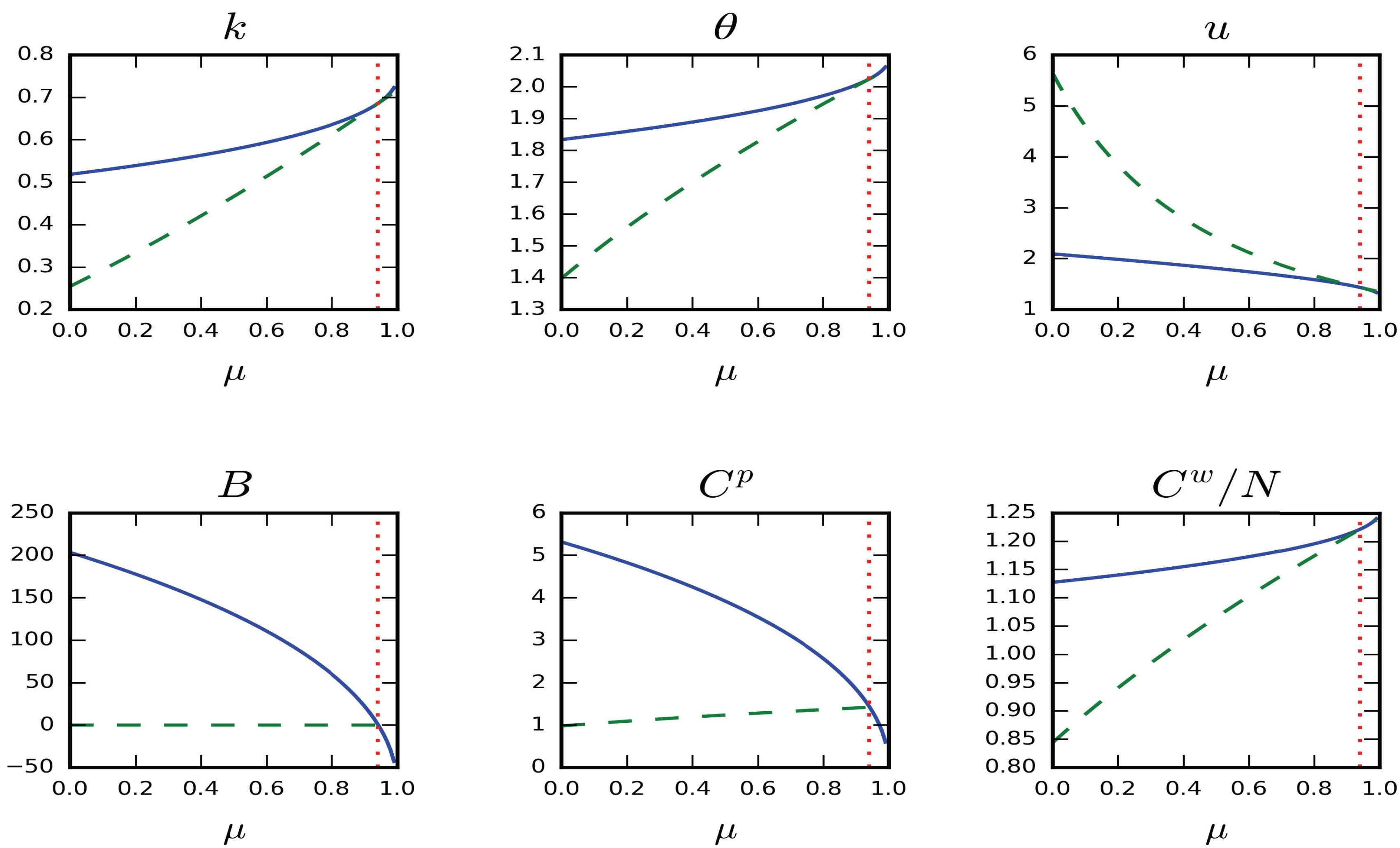

Figure 3: Effects of credit constraints.

than that in the bubbly steady state, θ^{**}. Thus, the unemployment rate in the bubbly steady state, u^{*}, is lower than that in the bubbleless steady state, u^{**} (Proposition 4). The aggregate consumption of entrepreneurs and that of workers in the bubbly steady state, $C^{p,*}$ and $C^{w,*}$, respectively, are higher than those in the bubbleless steady state, $C^{p,**}$ and $C^{w,**}$, respectively (Propositions 5 and 6).

As shown in Figure 3, relaxing the credit constraints promotes capital accumulation in both steady states. However, the marginal effect of μ on the capital per worker in the bubbly steady state is always smaller than that in the bubbleless steady state. Furthermore, the capital per worker in the bubbly steady state is equal to that in the bubbleless steady state at $\mu = 0.94$. The presence of asset bubbles (partially) corrects allocative inefficiency and promotes capital accumulation, reallocating production resources from lower-productivity entrepreneurs to higher-productivity entrepreneurs. However, as μ increases, the value of the bubbly assets, B, decreases. Thus, the allocative-efficiency effect of the asset bubbles shrinks. Therefore, the marginal effect of μ on the capital per worker is always smaller in the bubbly steady state than it is in the bubbleless steady state. Eventually, the capital per worker in the bubbly steady state coincides with that in the bubbleless steady state at $\mu = 0.94$.

The graphs of the jobs-to-applicants ratio, θ, in the two steady states are similar to those of the capital per worker, k. This is because the jobs-to-applicants ratio has a one-to-one positive relationship with the capital per worker. As μ increases, the jobs-to-applicants ratio, θ, increases, and accordingly, the unemployment rate decreases in both steady states. Because more capital accumulates in both steady states as μ increases, the profits of firms that match with a worker increase. As a result, more firms enter the labor market. Thus, the jobs-to-applicants ratio, θ, increases and the unemployment rate, u, decreases in both steady states.

As in the case of the jobs-to-applicants ratio, the graphs of per worker consumption, C^{w}/N, in both steady states are similar to those of the capital per worker, k.

This is again because the per worker consumption has a one-to-one positive relationship with the capital per worker. On the other hand, as μ increases, the aggregate consumption of entrepreneurs in the bubbly steady state, $C^{p,*}$, decreases, whereas that in the bubbleless steady state, $C^{p,**}$, increases. As noted previously, when the credit constraints are severe, the value of the bubbly assets is high. In this case, the aggregate net worth that the entrepreneurs hold is high. Because the marginal propensity to consume is constant over time, the larger aggregate net worth increases the aggregate consumption. However, as the credit constraint is relaxed, the wealth effect of the bubbles shrinks and the aggregate consumption of entrepreneurs in the bubbly steady state decreases. Thus, the decreasing trend of $C^{p,*}$ is produced in Figure 3. The wealth effect is indicated by the fact that the graph of C^p in the bubbly steady state has a similar feature to that of B.

5.2.2 Welfare effect

Here, we compare the welfare effects of changes in the credit constraints in the bubbly and bubbleless steady states. Suppose that the economy is in one of the steady states. Then, as shown in Appendix E, the expected lifetime utility of an entrepreneur with income, I_t, is given by

$$\begin{aligned} V(I_t) := & \frac{1}{1-\beta}\ln I_t + \frac{\ln\left[(1-\beta)^{1-\beta}\beta^{\beta}\right]}{(1-\beta)^2} + \frac{\ln(1-\tau)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left[(1-\mu\phi)\ln\left(\frac{1-\mu\phi}{1-\mu}\right)\right] \\ + & \frac{\beta}{(1-\beta)^2}\left[\ln(\rho A) + \mu\phi\ln\phi - (1-\phi)\right], \end{aligned} \tag{41}$$

where $\phi = \phi^*$ or ϕ^{**} and ρ is the steady-state value of ρ_t.

