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Optimal Taxation in an Endogenous Fertility Model with Non-Cooperative Couples

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Optimal Taxation in an Endogenous Fertility Model with Non-Cooperative Couples*

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Abstract

This study examines the optimal tax structure in an endogenous fertility model with non-cooperative couples. In the model, both the quality and number of children are suboptimal because of the non-cooperative behavior of couples. Moreover, we consider the external effects of children on society and center-based childcare services. In such a unified model, we characterize the formulae for optimal income tax rates, child tax/subsidy rates, and tax/subsidy rates on center-based childcare services. We find that income taxation, but not a child subsidy, corrects the suboptimal low fertility rate caused by the non-cooperative behavior of couples. To alleviate the deadweight loss from income taxation, a child tax is useful. The child tax (subsidy) becomes optimal if the required tax revenue is sufficiently large (small) or if the external effects are sufficiently small (large). The subsidy for external childcare services corrects the external effects of children, not the non-cooperative behavior. These results are reinforced by the numerical analysis.

Keywords: Non-Cooperative Couple, Endogenous Fertility, Optimal Income Tax, Optimal Child Tax/Subsidy

Classification: H21, J13, J16

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1 Introduction

Most Organisation for Economic Co-operation and Development (OECD) countries face a striking decrease in fertility rates: on average, the Total Fertility Rate (TFR) has been below replacement levels for about three decades, as shown in Figure 1.[1] Since this demographic trend may have a substantial adverse impact on economic growth, OECD governments have designed a variety of pro-natalist policies to affect the willingness of families to raise children, such as a direct child subsidy, a subsidy for center-based childcare services, income tax deduction, a childbearing leave program, and enhancement of childcare facilities (e.g., Eydal and Rostgaard, 2018).

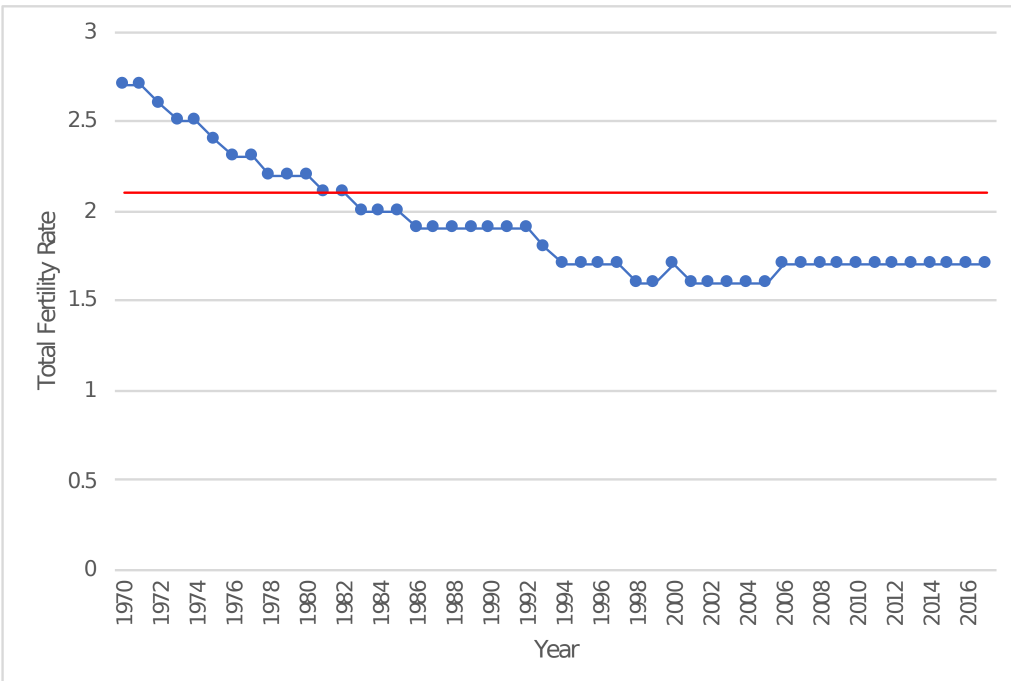

Figure 1: The OECD's average total fertility rate over 1970-2017

If the demographic change stems from the sub-optimality of the family's fertility choice, it is crucial to clarify the mechanism of inefficient fertility choices and to implement effective family policies. This study sheds light on two driving forces behind downward pressure on fertility choice within a household: the external effects of children on society; and couples' non-cooperative behavior for the provision of childcare. This study shows which family policies are best for improving the fertility decisions that yield downward pressure on the fertility rate.

[1]The TFR is the number of children that would be born to a woman if she were to live to the end of her childbearing years and give birth to children according to current age-specific fertility rates. Data on the TFR used in Figure 1 are from OECD Data (https://data.oecd.org/pop/fertility-rates.htm).

The externality of children on society has been treated as a major driving force behind inefficiently low fertility in the modern economy. The previous literature has mainly considered the external effect of children to be a positive fiscal externality generated under pay-as-you-go (PAYG) pension systems (e.g., Cigno, 1992; Sinn, 2001).[2] The literature on this subject concludes that child subsides have substantial capacity to achieve first-best investment in fertility. However, the TFR has remained below replacement level even when childbearing has been highly subsidized (e.g., Germany). It seems that there is very little effect of child subsidies correcting such a positive fiscal externality of children on fertility choice, since the process of fertility reductions is unchanged over time, not reversed.

This study provides another reason for the sub-optimal number of children from the viewpoint of non-cooperative household behavior by a couple. In an economy consisting of a representative couple, we introduce the strategic interaction between a husband and wife in providing childcare, which leads to underinvestment in childcare (and hence, a suboptimal low level of child quality).[3] The strategic interaction across partners is supported by recent econometric evidence by Del Boca and Flinn (2012), who show that one-fourth of couples under-provide household public goods because of non-cooperative behavior. Regarding childcare decisions, the literature has long recognized a couple's inability to reach legally enforceable agreements about their investments in children because of non-observability by third parties (Rasul, 2008),[4] and that their provision of childcare is not always observable due to the lack of effective monitoring between partners (Pailhé and Solaz, 2008). Thus, the commitment of the previously determined time investment in childcare is not credible.[5] Allowing for this fact, the present study theoretically considers that households do not commit to their decisions regarding the time supplied for childcare and non-cooperatively determine the time.[6] In our model, the time invested by parents does not mean the time that parents merely spend with their children but rather represents the time devoted to improving the quality of, for example, their children's non-cognitive and cognitive skills. In other words, childcare time includes the aspect of educational investment in children (Del Boca et al., 2014).[7] Since the quality of children is generally private provision of a household public

[2] Cigno (1992) states that one motive for having children is to secure the risk of old-age consumption. Since PAYG pension systems secure the risk of old-age consumption, public insurance induces people to have fewer children, and thus, fertility rates fall. Sinn (2001) estimates that an additional child in Germany brings a net benefit to the pension system of about 90,000 euros.

[3] Following previous literature as in de la Croix and Doepke (2003) and Gobbi (2018), we consider the quality of children as a household public good.

[4] Rasul (2008) empirically proves that spouses cannot commit to household chores.

[5] Akerlof (1991) argues that a major function of management is to monitor accomplishment so as to prevent procrastination in both project initiation and project termination.

[6] Browning et al. (2014) state that the behavior of spouses must be observable for each other to achieve a Pareto-efficient allocation.

[7] Del Boca et al. (2014) indicate that the time inputs of not only mothers but also fathers are extremely important in the cognitive development process, particularly for young children.

good, the free-rider problem arises in our model.

Another feature of this study is that it separates the quality per child and the number of children. Our model allows couples to collectively decide the number of children at the stage prior to non-cooperative determination of time inputs supplied by each spouse.[8] Under this setting, we find that non-cooperative behavior for the provision of childcare leads to an inefficient level of fertility deviating from the Pareto-efficient level, even though fertility choice is made collectively. This is consistent with Doepke and Kindermann (2019), who empirically conclude that non-cooperation of a couple due to lack of commitment leads to a low fertility rate. To the best of our knowledge, this study is the first to theoretically derive the result that the non-cooperative behavior of spouses leads to inefficiently low fertility. Based on our finding, we propose a novel channel through which the government can enhance low fertility rates by employing an appropriate choice of family policies. In addition, we allow the quality per child as well as the number of children to have external effects on society. Heckman (2006) and Heckman and Masterov (2007) empirically demonstrate that the increase in quality of children improves the health conditions in local area, promotes social skills, and reduces the crime rate and high school dropout rates, which implies that the quality of children has external effects on society.

In addition, we introduce external childcare services offered by centers, which can be substituted for childcare time of the spouses. Examples of such services are external early childhood education facilities, preschools, and cram schools. In this extensive model, we compare the effectiveness of the subsidy for center-based childcare services to that of the direct child subsidy.

To clarify how the government should design tax structure to correct the sub-optimally low fertility level, we allow the government to employ a commodity tax, income taxes, a (direct) child tax/subsidy (tax/subsidy on/for a child), and a tax/subsidy on/for center-based childcare services. We note that many countries face the problem of how to secure tax revenue because of cumulative budget deficits and increasing social security expenditure. In addition, a revenue source of subsidies for childcare should be allowed with the exception of a lump-sum tax in the real-world tax system. Therefore, this study adopts the revenue-constrained optimal tax framework, which originated with Ramsey (1927) and was extended by Diamond and Mirrlees (1971a, b) and Mirrlees (1971). This study allows policymakers to employ differential income tax rates on the husband and wife, that is, so-called gender-based income taxation.[9] We also analyze the case of a common income tax rate on the husband

[8]The justification of our setting is that, even though fertility and time devoted to childcare are collectively determined simultaneously, it is possible to change time devoted to childcare from what is planned today, which means that a lack of commitment occurs (Rasul, 2008). In addition, there is no way of monitoring a certain amount of childcare duties provided by the other partner (Pailhé and Solaz, 2008). Therefore, we assume such a setting.

[9]We provide some citations of previous studies examining gender-based income taxation in the last para-

and wife.

We demonstrate that, as the results of the comparative statics of a couple's behavior due to changes in taxes, the increase in the labor income tax enhances fertility. This theoretical result is consistent with the empirical result by Baughman and Dickert-Conlin (2009), showing that reduction of income taxes decreases the fertility rate. Under our optimal tax framework, the income taxes, but not the child tax/subsidy, improve the low fertility caused by spouses' non-cooperative behavior. In other words, the income taxes have a double dividend in that they increase tax revenue and correct the low fertility caused by non-cooperative behavior. The child tax/subsidy acts as a device to mitigate the income tax-induced deadweight loss, to cover the exogenous revenue requirement taking account of own price-induced deadweight loss, and to correct the external effects of children on society. In particular, under the availability of lump-sum taxes and no externality of children on society, the optimal intervention for children is to impose a tax, which can alleviate the distortion on labor supply from income taxation for correcting the non-cooperative behavior. However, if the external effect of children on society is sufficiently large, the optimal intervention for children is to provide a subsidy. These results hold regardless of whether the spouse uses center-based childcare services. As other important results, the subsidy for center-based childcare services becomes optimal as long as there is an externality of children on society and the difference of bargaining power of spouses is not so large. The role of the subsidy for the services is to correct the externality of children on society, but not to improve the low fertility associated with non-cooperative behavior.

This study also employs numerical analysis to investigate the effects of changes in several parameters on optimal tax rates. The theoretical results and interpretation are confirmed by numerical simulations. As a major concern, we investigate the ranking of the direct child subsidy and the subsidy for center-based childcare services. The result shows that the subsidy for center-based childcare services is more likely to be higher than the direct child subsidy as the required tax revenue becomes larger. The intuition behind this result is provided by the theory. We also numerically confirm that the introduction of a childcare facility always improves welfare, increases the number of children, and raises the quality per child under the optimal tax framework. In addition, we examine how the difference in wage rate, the variation in childcare productivity, and the difference in bargaining power between spouses affect income tax rates on the husband and wife; this corresponds to the analysis of gender-based taxation.

Based on our theoretical and numerical results, we suggest the following policy implications for family policies to improve the sub-optimally low fertility rate under a revenue constraint. First, if the low fertility rate in most OECD countries critically arises from the

graph of Section 2.

non-cooperative behavior of households, we recommend a downward shift of the direct child subsidy as a policy reform, which may lead to a direct child tax. This is a novel conclusion in our model that contrasts with findings from the previous literature emphasizing that Pigouvian corrective child subsidies are desirable. We show that income taxation is effective for improving fertility rate rather than the direct child subsidy. This result is supported by some empirical evidence, as shown in Jones and Tertilt (2008) and Jones et al. (2010).[10] Second, the child subsidy is generally not a useful device for alleviating non-cooperative behavior; it is required if the externality of children on society is sufficiently large. When policymakers aim to improve the TFR by correcting the externality arising from inefficient intra-household behavior, it should employ a combination of income taxation and a child tax. Third, the introduction of a childcare facility enhances the fertility rate and welfare. Our theoretical and numerical results imply that the top priority of family policies for improving low fertility is to ensure that childcare facilities are fully utilized. After improving and enhancing childcare facilities, the government should discuss the design of a child tax/subsidy. This is because, as shown by our numerical results, a child tax/subsidy implemented prior to expanding childcare facilities worsens welfare and fertility rates. In 2009, the direct child subsidy was a noticeable policy in Hatoyama administration in Japan. Commenting on this policy, Ángel Gurria, secretary-general of the OECD, recommended that it was more important for Japan to enhance childcare facilities than to provide a direct child subsidy. In addition, the fertility rate has improved in countries that provide more public childcare, such as France, Belgium, and Norway, while the fertility rate is still at low levels in countries with high subsidies for childbearing, such as Germany (Doepke and Kindermann, 2019). Our findings support the OECD's policy recommendation and the policies implemented by France, Belgium, and Norway. Fourth, under a high (low) revenue requirement, it is desirable that the subsidy rate for center-based childcare services is higher (lower) than the direct child subsidy rate, even if the direct child subsidy is optimal. Such policies may pertain to the optimal design of a tax/subsidy system in developed countries (developing countries) where a relatively large (small) amount of tax revenue is needed. Based on the third and fourth policy implications, developed (developing) countries should expand childcare facilities and then implement the subsidy for center-based childcare services (the direct child subsidy), rather than the direct child subsidy (the subsidy for center-based child care services).

The remainder of the paper is structured as follows. The next section discusses the related literature. Section 3 describes our model and Section 4 provides solutions of our model. The optimal taxation is analyzed in Section 5 and a childcare facility is introduced as an extension to the model in Section 6. Numerical analysis is undertaken in Section 7. Section 8 concludes.

[10] Jones and Tertilt (2008) and Jones et al. (2010) empirically show that fertility is negatively related to the wage rate in most countries at most times.

2 Related Literature

This study constructs a model with non-cooperative couples who under-invest in the quality of children and examines the optimal structure that plays a corrective role as an efficiency-enhancing device under a revenue constraint. In this respect, our study is mainly related to four strands of research. First, various previous works have investigated the structure of the household's decision-making. In the traditional framework, households are considered as a single decision-making agent, known as the "unitary" approach initiated by Samuelson (1956) and Becker (1974). Owing to a lack of empirical support for the unitary model of households, Apps and Rees (1988) and Chiappori (1988, 1992) propose a "collective" approach, allowing for bargaining power between spouses and assuming that households achieve a Pareto-efficient allocation.[11] A common assumption of the unitary and collective approach is that intra-household behavior is efficient. However, recent literature has increasingly employed the non-cooperative model in which the allocation is not fully efficient (Konrad and Lommerud, 1995; Cigno, 2012; Gobbi, 2018).[12] The non-cooperative model is supported by empirical evidence. For example, Del Boca and Flinn (2012) estimate household time allocation between labor market work and production of a public good, and find that about one-fourth of households act non-cooperatively. Our analysis builds on the literature on the non-cooperative model of households.

The second relevant strand of literature concerns the design of optimal taxation for households consisting of two or more agents.[13] In particular, using the self-selection approach (Stiglitz, 1982), Balestrino et al. (2002) develop a two-type model with non-linear labor income taxation, non-linear child taxes/subsidies, and linear commodity taxation, when households differ in ability in the labor market and ability in household production. Like our model, these models consider that both fertility and child quality are endogenously determined.[14] In their model, the government's intervention is justified by not only equity considerations (redistribution from rich to poor) but also allocative efficiency considerations (specialization according to comparative advantage). The justification for the government's intervenstion stems from differences in the two kinds of abilities between households. The

[11] The unitary model ensures an income-pooling result, in which a change in the source of household income does not affect demand if total income is constant. For example, this is empirically rejected by Browning and Chiappori (1998).

[12] Non-cooperative family decision-making has been adopted in the theoretical, empirical, and experimental literature. In particular, as with our model, Konrad and Lommerud (1995), Cigno (2012), and Gobbi (2018) use a non-cooperative model for childcare decisions. See other related literature of the non-cooperative model, for example, Lundberg and Pollak (1993), Anderberg (2007), Lechene and Preston (2011), Cochard et al. (2016), Doepke and Tertilt (2019), and Heath and Tan (2020).

[13] There is a growing body of literature analyzing the optimal family taxation scheme; see, for example, Cremer et al. (2003, 2011b, 2016, 2020), Schroyen (2003), Brett (2007), Kleven and Kreiner (2007), Kleven et al. (2009), Meier and Wrede (2013), Frankel (2014), and Apps and Rees (2018).

[14] There are other related works exploring the optimal system of policy instruments under endogenous fertility and child quality (e.g., Cigno, 2001; Cigno and Pettini, 2002).

model of Balestrino et al. (2002) falls within the "unitary" approach that supports Pareto efficiency. Thus, the fertility rate is initially efficient in their model. By contrast, we consider another justification for the government's intervention, which is to correct the externality arising from non-cooperative household behavior. Moreover, using the Ramsey tax framework, our study analyzes optimal tax policies for improving the sub-optimally low fertility induced by non-cooperative couples. In a recent contribution, Meier and Rainer (2015) study gender-based income taxation in the model with a non-cooperatively provided household public good, which leads to under-provision of the public good, and find that marginal income tax rates should be differentiated by gender based on both the Pigou and the Ramsey considerations. In the model, the production of household public goods is under-provided due to non-cooperative behavior. Even though their setting is similar to ours, our study differs in three ways from their framework. First, we separate the household public good into two factors: quality per child and number of children. Second, our model allows the government to employ a child tax/subsidy as a direct intervention on a public good. Third, we introduce center-based childcare services, which can be substituted for childcare time of the spouses.