Figure 4 provides the expected lifetime utilities of ex-ante homogeneous entrepreneurs in period zero in the bubbly and the bubbleless steady states. We set the entrepreneur's initial endowment at $I_0 = e_0 = 20$. As seen in the northwest panel, the lifetime utility in the bubbly steady state is higher than that in the bubbleless

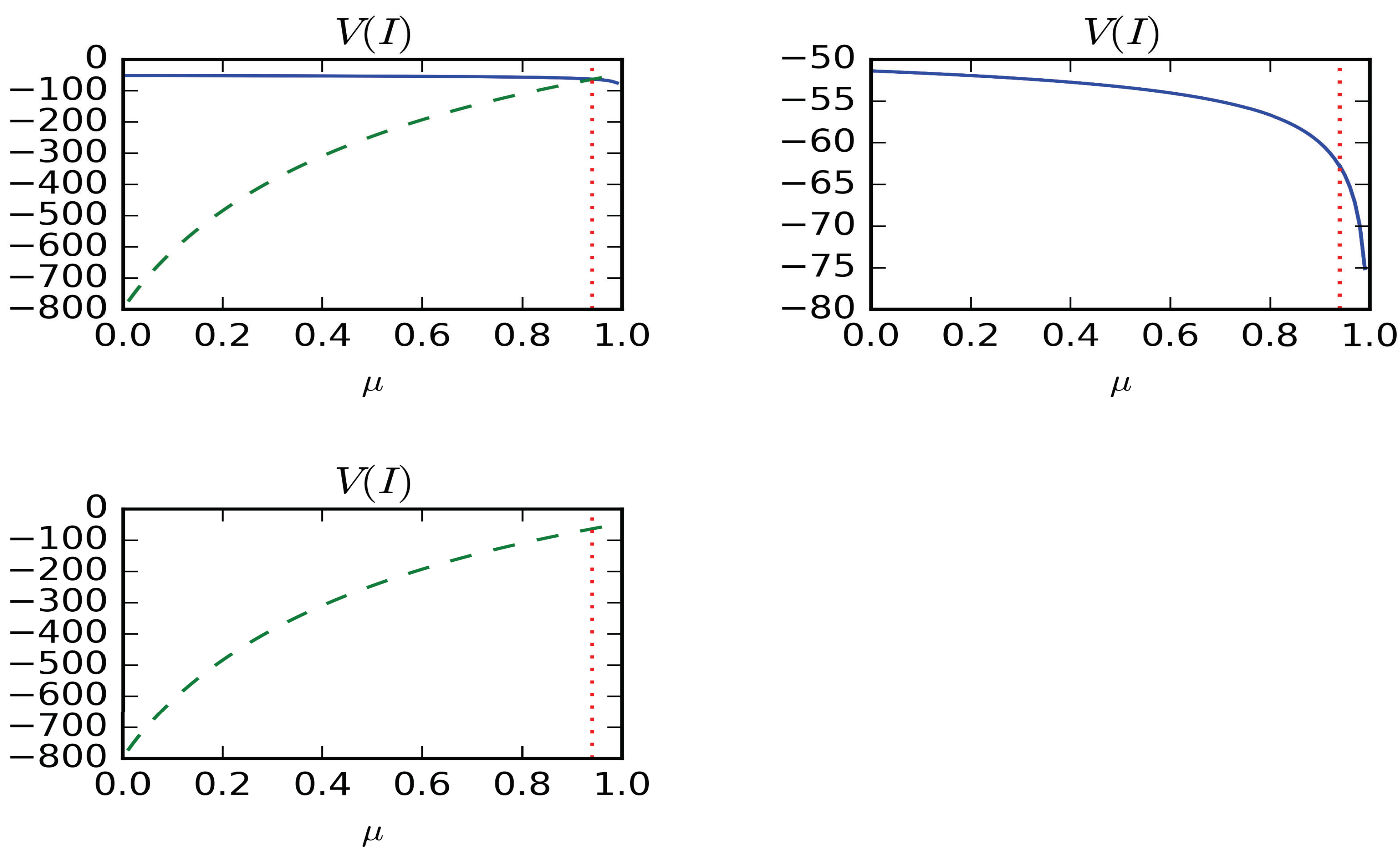

Figure 4: Effects of the credit constraints on the entrepreneur's welfare.