The third strand of literature discusses the driving force behind the low fertility rate in an economy and then establishes Pareto-improving family policies that correct the inefficiency. One major explanation for the decrease in the number of children is that children involve a positive fiscal externality when the government redistributes from the young to the old (e.g., PAYG transfers). As argued in Cigno (1992), the PAYG transfers lead to a sub-optimal number of children, since children, who parents consider as assets, are no longer needed to secure consumption in retirement. Groezen et al. (2003) analyze the role of a child allowance scheme when fertility is socially inefficient owing to the PAYG transfers.[15] They show that the child allowance system ensures the first-best outcome under lump-sum transfers. Compared to these studies, the present study proposes a theoretical framework that describes inefficiently low fertility due to limited commitment leading to non-cooperative behavior by a couple in addition to the external effect of children on society, and then provides the optimal structure of family policy measures.

The fourth strand of literature is gender-based taxation, which allows tax rates to differ between the husband and wife. Rosen (1977) is the first to argue the efficiency gains from employing differential taxation based on gender, while Akerlof (1978) shows that the use of categorical information, such as age, gender, and disability status, which is called "tagging," is welfare improving from the viewpoint of utilitarianism.[16] Therefore, if the government

[15]Cigno et al. (2003), Fenge and Meier (2005), and Cremer et al. (2008, 2011a) are among the related literature on family policy in the presence of fiscal externalities.

[16]It is well known that tagging violates the principle of horizontal equity and therefore, is limited in practice.

reflects observable characteristics in the tax system, it can reinforce the redistributive tax system. A number of studies explore the gender-based taxation system, see, for example, Boskin and Shesinski (1983), Piggott and Whalley (1996), Apps and Rees (1999a, b, 2011), Cremer et al. (2010), Alesina et al. (2011), Bastani (2013), Meier and Rainer (2015), and Komura et al. (2019).

3 Model

A household consists of a wife (m), husband (f), and children. The wife and husband collectively decide how many children to have, while each spouse non-cooperatively decides his/her two kinds of private consumption, labor supply in the external market, and time spent on childcare activities. The parental time investment in childcare enhances the quality of children, such as their non-cognitive and cognitive skills. Children positively affect the utility of both spouses as a household public good, which may give rise to a free-rider problem between the spouses in the process of enhancing the quality of children. Furthermore, we allow children to positively affect society as externalities.

The government corrects the free-rider problem and the externalities on society, while facing a revenue constraint. The government imposes taxes on income for each spouse and implements a tax/subsidy on/for a child. Here, a tax/subsidy on/for a child is called a (direct) child tax/subsidy, where the recipients of the tax burden/subsidy proportionally increase with the number of children. This study considers the case in which the income tax rates on husband and wife are allowed to differ: so-called "gender-based taxation."[17]

We consider the following sequential decisions of the government, the couple, and each spouse. First, the government determines the tax rates to collect a given level of tax revenue and to correct the sub-optimally low fertility level. Second, the wife and husband collectively decide on the number of children. Third, each spouse non-cooperatively decides his/her two kinds of private consumption: labor supply in the external market, and time spent on childcare.

3.1 Third Stage: Each Spouse

Each spouse non-cooperatively decides the working time in outside labor market l_i, the time spent by each spouse on childcare activities h_i, and private consumption of the numeraire z_i and other commodity y_i. We suppose that children provide direct utility benefits, that

[17]Gender-based taxation is equivalent to the combination of a common tax rate on both genders and a tax rate deduction on women. Only women bear burdens over the fertility period, such as the time devoted to pregnancy, childbirth, and lactation. It is plausible that a subsidy or tax deduction in allowance would be provided for these burdens.

is, children are a consumption good. Spouse i's utility U_i is given by

$$U_m = z_m + \frac{y_m^{\varphi}}{\varphi} - \frac{(l_m + h_m)^{1+\phi}}{1+\phi} + NQ, \tag{1}$$

$$U_f = z_f + \frac{y_f^{\varphi}}{\varphi} - \frac{(l_f + h_f)^{1+\phi}}{1+\phi} + NQ - c(N),$$

where $\varphi(< 1)$ is the curvature of the utility of commodity y, $\phi(> 0)$ is that of the disutility of total time use, N is the number of children, and Q is the quality per child. $c(N)$ is a cost that only wives bear, depending on the number of children. This cost includes that arising from the biology of child rearing over the fertility period, such as the time devoted to pregnancy, childbirth, and lactation (Rasul, 2008).[18] The cost function is assumed to satisfy $c' > 0$ and $c'' > 0$. NQ positively and equally affects the two spouses as a household public good.[19]

The quality function is given by

$$Q = \frac{\left(s_m \frac{h_m}{N}\right)^{\sigma}}{\sigma} + \frac{\left(s_f \frac{h_f}{N}\right)^{\sigma}}{\sigma} = N^{-\sigma}\left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma}\right], \tag{2}$$

where s_i denotes the productively of spouse i for child quality and σ, which satisfies $0 < \sigma < 1$, is the curvature of the quality function.[20] h_i/N is the childcare time per child. As an aspect of cost, h_i includes not only the time used for domestic childcare, but also the intensity and quality per unit of time. As an aspect of benefit, childcare time h_i includes not only the mere time spent raising a child, but also the time for improving Q, such as the time spent reading books to children, the time spent on early childhood education at home, and the cost of effort to discipline children. Childcare time h_i can be divided into two components: $h_i = \widetilde{h}_i + \tau N$, where $\widetilde{h}_i$ is the time spent enhancing the quality of children, and τ is the (constant) minimal amount of time spent raising a child; hence, τN is the total minimal amount of time spent bringing up children. If τ is exogenous, the theoretical results obtained in this study are unaffected as long as the equilibrium is an interior solution

[18] Rasul (2008) also assumes that only a wife bears the cost function.

[19] Although sub-utility functions in our model are specified, the generalization of the sub-utility function, such as

$$U_m = z_m + \varkappa_m(y_m) + \varpi_m(l_m + h_m) + \epsilon_m(N, Q),$$
$$U_f = z_f + \varkappa_f(y_f) + \varpi_f(l_f + h_f) + \epsilon_f(N, Q) - c(N),$$

does not affect the optimal tax/subsidy expressions provided in Propositions 4–7, which are the main theoretical results in our study. This is because our optimal tax/subsidy rates are expressed in terms of price elasticities.

[20] Meier and Rainer (2015) use a similar function for the household public good. However, in contrast to their setting, we divide the number of children from the quality per child, and the number of children and quality level are determined in the different stages.

($\widetilde{h}_i > 0$). Thus, our setting presented by (1) and (2) is not restrictive.[21]

In our model, both spouses provide the childcare time. Indeed, Del Boca et al. (2014) and Lundborg et al. (2014) empirically prove that the time invested by both a husband and wife is important for human capital formation of children. In particular, Del Boca et al. (2014) indicate that the time inputs of both parents are extremely important in the cognitive development process, particularly for young children, and we assume that the time fathers spend with children positively affects child quality. In addition, our quality function (2) does not include some commodities as inputs. This assumption also follows Del Boca et al. (2014), who empirically find that the effect of money on child outcomes is much more limited than the effect of parental time with children.

The budget constraint of each spouse is

$$z_i + (1 + t_y)p_y y_i + \gamma_{xi} p_N x + \gamma_{Ni} \kappa_N N = (1 - t_i) w_i l_i, \quad i = m, f, \tag{3}$$

where t_y is the commodity tax rate on y_i, p_y is the price of y_i, γ_{xi} is the share of spouse i on the purchase of the fertility good, x is the amount of a fertility good that a couple purchases, p_N is the price of x, γ_{Ni} is the share of spouse i in the child tax payment or child subsidy receipt, κ_N is the child tax/subsidy, t_i (for $i = m, f$) is the income tax rate on the labor income of spouse i, and w_i is the wage rate of spouse i. The before-tax prices of the numeraire good are normalized by one without loss of generality. The shares of the purchase of a fertility good and child tax payment (child subsidy receipt) are given for each spouse at a certain level, satisfying $\gamma_{xm} + \gamma_{xf} = 1$ and $\gamma_{Nm} + \gamma_{Nf} = 1$.

The required amount of the fertility good is given by the following function:

$$x = \upsilon N, \tag{4}$$

for a scalar υ. The cost includes food, clothing, medical expense, and overhead expenses needed for compulsory education.[22] The ratio of expenditure on nursery schools, tutors, and cram education to the cost of bringing up a child seems to be very large, particularly in developed countries. However, this expenditure is related to improving the quality per child. The extensive case in which such expenditure affects the quality per child is dealt with in Section 6. To make the analysis simpler but without loss of generality, we assume that one unit of the fertility good is needed to bring up a child, that is, $\upsilon = 1$, (Groezen et al., 2003),[23] and that the shares of the purchase of a fertility good and child tax payment

[21] Note that the first-order conditions of h_i and N are identical to (12) and (26), respectively, as long as interior solutions are ensured, that is, $h_i > \tau N$.

[22] Compulsory education includes lunch fees, material fees, stationery fees, field trip fees, study tour fees, and school excursion fees.

[23] This simple assumption is also adopted by Groezen et al. (2003).

are equal, that is, $\gamma_{xi} = \gamma_{Ni} (\equiv \gamma_i)$ for $i = m, f$.[24]

Under these assumptions, (3) can be rewritten as

$$z_i + (1 + t_y) p_y y_i + \gamma_i (1 + t_N) p_N N = (1 - t_i) w_i l_i, \quad i = m, f, \tag{5}$$

where $t_N (\equiv \kappa_N / p_N)$ is the tax rate on the number of children in terms of the price of the fertility good. Each spouse has a different budget constraint, which is based on the non-cooperative couple model (Lundberg and Pollak, 1993; Konrad and Lommerud, 1995; Anderberg, 2007; Lechene and Preston, 2011; Cigno, 2012; Meier and Rainer, 2015; Doepke and Tertilt, 2019; Heath and Tan, 2020).[25] Denoting γ_m as γ and hence, γ_f as $1 - \gamma$ and making use of (2) and (5), (1) can be rewritten as

$$\begin{aligned} U_m &= (1 - t_m) w_m l_m - (1 + t_y) p_y y_m - \gamma (1 + t_N) p_N N + \frac{y_m^{\varphi}}{\varphi} \\ &\quad - \frac{(l_m + h_m)^{1+\phi}}{1 + \phi} + N^{1-\sigma} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} \right], \end{aligned} \tag{6}$$

$$\begin{aligned} U_f &= (1 - t_f) w_f l_f - (1 + t_y) p_y y_f - (1 - \gamma)(1 + t_N) p_N N + \frac{y_f^{\varphi}}{\varphi} \\ &\quad - \frac{(l_f + h_f)^{1+\phi}}{1 + \phi} + N^{1-\sigma} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} \right] - c(N). \end{aligned}$$

Each spouse decides its own labor supply and time use for childcare, taking childcare time of the partner and the number of children as given. Spouse i does not consider that his or her own childcare time positively affects the partner's utility. As for childcare, as shown by Rasul (2008), partners do not commit to supplying a certain amount of childcare duties (i.e., there is no clause in the marriage contract regarding how much time each parent should spend with children). Moreover, actions are unobservable, and there is no way of monitoring the time the other partner spends with the children (Pailhé and Solaz, 2008). Hence, I resort to a Cournot–Nash non-cooperative game to model the third stage, which leads to a sub-optimally low quality of children owing to the free-rider problem.[26]

[24] Even if these two assumptions $v = 1$ and $\gamma_{xi} = \gamma_{Ni}$ are relaxed, our theoretical results are unaffected based on the following argument. The third and fourth terms on the left-hand side in (3) can be rewritten as $\left(1 + \frac{\gamma_{Ni}\kappa_N}{\gamma_{xi} p_N v}\right) \gamma_{xi} p_N v N$. Defining $\widetilde{t}_N \equiv \gamma_{Ni}\kappa_N / \gamma_{xi} p_N v$ and $\widetilde{p}_N \equiv \gamma_{xi} p_N v$, the expression becomes $\left(1 + \widetilde{t}_N\right) \widetilde{p}_N N$, which is the same form as the third term on the left-hand side in (5).

[25] Substantial evidence that each spouse has his or her own budget constraint has been documented by Pahl (1983, 1995, 2008), Kenney (2006), and Lauer and Yodanis (2014).

[26] This assumption is supported by recent econometric evidence from Del Boca and Flinn (2012), showing that one-fourth of couples under-provide household public goods because of non-cooperative behavior.

3.2 Second Stage: Couple

In the second stage, the number of children N is collectively determined as a decision of the couple. In this decision, the couple takes the income tax rates and the child tax/subsidy rate as given. The couple's utility function is a weighted average of the utility of the spouses:

$$U = \rho U_m + (1-\rho)U_f, \tag{7}$$

where ρ is the bargaining power of the husband and satisfies $0 \leq \rho \leq 1$. The value of the bargaining power ρ is assumed constant in our model.[27] If the couple considers the cost $c(N)$ as an important factor, ρ would be less than 0.5, which leads to low fertility.[28] The couple maximizes U, allowing for l_i and h_i to be functions of N, which is formulated in the third decision stage.[29] This means that the decision about the number of children is made prior to the non-cooperative decisions regarding l_i, h_i, z_i, and y_i. Even though the number of children and time devoted to childcare are collectively determined, it is possible to deviate from what is planned today and to non-cooperatively determine the amount of childcare due to a lack of commitment and effective monitoring, as explained above. Thus, we postulate that couples collectively determine the number of children prior to time inputs made non-cooperatively by both a husband and wife. In the setting, since the couple collectively decide the number of children, the determination process of N is efficient. However, the number of children is inefficiently under-provided, because the couple knows that the quality per child Q is under-provided in the next stage. This is discussed in Subsection 4.3.

This setting is applicable to housing and healthcare. For example, a couple collectively determines the area size, design, and floor plan of a house, and then, each spouse non-cooperatively provides housing maintenance. As an alternative example, the couple collectively decides their medical insurance, and then, each spouse non-cooperatively maintains his/her own health.

The number of children N can be divided into two components, $N = \overline{N} + \widetilde{N}$, where $\overline{N}$ is the preliminarily determined number of children, which can be the number of children desired by the spouse who wants the lower number of children,[30] and $\widetilde{N}$ is the endogenously determined number. For example, $\overline{N}$ is the number of children that a couple determines or promises before marriage and $\widetilde{N}$ is the number after marriage. If each spouse intends to

[27] Komura et al. (2019) consider endogenous bargaining power depending on the relative income difference between a husband and wife.

[28] As long as $\rho = 0.5$, even if the husband also bears this type of cost or even if the cost is shared by the spouses, the theoretical results obtained in this study are unaffected, because the number of children is collectively determined.

[29] Note that this optimization allows for the budget constraint of each spouse, because U_i $(i = m, f)$ in (7) corresponds to each spouse's utility given by (6).

[30] Let N^i (for $i = m, f$) to be the number of children that spouse i wants. $\overline{N}$ can be regarded as $\min[N^m, N^f]$.

have at least one child before marriage, that is, $\overline{N} = 1$, $\widetilde{N}$ can be interpreted as subsequent children determined by the spouses. Throughout this study, the change in N may be interpreted as that in $\widetilde{N}$.

3.3 First Stage: Government

The government maximizes its social welfare under a revenue constraint by manipulating income taxes and the child tax/subsidy. We suppose that the objective function of the government is the utilitarian optimum with equal weights between the husband and wife, while a possibly larger evaluation is given by the number of children multiplied by the quality per child (Bastani et al., 2017).[31] The government's welfare function is given by

$$W = U_m + U_f + \mu NQ, \tag{8}$$

where $\mu(\geq 0)$ is the weight on the children multiplied by the quality per child.[32] If $\mu > 0$, the government regards the children multiplied by the quality per child as more important than the arguments other than NQ in the spouses' utility. In this case, the country puts weight on its own future in respect of economic growth, presence, tax revenue. In particular, the promotion of human capital investment potentially increases the future tax base, which leads to a reduction of tax burdens on future generations. This term can be also interpreted as the externality of children on society, in the sense that a couple does not consider the effects of their children on society. For example, the improvement of Q enhances the local security level, promotes social skills, and reduces adverse health conditions as external effects (Heckman, 2006; Heckman and Masterov, 2007). In addition, as the quality per child improves, peer effects that children produce positive learning spillovers in school life increase. As the external effects of N, children can learn sociality from the community of children, and parents can also learn about childcare and receive information about education and medical care from other couples with children. The increase in N generates synergy effects if Q has peer effects. Moreover, under PAYG social security systems, the size of a person's pension benefits relies on the number of all households' children, as considered in many previous studies.

The revenue constraint of the government is

$$R = t_m w_m l_m + t_f w_f l_f + t_y p_y (y_m + y_f) + t_N p_N N, \tag{9}$$

[31] Bastani et al. (2017) also incorporate an additively separable term expressing such positive externalities in the social welfare function, like our social welfare function.

[32] An alternative expression of the government's welfare function is that $\widetilde{W} = \varsigma\left(\frac{1}{2}U_m + \frac{1}{2}U_f\right) + (1-\varsigma)NQ$, in which the husband and wife are associated with the same weight, and ς adjusts the difference between $U_m + U_f$ and NQ. However, this function can be reformulated by $\left(\frac{2}{\varsigma}\right)\widetilde{W} = U_m + U_f + \psi NQ$, where $\psi \equiv 2(1-\varsigma)/\varsigma$, which is the same formula as (8).

where R is the required tax revenue and its level is assumed to be constant. The government maximizes (8) with respect to t_m, t_f, t_y, and t_N, subject to (9).[33] To make the analysis meaningful, throughout this study, we assume that the required tax revenue exceeds the revenue collected from the externality-correcting tax systems.