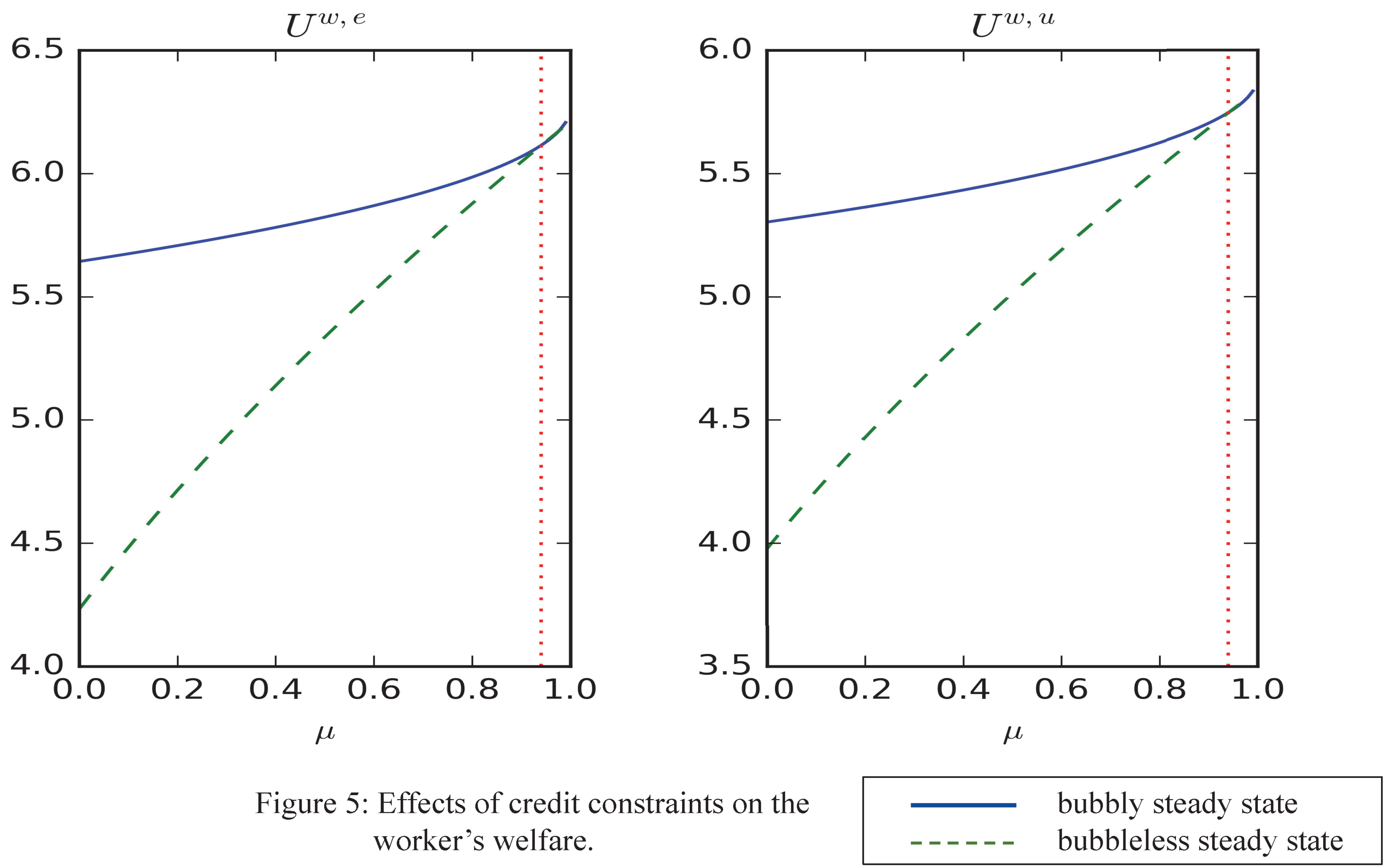

Figure 5: Effects of credit constraints on the worker's welfare.

steady state. In order to see the detail of the effect of the credit constraint on welfare in the bubbly steady state, we depict an enlarged graph of the lifetime utility in this steady state in the northeast panel (the lifetime utility in the bubbleless steady state is depicted in the southwest panel). As shown in the northeast panel, the lifetime utility in the bubbly steady state decreases monotonically as μ increases from 0 to 0.94. This is because as μ increases, the wealth effect of the bubbles on the entrepreneurs' consumption shrinks, as discussed in the previous section. In contrast, the lifetime utility in the bubbleless steady state increases, as in the case of aggregate consumption.

Next, we consider the expected lifetime utilities of both employed and unemployed workers in the bubbly and bubbleless steady states. As shown in Appendix E, the employed and unemployed workers' expected utility functions are given by

$$U^{w,e} := \left[1 + \frac{\tilde{\beta}}{1-\tilde{\beta}}\theta q(\theta) + \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\theta q(\theta))\gamma\right] w(1-\tau^w),$$

and

$$U^{w,u} := \left[\gamma + \frac{\tilde{\beta}}{1-\tilde{\beta}}\theta q(\theta) + \frac{\tilde{\beta}}{1-\tilde{\beta}}(1-\theta q(\theta))\gamma\right] w(1-\tau^w),$$

respectively, where $w = \Theta(1-\alpha)\bar{A}k^{\alpha}$, $\Theta := \epsilon/\{1-(1-\epsilon)\gamma\}$, and $\tau^w = [\gamma(1-\Psi(k)) - \alpha\tau\Gamma(\phi)\Psi(k)/(\Theta(1-\alpha))]/[(1-\gamma)\Psi(k)+\gamma]$. Figure 5 shows the lifetime utilities of employed and unemployed workers in the bubbly and bubbleless steady states. The lifetime utility in the bubbly steady state is always higher than that in the bubbleless steady state for all workers, and the lifetime utilities in both steady states increase monotonically as μ increases. Note that the effects of an increase in μ on the lifetime utilities of employed and unemployed workers (Figure 5) are similar to those on the capital per worker (Figure 3). This is because the lifetime utilities of employed and unemployed workers have a positive one-to-one relationship with the capital per worker, as in the case of per worker consumption.

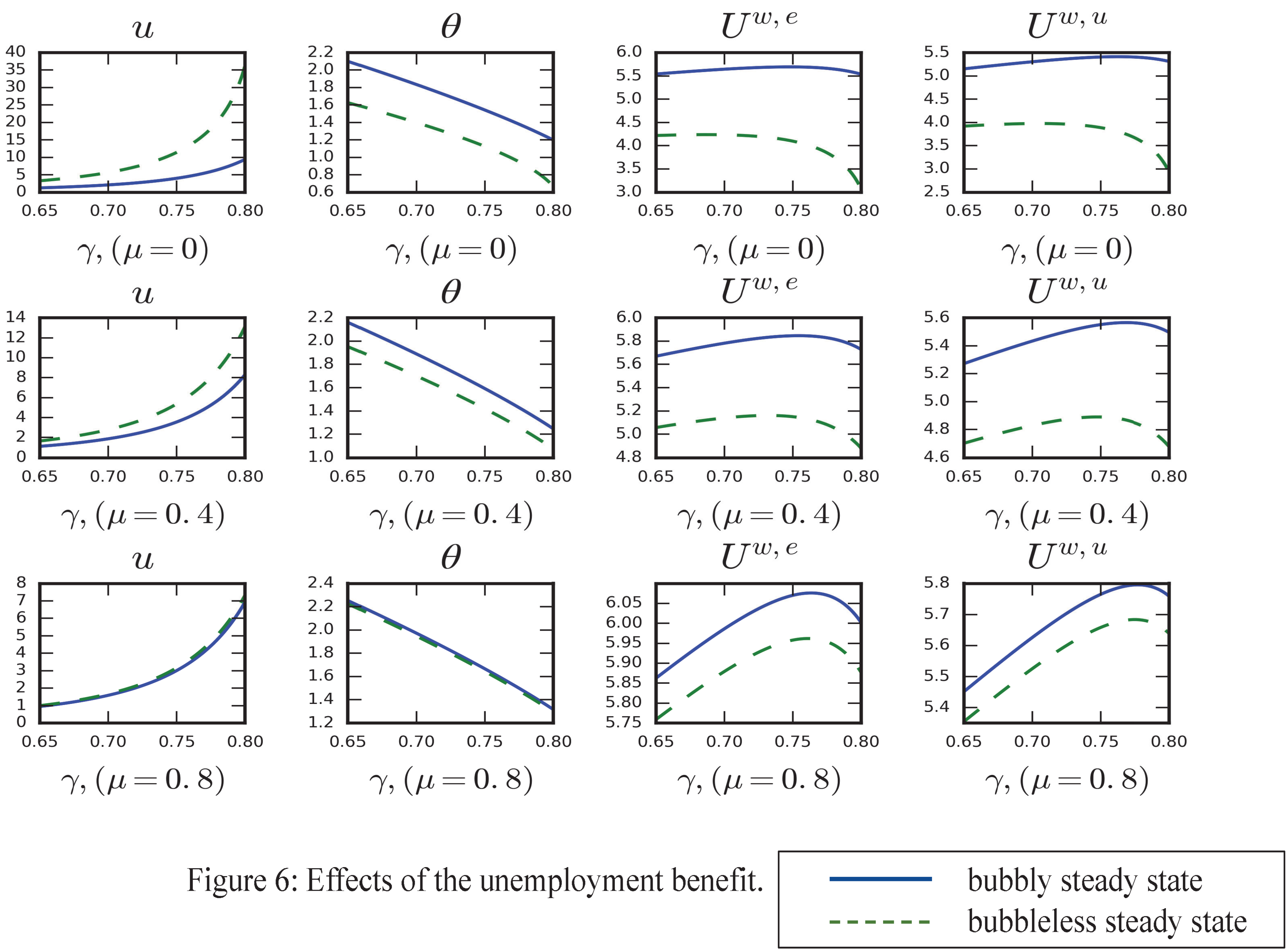

Figure 6: Effects of the unemployment benefit.

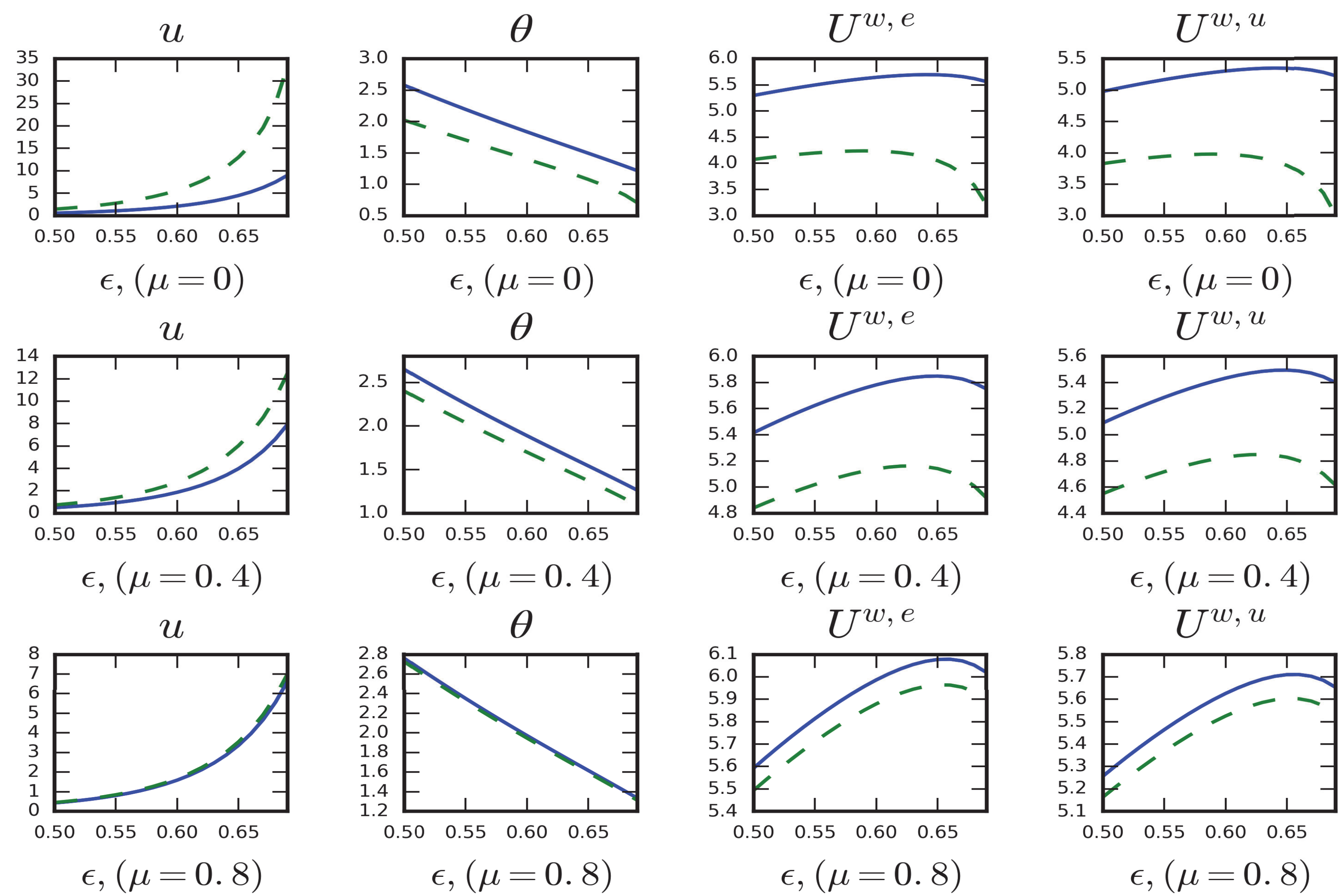

Figure 7: Effects of the worker's bargaining power.

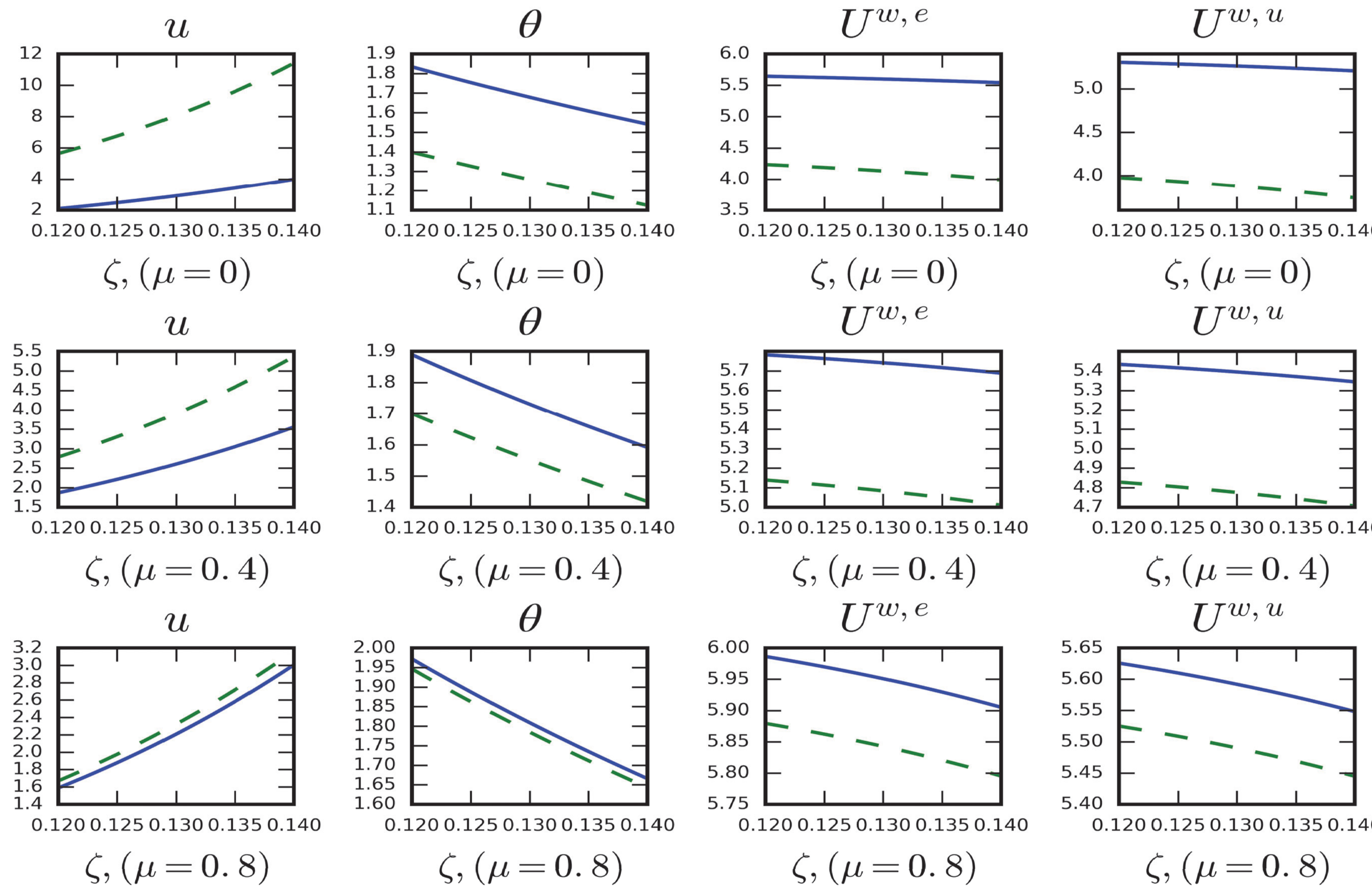

Figure 8: Effects of the search cost.