4 Solutions of the Model

4.1 Spouse

In this section, we analyze the solutions of utility maximization of each spouse and the properties of the labor supply function and the childcare function. From (6), we obtain the first-order conditions for the utility maximization of each spouse with respect to y_i, l_i, and h_i:

$$0 = \frac{\partial U_i}{\partial y_i} = -(1+t_y)p_y + y_i^{\varphi-1}, \quad i = m, f, \tag{10}$$

$$0 = \frac{\partial U_i}{\partial l_i} = (1-t_i)w_i - (l_i + h_i)^{\phi}, \quad i = m, f, \tag{11}$$

$$0 = \frac{\partial U_i}{\partial h_i} = -(l_i + h_i)^{\phi} + N^{1-\sigma} s_i^{\sigma} h_i^{\sigma-1}, \quad i = m, f. \tag{12}$$

Defining the after-tax wage rate as $\omega_i (\equiv (1-t_i)w_i)$, (10), (11), and (12) immediately yield

$$y_i(t_y) = [(1+t_y)\, p_y]^{\frac{1}{\varphi-1}}, \quad i = m, f, \tag{13}$$

$$h_i(t_i, N; w_i, s_i) = \omega_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N, \quad i = m, f, \tag{14}$$

$$l_i(t_i, N; w_i, s_i) = \omega_i^{\frac{1}{\phi}} - \omega_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N, \quad i = m, f, \tag{15}$$

$$h_i(t_i, N; w_i, s_i) + l_i(t_i, N; w_i, s_i) = \omega_i^{\frac{1}{\phi}}, \quad i = m, f. \tag{16}$$

The aggregate time for the external labor market and domestic childcare, given by (16), depends only on the after-tax wage rate ω_i and the parameter of the sub-utility function ϕ, because of a quasi-linear utility functional form. From (14) and (15), the time spent on domestic childcare and the external labor market is affected by the productivity of household

[33] We implicitly assume that the government uses its tax revenue to purchase a public good g, satisfying $g = R$, and provides it to consumers. In addition, we assume that the public good is additively separable in each spouse's utility; that is, $U_i + g$. Then, the precise expressions for the couple's utility function and the government's objective function are $U+g$ and $W+2g$, respectively. From these functional forms and constant g, owing to the fixed revenue requirement, we find that the optimal conditions presented hereafter are not affected by g. Therefore, our results remain valid even if the constant public good is explicitly introduced to the utility functions. Furthermore, if there are many identical couples in this economy, the amount of the public good $G = Kg = KR$, where K is the number of couples and R can be interpreted as the tax revenue collected from couples. Even in this case, our results remain valid, because G is additively separable in each spouse's utility function and G is constant.

production s_i, the number of children N, as well as the after-tax wage rate ω_i, because the per-child quality depends on h_i, s_i, and N (Equation (2)). From (13), the commodity y_i depends only on the tax-inclusive price and the parameter of the sub-utility function φ.

From (14) and (15), we obtain

$$h_{iN}\left(\equiv \frac{\partial h_i}{\partial N}\right) = -l_{iN}\left(\equiv \frac{\partial l_i}{\partial N}\right) = \omega_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} > 0, \quad i = m, f, \tag{17}$$

$$h_{is_i}\left(\equiv \frac{\partial h_i}{\partial s_i}\right) = -l_{is_i}\left(\equiv \frac{\partial l_i}{\partial s_i}\right) = \left(\frac{\sigma}{1-\sigma}\right) \omega_i^{-\frac{1}{1-\sigma}} s_i^{\frac{-1+2\sigma}{1-\sigma}} N > 0, \quad i = m, f, \tag{18}$$

$$h_{i\omega_i}\left(\equiv \frac{\partial h_i}{\partial \omega_i}\right) = -\left(\frac{1}{1-\sigma}\right) \omega_i^{\frac{-2+\sigma}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N < 0, \quad i = m, f, \tag{19}$$

$$l_{i\omega_i}\left(\equiv \frac{\partial l_i}{\partial \omega_i}\right) = \frac{1}{\phi}\omega_i^{\frac{1-\phi}{\phi}} + \left(\frac{1}{1-\sigma}\right) \omega_i^{\frac{-2+\sigma}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N > 0, \quad i = m, f. \tag{20}$$

(17) and (18) show that time spent on childcare increases and time spent on the external labor market decreases with the number of children N and with the productivity of childcare s_i. These results are very intuitive. The increase in N obviously requires more time to be spent on childcare. The increase in the productivity of childcare s_i enhances the marginal utility of h_i through the change in Q and hence, the time spent on childcare increases with s_i. The amount of the increase in h_i is the same as that of the decrease in l_i, because N and s_i do not affect aggregate time $h_i + l_i$, that is, $h_{iN} + l_{iN} = 0$ and $h_{is_i} + l_{is_i} = 0$, as shown by (17) and (18). This is confirmed by (16). ω_i also gives the opposite effects on l_i and h_i: time spent on childcare decreases and that spent on the external labor market increases, while time spent on the external labor market increases with the after-tax wage ω_i. However, the amount of the increase in l_i exceeds that of the decrease in h_i. From (19) and (20), we have

$$h_{i\omega_i} + l_{i\omega_i} = \frac{1}{\phi}\omega_i^{\frac{1-\phi}{\phi}} > 0, \quad i = m, f, \tag{21}$$

which shows that the income taxation yields the price-distortions.

The comparison of time allocation between the wife and husband is also obtained from (17)–(20). The results are summarized as the following proposition.

Proposition 1. *Suppose that $\omega_i \geq \omega_j$ and $s_i \leq s_j$ with at least one strict inequality. Then, (i) $l_i > l_j$, (ii) $h_i < h_j$, (iii) $h_{iN} < h_{jN}$, and (iv) $-l_{iN} < -l_{jN}$.*

Proposition 1(i) is obtained from (18) and (20), and (ii) from (18) and (19). We also confirm these results from (14) and (15). Proposition 1(i) and (ii) show that the time allocation of the couple is similar to Ricardo's comparative advantage in international trade theory. Proposition 1(iii) and (iv) are obtained from (17). They imply that the effects of abilities w_i and s_i on time allocation increase with a larger number of children. In other words, the existence of children strengthens the movement toward a complete division of labor between the external labor market and domestic childcare if there are gender differences in productivity, w_i and s_i. In our model, a corner solution in which $N = 0$ is possible: however, to obtain fruitful suggestions, we assume that $N > 0$ under the optimal taxation. Our numerical examples, provided in Section 7, ensure that $N > 0$.

Finally, we show that the income taxation yields the price-distortions on time allocation between h_i and l_i. By noting that $l_{it_i} = -w_i l_{i\omega_i}$ and $h_{it_i} = -w_i h_{i\omega_i}$ for $i = m, f$, (19), (20), and (21) lead to

$$l_{it_i}\left(\equiv \frac{\partial l_i}{\partial t_i}\right) = -\frac{1}{\phi} w_i \omega_i^{\frac{1-\phi}{\phi}} - \left(\frac{1}{1-\sigma}\right) w_i \omega_i^{\frac{-2+\sigma}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N < 0, \quad i = m, f, \tag{22}$$

$$h_{it_i}\left(\equiv \frac{\partial h_i}{\partial t_i}\right) = \left(\frac{1}{1-\sigma}\right) w_i \omega_i^{\frac{-2+\sigma}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N > 0, \quad i = m, f, \tag{23}$$

$$l_{it_i} + h_{it_i} = -\frac{1}{\phi} w_i \omega_i^{\frac{1-\phi}{\phi}} < 0, \quad i = m, f. \tag{24}$$

The income taxes can change the time allocation between the external labor market and domestic childcare: the labor supply of spouse i decreases, and his/her childcare time increases with the income tax rate on spouse i. Thus, income taxes can play the role of correcting the non-cooperative behavior of spouses. In other words, the optimal income taxation involves the Pigouvian tax consideration. However, since the income tax reduces the total amount of time spent on the labor market and domestic childcare, as shown by (24), it inevitably yields the price-distortions.

4.2 Couple

In this subsection, we consider the couple's decision on the number of children. Allowing for (13)–(15), the couple maximizes (7) with respect to N; this is the collective optimization problem of the couple. (7) is represented by

$$\begin{aligned}U &= \rho\Bigg[(1-t_m)w_m l_m(t_m,N)-(1+t_y)p_y y_m(t_y)+\frac{(y_m(t_y))^{\varphi}}{\varphi} \\ &\quad -\frac{(l_m(t_m,N)+h_m(t_m,N))^{1+\phi}}{1+\phi}\Bigg] \\ &\quad +(1-\rho)\Bigg[(1-t_f)w_f l_f(t_m,N)-(1+t_y)p_y y_f(t_y)+\frac{(y_f(t_y))^{\varphi}}{\varphi} \\ &\quad -\frac{(l_f(t_f,N)+h_f(t_f,N))^{1+\phi}}{1+\phi}-c(N)\Bigg] \\ &\quad -\left[(1-\rho)(1-\gamma)+\rho\gamma\right](1+t_N)p_N N \\ &\quad +N^{1-\sigma}\left[\frac{(s_m h_m(t_m,N))^{\sigma}}{\sigma}+\frac{(s_f h_f(t_f,N))^{\sigma}}{\sigma}\right].\end{aligned} \tag{25}$$

Allowing for (11), (12), and (17), the first-order condition with respect to N is given by[34]

$$\begin{aligned}0 &= \frac{\partial U}{\partial N}=-\left[(1-\rho)(1-\gamma)+\rho\gamma\right](1+t_N)p_N-(1-\rho)c'(N) \\ &\quad +\left(\frac{1-\sigma}{\sigma}+1-\rho\right)(1-t_m)w_m h_{mN}(t_m) \\ &\quad +\left(\frac{1-\sigma}{\sigma}+\rho\right)(1-t_f)w_f h_{fN}(t_f).\end{aligned} \tag{26}$$

(See Appendix A). This implies that

$$N = N(t_N, t_m, t_f). \tag{27}$$

Although the number of children is collectively determined, it downwardly deviates from an efficient level. This is specifically discussed in Subsection 4.3. In this subsection, we provide an intuition for this result. From (2), we observe that $NQ = (N^{1-\sigma}/\sigma)\left[(s_m h_m)^{\sigma}+(s_f h_f)^{\sigma}\right]$, which shows that the smaller h_i leads to the lower marginal utility of N. Because the spouses non-cooperatively take care of their children in the third stage, the amount of h_i is under-provided. Thus, the number of the children is also under-provided.

Totally differentiating (26) with respect to N, t_m, t_f, and t_N and making use of (17) yields the following results

$$N_{t_m}\left(\equiv\frac{\partial N}{\partial t_m}\right)=\frac{\left[1+(1-\rho)\left(\frac{\sigma}{1-\sigma}\right)\right]w_m\omega_m^{-\frac{1}{1-\sigma}}s_m^{\frac{\sigma}{1-\sigma}}}{(1-\rho)c''}>0, \tag{28}$$

[34]From (17), we observe that $h_{iN}(t_i)$ for $i=m,f$.

$$N_{t_f}\left(\equiv\frac{\partial N}{\partial t_f}\right)=\frac{\left[1+\rho\left(\frac{\sigma}{1-\sigma}\right)\right]w_f\omega_f^{-\frac{1}{1-\sigma}}s_f^{\frac{\sigma}{1-\sigma}}}{(1-\rho)c''}>0, \tag{29}$$

$$N_{t_N}\left(\equiv\frac{\partial N}{\partial t_N}\right)=-\frac{[(1-\rho)(1-\gamma)+\rho\gamma]\,p_N}{(1-\rho)c''}<0. \tag{30}$$

From (28)–(30), we obtain the following proposition.

Proposition 2. *(i) $N_{t_i}>0$ for $i=m,f$, and $N_{t_N}<0$. (ii) Suppose that $\rho=0.5$. Then, if $w_m \gtreqless w_f$ and $s_m=s_f$, $N_{t_m}\lesseqgtr N_{t_f}$, and if $w_m=w_f$ and $s_m\lesseqgtr s_f$, $N_{t_m}\lesseqgtr N_{t_f}$. (iii) Suppose that $w_m=w_f$ and $s_m=s_f$. Then, if $\rho\gtreqless 0.5$, $N_{t_m}\lesseqgtr N_{t_f}$.*

Proposition 2(i) shows that the number of children increases with the income tax rates. As mentioned above, children are under-provided because both spouses are aware of non-cooperative behavior for childcare in the next stage. Since time spent on childcare increases with the income tax, which is shown by (23), the quality per child is improved with the income tax rates and then the number of children is also increased. This result has a very interesting policy implication. The income taxation both raises tax revenue and improves low fertility. There is overwhelming empirical evidence that fertility is negatively related to the wage rate in most countries at most times (Jones and Tertilt, 2008; Jones et al., 2010), which supports our theoretical results. Although income effects due to income reduction have negative effects on the fertility rate, the decrease in after-tax wage lowers the opportunity cost of having children. As a result, the former effect is not actually so large and hence, the increase in income tax can raise the fertility rate.

As discussed in the last part of Subsection 3.2, if $\overline{N}=1$, the change in N can be interpreted as the change in subsequent children after the first child. Baughman and Dickert-Conlin (2009) empirically show that the income tax deduction decreases the number of subsequent children after the first child, which supports the first result in Proposition 2(i).

The second result in Proposition 2(i) shows that the direct child subsidy unambiguously raises the fertility rate. The intuition of this result is straightforward. Proposition 2(i) shows that both the high-income tax rate and low child tax (or child subsidy) rate increases the number of children. Here, an important question arises: which of these two instruments plays the role of correcting low fertility caused by non-cooperative behavior in a revenue-constrained optimal tax framework? This is examined in Section 5.

From Proposition 2(ii), we observe that the income tax imposed on the spouse with the lower productivity in the external labor market, or with the higher productivity of childcare, yields a higher birthrate-improvement effect. This is because, as shown by Proposition 1(iii), the income tax imposed on such a spouse yields larger marginal effects on childcare time. Proposition 2(iii) shows that an increase in the income tax rate on a spouse with lower bargaining power induces a couple to have more children. In other words, although income taxes improve the number of children, the effect of income taxes on a spouse with higher bargaining power is limited. Without loss of generality, we consider a case in which the bargaining power of the husband is larger (i.e., $\rho > 0.5$). Notice that the increase in t_m directly decreases the disposable income of the husband, although the increase in t_f does not directly affect the disposable income. Given this fact and $l_{mN} < 0$, the husband desires fewer children to mitigate the reduction of his private consumption when t_m increases than when t_f increases. Thus, since the couple's decision about N regards the husband's utility as being more important, the increase in N is more mitigated when t_m increases than when t_f increases.

Before analyzing the government's optimization, we provide the functions of l_i and h_i, which allows for (14), (15), and (27) as

$$l_i(t_i, N(t_N, t_m, t_f)), \quad h_i(t_i, N(t_N, t_m, t_f)), \quad i = m, f. \tag{31}$$

These functions involve information about the decision in the second and third stages. Allowing for (31), the government maximizes social welfare subject to the tax revenue constraint.

4.3 First-best Allocation of Time and Number of Children

The objective of this subsection is to justify the government's intervention for correcting the inefficiently low fertility due to non-cooperative behavior by a couple. Thus, we compare two allocations without the government's intervention: the first-best Pareto-efficient allocation and a household allocation in our non-cooperative decision-making model. If the number of children under the non-cooperative setting deviates from the (first-best) socially efficient level, then efficiency-enhancing policy intervention is desirable. First, we derive the first-best Pareto efficient allocation without the government's intervention, which corresponds to a maximization problem of one partner's utility subject to a given level of the other and the resource constraint. The Lagrangian is expressed by

$$\begin{aligned}
\max_{\substack{z_m,\, z_f,\, y_m,\, y_f,\, l_m,\\ l_f,\, h_m,\, h_f,\, N}} \quad \mathcal{L} \quad = \quad & z_m + \frac{y_m^{\varphi}}{\varphi} - \frac{(l_m + h_m)^{1+\phi}}{1+\phi} + N^{1-\sigma}\left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma}\right] \\
& + \iota \left\{ z_f + \frac{y_m^{\varphi}}{\varphi} - \frac{(l_f + h_f)^{1+\phi}}{1+\phi} \right. \\
& \left. + N^{1-\sigma}\left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma}\right] - c(N) - \overline{U}_f \right\} \\
& + \zeta \left(w_m l_m + w_f l_f - z_m - z_f - p_y(y_m + y_f) - p_N N \right),
\end{aligned} \tag{32}$$

where, ι and ζ are Lagrange multipliers.[35] The first-order conditions are

$$0 = \frac{\partial \mathcal{L}}{\partial z_m} = 1 - \zeta, \tag{33}$$

$$0 = \frac{\partial \mathcal{L}}{\partial z_f} = \iota - \zeta, \tag{34}$$

$$0 = \frac{\partial \mathcal{L}}{\partial y_m} = y_m^{\varphi - 1} - \zeta p_y, \tag{35}$$

$$0 = \frac{\partial \mathcal{L}}{\partial y_f} = \iota y_f^{\varphi - 1} - \zeta p_y, \tag{36}$$

$$0 = \frac{\partial \mathcal{L}}{\partial l_m} = -\left(l_m + h_m\right)^{\phi} + \zeta w_m, \tag{37}$$

$$0 = \frac{\partial \mathcal{L}}{\partial l_f} = -\iota \left(l_f + h_f\right)^{\phi} + \zeta w_f, \tag{38}$$

$$0 = \frac{\partial \mathcal{L}}{\partial h_m} = -\left(l_m + h_m\right)^{\phi} + (1+\iota) N^{1-\sigma} s_m^{\sigma} h_m^{\sigma - 1}, \tag{39}$$

$$0 = \frac{\partial \mathcal{L}}{\partial h_f} = -\iota \left(l_f + h_f\right)^{\phi} + (1+\iota) N^{1-\sigma} s_f^{\sigma} h_f^{\sigma - 1}, \tag{40}$$

$$0 = \frac{\partial \mathcal{L}}{\partial N} = (1+\iota)(1-\sigma) N^{-\sigma} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} \right] - \iota c'(N) - \zeta p_N. \tag{41}$$

Before comparing the number of children between the two cases, to avoid confusion, we denote the number of children under the Pareto-efficient allocation by N^{PE} and that under the non-cooperative case by N^{NC}.[36] Using (33)–(40), (41) can be rewritten as

$$N^{PE} : 0 = 2^{\frac{\sigma}{1-\sigma}} \left(\frac{1-\sigma}{\sigma} \right) \left(w_m^{\frac{-\sigma}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}} + w_f^{\frac{-\sigma}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}} \right) - \frac{1}{2} c'(N^{PE}) - \frac{1}{2} p_N. \tag{42}$$

(see Appendix B). (42) determines N^{PE}. We next derive the condition that determines the number of children under the non-cooperative case. Given $t_i = 0$ for $i = m, f$ and $t_N = 0$, substituting (17) for h_{iN} in (26) yields

$$\begin{aligned} N^{NC} : 0 = & -(1-\rho) c'(N^{NC}) - \left[(1-\rho)(1-\gamma) + \rho\gamma\right] p_N \\ & + \left(\frac{1-\sigma}{\sigma} + 1 - \rho \right) w_m^{\frac{-\sigma}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}} + \left(\frac{1-\sigma}{\sigma} + \rho \right) w_f^{\frac{-\sigma}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}}, \end{aligned} \tag{43}$$

which determines N^{NC}.