5.3 Labor market conditions

Figure 6 shows the effects of the unemployment benefit, γ, on θ, u, $U^{w,e}$, and $U^{w,u}$ in both the bubbly and bubbleless steady states when μ takes values of 0, 0.4, and 0.8. In our model, the labor market conditions (e.g., ϵ, γ, and ζ) have no impact on the capital per worker because the tax rate imposed on the entrepreneur's income is fixed. Thus, the optimal behavior of entrepreneurs is not affected by the labor market conditions.

As shown in Figure 6, the jobs-to-applicants ratio in the bubbly steady state is always higher than that in the bubbleless steady state, and the unemployment rate in the bubbly steady state is always lower than that in the bubbleless steady state, for any value of γ. Additionally, the lifetime utilities of the employed and unemployed (i.e., $U^{w,e}$ and $U^{w,u}$, respectively) in the bubbly steady state are higher than those in the bubbleless steady state. However, from the three cases of $\mu = 0, 0.4$, and 0.8, we find that although asset bubbles mitigate the negative effects of the unemployment benefit on the jobs-to-applicants ratio, unemployment rate, and welfare, this mitigation becomes limited as the credit constraint is relaxed.

Figure 6 also indicates that the jobs-to-applicants ratio decreases and the unemployment rate increases in both steady states as γ increases from 0.65 to 0.8. When the unemployment benefit, γ, increases, the worker's output share, Θ, also increases. Then, the profits yielded by a match between a firm and a worker decrease. Thus, the firms are incentivized to not enter the labor market. As a result, the jobs-to-applicants ratio decreases and the unemployment rate increases in both steady states. The graphs of $U^{w,e}$ and $U^{w,u}$ in Figure 6 have an inverted U-shape. As γ increases, the employed and unemployed workers obtain a higher wage income and higher unemployment benefits, respectively, which positively affect $U^{w,e}$ and $U^{w,u}$, respectively. On the other hand, as γ increases, the probability of a worker being unemployed is higher, which means that γ has a negative effect on both $U^{w,e}$ and $U^{w,u}$. Moreover, γ

negatively affects $U^{w,e}$ and $U^{w,u}$ indirectly through τ^w. These effects are mixed, and thus, $U^{w,e}$ and $U^{w,u}$ have an inverted U-shape.

Figure 7 examines the effects of the worker's bargaining power, ϵ, on u, θ, $U^{w,e}$, and $U^{w,u}$ in both steady states when ϵ varies from 0.5 to 0.7. As seen in the figure, ϵ's effects on each variable are similar to those of γ because ϵ affects each variable in almost the same manner as γ does.

Figure 8 shows the effects of the search cost, ζ, on u, θ, $U^{w,e}$, and $U^{w,u}$ in both steady states. As in the cases of γ and ϵ, the presence of an asset bubble mitigates the negative effects of the search cost on the jobs-to-applicants ratio, unemployment rate, and welfare, but this mitigation becomes limited as the credit constraint is relaxed. As ζ varies from 0.12 to 0.14, the jobs-to-applicants ratio, θ, decreases and the unemployment ratio, u, increases in both steady states because the increase in ζ impedes the entry of firms with vacancies into the labor market. Moreover, an increase in the search cost monotonically reduces the lifetime utilities of the employed and unemployed in both steady states. This is because the unemployment rate increases as ζ increases, which has a negative effect on the lifetime utilities of all workers.

5.4 Tax on the entrepreneur's income

Thus far, we have fixed the tax rate imposed on the entrepreneur's income. In this subsection, we examine the effects of changes in τ. The tax revenue collected from entrepreneurs is used to pay for the unemployment benefits. Thus, the taxation that we analyze is a redistribution policy from entrepreneurs to workers. As shown in Figure 9, the negative effects that the income tax has on u, θ, $U^{w,e}$, and $U^{w,u}$ are weakened by the presence of asset bubbles. However, as in the previous cases, this mitigation becomes weaker as the credit constraint is relaxed.

Figure 9 shows that as τ increases from 0 to 0.1, the jobs-to-applicants ratio, θ, decreases and the unemployment rate, u, increases in both steady states. This