To clarify the effect of non-cooperative household behavior on the number of children,

[35]Note that Q is replaced by the right-hand side in (2).

[36]Note that N^{NC} is the number of children when there are no taxes and subsidies.

we consider $\rho = 0.5$, that is, we eliminate the difference of bargaining power across spouses. In this case, (43) can be rewritten as

$$N^{NC} : 0 = \left(\frac{1-\sigma}{\sigma} + \frac{1}{2}\right)\left(w_m^{\frac{-\sigma}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}} + w_f^{\frac{-\sigma}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}}\right) - \frac{1}{2}c'(N^{NC}) - \frac{1}{2}p_N. \tag{44}$$

Noting that $c(N)$ is a strictly convex function, from (42) and (44), we observe that $N^{PE} > N^{NC}$ if $2^{\frac{\sigma}{1-\sigma}}\left(\frac{1-\sigma}{\sigma}\right) - \frac{1-\sigma}{\sigma} - \frac{1}{2} (\equiv \pi(\sigma)) > 0$. We can prove that $\pi(\sigma) > 0$ for any $0 < \sigma < 1$ (see Appendix C). Therefore, $N^{PE} > N^{NC}$. This is summarized as the following proposition.

Proposition 3. *Under $\rho = 0.5$, if a couple non-cooperatively provides the time invested in children, the number of children is under-provided, that is, $N^{PE} > N^{NC}$.*

Although N^{PE} is realized under a situation in which the time devoted to childcare and the number of children are determined simultaneously, the sequential decision, that the number of children is determined prior to consumption and time allocation, does not result in under-investment in fertility. Indeed, the number of children, which is determined prior to collective decisions concerning Q, is at the same level as that under the Pareto-efficient allocation, that is, $N^{PE} = N^C$, where N^C denotes the number of children under the collective case in the sequential decision (see Appendix D). Thus, when $\rho = 0.5$ holds, the low fertility stems from only the non-cooperative household behavior. Furthermore, this argument holds even under the introduction of a childcare facility in Section 6.[37] This implies that, the time children spend in a childcare facility does not solve parental under-investment in childcare owing to the non-cooperative household behavior, although it increases the quality per child.

5 Optimal Taxation

In this section, we examine the optimal structure of the income taxes and the child tax/subsidy. The income tax rates can be differentiated across genders, which is so-called "gender-based taxation." The gender-based taxation is a generalization of the linear income taxation on a couple. The case with the common income tax rate on a couple, which is more restrictive and realistic tax system, essentially yields the same results as the case with gender-based taxation (see Appendix E). By allowing for (13), (27), and (31), the government's welfare

[37] We provide an outline of the proof. First, we conclude that $N^{PE} > N^{NC}$ holds even in the presence of a childcare facility using $\pi(\sigma) > 0$ for any $0 < \sigma < 1$, which is shown in Appendix C. Furthermore, using a similar method in Appendix D, we show that $N^{PE} = N^C$ holds even under a childcare facility.

function and tax revenue constraint are represented by

$$\begin{aligned} W &= (1-t_m)w_m l_m(t_m, N(t_N, t_m, t_f)) + \frac{(y_m(t_y))^\varphi}{\varphi} \\ &- \frac{(l_m(t_m, N(t_N, t_m, t_f)) + h_m(t_m, N(t_N, t_m, t_f)))^{1+\phi}}{1+\phi} \\ &+ (1-t_f)w_f l_f(t_f, N(t_N, t_m, t_f)) + \frac{(y_f(t_y))^\varphi}{\varphi} \\ &- \frac{(l_f(t_f, N(t_N, t_m, t_f)) + h_f(t_f, N(t_N, t_m, t_f)))^{1+\phi}}{1+\phi} \\ &- c(N(t_N, t_m, t_f)) - (1+t_N)p_N N(t_N, t_m, t_f) \\ &- (1+t_y)p_y(y_m(t_y) + y_f(t_y)) + (2+\mu)(N(t_N, t_m, t_f))^{1-\sigma} \\ &\cdot \left[\frac{(s_m h_m(t_m, N(t_N, t_m, t_f)))^\sigma}{\sigma} + \frac{(s_f h_f(t_f, N(t_N, t_m, t_f)))^\sigma}{\sigma}\right], \end{aligned} \tag{45}$$

$$\begin{aligned} R &= t_m w_m l_m(t_m, N(t_N, t_m, t_f)) + t_f w_f l_f(t_f, N(t_N, t_m, t_f)) \\ &+ t_y p_y \left(y_m(t_y) + y_f(t_y)\right) + t_N p_N N(t_N, t_m, t_f), \end{aligned} \tag{46}$$

respectively. The government maximizes welfare (45) under the tax revenue constraint (46) by manipulating t_y, t_m, t_f, and t_N. Let us define the Lagrange function as L and the Lagrange multiplier on the revenue constraint as λ. Allowing for (17), the first-order conditions with respect to t_y, t_m, t_f, and t_N are given by

$$\begin{aligned} 0 &= \frac{\partial L}{\partial t_y} = -p_y y_m - (1+t_y)p_y y_m' - p_y y_f - (1+t_y)p_y y_f' \\ &+ y_m^{\varphi-1}y_m' + y_f^{\varphi-1}y_f' - \lambda[y_m + y_f + t_y(y_m' + y_f')]p_y, \end{aligned} \tag{47}$$

$$\begin{aligned} 0 &= \frac{\partial L}{\partial t_m} = -w_m l_m + (1-t_m)w_m l_{mt_m} + (1-t_m)w_m l_{mN} N_{t_m} \\ &- (l_m + h_m)^\phi \left(l_{mt_m} + h_{mt_m}\right) + (1-t_f)w_f l_{fN} N_{t_m} - c' N_{t_m} \\ &- (1+t_N)p_N N_{t_m} + (2+\mu)\left(1-\sigma\right) N^{-\sigma} N_{t_m} \left[\frac{(s_m h_m)^\sigma}{\sigma} + \frac{(s_f h_f)^\sigma}{\sigma}\right] \\ &+ (2+\mu)N^{1-\sigma}\left[s_m^\sigma h_m^{\sigma-1}\left(h_{mt_m} + h_{mN}N_{t_m}\right) + s_f^\sigma h_f^{\sigma-1} h_{fN} N_{t_m}\right] \\ &- \lambda\left(w_m l_m + t_m w_m l_{mt_m} + t_m w_m l_{mN} N_{t_m} + t_f w_f l_{fN} N_{t_m} + t_N p_N N_{t_m}\right), \end{aligned} \tag{48}$$

$$
\begin{aligned}
0 \quad = \quad \frac{\partial L}{\partial t_f} &= (1-t_m)w_m l_{mN} N_{t_f} - w_f l_f + (1-t_f) w_f l_{ft_f} + (1-t_f) w_f l_{fN} N_{t_f} \\
&- (l_f + h_f)^{\phi} \left(l_{ft_f} + h_{ft_f} \right) - c' N_{t_f} - (1+t_N) p_N N_{t_f} \\
&+ (2+\mu)(1-\sigma) N^{-\sigma} N_{t_f} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} \right] \\
&+ (2+\mu) N^{1-\sigma} \left[s_m^{\sigma} h_m^{\sigma-1} h_{mN} N_{t_f} + s_f^{\sigma} h_f^{\sigma-1} (h_{ft_f} + h_{fN} N_{t_f}) \right] \\
&- \lambda \left(t_m w_m l_{mN} N_{t_f} + w_f l_f + t_f w_f l_{ft_f} + t_f w_f l_{fN} N_{t_f} + t_N p_N N_{t_f} \right),
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
0 \quad = \quad \frac{\partial L}{\partial t_N} &= (1-t_m)w_m l_{mN} N_{t_N} + (1-t_f) w_f l_{fN} N_{t_N} - c' N_{t_N} - p_N N \\
&- (1+t_N) p_N N_{t_N} + (2+\mu)(1-\sigma) N^{-\sigma} N_{t_N} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} \right] \\
&+ (2+\mu) N^{1-\sigma} \left(s_m^{\sigma} h_m^{\sigma-1} h_{mN} N_{t_N} + s_f^{\sigma} h_f^{\sigma-1} h_{fN} N_{t_N} \right) \\
&- \lambda \left(t_m w_m l_{mN} N_{t_N} + t_f w_f l_{fN} N_{t_N} + p_N N + t_N p_N N_{t_N} \right).
\end{aligned} \tag{50}
$$

From these conditions, we first provide the optimal tax expressions and then discuss the optimal tax structure.

First, we examine the optimal tax rate on commodity y_i. Using (10) and (47), we immediately observe that

$$
r_y \left(\equiv \frac{t_y}{1+t_y} \right) = \frac{\beta}{\Xi}, \tag{51}
$$

where $\beta \equiv \frac{1+\lambda}{\lambda}$ and $\Xi \equiv -\frac{(1+t_y)(y'_m + y'_f)}{(y_m + y_f)}$.[38] This is a standard Ramsey tax expression, and the optimal tax rate on commodity y_i follows the well-known inverse elasticity rule. Let us consider the case in which the lump-sum tax is available for the government; that is, the government does not virtually face the revenue constraint. Let us denote the lump-sum tax by t_L. Since a couple consists of two spouses, $2t_L$ is subtracted from the government's welfare and is added to the revenue constraint. Thus, the first-order condition with respect to t_L is that $t_L : 0 = -2 - 2\lambda$, which leads to $\lambda = -1$ and hence, $\beta (\equiv (1+\lambda)/\lambda) = 0$. Therefore, the optimal tax rate on commodity y is zero. This is a natural consequence of the optimal tax theory under a revenue constraint. However, this consequence does not hold for the optimal income taxes and the child tax/subsidy in our model, as shown below.

Using (11), (12), and (50), after some manipulation, (48) and (49) can be rewritten as the following simple conditions (see Appendix F):

$$
\begin{aligned}
t_m \quad : \quad 0 = &-(1+\lambda) w_m l_m - \lambda t_m w_m l_{mt_m} \\
&+ (1+\mu)(1-t_m) w_m h_{mt_m} + (1+\lambda) p_N N N_{t_N}^{-1} N_{t_m},
\end{aligned} \tag{52}
$$

[38] Note that $y'_i (\equiv \partial y_i / \partial t_y) = p_y (\partial y_i / \partial (1+t_y) p_y)$. Thus, Ξ is the own price elasticity of commodity y.

$$t_f \quad : \quad 0 = -(1+\lambda) w_f l_f - \lambda t_f w_f l_{ft_f} + (1+\mu)(1-t_f) w_f h_{ft_f} + (1+\lambda) p_N N N_{t_N}^{-1} N_{t_f}. \tag{53}$$

These conditions explain the effects of the income taxes clearly and intuitively. The first two terms reflect the price-distortion effects on resource allocation between the working time and the consumption of the numeraire. These terms are related to a standard Ramsey tax implication. In our model, the income taxes also alter the time allocation from hence he external labor market to childcare time. This effect is described by the third term, which involves the corrective taxes for non-cooperative behavior, taking account of the weight μ on NQ. The fourth term reflects the effects on the number of children. Although the number of children is inefficiently under-provided in our model, this term does not reflect the externality-correcting term but is related to the tax-induced price-distortions under a revenue constraint. To confirm this, we consider the case in which the lump-sum tax is available; that is, there is virtually no revenue constraint. As shown above, the availability of the lump-sum tax leads to $\lambda = -1$ and hence, the fourth term vanishes.

Before presenting the optimal income tax expression, we define some elasticities as

$$\eta_i \equiv \frac{\omega_i l_{i\omega_i}}{l_i} = -\frac{\omega_i l_{it_i}}{l_i w_i} = -\frac{(1-t_i) l_{it_i}}{l_i} > 0, \quad i = m, f, \tag{54}$$

$$\varepsilon_i \equiv -\frac{\omega_i h_{i\omega_i}}{h_i} = \frac{\omega_i h_{it_i}}{h_i w_i} = \frac{(1-t_i) h_{it_i}}{h_i} > 0, \quad i = m, f,$$

$$\theta_i \equiv -\frac{\omega_i N_{\omega_i}}{N} = \frac{\omega_i N_{t_i}}{N w_i} = \frac{(1-t_i) N_{t_i}}{N} > 0, \quad i = m, f,$$

$$\delta \equiv -\frac{(1+t_N) N_{t_N}}{N} > 0,$$

where we use the definition of $\omega_i (\equiv (1-t_i) w_i)$.[39] η_i is the (after-tax) wage elasticity of labor supply and ε_i the (after-tax) wage elasticity of childcare time. Since ω_i is the opportunity cost of h_i, the definition of ε_i is multiplied by -1. θ_i is the wage elasticity of the number of children and involves the effect of ω_i on N through the determination of N in the second stage. δ can be interpreted as the price elasticity of the number of children through the determination of N in the second stage. Note that all elasticities are defined as positive values in this study. In addition, we adopt the following definitions:

$$\alpha_{hl}^i \equiv \frac{(1-t_i) w_i h_i}{(1-t_i) w_i l_i}, \quad \alpha_{Nl}^i \equiv \frac{(1+t_N) p_N N}{(1-t_i) w_i l_i}, \quad i = m, f. \tag{55}$$

α_{hl}^i is the ratio between the after-tax labor income and the value of childcare evaluated by

[39]Note that $N_{t_N} (\equiv \partial N / \partial t_N) = p_N (\partial N / \partial (1+t_N) p_N)$. Thus, δ is the price elasticity of the number of children.

the opportunity cost, and α^i_{Nl} is the expenditure share of the childcare expenses on the after-tax labor income. The tax rates are defined by

$$r_i \equiv \frac{t_i}{1-t_i}, \quad i = m, f, \qquad r_N \equiv \frac{t_N}{1+t_N}. \tag{56}$$

Note the following three points concerning the definitions of r_i. First, from the definition of r_i, we observe that $\partial r_i/\partial t_i = 1/(1-t_i)^2 > 0$ and that $\partial r_N/\partial t_N = 1/(1+t_N)^2 > 0$. Second, allowing for the first property, we observe that $t_m \gtreqless t_f \Longleftrightarrow r_m \gtreqless r_f$. Third, the sign of r_i is the same as that of t_i since $t_i < 1$ for $i = m, f$ while the sign of r_N is the same as that of t_N since $t_N > -1$. In addition, since the optimal tax expressions of r_i and r_N are very simple and intuitive, we treat the optimal tax expressions of r_i and r_N to examine the properties and structure of the optimal taxation.

Using (54)–(56), (52) and (53) are transformed by the following optimal tax formula, respectively (see Appendix G).

Proposition 4. *In the endogenous fertility model, the optimal income tax rates are given by*

$$r_m = \frac{\beta\left(1+\alpha^m_{Nl}\frac{\theta_m}{\delta}\right) + (1+\mu)(1-\beta)\,\alpha^m_{hl}\varepsilon_m}{\eta_m}, \tag{57}$$

$$r_f = \frac{\beta\left(1+\alpha^f_{Nl}\frac{\theta_f}{\delta}\right) + (1+\mu)(1-\beta)\,\alpha^f_{hl}\varepsilon_f}{\eta_f}, \tag{58}$$

where $\beta \equiv \frac{1+\lambda}{\lambda}$ *and hence,* $1-\beta \equiv -\frac{1}{\lambda}$.[40]

The elasticity η_i, which is in the denominator, is related to the price-distortions between the consumption of the numeraire and the working time. The optimal income tax rate r_i becomes lower as the value of η_i increases relative to the other elasticities and the expenditure shares. To clarify this, let us consider the case in which $\theta_m = \theta_f$, $\varepsilon_m = \varepsilon_f$, $\alpha^m_{Nl} = \alpha^f_{Nl}$, and $\alpha^m_{hl} = \alpha^f_{hl}$. In this case, from (57) and (58), we observe that $r_m \gtreqless r_f \Longleftrightarrow \eta_m \lesseqgtr \eta_f$: the higher tax rate should be imposed on the income of the spouse with the smaller wage elasticities of labor supply, which implies that the optimal gender-based taxation involves Ramsey's inverse elasticity rule (Boskin and Sheshinski, 1983).

ε_i is related to the corrective effects on under-investment in childcare and the sub-optimally low fertility, since an improvement of the childcare time improves the inefficient fertility level. The income taxes on spouse i correct the inefficiently low childcare time due to non-cooperative behavior, as shown by (23). This means that the income taxation has a double dividend: it can increase tax revenue as well as correct the low fertility caused

[40] Note that $1-\beta > 0$, since $\lambda < 0$. We numerically confirm that β is also positive in the numerical examples provided in Section 7, regardless of the availability of a childcare facility.

by non-cooperative behavior. Thus, as shown by (57) and (58), the optimal income tax rate r_i becomes higher as ε_i is larger, *ceteris paribus*. Concerning the relative tax rates, we observe that $r_m \gtreqless r_f \iff \varepsilon_m \gtreqless \varepsilon_f$ if the other elasticities and all the shares are equal between a wife and husband. Another implication is that the corrective effect of income taxes should be considered as being more important as μ increases. Because NQ becomes a more important factor in social welfare when the value of μ is larger, more time spent on childcare should be induced by the higher income taxes to improve Q and then N. When the value of μ is larger, NQ becomes a more important factor in social welfare. This implies that the policymaker considers that inefficiencies arising from non-cooperative behavior are their primary target. Thus, more time on childcare should be induced by the higher income taxes to improve Q and then N. This differs from the result under the cooperative setting in that the optimal design of income taxation takes account of the external effect of children on society. Indeed, if $\varepsilon_i = 0$ (i.e., it corresponds to the cooperative setting), the externality (μ) does not affect the optimal income tax rate on each spouse.

Next, we discuss the relationship between the optimal income taxes and the elasticity θ_i/δ. Since θ_i expresses the effect of the income tax on the number of children and $1/\delta$ the effect of the number of children on the child tax/subsidy, we observe that θ_i/δ reflects the effects of the income tax (t_i for $i = m, f$) on the child tax/subsidy (t_N) through the change in the number of children (N). As shown by (28) and (29), the increase in t_i $(i = m, f)$ raises N, which leads to the increase in t_N owing to the rise in tax base of t_N. Thus, we observe that the increase in t_N partially reduces N and hence, can mitigate the deadweight loss of labor supply from (17). Therefore, as θ_i/δ becomes larger, the optimal income tax rate t_i becomes higher. If the lump-sum tax is available (i.e., $\beta = 0$), the consideration is not needed under optimal taxation.

We next provide the expression on the optimal child tax/subsidy. Before doing so, let us define

$$\chi_i \equiv -\frac{N l_{iN}}{l_i} > 0, \quad i = m, f. \tag{59}$$

χ_i denotes the elasticity of working time in the outside labor market with respect to the number of children. Using (26), (50) can be rewritten as

$$\begin{aligned} t_N \quad : \quad 0 = & -(1+\lambda)p_N N N_{t_N}^{-1} - \lambda(t_m w_m l_{mN} + t_f w_f l_{fN} + t_N p_N) \\ & + (1-t_m)w_m l_{mN} + (1-t_f)w_f l_{fN} - (1+t_N)p_N \\ & - (2+\mu)\rho(1-t_m)w_m l_{mN} - (2+\mu)(1-\rho)(1-t_f)w_f l_{fN} \\ & + [(2+\mu)(1-\rho)-1]c' + (2+\mu)[(1-\rho)(1-\gamma)+\rho\gamma](1+t_N)p_N. \end{aligned} \tag{60}$$

Applying (54)–(56) and (59) to (60), we present the optimal child tax/subsidy expression in the following proposition (see Appendix H).

Proposition 5. *In the endogenous fertility model, the optimal child subsidy/tax is given by*

$$r_N = \frac{\beta}{\delta} + \frac{r_m \chi_m}{\alpha_{Nl}^m} + \frac{r_f \chi_f}{\alpha_{Nl}^f} + (1-\beta)\Lambda, \tag{61}$$

where

$$\begin{aligned} \Lambda \quad \equiv \quad & \left\{ [1-(2+\mu)\rho] \frac{\chi_m}{\alpha_{Nl}^m} + [1-(2+\mu)(1-\rho)] \frac{\chi_f}{\alpha_{Nl}^f} \right\} \\ & + \frac{[1-(2+\mu)(1-\rho)]\, c'}{(1+t_N)p_N} + \{1-(2+\mu)\,[(1-\rho)(1-\gamma)+\rho\gamma]\}\,. \end{aligned} \tag{62}$$

The first term β/δ in (61) reflects its own price distortion on the number of children, that is, the Ramsey tax implication: the direct child tax/subsidy rate should be inversely proportional to the own-price elasticity δ. The second and third terms are related to the effect of t_N on the deadweight loss created by income taxes t_m and t_f, respectively, through the change in the number of children. Noticing that $\chi_i (\equiv -N l_{iN}/l_i)$ includes l_{iN} and that χ_i is multiplied by the tax rate r_i, we observe that $r_i \chi_i / \alpha_{Nl}^i$ reflects the effects of t_N on the price-distortions on the labor supply of spouse i through the change in the number of children. Since the larger value of χ_i reflects the larger response of l_i due to the change in N, the larger χ_i implies that the increase in the child tax makes the income tax-induced deadweight loss smaller. Thus, as the second and third terms increase, child taxes tend to become more desirable. The last term Λ allows for bargaining power between the spouses ρ and the government's weight on children μ. As β, which is related to the required tax revenue, increases (decreases), the first term should be valued more (less) than the fourth term for characterizing the optimal child tax/subsidy.

To obtain an intuition of the optimal child tax/subsidy more clearly, let us consider the case in which $\rho = 0.5$ and $\mu = 0$. In this case, $\Lambda|_{\rho=0.5,\ \mu=0} = 0$; that is, Λ is generated when the weights are different between the spouses in the couple's utility function ($\rho \neq 0.5$) or when the government regards children as being more important than parents' utility ($\mu \neq 0$). The role of ρ and μ in characterizing the optimal tax structure is explained later. When $\rho = 0.5$ and $\mu = 0$, (61) is reduced to

$$r_N = \frac{\beta}{\delta} + \frac{r_m \chi_m}{\alpha_{Nl}^m} + \frac{r_f \chi_f}{\alpha_{Nl}^f}. \tag{63}$$

As shown above, if the lump-sum tax is available for the government, $\beta (\equiv (1+\lambda)/\lambda) = 0$ and hence, β/δ in (63) vanishes. This is because, as mentioned above, β/δ is related to the price-distortion effect of the number of children under a revenue constraint. However, even if the lump-sum tax is available, (63) shows that the child tax/subsidy is not zero: the optimal intervention for a child is unambiguously to impose a tax. The income taxes act as a device to correct under-investment in childcare and hence, improve the sub-optimally low fertility through the enhancement of the quality per child. However, income taxation reduces the aggregate working time $l_i + h_i$, which implies the occurrence of the deadweight loss. To partially repress the distortions, the optimal intervention for children is to impose a tax, because the child tax lowers N, as shown by (30), and the decreases in N raises l_i, as shown by (17).

Here, we clarify the roles of the income taxes and the direct child tax/subsidy to correct the inefficiently low fertility caused by non-cooperative behavior of the spouses. Assuming that $\mu = 0$, $\rho = 0.5$, and the lump-sum tax is available ($\beta = 0$), (57), (58), and (61) can be rewritten as

$$r_i = \frac{\alpha_{hl}^i \varepsilon_i}{\eta_i} > 0, \quad i = m, f, \qquad r_N = \frac{r_m \chi_m}{\alpha_{Nl}^m} + \frac{r_f \chi_f}{\alpha_{Nl}^f} > 0. \tag{64}$$

From this result and Proposition 2(i), we undoubtedly observe that income taxation, but not direct child subsidy, plays the role of correcting the low fertility arising from under-investment in childcare due to non-cooperative behavior of the spouses. Income taxes can directly correct the inefficient decision on childcare time, h_i, and hence, enhance the low fertility, N, by improving the quality per child, Q. However, the direct child subsidy cannot create such effects. Thus, the income taxes are more effective policy instruments for improving low fertility stemming from non-cooperative behavior of spouses.

Now, we turn to exploring the implication of the last term Λ, which is related to the bargaining power between the spouses ρ and the government's weight on children μ. First, to focus on the role of the bargaining power, suppose that $\mu = 0$. Then, (62) can be rewritten as

$$\Lambda|_{\mu=0} = 2\,(\rho - 0.5)\left[\left(\frac{\chi_f}{\alpha_{Nl}^f} - \frac{\chi_m}{\alpha_{Nl}^m}\right) + \frac{c'}{(1+t_N)p_N} + 2\,(0.5 - \gamma)\right]. \tag{65}$$

Totally differentiating (26) with respect to ρ and N and making use of (17), (55), and (59), we obtain[41]

$$\frac{\partial N}{\partial \rho} = \left[\frac{(1+t_N)p_N}{c''(1-\rho)}\right]\left[\left(\frac{\chi_f}{\alpha_{Nl}^f} - \frac{\chi_m}{\alpha_{Nl}^m}\right) + \frac{c'}{(1+t_N)p_N} + 2\,(0.5 - \gamma)\right]. \tag{66}$$

(see Appendix I). Noting that $1 - \rho > 0$, $1 + t_N > 0$, and $c'' > 0$, from (65) and (66), we observe that

$$\Lambda|_{\mu=0} \gtreqless 0 \Longleftrightarrow (\rho - 0.5)\left(\frac{\partial N}{\partial \rho}\right) \gtreqless 0. \tag{67}$$

First, we clarify the meaning of (67). Without loss of generality, we assume that $\rho > 0.5$: the bargaining power of the husband is larger than that of the wife. In this case, the government wants to encourage distributions from the husband to the wife because the weights on the

[41] $\partial N/\partial \rho$ is independent of μ, because N is determined by the couple ignoring μ.

spouses in the government's welfare function are equal. If $\partial N/\partial\rho > (<)0$, the husband wants to increase (decrease) the number of children. Allowing for this fact, the government increases (decreases) the child tax rate, in the view of distribution between the spouses. Thus, from (61), the optimal child tax (subsidy) increases with the absolute value of Λ if $\Lambda > (<)0$. This argument holds even in the case in which $\rho < 0.5$, that is, the wife has more bargaining power than the husband. When $\rho = 0.5$, since the weights on the spouses are equal between the couple's utility and the government's welfare functions, the government does not need to encourage distribution caused by the difference of the weights. Thus, if $\rho = 0.5$, $\Lambda|_{\mu=0} = 0$.

Next, we explore the determinants of the sign of $\partial N/\partial\rho$. The sign of $\partial N/\partial\rho$ depends on the three terms $(\chi_f/\alpha^f_{Nl} - \chi_m/\alpha^m_{Nl})$, $c'/(1+t_N)p_N$, and $2\,(0.5-\gamma)$ in (66) since $c''(1-\rho)(1+t_N)p_N > 0$. First, we consider the meaning of the term $\chi_f/\alpha^f_{Nl} - \chi_m/\alpha^m_{Nl}$. Roughly speaking, if the increase in the number of children N reduces labor supply of spouse i, χ_i/α^i_{Nl} becomes large, because χ_i includes l_{iN}. The labor supply reduction of each spouse is harmful for him/her because it reduces his/her private consumption, while a part of labor supply reduction is used for childcare and leads to the improvement of the quality of children, which is beneficial to both spouses. Consider that the condition that $\chi_f/\alpha^f_{Nl} - \chi_m/\alpha^m_{Nl} > 0$ holds. This implies a larger reduction of the labor supply of the wife. Under this condition, the husband wants to increase the number of children in the second stage because the increase in N benefits him without a large reduction of his private consumption. Thus, an increase in the husband's bargaining power under the condition that $\chi_f/\alpha^f_{Nl} - \chi_m/\alpha^m_{Nl} > 0$ contributes to an increase in the number of children. The second term $c'/(1+t_N)p_N$ describes the allowance for the cost incurred by a wife. As the bargaining power of the husband ρ is large, the number of children increases, because the cost $c(N)$ is irrelevant to the husband.[42] The final term relates to the cost burden of bringing up children. If $0.5 > \gamma$ (i.e., if the smaller cost burden for the expense of bringing up children is imposed on the husband), then the husband wants to increase the number of children and the increase in ρ leads to the increase of N.

Finally, we discuss the parameter of the externality of children on society μ in the optimal child tax/subsidy. To clarify the effects of μ in (61), we consider the case in which $\rho = 0.5$ but $\mu \neq 0$. In this case, (61) is reduced to

$$r_N = \frac{\beta}{\delta} + \left(\frac{r_m\chi_m}{\alpha^m_{Nl}} + \frac{r_f\chi_f}{\alpha^f_{Nl}}\right) - \frac{(1-\beta)\mu}{2}\left[\frac{\chi_m}{\alpha^m_{Nl}} + \frac{\chi_f}{\alpha^f_{Nl}} + 1 + \frac{c'}{(1+t_N)\,p_N}\right]. \tag{68}$$

[42]As shown in the third term in (68) and the fourth term in (94), the marginal cost of the number of children c' contributes to the lower tax rate (higher subsidy rate). To improve the fertility rate, the direct child tax should be reduced. We confirm that the optimal child tax rate decreases with c' and it can become a subsidy if c' is sufficiently large.

Except for the second term whose sign is positive, we observe that the direct child tax/subsidy depends on the Ramsey consideration, which allows for the price distortion in the revenue-constrained optimal tax framework, and the Pigou consideration, which allows for the externality correction. The former is shown by the first term, which includes β, and the latter is shown by the third term, which includes μ. The first term is positive, while the third term is negative. Thus, if the required tax revenue is larger than the external effect of children on society (i.e., the Ramsey consideration dominates the Pigou consideration), the first term becomes larger than the third term and hence, the direct child tax is likely to be optimal. However, even if the required tax revenue is smaller than the external effect of children on society (i.e., the Ramsey consideration is dominated by the Pigou consideration), it does not necessarily lead to the direct child subsidy owing to the presence of the second term.

From (68), we observe that if μ is sufficiently large, the sign of r_N is also likely to be negative: the optimal intervention for children is likely to be a subsidy as μ becomes larger. If the government can employ a lump-sum tax, we have $\beta = 0$. Then, (68) can be rewritten as

$$r_N = -\left(\frac{\mu}{2} - r_m\right)\frac{\chi_m}{\alpha^m_{Nl}} - \left(\frac{\mu}{2} - r_f\right)\frac{\chi_f}{\alpha^f_{Nl}} - \frac{\mu}{2}\left[1 + \frac{c'}{(1+t_N)\,p_N}\right]. \tag{69}$$

If $\mu/2 \geq \max[r_m, r_f]$, the optimal childcare policy ambiguously becomes a subsidy.

6 Childcare Facility

In this section, we introduce center-based childcare services, such as early childhood education facilities, preschools, and cram schools, as an educational investment by a couple. This can be substituted for the childcare time of each spouse. Let us denote the number of hours that children spend in a childcare facility by h_c. In this model, although the couple collectively decides the time children spend in the childcare facility as well as the number of children, those decisions are not made simultaneously.[43] We modify the sequential decisions of the government, the couple, and each spouse, as follows: first, the government determines the tax rates; second, the couple collectively decides on the number of children; third, the couple collectively decides the time children spend in the childcare facility; and finally, each spouse non-cooperatively decides his/her two kinds of private consumption, labor supply in the external market, and time spent on domestic childcare.[44]

[43]Due to the long-term nature of bringing up children, the decision on the number of children that a couple will have is made prior to the use of a childcare facility. This leads to the possibility that time use of the childcare facility may deviate from what is planned today, that is, couples do not commit to one deemed optimal from the current perspective. Therefore, we consider that the couple decides time use of the childcare facility at the next stage after they choose the number of children.

[44]Even if we change the order of the third (h_c) and fourth decisions (l_i and h_i) is reversed, our main qualitative results are unaffected. This is because l_i and h_i ($i = m, f$) are separable with h_c in the utility functions.

The function of the quality per child Q is modified by

$$Q = \frac{\left(s_m \frac{h_m}{N}\right)^\sigma}{\sigma} + \frac{\left(s_f \frac{h_f}{N}\right)^\sigma}{\sigma} + \frac{\left(s_c \frac{h_c}{N}\right)^\sigma}{\sigma} = N^{-\sigma}\left[\frac{(s_m h_m)^\sigma}{\sigma} + \frac{(s_f h_f)^\sigma}{\sigma} + \frac{(s_c h_c)^\sigma}{\sigma}\right], \quad (70)$$

where s_c is the productivity of a childcare facility.[45] To simplify the analysis, we assume that the curvature of the quality function σ on the time children spend in the childcare facility is the same as that on childcare time spent by each spouse.[46] The budget constraint of each spouse is modified by

$$z_i + (1+t_y)p_y y_i + \gamma_i(1+t_N)p_N N + \nu_i(1+t_c)p_c h_c = (1-t_i)w_i l_i, \quad i = m, f. \quad (71)$$

The expenditure on the childcare facility is given by the fourth term on the left-hand side, in which ν_i is the share of spouse i on the expense of the childcare facility, p_c is the hourly price, and t_c is the tax/subsidy rate on the use of the childcare facility. Defining $\nu_m \equiv \nu$ (and hence, $\nu_f \equiv 1-\nu$) and substituting (70) for Q in (1) and (71) for z_i in (1) yields

$$\begin{aligned} U_m &= (1-t_m)w_m l_m - (1+t_y)p_y y_m - \gamma(1+t_N)p_N N + \frac{y_m^\varphi}{\varphi} - \nu(1+t_c)p_c h_c \quad (72) \\ &\quad - \frac{(l_m+h_m)^{1+\phi}}{1+\phi} + N^{1-\sigma}\left[\frac{(s_m h_m)^\sigma}{\sigma} + \frac{(s_f h_f)^\sigma}{\sigma} + \frac{(s_c h_c)^\sigma}{\sigma}\right], \end{aligned}$$

$$\begin{aligned} U_f &= (1-t_f)w_f l_f - (1+t_y)p_y y_f - (1-\gamma)(1+t_N)p_N N + \frac{y_f^\varphi}{\varphi} - (1-\nu)(1+t_c)p_c h_c \\ &\quad - \frac{(l_f+h_f)^{1+\phi}}{1+\phi} + N^{1-\sigma}\left[\frac{(s_m h_m)^\sigma}{\sigma} + \frac{(s_f h_f)^\sigma}{\sigma} + \frac{(s_c h_c)^\sigma}{\sigma}\right] - c(N). \end{aligned}$$

The difference of each spouse's utility function between the cases with and without the childcare facility is the expenditure on the childcare facility (fifth term) and the contribution of the childcare facility on the quality per child (third term in square brackets). We should note that, because y_i, l_i and h_i are additively separable with respect to h_c in each spouse's utility function, the first-order conditions of each spouse with respect to y_i, l_i, and h_i are identical to (10), (11), and (12) and hence, equations (13)–(24) hold even in the model with the childcare facility. This fact is used in the following analysis.

[45] Bastani et al. (2020) consider the quality of a childcare facility as a choice variable of parents. For simplicity, we do not endogenously treat the choice of quality of formal care. If our model extends to such a setting, s_c is an endogenous variable and p_c is a strictly increasing convex function of s_c.

[46] The difference between home care productivity for each spouse and the quality of the childcare facility is indicated by the difference between s_i and s_c. Although we check numerical results for how an increase in s_c affects optimal tax/subsidy structures while keeping $s_i$constant, we omit the results because the intuition of the results is very straightforward.