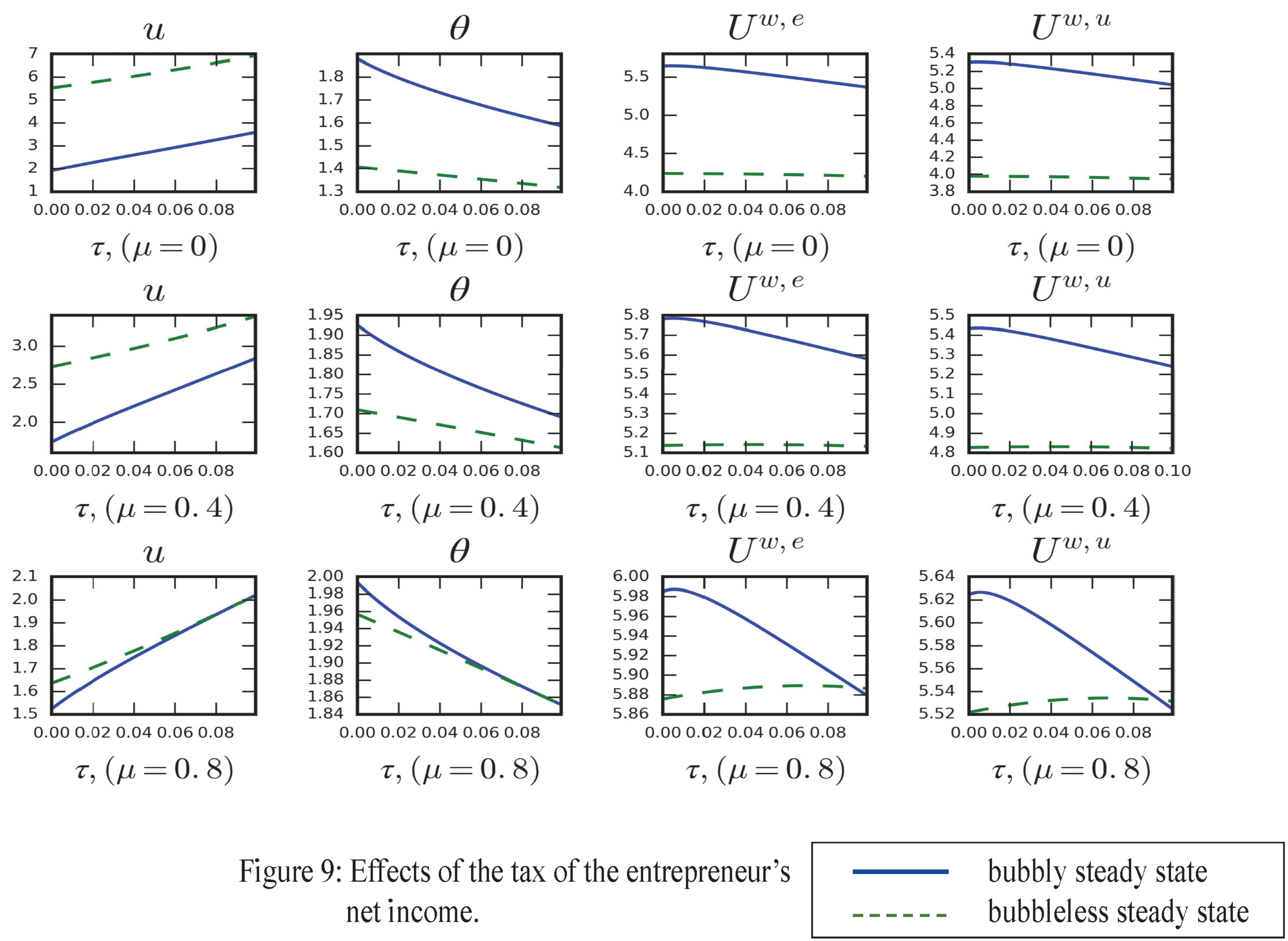

Figure 9: Effects of the tax of the entrepreneur's net income.

is because as τ increases, the investment by entrepreneurs decreases, and thus, the capital per worker, k, decreases. As a result, the profit yielded by a match between a firm and a worker decreases and the number of firms that enter the job market decreases. The graphs of $U^{w,e}$ and $U^{w,u}$ are downward sloping when $\mu = 0$ and 0.4, but the graph when $\mu = 0.8$ has an inverted U-shape in both steady states. As τ increases, the probability of a worker being unemployed becomes higher, which negatively affects both $U^{w,e}$ and $U^{w,u}$. On the other hand, an increase in τ promotes the redistribution from entrepreneurs to workers. Then, both $U^{w,e}$ and $U^{w,u}$ increase as τ increases. Owing to these conflicting effects of τ, the graphs of $U^{w,e}$ and $U^{w,u}$ have an inverted U-shape.

6 Conclusion

Our analysis has shown that the presence of asset bubbles mitigates the negative effects of taxation and unemployment benefits on unemployment rates and welfare. As the credit constraint is relaxed, this mitigation becomes limited. This means that the presence of asset bubbles is more beneficial in an economy with severe credit constraints.

Although the presence of asset bubbles increases the aggregate productivity in the economy by excluding less productive entrepreneurs from production activities, only the second-best outcome can be attained, as in the model of Bewley (1980). This is because, in our model, not only the most productive entrepreneurs but also the relatively less productive entrepreneurs engage in production when asset bubbles are present. As a result, the unemployment rate when asset bubbles occur is not as low as that in the first-best outcome. Therefore, a government policy is necessary for the economy to be Pareto-improved, even though the presence of asset bubbles reduces the unemployment rate. An analysis of such a government policy is left for future research.

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Appendices

Appendix A: Proof of Propositions 1 and 2

To prove Propositions 1 and 2, we obtain the following useful lemma.

Lemma 3 *Define ϕ^* and ϕ^{**} such that*

$$\frac{\beta(1-\tau)H(\phi^*)}{1-\mu-\beta(1-\tau)(G(\phi^*)-\mu)}=\phi^*,$$

and

$$G(\phi^{**})=\mu.$$

Then, both ϕ^* and ϕ^{**} are uniquely determined.

Proof: Because $G(\Phi)$ is a strictly increasing function over the support of Φ, ϕ^{**} uniquely determined. With regard to ϕ^*, note that $T(\phi) := \beta(1-\tau)H(\phi)/\phi - [1-\mu-\beta(1-\tau)(G(\phi)-\mu)]$ is strictly decreasing in $(0,\eta)$ because over the support, $T'(\phi) = \beta(1-\tau)\left[-H(\phi)/\phi^2 - dG(\phi)/d\phi\right] + \beta(1-\tau)dG(\phi)/d\phi = -\beta(1-\tau)H(\phi)/\phi^2 < 0$, and in the area other than the support, both $H(\phi)$ and $G(\phi)$ are constant. In addition, $\lim_{\phi\to 0} T(\phi) = \infty$ and $\lim_{\phi\to\eta} T(\phi) = -\left[\beta(1-\mu)(1-\tau)\right] < 0$. Hence, ϕ^*, which is the solution of $T(\phi) = 0$, is uniquely determined. □

Appendix A.1: Proof of Proposition 1

As shown by Lemma 3, ϕ^* is given by the solution of $T(\phi) = 0$, where $T(\phi) = \beta(1-\tau)H(\phi)/\phi - [1-\mu-\beta(1-\tau)(G(\phi)-\mu)]$ is a decreasing function with respect to ϕ. Eq. (28) has two steady-state equilibria, ϕ^* and ϕ^{**}, if and only if ϕ^* is strictly greater than ϕ^{**}. This is because the domain of the dynamical system in Eq. (28) is $[\phi^{**}, \eta]$ because of the free disposability of the bubbly assets. ϕ^* is strictly greater than ϕ^{**} if and only if $T(\phi^{**}) > 0$ or, equivalently, $(1-\mu)\phi^{**} < \beta(1-\tau)H(\phi^{**})$ because $0 = T(\phi^*) < T(\phi^{**})$ and $T(\phi)$ is a decreasing function with respect to ϕ. In this case, the asset bubble has a positive value in the steady state ϕ^*. At the same time, Eq. (28) has only one steady-state equilibrium if and only if $(1-\mu)\phi^{**} \geq \beta(1-\tau)H(\phi^{**})$.
□

Appendix A.2: Proof of Proposition2

The linear approximation of the dynamical system around a steady state is computed from Eqs. (28) and (32), as follows:

$$\begin{pmatrix} k_{t+1} - \hat{k} \\ \phi_{t+1} - \hat{\phi} \end{pmatrix} = \begin{pmatrix} \kappa_1(\hat{k}, \hat{\phi}) & x(\hat{k}, \hat{\phi}) \\ 0 & \kappa_2(\hat{\phi}) \end{pmatrix} \begin{pmatrix} k_t - \hat{k} \\ \phi_t - \hat{\phi} \end{pmatrix}, \tag{A.1}$$

where $(\hat{k},\hat{\phi})$=(k^*,ϕ^*) or (k^{**},ϕ^{**}),

$$\kappa_1 = \frac{1}{\Psi(\hat{k})+\hat{k}\Psi'(\hat{k})}\frac{A\beta(1-\tau)H(\hat{\phi})}{1-\mu(1-\beta)\left(G(\hat{\phi})-\mu\right)}\left(f''(\hat{k})\hat{k}\Psi(\hat{k})+f'(\hat{k})\Psi(\hat{k})+f'(\hat{k})\hat{k}\Psi'(\hat{k})\right),$$

$$\kappa_2 = \frac{G'(\hat{\phi})(1-\mu)}{\left(1-\mu-\beta(1-\tau)\left(G(\hat{\phi})-\mu\right)\right)^2},$$

and

$$x = \frac{1}{\Psi(\hat{k})+\hat{k}\Psi'(\hat{k})}\frac{\beta(1-\tau)H'(\hat{\phi})\left(1-\mu-\beta(1-\tau)(G(\hat{\phi})-\mu)\right)+\beta^2(1-\tau)^2H(\hat{\phi})G'(\hat{\phi})}{\left(1-\mu-\beta(1-\tau)(G(\hat{\phi})-\mu)\right)^2}.$$