Substituting (72) for U_i in (7) and allowing for (13)–(15), we obtain the couple's utility function:

$$\begin{aligned} U \;=\; & \rho\left[(1-t_m)w_m l_m(t_m,N)-(1+t_y)p_y y_m(t_y)+\frac{(y_m(t_y))^{\varphi}}{\varphi}\right. \\ & \left.-\frac{(l_m(t_m,N)+h_m(t_m,N))^{1+\phi}}{1+\phi}\right] \\ & +(1-\rho)\left[(1-t_f)w_f l_f(t_f,N)-(1+t_y)p_y y_f(t_y)+\frac{(y_f(t_y))^{\varphi}}{\varphi}\right. \\ & \left.-\frac{(l_f(t_f,N)+h_f(t_f,N))^{1+\phi}}{1+\phi}-c(N)\right] \\ & -\left[(1-\rho)(1-\gamma)+\rho\gamma\right](1+t_N)p_N N-\left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c h_c \\ & +N^{1-\sigma}\left[\frac{(s_m h_m(t_m,N))^{\sigma}}{\sigma}+\frac{(s_f h_f(t_f,N))^{\sigma}}{\sigma}+\frac{(s_c h_c)^{\sigma}}{\sigma}\right]. \end{aligned} \tag{73}$$

As mentioned above, the spouses collectively maximize U firstly with respect to N and next with respect to h_c. We first show the determination of h_c of the couple. The first-order condition of (73) with respect to h_c is

$$0=\frac{\partial U}{\partial h_c}=-\left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c+N^{1-\sigma}s_c^{\sigma}h_c^{\sigma-1}. \tag{74}$$

Solving this equation with respect to h_c, we immediately obtain the following function:

$$h_c(t_c,N)=\left\{\left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c\right\}^{-\frac{1}{1-\sigma}}s_c^{\frac{\sigma}{1-\sigma}}N. \tag{75}$$

From (75), we obtain

$$h_{ct_c}\left(\equiv\frac{\partial h_c}{\partial t_c}\right)=-\left(\frac{1}{1-\sigma}\right)h_c(1+t_c)^{-1}<0, \tag{76}$$

$$h_{cN}\left(\equiv\frac{\partial h_c}{\partial N}\right)=\left\{\left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c\right\}^{-\frac{1}{1-\sigma}}s_c^{\frac{\sigma}{1-\sigma}}>0. \tag{77}$$

The intuitions for the two results are very straightforward.

We now turn to the couple's decision about the number of children. Allowing for $h_c = h_c(t_c, N)$, the couple maximizes the utility function (73) with respect to N. The first-order condition with respect to N is that

$$\begin{aligned} 0 \quad = \quad \frac{\partial U}{\partial N} = & -\left[(1-\rho)(1-\gamma)+\rho\gamma\right](1+t_N)p_N - (1-\rho)c'(N) \\ & + \left(\frac{1-\sigma}{\sigma}+1-\rho\right)(1-t_m)w_m h_{mN}(t_m) \\ & + \left(\frac{1-\sigma}{\sigma}+\rho\right)(1-t_f)w_f h_{fN}(t_f) \\ & + \left(\frac{1-\sigma}{\sigma}\right)\left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c h_{cN}(t_c), \end{aligned} \tag{78}$$

where we use (14), (15), (17), (74), and (77) for the derivation of this equation (see Appendix J). (78) implies

$$N = N(t_c, t_N, t_m, t_f). \tag{79}$$

Here, we propose the effects of each tax rate on the number of children. Totally differentiating (78) with respect to t_m, t_f, t_N, and t_c yields

$$N_{t_m}\left(\equiv \frac{\partial N}{\partial t_m}\right) = \frac{\left[1+(1-\rho)\left(\frac{\sigma}{1-\sigma}\right)\right] w_m \omega_m^{-\frac{1}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}}}{(1-\rho)c''} > 0, \tag{80}$$

$$N_{t_f}\left(\equiv \frac{\partial N}{\partial t_f}\right) = \frac{\left[1+\rho\left(\frac{\sigma}{1-\sigma}\right)\right] w_f \omega_f^{-\frac{1}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}}}{(1-\rho)c''} > 0, \tag{81}$$

$$N_{t_N}\left(\equiv \frac{\partial N}{\partial t_N}\right) = -\frac{\left[(1-\rho)(1-\gamma)+\rho\gamma\right] p_N}{(1-\rho)c''} < 0, \tag{82}$$

$$N_{t_c}\left(\equiv \frac{\partial N}{\partial t_c}\right) = -\frac{\left[(1-\rho)(1-\nu)+\rho\nu\right]^{-\frac{\sigma}{1-\sigma}} p_c \omega_c^{-\frac{1}{1-\sigma}} s_c^{\frac{\sigma}{1-\sigma}}}{(1-\rho)c''} < 0, \tag{83}$$

where $\omega_c \equiv (1+t_c)p_c$. To derive these four equations, we use (17) and (77). (80), (81), and (82) coincide with (28), (29), and (30), respectively. The intuition of these results is discussed below (28), (29), and (30), respectively. The intuition of (83) is very straightforward, because the increase in t_c reduces the time children spend in the childcare facility, which means that it worsens the quality per child and then leads to lower fertility.

Substituting (79) for N in (14), (15), and (75) yields

$$\begin{gathered} l_i(t_i, N(t_c, t_N, t_m, t_f)), \quad h_i(t_i, N(t_c, t_N, t_m, t_f)), \quad i = m, f, \\ h_c(t_c, N(t_c, t_N, t_m, t_f)). \end{gathered} \tag{84}$$

These functions involve information of the decision process in the second, third, and fourth stages.

Substituting (72) for U_i in (8) and allowing for (13), (79), and (84), we obtain the government's welfare function:

$$\begin{aligned} W =\; & (1-t_m)w_m l_m(t_m, N(t_c,t_N,t_m,t_f)) + \frac{(y_m(t_y))^{\varphi}}{\varphi} \\ & - \frac{(l_m(t_m, N(t_c,t_N,t_m,t_f)) + h_m(t_m, N(t_c,t_N,t_m,t_f)))^{1+\phi}}{1+\phi} \\ & + (1-t_f)w_f l_f(t_f, N(t_c,t_N,t_m,t_f)) + \frac{(y_f(t_y))^{\varphi}}{\varphi} \\ & - \frac{(l_f(t_f, N(t_c,t_N,t_m,t_f)) + h_f(t_f, N(t_c,t_N,t_m,t_f)))^{1+\phi}}{1+\phi} \\ & - (1+t_y)p_y(y_m(t_y)+y_f(t_y)) - c(N(t_c,t_N,t_m,t_f)) \\ & - (1+t_N)p_N N(t_c,t_N,t_m,t_f) - (1+t_c)p_c h_c(t_c, N(t_c,t_N,t_m,t_f)) \\ & + (2+\mu)(N(t_c,t_N,t_m,t_f))^{1-\sigma}\left[\frac{(s_m h_m(t_m, N(t_c,t_N,t_m,t_f)))^{\sigma}}{\sigma} \right. \\ & \left. + \frac{(s_f h_f(t_f, N(t_c,t_N,t_m,t_f)))^{\sigma}}{\sigma} + \frac{(s_c h_c(t_c, N(t_c,t_N,t_m,t_f)))^{\sigma}}{\sigma}\right]. \end{aligned} \tag{85}$$

The revenue constraint of the government is modified by

$$\begin{aligned} R =\; & t_m w_m l_m(t_m, N(t_c,t_N,t_m,t_f)) + t_f w_f l_f(t_f, N(t_c,t_N,t_m,t_f)) \\ & + t_y p_y\left(y_m(t_y)+y_f(t_y)\right) + t_N p_N N(t_c,t_N,t_m,t_f) + t_c p_c h_c(t_c, N(t_c,t_N,t_m,t_f)), \end{aligned} \tag{86}$$

where the fifth term is the tax revenue from the tax/subsidy on the use of the childcare facility. From the government's social welfare maximization subject to the revenue constraint, we obtain the optimal tax expressions for t_y, t_m, t_f, t_N, and t_c. Before characterizing them, we provide the following definitions:

$$\begin{aligned} & \delta_c \equiv -\frac{(1+t_c)h_{ct_c}}{h_c} > 0, \quad \xi \equiv -\frac{(1+t_c)N_{t_c}}{N} > 0, \\ & \alpha_{Nh_c} \equiv \frac{(1+t_N)p_N N}{(1+t_c)p_c h_c} > 0, \quad \chi_c \equiv \frac{h_{cN}N}{h_c} = 1, \quad r_c \equiv \frac{t_c}{1+t_c}. \end{aligned} \tag{87}$$

δ_c is the price elasticity of time use of a childcare facility, ξ is the elasticity of the number of children with respect to the price of external childcare services, α_{Nh_c} is the ratio between the expenditure on fertility good and childcare expenditure, and χ_c is the elasticity of time use of a childcare facility with respect to the number of children.[47]

From the conditions and equations shown in this section, we provide the following opti-

[47]Note that $h_{ct_c}(\equiv \partial h_c/\partial t_c) = p_c(\partial h_c/\partial(1+t_c)p_c)$. Thus, δ_c is the price elasticity of external childcare services. Similarly, $N_{t_c}(\equiv \partial N/\partial t_c) = p_c(\partial N/\partial(1+t_c)p_c)$. In other words, ξ is the elasticity of the number of children with respect to the price of external childcare services.

mal tax formulae in the case with a childcare facility (see Appendix K).

Proposition 6. *In the endogenous fertility model with a childcare facility, the optimal taxes are characterized by*

$$r_y = \frac{\beta}{\Xi}, \tag{88}$$

$$r_m = \frac{\beta\left(1 + \alpha_{Nl}^{m}\frac{\theta_m}{\delta}\right) + (1+\mu)(1-\beta)\,\alpha_{hl}^{m}\varepsilon_m}{\eta_m}, \tag{89}$$

$$r_f = \frac{\beta\left(1 + \alpha_{Nl}^{f}\frac{\theta_f}{\delta}\right) + (1+\mu)(1-\beta)\,\alpha_{hl}^{f}\varepsilon_f}{\eta_f}, \tag{90}$$

$$r_N = \frac{\beta}{\delta} + \frac{r_m\chi_m}{\alpha_{Nl}^{m}} + \frac{r_f\chi_f}{\alpha_{Nl}^{f}} - \frac{r_c\chi_c}{\alpha_{Nh_c}} + (1-\beta)\Omega, \tag{91}$$

$$r_c = \frac{\beta\left(1 - \alpha_{Nh_c}\frac{\xi}{\delta}\right)}{\delta_c} + (1-\beta)\left\{1 - (2+\mu)\left[(1-\rho)(1-\nu) + \rho\nu\right]\right\}, \tag{92}$$

where

$$\Omega \equiv \Lambda + \left\{1 - (2+\mu)\left[(1-\rho)(1-\nu) + \rho\nu\right]\right\}\frac{\chi_c}{\alpha_{Nh_c}}. \tag{93}$$

Comparing Proposition 6 with Propositions 4, we observe that the optimal commodity and income tax expressions are identical to the expressions in the case without a childcare facility. The fourth term in (91) newly appears in the optimal child tax/subsidy expression. The interpretation of the term is similar to that of the second and third terms, whose intuition is discussed below Proposition 5. $r_c\chi_c/\alpha_{Nh_c}$ reflects the effects of t_N on the price-distortions on h_c through the change in N. If $r_c < (>)0$, as the fourth term is larger, the child taxes (subsidies) tend to be desirable. However, since $\chi_c = 1$, the absolute value of the fourth term depends on r_c, *ceteris paribus* α_{Nh_c}. The second term in Ω also newly appears in the optimal child tax/subsidy expression and its intuition is similar to the first term in Λ, which is given by (62).

Next, we check the expression in the optimal tax/subsidy for center-based childcare services. The denominator in the first term shows the Ramsey tax implication. If $\xi = 0$, the first term is reduced to the standard Ramsey expression β/δ_c. ξ/δ describes the effect of the change in t_c on t_N through the change in N. The increase in t_c lowers N, which leads to the decrease in t_N because of the reduction of tax base for t_N. The decrease in t_N raises N again and hence, exacerbates the deadweight loss in labor supply. Therefore, as ξ/δ becomes larger, the optimal subsidy (tax) for center-based childcare services becomes higher (lower). The second term in (92) is related to bargaining power between the spouses and the weight on the children. Although this term reflects the corrections for h_c deviating from a socially desirable level due to the external effect of children or the difference of the weights on the spouses between the social welfare function and the couple's utility function, the intuition of this term is similar to that of the third term of Λ in expression (61), which is discussed below Proposition 5.

Some important and suggestive results are obtained from the optimal tax/subsidy expression on center-based childcare services. Consider the case in which $\nu = \gamma$, that is, the shares of each spouse are equal between two kinds of child expenditure. In this case, we obtain the following optimal tax expressions (see Appendix L).

Proposition 7. *If $\nu = \gamma$, the optimal taxes satisfy*

$$r_N = \frac{\beta}{\delta} + \frac{r_m \chi_m}{\alpha^m_{Nl}} + \frac{r_f \chi_f}{\alpha^f_{Nl}} + (1-\beta)\Lambda, \tag{94}$$

$$r_c = (1-\beta)\{1 - (2+\mu)[(1-\rho)(1-\nu) + \rho\nu]\}. \tag{95}$$

Comparing Proposition 7 with (61) in Proposition 5, we observe that if $\nu = \gamma$, the optimal child tax/subsidy expression is identical irrespective of whether there is a childcare facility. When the condition is satisfied, the fourth term in (91) and the second term in (93) cancel out, which stems from the fact that the first term on the right-hand side of (92) vanishes. As shown in Appendix L, if $\nu = \gamma$, the first term in (92) disappears, since the positive effect of t_c on revenue requirement (β) offsets the negative effect of t_c on labor supply through the change in N $(-\beta \alpha_{Nh_c} \frac{\xi}{\delta})$.[48] As a result, we obtain (95). In this case, we observe that the fourth term in (91) and the second term in (93) cancel out. Thus, the optimal child tax/subsidy expression in (61) is replicated. This result implies that the only role of the direct child subsidy allowing for a choice of center-based childcare services is to alleviate the inefficiently downward distortion on h_c due to the effect of t_c as revenue-raising taxes, where we consider that the first term in (92) is positive.

Meanwhile, if $\nu = \gamma$, the optimal tax formula for r_c is expressed by (95). Furthermore, if $\rho = 0.5$ or $\nu = 0.5$ in addition to $\nu = \gamma$, the formula reduces to

$$r_c = -\frac{(1-\beta)\mu}{2} \leq 0. \tag{96}$$

The optimal intervention in the childcare facility is to unambiguously provide a subsidy to correct the external effect of children on society (NQ) as long as $\mu > 0$. As the subsidy for center-based childcare services increases the time devoted to a childcare facility, it improves the quality per child and hence, enhances the number of children. This is consistent with Bastani et al. (2017), and shows that the subsidy for external childcare services acts as a means for internalizing externalities associated with the external effect of children on society and then improves the human capital accumulation of children.

[48] Appendix L shows that if $\nu = \gamma$, then $\alpha_{Nh_c} \frac{\xi}{\delta} = 1$.

Noting that $\nu = \gamma$, from (94) and (95), we observe that

$$r_N - r_c = \frac{\beta}{\delta} + \frac{r_m \chi_m}{\alpha^m_{Nl}} + \frac{r_f \chi_f}{\alpha^f_{Nl}} + (1-\beta)\left\{ [1-(2+\mu)\rho] \frac{\chi_m}{\alpha^m_{Nl}} + [1-(2+\mu)(1-\rho)] \frac{\chi_f}{\alpha^f_{Nl}} + \frac{[1-(2+\mu)(1-\rho)]c'}{(1+t_N)p_N} \right\}. \tag{97}$$

First, we observe that a high revenue requirement raises the weight on the first term on the right-hand side (β) and decreases the weight on the fourth term $(1-\beta)$, *ceteris paribus*. In particular, if $\rho = 0.5$, the fourth term increases with β, since the sign of the term is negative. In addition, since the optimal income tax rates depend on β from (89) and (90), the change in the revenue requirement affects the second and third terms on the right-hand side. However, it is ambiguous whether the revenue-raising position of the government increases or decreases the second and third terms. This is because, in our model, income taxes are not only revenue-raising taxes but also Pigou taxes. An increase in β strengthens the revenue-raising purpose and weakens the corrective-raising purpose, which means that it is unclear whether income tax rates increase or decrease. Thus, if $\rho = 0.5$ and income tax rates increase with β, the optimal tax/subsidy structure is more likely to be such that $r_N > (<) r_c$ as the required tax revenue is higher (lower). In addition, we observe that the second and third terms increase and the fourth term decreases with the external effect of children on society μ, *ceteris paribus*. This means that an increase in μ not only strongly requires the role of t_N as a subsidy to correct the externality of children on society but also reinforces the role of t_N as a tax to mitigate the distortionary impact on labor supply associated with the income taxes, as shown by (89) and (90). As a result, whether t_N or t_c should be taxed/subsidized at a higher rate depends on the two effects of μ working in opposite directions. These results are confirmed in the numerical analysis in Section 7.

Finally, we clarify the role of each policy instrument to correct the inefficiently low fertility arising from non-cooperative behavior within a couple. Propositions 6 and 7 show that, if $\mu = 0$, $\rho = 0.5$, and the lump-sum tax is available ($\beta = 0$), then

$$r_i = \frac{\alpha^i_{hl}\varepsilon_i}{\eta_i} > 0, \quad i = m, f, \quad r_N = \frac{r_m \chi_m}{\alpha^m_{Nl}} + \frac{r_f \chi_f}{\alpha^f_{Nl}} > 0, \quad r_c = 0. \tag{98}$$

The optimal income taxes and the direct child tax/subsidy are identical to those in (64), and the intuitions follow those below (64). Note that no intervention in the childcare facility is optimal. As shown in (83), the subsidy for center-based childcare services can improve fertility, but it is not necessary under optimal taxation. Both income taxes and the subsidy for center-based childcare services improve the quality per child and then enhance the number of children. However, the income taxes directly correct the inefficient choice of h_i, while the subsidy for center-based childcare services intervenes in the choice of h_c efficiently decided by the couple. In other words, the subsidy for the services merely distorts the quality per child upwardly. Therefore, income taxes are more effective than the subsidy for center-based childcare services under optimal taxation.

Furthermore, notice that the direct child tax is needed to mitigate income tax-induced deadweight loss whereas the tax on external childcare services is redundant. The intuitive interpretation for this result is as follows. From (17), (82), and (83), both taxes on the number of children and external childcare services enable the government to mitigate price-distortions on labor supply induced by income taxation through the change in the number of children. However, from (76), the tax on external childcare services directly distorts the time use of center-based childcare services efficiently decided by the couple. For this reason, the tax on external childcare services is not required to mitigate income tax-induced deadweight loss.

7 Numerical Analysis

This section numerically examines the optimal tax structure in the presence of the childcare facility when some important and suggestive parameters vary. The numerical analysis reinforces the intuition of our theoretical results and provides important policy implications. We consider the variations of the parameters μ, $s(\equiv s_m = s_f)$, R, w_m, and ρ, where a change in s means that s_m and s_f change simultaneously keeping $s_m = s_f$. The variations of μ, s, and R clarify the structure of the direct child tax/subsidy and the tax/subsidy for the center-based childcare services, and those of w_m and ρ are undertaken to examine the gender-based income taxation under the asymmetric spouses. To make the analysis tractable, we specify the function $c(N)$ and the parameters as follows: $c(N) = N^2/2$, $\varphi = 0.2$, $\sigma = \gamma = \nu = 0.5$, $\phi = 1.0$, $w_f = p_c = 4.0$, $s_c = 1.2$ and $p_y = p_N = 1.0$.[49] In this case, t_N and t_c under the optimal taxation are given by (94) and (96), respectively. Then, (78) yields

$$N = -\frac{1+t_N}{2(1-\rho)} + \frac{(2-\rho)\,s}{(1-\rho)(1-t_m)w_m} + \frac{(1+\rho)\,s}{4(1-\rho)(1-t_f)} + \frac{3}{5(1-\rho)(1+t_c)}, \qquad (99)$$

and (80)–(83) can be rewritten as

$$N_{t_m} = \frac{(2-\rho)\,s}{(1-\rho)(1-t_m)^2 w_m}, \quad N_{t_f} = \frac{(1+\rho)\,s}{4(1-\rho)(1-t_f)^2}, \qquad (100)$$
$$N_{t_N} = -\frac{1}{2(1-\rho)}, \quad N_{t_c} = -\frac{3}{5(1-\rho)(1+t_c)^2}.$$

[49] Since the labor-intensity of center-based childcare services are very high, we assume that p_c equals the wage rate w_i.

As a benchmark case, we consider the symmetric case between the spouses, in which $\mu = 0.2$, $R = 5.0$, $w_m = 4$, $s = 1.2$, and $\rho = 0.5$. Unless otherwise noted, we consider these values under which the spouses are symmetric. We make use of these numerical values of the parameters, (99), and (100), in numerically deriving the optimal tax formulae in Proposition 7.

7.1 Child Subsidy

We first investigate how the external effects of children on society μ affect the optimal tax structure. Table 1 demonstrates the optimal tax rates when μ takes the values from 0 to 0.3 with an interval of 0.05. The optimal income tax rates are always the same between the spouses, $t_m = t_f$, because of spousal symmetry. As the external effect of children on society becomes large, the income taxes, child tax/subsidy, and tax/subsidy on/for external childcare services should play a stronger role in improving child quality and hence, increasing the number of children. To do so, the income tax rates increase with the externality to increase childcare time and hence, improve child quality; the subsidy rate for center-based childcare services increases with the externality to promote time use of childcare facilities and hence, improves child quality; then, the child tax rate decreases with the externality to directly enhance the number of children. To secure tax revenues for subsidies for external childcare services and to compensate for a deficit in revenue due to a decrease in the child tax, commodity taxes as well as income taxes increase with the external effect of children.

Table 1. Optimal Tax Rates : Changes in μ

μ	0	0.05	0.1	0.15	0.2	0.25	0.3
t_y	0.083	0.083	0.084	0.085	0.089	0.093	0.100
t_m	0.167	0.177	0.189	0.203	0.220	0.238	0.259
t_f	0.167	0.177	0.189	0.203	0.220	0.238	0.259
t_N	0.339	0.269	0.202	0.138	0.080	0.030	−0.009
t_c	0	−0.022	−0.043	−0.063	−0.082	−0.100	−0.117

Table 1 shows that the optimal intervention for children tends to be a subsidy if the externality of children on society is sufficiently large. The condition implies that the externality of children on society is a more important factor of sub-optimally low fertility than the non-cooperative behavior of the couple. By contrast, if the non-cooperative behavior of the spouses is the main causes of under-providing for children, the direct child subsidy worsens welfare. Although the direct child tax decreases and the subsidy for center-based childcare services increases with the external effect of children, the difference between these tax/subsidy rates becomes smaller as μ becomes larger. The related discussion is provided below Table 3.

Table 2 shows the proportional changes in s from $s_m = s_f$ on the optimal tax structure. The increase in s leads to more inefficiencies due to the non-cooperative behavior of the couple. Since each spouse does not consider that his/her own childcare time positively affects the partner's utility, a higher level of s implies a larger loss arising from the external effect of the couple. Table 2 shows that the optimal income tax rates increase with s: the low fertility caused by non-cooperative behavior should be improved by the income taxation. The commodity tax rate decreases with s. This stems from the double dividend of income taxation, which increases tax revenue as well as corrects the non-cooperative behavior. The numerical results are consistent with the interpretation of Proposition 4.

Table 2. Optimal Tax Rates: Change in s

s	0.9	1.0	1.1	1.2	1.3	1.4	1.5
t_y	0.135	0.120	0.105	0.089	0.073	0.057	0.044
t_m	0.195	0.201	0.209	0.220	0.234	0.255	0.283
t_f	0.195	0.201	0.209	0.220	0.234	0.255	0.283
t_N	0.112	0.105	0.095	0.080	0.066	0.058	0.071
t_c	−0.078	−0.080	−0.081	−0.082	−0.084	−0.085	−0.087

Next, we examine the effects of the required tax revenue level on the optimal tax structure. Since $\gamma = \nu = 0.5$, the optimal child tax/subsidy is given by (94). As discussed below (94), if the required tax revenue is relatively small (large), the direct child subsidy (tax) is likely to be optimal. Table 3 demonstrates the optimal tax rates when R takes the values from 3.5 to 5.3 with an interval of 0.3. The optimal income taxes and the direct child tax increase, and the subsidy rate for external childcare services decreases with the required tax revenue. The tax changes are intuitive.

The numerical results in Tables 1 and 3 yield important policy implications for the child tax/subsidy. If the required tax revenue becomes sufficiently large or the external effect of children on society becomes sufficiently small, the price-distortion effect in an optimal tax framework is more likely to dominate the corrective effect on sub-optimally low fertility stemming from the externality of children on society. Thus, the direct child subsidy becomes optimal if the required tax revenue is relatively small or if the external effect of children on society is relatively large.

Table 3. Optimal Tax Rates: Change in R

R	3.5	3.8	4.1	4.4	4.7	5.0	5.3
t_y	0.031	0.041	0.051	0.063	0.075	0.089	0.103
t_m	0.174	0.183	0.192	0.201	0.210	0.220	0.230
t_f	0.174	0.183	0.192	0.201	0.210	0.220	0.230
t_N	−0.155	−0.112	−0.067	−0.020	0.029	0.080	0.134
t_c	−0.088	−0.087	−0.086	−0.085	−0.084	−0.082	−0.081

The most important finding is as follows. The ranking of the direct child subsidy rate and the subsidy rate for center-based childcare services are switched with the required tax revenue. As shown by (97), the sign of $(t_N - t_c)$ is determined by price-distortions under a revenue constraint (first term), the income tax-induced distortion on labor supply (second and third terms), and the external effect of children on society (fourth term). As the requited tax revenue becomes large, the optimal taxes/subsidies must put weight on price-distortions under a revenue constraint and the income tax-induced distortion on labor supply more than the external effect of children on society. Thus, as the required tax revenue becomes large, the ranking of these subsidy rates is switched. As a policy recommendation, the welfare state in a developed country, in which a huge amount of tax revenue is needed because the government size is generally large, should design its tax/subsidy system so that the subsidy rate for center-based childcare services is higher than the direct child subsidy rate, even if the direct child subsidy is optimal. However, in developing countries, where the government's size is generally small, the direct child subsidy rate should be higher than the subsidy rate for center-based childcare services.

Finally, Table 4 shows the rate of welfare gain and the increase in fertility rate owing to the full utilization of childcare facilities. Unambiguously, they are improved by the introduction of childcare facilities. The result is confirmed by almost all values of the parameters. The improvement and expansion of childcare facilities are effective for enhancing the fertility rate and welfare. Allowing for the result in Table 1, we observe that the improvement and expansion of childcare facilities have higher priority than the child tax/subsidy, and the government should implement the subsidy for center-based childcare services after developing a large number of childcare facilities. If non-cooperative behavior of spouses is the main cause of under-provision for children, then the direct child subsidy worsens welfare.

Table 4. The Effects of Childcare Facility on Welfare and Fertility Rate

μ	0	0.05	0.1	0.15	0.2	0.25	0.3
$\widehat{W}$	0.104	0.115	0.126	0.139	0.152	0.166	0.181
$\widetilde{N}$	1.142	1.207	1.276	1.345	1.416	1.185	1.554

$\widehat{W}$: the rate of welfare gain, $\widetilde{N}$: the difference in the number of children

7.2 Gender-Based Income Taxation

Here, we treat asymmetric cases between spouses to examine gender-based taxation. The wage rate and bargaining power of spouses vary. First, we consider the variation of the wage rate of the husband, while keeping the wage rate of the wife constant: w_m takes the values from 3.4 to 4.6 with an interval of 0.2. The case of $w_m = 4$ is the benchmark case, as shown in the third column from the right of Table 1. The optimal tax rates in this case are given in Table 5.

Table 5. Optimal Tax Structure under Different Wage Rates

w_m	3.4	3.6	3.8	4.0	4.2	4.4	4.6
t_y	0.098	0.093	0.090	0.089	0.087	0.085	0.082
t_m	0.336	0.289	0.250	0.220	0.196	0.177	0.161
t_f	0.243	0.233	0.225	0.220	0.214	0.209	0.205
t_N	0.266	0.173	0.117	0.080	0.053	0.032	0.013
t_c	−0.0816	−0.0820	−0.0822	−0.0824	−0.0826	−0.0827	−0.0829

All tax rates decrease with the wage rate of the husband. The increase in the wage rate w_m implies the expansion of the tax base and hence, the required tax revenue can be attained at the lower tax rates. Another finding is that the optimal income tax rate on the husband is lower (higher) than that on the wife if $w_m > (<) w_f$. This is contrary to the Ramsey inverse elasticity rule, which is that the higher tax rate should be imposed on the spouse with smaller wage elasticity, that is, with higher productivity (Boskin and Sheshinski, 1983). In the model with time spent on childcare, the income taxation motivates workers to reduce more labor supply in the external market. If the husband has higher productivity than the wife, the government has an incentive for the husband to work more in the external labor market to enhance economic efficiency, while the wife engages more in childcare activities (Meier and Rainer, 2015). Thus, the optimal income tax rate on the husband is lower (higher) than that of the wife if $w_m > (<) w_f$.[50]

[50]We examine the optimal tax structure when the childcare productivity of husband s_m varies from 0.9 to 1.5 with interval 0.1, keeping s_f constant. The increase (decrease) in s_m has the reverse impact of the increase (decrease) in w_m on time allocation of l_i and h_i. Therefore, the relative size between the optimal

Next, we consider the variation of the bargaining power of the spouse ρ in the decision about the number of children. Table 6 shows the optimal tax rates in the case in which ρ takes from 0.65 to 0.35 with interval 0.05. As the value of ρ becomes smaller, the optimal income tax rates on both spouses increases, while the optimal child tax rate decreases.

Table 6. Optimal Tax Structure under Higher Bargaining Powers of a Wife

ρ	0.65	0.6	0.55	0.5	0.45	0.4	0.35
t_y	0.010	0.035	0.062	0.089	0.113	0.135	0.154
t_m	0.1550	0.1798	0.2017	0.220	0.233	0.244	0.252
t_f	0.1557	0.1812	0.2029	0.220	0.231	0.240	0.245
t_N	0.548	0.361	0.206	0.080	−0.022	−0.109	−0.183
t_c	−0.090	−0.087	−0.085	−0.082	−0.080	−0.078	−0.077

As ρ becomes smaller (i.e., $(1-\rho)$ becomes larger), the cost $c(N)$ becomes a more important factor in the couple's decision about the number of children. As children impose a burden on the wife, the larger bargaining power of the wife leads to fewer children.[51] To repress this effect and to increase the number of children, the child tax rate decreases with ρ and it becomes a subsidy if ρ is sufficiently small. The optimal income tax rates on both spouses increases with ρ to improve the number of children from the above reason and to secure funds for the child subsidy.

Another feature of income tax rates is as follows: the spouse with higher bargaining power should be taxed at a lower rate. Without loss of generality, consider a situation in which ρ is greater than 0.5. In this case, the government intends to decrease the number of children, since the couple decides to have more children because they disregard the cost $c(N)$ for a wife. To this end, from Proposition 2(iii), it is more effective to reduce the income tax rates on the wife more than that on husband. However, there are disadvantages of lower income taxes on wife: Proposition 2(iii) shows that the couple decides to have fewer children and hence, this greatly reduces parents' childcare time from (17). This means that under-investment in childcare worsens. Thus, the lower income tax rate on the wife brings stronger downward pressure on child quality than a lower income tax rate on the husband does. As a result, the government must allow for the two forces working in opposite directions when differentiating the income tax rates between spouses. Table 6 suggests that the government should set lower income tax rates on the husband, which implies that the government should emphasize maintaining the quality of children over correcting for the intra-family distribution through the decline in fertility. Lise and Yamada (2019) empirically show that the bargaining power of men is higher than that of women, $\rho > 0.5$. In that case, our numerical result shows that a higher income tax rate should be

tax rates t_m and t_f is opposite to that in Table 5.

[51]This is consistent with the empirical results of Ashraf et al. (2014).

imposed on the wife than on the husband.

8 Conclusion

This study analyzes the optimal taxation in an economy with non-cooperative couples, including gender-based income taxation, commodity tax, child tax/subsidy, and a tax/subsidy on/for external childcare services. The number of children is at a sub-optimally low level in the economy for two reasons: the first is the externality of children on society and the second is non-cooperative household behavior. To model our scenario, we separate the quality per child and the number of children. As the time spent on childcare cannot be credibly committed across spouses and hence, the behavior of a couple becomes non-cooperative, the quality per child is sub-optimally low. Meanwhile, the number of children and the time children spend in a childcare facility are collectively decided by a couple.

This study proves that non-cooperative household behavior regarding the amount of childcare leads to a sub-optimally low number of children, which is consistent with the empirical evidence (Doepke and Kindermann, 2019). This study makes the following suggestions to improve the low fertility rate under a revenue constraint. First, sub-optimally low fertility stemming from non-cooperative behavior should be corrected by income taxes, not a child subsidy. As long as the externality of children on society is sufficiently small, a child tax is desirable to mitigate the distortionary impact of income taxes on labor supply. In this situation, the child subsidy should be reduced or removed, since it worsens welfare. Second, as the externality of children on society becomes larger, the income tax rate and the subsidy for external childcare service increase, while the direct child tax decreases. According to the first and second arguments, if a low fertility rate is caused by both the non-cooperative behavior of spouses and the externality of children on society, the government faces the problem of designing appropriate family policies corresponding to the two driving forces behind the inefficiently low fertility. As the external effect of children on society becomes less (more) serious, the government should design the tax system with a lower (higher) income tax rate, a child tax (subsidy), and a lower (higher) subsidy for external childcare services to implement effective policies. Third, our numerical analysis shows that the full utilization of childcare facilities is an effective policy, rather than the child tax/subsidy, to improve the fertility rate and welfare. This finding supports policies that provide public childcare, which are notably implemented by the countries with higher fertility rates (e.g., France, Norway, and Belgium). Finally, we recommend that developed countries should employ higher subsidy rates for center-based childcare services than direct child subsidy rates, while developing countries should implement the opposite policy.

Some extensions are left for our future research. First, since our model considers a representative household and linear tax/subsidy instruments, it does not clarify whether all tax/subsidy instruments, including gender-based income taxation, the direct child tax/subsidy, and the tax/subsidy on/for center-based childcare services, should be regressive, proportional, or progressive with respect to family size and earnings. To explore the optimal design of such policies under non-cooperative couple's behavior, we aim to extend our model to the Mirrleesian framework with non-linear schedules of these tax/subsidy instruments. Second, we abstract from the effect of parents' human capital accumulation on the amount of time they spend with their children. As mentioned in Gobbi (2018), the American Time Use Survey for the years 2003–2013 shows that the amount of the time invested by parents increases with their education. This may imply that subsidies for higher education have significant returns on children's human capital being inefficiently low due to non-cooperative behavior within a couple. Thus, it may be valuable to take into account the effect of education subsidies through such a channel on children's human capital in order to suggest implications for applied tax/subsidy policies. Finally, we adopt a quasilinear utility function to avoid analytical complexity. It would be interesting to derive policy implications under a utility function with income effect, which may reduce the fertility rate by increasing the income tax rate.[52]

Appendix A: The Derivation of (26)

The first-order condition of (25) with respect to N is

$$\begin{aligned} 0 \;=\; & \rho(1-t_m)w_m l_{mN} + (1-\rho)(1-t_f)w_f l_{fN} - (1-\rho)c' \\ & - [(1-\rho)(1-\gamma)+\rho\gamma](1+t_N)p_N + \left(\frac{1-\sigma}{\sigma}\right) N^{-\sigma}\left[(s_m h_m)^\sigma + (s_f h_f)^\sigma\right] \\ & + N^{1-\sigma}\left(s_m^\sigma h_m^{\sigma-1} h_{mN} + s_f^\sigma h_f^{\sigma-1} h_{fN}\right). \end{aligned} \tag{A1}$$

From (11) and (12), we have

$$\begin{gathered} (1-t_i)w_i = N^{1-\sigma} s_i^\sigma h_i^{\sigma-1}, \quad i=m,f, \\ \text{that is, } (1-t_i)w_i h_i = N^{1-\sigma} s_i^\sigma h_i^\sigma, \quad i=m,f. \end{gathered} \tag{A2}$$

From (14) and (17), we observe that $h_{iN} = h_i N^{-1}$. Using this and (A2), (A1) can be rewritten as (26).