The eigenvalues of the local dynamical system associated with Eq. (A.1) around the bubbly steady state, (k^*,ϕ^*), are given by

$$\kappa_1(k^*,\phi^*) = \frac{\phi^*A}{\Psi(k^*)+k^*\Psi'(k^*)}\left(f''(k^*)k^*\Psi(k^*)+f'(k^*)\Psi(k^*)+f'(k^*)k^*\Psi'(k^*)\right),$$

$$\kappa_2(\phi^*) = \left(\frac{G(\phi^*)-\mu}{\phi^*G'(\phi^*)H(\phi^*)+(\phi^*)^2G'(\phi^*)(G(\phi^*)-\mu)}+1\right).$$

Using $[k_tf'(k_t)]' > 0$ and $1 = \phi^*Af'(k^*)$, it follows that $0 < \kappa_1(k^*,\phi^*) < 1 < \kappa_2(k^*,\phi^*)$. Thus, the bubbly steady state, (k^*,ϕ^*) is a saddle point.

The eigenvalues of the local dynamical system associated with Eq. (A.1) around the bubbleless steady state, (k^{**},ϕ^{**}), are given by

$$\kappa_1(k^{**},\phi^{**}) = \frac{A\beta(1-\tau)H(\phi^{**})}{(1-\mu)(\Psi(k^{**})+k^{**}\Psi'(k^{**}))}(f''(k^{**})k^{**}\Psi(k^{**})+f'(k^{**})\Psi(k^{**})$$
$$+f'(k^{**})k^{**}\Psi'(k^{**})),$$

$$\kappa_2(\phi^{**}) = \frac{(1-\mu)\phi^{**}}{\beta(1-\tau)H(\phi^{**})}.$$

From proposition 1, $(1-\mu)\phi^{**} \geq \beta(1-\tau)H(\phi^{**})$ is satisfied, and from the assumption

$[k_t f'(k_t)]' > 0$, we obtain $0 < \kappa_1(k^{**}, \phi^{**}), \kappa_2(k^*, \phi^*) < 1$. Therefore, the bubbleless steady state (k^{**}, ϕ^{**}) is totally stable. $\square$

Appendix B: Proof of Lemma 1

From Eqs. (33) and (35), $S(\phi^*) = \phi^*$ and $S(\phi^{**}) = \beta(1-\tau)H(\phi^{**})/(1-\mu)$. The differentiation of $S(\phi)$ is given by

$$S'(\phi) = T(\phi)\frac{\beta(1-\tau)G'(\phi)\phi}{[1-\mu-\beta(1-\tau)(G(\phi)-\mu)]^2}.$$

In the above equation $T(\phi) := \beta(1-\tau)H(\phi)/\phi - [1-\mu-\beta(1-\tau)(G(\phi)-\mu)]$, which is obtained in the proof of Lemma 3. $T(\phi)$ is decreasing in $(0, \eta)$, as shown by Lemma 3. It follows that $\lim_{\phi\to 0} T(\phi) = \infty$ and $\lim_{\phi\to\eta} T(\phi) = -[\beta(1-\mu)(1-\tau)] < 0$. Moreover, it follows that $S'(\phi^*) = 0$ holds because $T(\phi^*) = 0$. Then, it follows that $S'(\phi)$ is positive if $0 < \phi < \phi^*$ and is negative if $\phi^* < \phi < \eta$. Thus, $S(\phi^*)$ is a maximum, and $S(\phi^{**}) < S(\phi^*)$. $\square$

Appendix C: Proof of Lemma 2

Because the total income over all entrepreneurs is the total capital income plus the total income from holding the bubbly asset, and because the marginal propensity to consume is equal to $1-\beta$, the aggregate consumption of entrepreneurs is given as follows:

$$C_t^p := (1-\beta)(1-\tau)(\rho_t Z_t + r_t B_{t-1}). \tag{C.1}$$

Using Eqs. (12) and (26), we obtain the after-tax aggregate income from savings in the economy as $(1-\tau)(\rho_t Z_t + r_t B_{t-1}) = (1-\tau)(1-\mu)\rho_t Z_t/[1-\mu-\beta(1-\tau)(G(\phi_t)-\mu)]$. Using Eqs. (13), (25), and (30), we have $\rho_t Z_t = f'(k_t)k_t\Psi(k_t)N$. Thus, the aggregate consumption of entrepreneurs is obtained as follows:

$$
\begin{aligned}
C_t^p &= \frac{(1-\beta)(1-\tau)(1-\mu)}{1-\mu-\beta(1-\tau)(G(\phi_t)-\mu))} f'(k_t)k_t\Psi(k_t)N \\
&= (1-\beta)(1-\tau)\Gamma(\phi_t)f'(k_t)k_t\Psi(k_t)N, \qquad \text{(C.2)}
\end{aligned}
$$

where $\Gamma(\phi) = (1-\mu)/[1-\mu-\beta(1-\tau)(G(\phi)-\mu)]$. Substituting the steady-state values into Eq. (C.2) yields Eqs. (37) and (38). Then, it is straightforward to obtain the aggregate consumption of workers, as follows:

$$
C_t^w = (1-\tau_t^w)(w_t(1-u_t)N + \gamma_t u_t N). \qquad \text{(C.3)}
$$

The aggregate tax revenue from entrepreneurs is given by $\tau(\rho_t Z_t + r_t B_{t-1}) = \tau(1-\mu)\rho_t Z_t/[1-\mu-\beta(1-\tau)(G(\phi_t)-\mu)]$. From Eqs. (14), (17), (22), $\rho_t Z_t = f'(k_t)k_t\Psi(k_t)N$, and the aggregate tax revenue from entrepreneurs, Eq. (C.3) can be rewritten as

$$
\begin{aligned}
C_t^w &= \left(\Theta(f(k_t)-f'(k_t)k_t) + \frac{\tau(1-\mu)}{1-\mu-\beta(1-\tau)(G(\phi_t)-\mu)} f'(k_t)k_t\right)\Psi(k_t)N \\
&= \left[\Theta(f(k_t)-f'(k_t)k_t) + \tau\Gamma(\phi_t)f'(k_t)k_t\right]\Psi(k_t)N. \qquad \text{(C.4)}
\end{aligned}
$$

Substituting the steady-state values into Eq. (C.4) yields Eqs. (39) and (40). □

Appendix D: Steady states in the numerical analysis

Under the functional form setting in Section 5, each macroeconomic variable can be computed as follows. In the bubbly steady state, we have

$$
k^* = (\phi^* A\alpha\bar{A})^{\frac{1}{1-\alpha}},
$$

$$
u^* = 1 - \left(1 - \left(\frac{\zeta}{(1-\Theta)(1-\alpha)\bar{A}(k^*)^\alpha}\right)^\sigma\right)^{\frac{1}{\sigma}},
$$

$$\theta^* = \left(\frac{(1-u^*)^\sigma}{1-(1-u^*)^\sigma} \right)^{1/\sigma},$$

$$B^* = \frac{\beta(1-\tau)(\phi^*-\mu)}{1-\mu-\beta(1-\tau)(\phi^*-\mu)} \alpha \bar{A}(k^*)^\alpha (1-u^*)N,$$

$$C^{p,*} = (1-\beta)(1-\tau)\Gamma(\phi^*)\alpha\bar{A}(k^*)^\alpha(1-u^*)N,$$

and

$$C^{w,*} = [\Theta(1-\alpha)\bar{A}(k^*)^\alpha + \tau\Gamma(\phi^*)\alpha\bar{A}(k^*)^\alpha](1-u^*)N,$$

where $k^*, u^*, B^*, C^{p,*}$, and $C^{w,*}$ are the capital per worker, unemployment rate, total value of bubbly assets, aggregate consumption of entrepreneurs, and aggregate consumption of workers, respectively.

Additionally, in the bubbleless steady state, we have

$$k^{**} = \left(\frac{\beta(1-\tau)(1-\mu^2)A\alpha\bar{A}}{2(1-\mu)} \right)^{\frac{1}{1-\alpha}},$$

$$u^{**} = 1 - \left(1 - \left(\frac{\zeta}{(1-\Theta)(1-\alpha)\bar{A}(k^{**})^\alpha} \right)^\sigma \right)^{\frac{1}{\sigma}},$$

$$\theta^{**} = \left(\frac{(1-u^{**})^\sigma}{1-(1-u^{**})^\sigma} \right)^{1/\sigma},$$

$$C^{p,**} = (1-\beta)(1-\tau)\alpha\bar{A}(k^{**})^\alpha(1-u^{**})N,$$

and

$$C^{w,**} = [\Theta(1-\alpha)\bar{A}(k^{**})^\alpha + \tau\alpha\bar{A}(k^{**})^\alpha](1-u^*)N.$$

Appendix E: Derivation of the indirect lifetime utility

The income of an entrepreneur is given by $I_t = \rho_t A \Phi_{t-1} i_{t-1} + r_t b_{t-1}$. From the optimization problem, we obtain the law of motion of I_t as $I_{t+1} = \beta(1-\tau)R_{t+1}I_t$ and, thus, the entrepreneur's optimal consumption is $c_t = (1-\beta)(1-\tau)I_t$. We derive the expected indirect lifetime utility by the guess-and-verify method. We guess the form of the expected indirect utility function as $V(I_t) = m \ln I_t + n$, where we assume that m and n are certain constants because the economy is in a steady state. Using the Bellman equation, $V(I_t) = \ln[(1-\beta)I_t] + \beta E_t V(I_{t+1})$, and $I_{t+1} = \beta(1-\tau)R_{t+1}I_t$, we obtain the following expression:

$$m \ln I_t + n = \ln[(1-\beta)(1-\tau)I_t] + m\beta \ln I_t + m\beta \ln[(1-\beta)(1-\tau)] + n\beta + m\beta E_t \ln R_{t+1}. \tag{E.1}$$

$E_t \ln R_{t+1}$ can be calculated as follows:

$$\begin{aligned} E_t \ln R_{t+1} &= \int_0^1 \ln \left[\left\{ \max r_{t+1}, \frac{\rho_{t+1} A \Phi_t - r_{t+1}\mu}{1-\mu} \right\} \right] d\Phi_t \\ &= \int_0^1 \ln \left[\left\{ \max \rho_{t+1} A \phi_t, \frac{\rho_{t+1} A \Phi_t - \rho_{t+1} A \phi_t \mu}{1-\mu} \right\} \right] d\Phi_t \\ &= \int_0^{\phi_t} \ln \left[\rho_{t+1} A \phi_t \right] d\Phi_t + \int_{\phi_t}^1 \ln \left[\frac{\rho_{t+1} A \Phi_t - \rho_{t+1} A \phi_t \mu}{1-\mu} \right] d\Phi_t \\ &= \ln(\rho_{t+1} A) + \mu\phi_t \ln \phi_t - (1-\phi_t) + (1-\mu\phi_t) \ln \left[\frac{1-\mu\phi_t}{1-\mu} \right]. \end{aligned} \tag{E.2}$$

Using Eqs. (E.1) and (E.2), we have

$$m = \frac{1}{1-\beta},$$

and

$$\begin{aligned} n &= \frac{\ln\left[(1-\beta)^{1-\beta}\beta^{\beta}\right]}{(1-\beta)^2} + \frac{\ln(1-\tau)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left[\ln(\rho A) + \mu\phi_t \ln \phi_t - (1-\phi_t)\right] \\ &+ \frac{\beta}{(1-\beta)^2}\left[(1-\mu\phi_t)\ln\left(\frac{1-\mu\phi_t}{1-\mu}\right)\right], \end{aligned}$$

where $\phi_t = \phi^*$ or ϕ^{**} and ρ is the steady-state value of ρ_t. Using m and n above, we have the expected indirect lifetime utility of the entrepreneur, $V(I_t)$, as follows:

$$V(I_t) = \frac{1}{1-\beta}\ln I_t + \frac{\ln\left[(1-\beta)^{1-\beta}\beta^{\beta}\right]}{(1-\beta)^2} + \frac{\ln(1-\tau)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left[(1-\mu\phi_t)\ln\left(\frac{1-\mu\phi_t}{1-\mu}\right)\right]$$
$$+ \frac{\beta}{(1-\beta)^2}\left[\ln(\rho A) + \mu\phi_t \ln\phi_t - (1-\phi_t)\right]. \quad \text{(E.3)}$$

By omitting the time subscripts from Eq. (E.3), we have Eq. (41), with $\phi_t = \phi^*$ or ϕ^{**}.

Next, we derive the lifetime utility functions of the employed and unemployed workers in a steady state. Using Eqs. (15) and (16), we obtain

$$U_t^{w,e} = w_t(1-\tau_t^w) + \sum_{s=1}^{\infty}\left(\tilde{\beta}^s[\theta_{t+s}q(\theta_{t+s})w_{t+s}(1-\tau_t^w) + (1-\theta_{t+s}q(\theta_{t+s}))\gamma_{t+s}(1-\tau_t^w)]\right),$$

and

$$U_t^{w,u} = \gamma_t(1-\tau_t^w) + \sum_{s=1}^{\infty}\left(\tilde{\beta}^s[\theta_{t+s}q(\theta_{t+s})w_{t+s}(1-\tau_t^w) + (1-\theta_{t+s}q(\theta_{t+s}))\gamma_{t+s}(1-\tau_t^w)]\right).$$

From $\gamma_t = \gamma w_t$, in a steady state, the above equations can be written as follows:

$$U^{w,e} = \left[1 + \frac{\beta}{1-\beta}\theta q(\theta) + \frac{\beta}{1-\beta}(1-\theta q(\theta))\gamma\right] w(1-\tau^w),$$

and

$$U^{w,u} = \left[\gamma + \frac{\beta}{1-\beta}\theta q(\theta) + \frac{\beta}{1-\beta}(1-\theta q(\theta))\gamma\right] w(1-\tau^w),$$

where γ, θ, τ^w, and w are the steady-state values.