[52] The negative relationship between wages and fertility is widespread across time and regions (Jones and Tertilt, 2008; Baughman and Dickert-Conlin, 2009; Jones et al., 2010) but is not universal. Several studies have reported exceptional findings. It is sometimes argued that in the early stage of the development process, there is a positive income–fertility relationship (e.g., Vogl, 2016). The cross-sectional relationship between fertility and women's education in the United States has recently become U-shaped (Hazan and Zoabi, 2015).

Appendix B: The Derivation of (42)

Using (33)-(40), we obtain

$$w_i = 2N^{1-\sigma} s_i^{\sigma} h_i^{\sigma-1}, \qquad i = m, f, \tag{B1}$$
$$\text{that is, } w_i h_i = 2N^{1-\sigma} s_i^{\sigma} h_i^{\sigma}, \qquad i = m, f.$$

Then, it yields

$$h_i = 2^{\frac{1}{1-\sigma}} w_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N, \qquad i = m, f, \tag{B2}$$

$$l_i = w_i^{\frac{1}{\phi}} - 2^{\frac{1}{1-\sigma}} w_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} N, \qquad i = m, f. \tag{B3}$$

Substituting (B1) into (41) and using $\zeta = \iota = 1$ from (33) and (34) yields

$$\left(\frac{1-\sigma}{2\sigma}\right) N^{-1} \left(w_m h_m + w_f h_f\right) - \frac{1}{2}c' - \frac{1}{2}p_N = 0. \tag{B4}$$

Moreover, substituting (B2) for h_i in (B4) yields (42).

Appendix C: The Proof of $\pi(\sigma) > 0$

To show that $\pi(\sigma) > 0$, we take the following two steps. First, we show that $\lim_{\sigma\to 0}\left[\frac{(1-\sigma)}{\sigma}2^{\frac{\sigma}{1-\sigma}} - \frac{(1-\sigma)}{\sigma}\right] = \ln 2 \approx 0.69$. Define $f(\sigma) \equiv (1-\sigma)(2^{\frac{\sigma}{1-\sigma}} - 1)$ and $g(\sigma) \equiv \sigma$. Obviously, $f(0) = g(0) = 0$ and $g'(\sigma) = 1$. Furthermore, $\lim_{\sigma\to 0}\left[\frac{f'(\sigma)}{g'(\sigma)}\right] = \ln 2$. Hence, using L'Hospital's rule, $\lim_{\sigma\to 0}\left[\frac{f(\sigma)}{g(\sigma)}\right] = \lim_{\sigma\to 0}\left[\frac{f'(\sigma)}{g'(\sigma)}\right]$. As a result, $\lim_{\sigma\to 0}\pi > 0$.

Second, we show that $\pi'(\sigma) > 0$ for any $0 < \sigma < 1$, where $\pi'(\sigma) = \frac{1+2^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{1-\sigma}\ln 2 - 1\right)}{\sigma^2}$. Noticing that $2^{\frac{\sigma}{1-\sigma}}\left(\frac{\sigma}{1-\sigma}\ln 2 - 1\right)$ $(\equiv \psi(\sigma))$ is -1 at $\sigma = 0$, it is sufficient to show that $\psi'(\sigma) > 0$ for any $0 < \sigma < 1$. After computation, we obtain $\psi' = 2^{\frac{\sigma}{1-\sigma}}(\ln 2)^2 \frac{\sigma}{(1-\sigma)^3} > 0$. Therefore, $\pi'(\sigma)$ is positive for any $0 < \sigma < 1$.

Appendix D: The Proof of $N^{PE} = N^C$

In a cooperative setting, the total after-tax income for each spouse is shared between the husband and wife such that i's consumption is

$$z_i + \gamma_i(1 + t_N)p_N N = \mu_i\left[(1 - t_m)w_m l_m + (1 - t_f)w_f l_f\right], \tag{D1}$$

where $\mu^m + \mu^f = 1$.

Given this sharing rule, cooperative couples maximize joint utility (7). Using (D1), the maximization problem at the third stage can be rewritten as

$$\max_{l_m,l_f,h_m,h_f} \quad U = (\rho\mu_m + (1-\rho)\mu_f)\left[(1-t_m)w_m l_m + (1-t_f)w_f l_f\right] \tag{D2}$$
$$- [(1-\rho)(1-\gamma)+\rho\gamma](1+t_N)p_N N - \rho\frac{(l_m+h_m)^{1+\phi}}{1+\phi}$$
$$- (1-\rho)\frac{(l_f+h_f)^{1+\phi}}{1+\phi} + N^{1-\sigma}\left[\frac{(s_m h_m)^\sigma}{\sigma} + \frac{(s_f h_f)^\sigma}{\sigma}\right]$$
$$- (1-\rho)c(N).$$

Denoting $\rho_m \equiv \rho$ and $\rho_f \equiv 1-\rho$, the first-order conditions for the utility maximization are

$$0 = \frac{\partial U}{\partial l_i} = (\rho_m\mu_m + \rho_f\mu_f)(1-t_i)w_i - \rho_i\left(l_i+h_i\right)^\phi, \quad i = m, f, \tag{D3}$$

$$0 = \frac{\partial U}{\partial h_i} = -\rho_i\left(l_i+h_i\right)^\phi + N^{1-\sigma}s_i^\sigma h_i^{\sigma-1}, \quad i = m, f. \tag{D4}$$

These two equations yield

$$(1-t_i)w_i = \Gamma N^{1-\sigma}s_i^\sigma h_i^{\sigma-1}, \quad i = m, f, \tag{D5}$$
$$\text{that is, } (1-t_i)w_i h_i = \Gamma N^{1-\sigma}s_i^\sigma h_i^{\sigma}, \quad i = m, f,$$

where $\Gamma \equiv \frac{1}{\rho\mu_m+(1-\rho)\mu_f}$. Then, these conditions yield

$$h_i^*(t_i, N) = \omega_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} \Gamma^{\frac{1}{1-\sigma}} N, \quad i = m, f, \tag{D6}$$

$$l_i^*(t_i, N) = (\Theta_i\omega_i)^{\frac{1}{\phi}} - \omega_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} \Gamma^{\frac{1}{1-\sigma}} N, \quad i = m, f, \tag{D7}$$

where $\Theta_i \equiv \frac{\rho\mu_m+(1-\rho)\mu_f}{\rho_i}$. Note that a similar condition to (17) holds:

$$h_{iN}^* = -l_{iN}^* = \omega_i^{-\frac{1}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}} \Gamma^{\frac{1}{1-\sigma}}, \quad i = m, f. \tag{D8}$$

At the second stage, cooperative couples maximize (D2), which is evaluated by h_i^* and l_i^*, with respect to N. Using (D8), we can obtain the following first-order condition with respect to N:

$$N \quad : \quad 0 = [\rho\mu_m + (1-\rho)\mu_f]\left[(1-t_m)w_m l^*_{mN} + (1-t_f)w_f l^*_{fN}\right] \tag{D9}$$
$$- [(1-\rho)(1-\gamma) + \rho\gamma](1+t_N)p_N - (1-\rho)c'(N)$$
$$+ (1-\sigma)N^{-\sigma}\left[\frac{(s_m h^*_m)^\sigma}{\sigma} + \frac{\left(s_f h^*_f\right)^\sigma}{\sigma}\right]$$
$$+ N^{1-\sigma}\left[s^\sigma_m (h^*_m)^{\sigma-1} h^*_{mN} + s^\sigma_f (h^*_f)^{\sigma-1} h^*_{fN}\right].$$

By allowing for (D5) and (D8), this can be rewritten as

$$N \quad : \quad 0 = -(1-\rho)c'(N) - [(1-\rho)(1-\gamma) + \rho\gamma](1+t_N)p_N \tag{D10}$$
$$+ \Gamma^{-1}\frac{(1-\sigma)N^{-1}}{\sigma}\left(\omega_m h^*_m + \omega_f h^*_f\right).$$

Using (D6), (D10) can be rewritten as

$$N \quad : \quad 0 = -(1-\rho)c'(N) - [(1-\rho)(1-\gamma) + \rho\gamma](1+t_N)p_N \tag{D11}$$
$$+ \frac{(1-\sigma)}{\sigma}\Gamma^{\frac{\sigma}{1-\sigma}}\left(\omega_m^{\frac{-\sigma}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}} + \omega_f^{\frac{-\sigma}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}}\right),$$

which yields the number of children under the cooperative case, which is denoted by N^C. If $\rho = 1/2$ and $t_i = t_N = 0$, (D11) coincides with (42), which means that $N^{PE} = N^C$.

Appendix E: Common Income Tax Rate

In this section, we consider a common income tax rate on the husband and wife instead of the gender-based taxation. Let us denote the common income tax rate by t. Hence, $t_m = t_f (\equiv t)$ and $dt_m = dt_f (\equiv dt)$. Except for (28) and (29), note that the equations in Section 5 are unaffected by deleting the index i of t_i. (28) and (29) are replaced by the following equation:

$$N_t = \frac{\left[1 + (1-\rho)\left(\frac{\sigma}{1-\sigma}\right)\right] w_m \omega_m^{-\frac{1}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}} + \left[1 + \rho\left(\frac{\sigma}{1-\sigma}\right)\right] w_f \omega_f^{-\frac{1}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}}}{(1-\rho)c''} > 0. \tag{E1}$$

Given these facts, the government maximizes (45) subject to (46) by choosing t and t_N. Using (17), the first-order conditions with respect to t and t_N are[53]

$$\begin{aligned} t \quad : \quad 0 = & -w_m l_m + (1-t)w_m l_{mt} + (1-t)w_m l_{mN} N_t - (l_m + h_m)^{\phi} (l_{mt} + h_{mt}) \\ & - w_f l_f + (1-t)w_f l_{ft} + (1-t)w_f l_{fN} N_t - (l_f + h_f)^{\phi} (l_{ft} + h_{ft}) \\ & - (1+t_N)p_N N_t - c' N_t + (2+\mu)(1-\sigma) N^{-\sigma} N_t \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} \right] \\ & + (2+\mu) N^{1-\sigma} \left[s_m^{\sigma} h_m^{\sigma-1} (h_{mt} + h_{mN} N_t) + s_f^{\sigma} h_f^{\sigma-1} (h_{ft} + h_{fN} N_t) \right] \\ & - \lambda (w_m l_m + w_f l_f + t w_m l_{mt} + t w_f l_{ft} + t w_m l_{mN} N_t + t w_f l_{fN} N_t + t_N p_N N_t), \end{aligned} \tag{E2}$$

$$\begin{aligned} t_N \quad : \quad 0 = & (1-t)w_m l_{mN} N_{t_N} + (1-t)w_f l_{fN} N_{t_N} - p_N N - (1+t_N)p_N N_{t_N} \\ & - c' N_{t_N} + (2+\mu)(1-\sigma) N^{-\sigma} N_{t_N} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} \right] \\ & + (2+\mu) N^{1-\sigma} \left(s_m^{\sigma} h_m^{\sigma-1} h_{mN} N_{t_N} + s_f^{\sigma} h_f^{\sigma-1} h_{fN} N_{t_N} \right) \\ & - \lambda (t w_m l_{mN} N_{t_N} + t w_f l_{fN} N_{t_N} + p_N N + t_N p_N N_{t_N}). \end{aligned} \tag{E3}$$

By noting the $t_m = t_f = t$, (26) is reduced to

$$\begin{aligned} N \quad : \quad 0 = & -[(1-\rho)(1-\gamma) + \rho\gamma](1+t_N)p_N - (1-\rho)c'(N) \\ & + \left(\frac{1-\sigma}{\sigma} + 1 - \rho \right)(1-t)w_m h_{mN}(t) + \left(\frac{1-\sigma}{\sigma} + \rho \right)(1-t)w_f h_{fN}(t). \end{aligned} \tag{E4}$$

Before providing the proof, we note that all conditions and equations obtained in Sections 3 and 4 are valid as long as the index i of t_i is deleted, except for N_t. Furthermore, we provide the following definitions:

$$\begin{aligned} \alpha_{Nl} \equiv \frac{(1+t_N)p_N N}{a_m + a_f}, \quad \theta_{\omega} \equiv \frac{(1+t)N_t}{N}, \quad \eta \equiv \frac{a_m \eta_m + a_f \eta_f}{a_m + a_f}, \\ \varepsilon \equiv \frac{a_m \alpha_{hl}^m \varepsilon_m + a_f \alpha_{hl}^f \varepsilon_f}{a_m + a_f}, \quad r \equiv \frac{t}{1-t}, \quad a_i \equiv (1-t)w_i l_i, \quad i = m, f. \end{aligned} \tag{E5}$$

Multiplying each term in (E3) by $-N_{t_N}^{-1} N_t$ and applying the resulting equation to (E2)

[53]We omit the derivation of the optimal commodity tax rate on y_i, since the same expression as (51) obviously holds even under the common income tax rate.

yields

$$t \quad : \quad 0 = -w_m l_m + (1-t) w_m l_{mt} - (l_m + h_m)^{\phi} (l_{mt} + h_{mt}) \\ - w_f l_f + (1-t) w_f l_{ft} - (l_f + h_f)^{\phi} (l_{ft} + h_{ft}) + p_N N N_{t_N}^{-1} N_t \\ + (2+\mu) N^{1-\sigma} \left(s_m^{\sigma} h_m^{\sigma-1} h_{mt} + s_f^{\sigma} h_f^{\sigma-1} h_{ft} \right) \\ - \lambda \left(w_m l_m + w_f l_f + t w_m l_{mt} + t w_f l_{ft} - p_N N N_{t_N}^{-1} N_t \right) \tag{E6}$$

Using (11), (A2), and the definition of β, (E6) can be rewritten as

$$t \quad : \quad 0 = -\beta (w_m l_m + w_f l_f) - (1-\beta)(1+\mu)(1-t)(w_m h_{mt} + w_f h_{ft}) \\ - t (w_m l_{mt} + w_f l_{ft}) + \beta p_N N N_{t_N}^{-1} N_t, \tag{E7}$$

which yields

$$\frac{t}{1-t} = -\frac{w_m l_m + w_f l_f}{(1-t)(w_m l_{mt} + w_f l_{ft})} \left\{ \beta \left(1 - \frac{p_N N N_{t_N}^{-1} N_t}{w_m l_m + w_f l_f} \right) + (1-\beta) \left[\frac{(1+\mu)(1-t)(w_m h_{mt} + w_f h_{ft})}{(w_m l_m + w_f l_f)} \right] \right\}. \tag{E8}$$

Using (54), (55), and (E5), we have

$$\frac{w_m l_m + w_f l_f}{(1-t)(w_m l_{mt} + w_f l_{ft})} = -\frac{1}{\frac{a_m \eta_m + a_f \eta_f}{a_m + a_f}}, \tag{E9}$$

$$\frac{(1-t)(w_m h_{mt} + w_f h_{ft})}{w_m l_m + w_f l_f} = \frac{a_m \alpha_{hl}^m \varepsilon_m + a_f \alpha_{hl}^f \varepsilon_f}{a_m + a_f}, \tag{E10}$$

$$\frac{p_N N N_{t_N}^{-1} N_t}{w_m l_m + w_f l_f} = -\frac{\alpha_{Nl} \theta_{\omega}}{\delta}. \tag{E11}$$

Using (E5) and (E9)–(E11), (E8) can be rewritten as

$$r = \frac{\beta \left(1 + \frac{\alpha_{Nl} \theta_{\omega}}{\delta} \right) + (1-\beta)(1+\mu)\varepsilon}{\eta}. \tag{E12}$$

The optimal income tax expression under the common tax rate (E12) is similar to the optimal income tax expressions under the gender-based taxation, which is provided in Proposition 4. η is the weighted average of the wage elasticities of the spouses and ε is the weighted average of $\alpha_{hl}^i \varepsilon_i$, with the weight being the disposable income share of the spouses. The intuition is similar to Proposition 4.

Multiplying each term in (E3) by $-N_{t_N}^{-1}$, we obtain

$$\begin{aligned} t_N \quad : \quad 0 = & -(1-t)w_m l_{mN} - (1-t)w_f l_{fN} + p_N N N_{t_N}^{-1} + (1+t_N)p_N \\ & + c' - (2+\mu)(1-\sigma)N^{-\sigma}\left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma}\right] \\ & - (2+\mu)N^{1-\sigma}\left(s_m^{\sigma} h_m^{\sigma-1} h_{mN} + s_f^{\sigma} h_f^{\sigma-1} h_{fN}\right) \\ & + \lambda\left(t w_m l_{mN} + t w_f l_{fN} + p_N N N_{t_N}^{-1} + p_N t_N\right). \end{aligned} \tag{E13}$$

Using (17) and (A2), this can be rewritten as

$$\begin{aligned} t_N \quad : \quad 0 = & -(1-t)w_m l_{mN} - (1-t)w_f l_{fN} + p_N N N_{t_N}^{-1} + (1+t_N)p_N + c' \\ & - (2+\mu)\left(\frac{1-\sigma}{\sigma}\right)N^{-1}(1-t)(w_m h_m + w_f h_f) \\ & + (2+\mu)(1-t)\left(w_m l_{mN} + w_f l_{fN}\right) \\ & + \lambda\left(t w_m l_{mN} + t w_f l_{fN} + p_N N N_{t_N}^{-1} + t_N p_N\right). \end{aligned} \tag{E14}$$

Multiplying each term in (E4) by $(2+\mu)$ and making use of (14) and (17) yields

$$\begin{aligned} N \quad : \quad 0 = & -(2+\mu)\left[(1-\rho)(1-\gamma) + \rho\gamma\right](1+t_N)p_N - (2+\mu)(1-\rho)c' \\ & + (2+\mu)\left(\frac{1-\sigma}{\sigma}\right)N^{-1}(1-t)(w_m h_m + w_f h_f) \\ & - (2+\mu)(1-\rho)(1-t)w_m l_{mN} - (2+\mu)\rho(1-t)w_f l_{fN}. \end{aligned}$$

Applying this to (E14), we obtain

$$\begin{aligned} t_N \quad : \quad 0 = & (1+\lambda)p_N N N_{t_N}^{-1} + \left\{1 - (2+\mu)\left[(1-\rho)(1-\gamma) + \rho\gamma\right]\right\}(1+t_N)p_N \\ & + [1 - (2+\mu)(1-\rho)]c' - \left[1 - (2+\mu)\rho\right](1-t)w_m l_{mN} \\ & - [1 - (2+\mu)(1-\rho)](1-t)w_f l_{fN} + \lambda\left(t w_m l_{mN} + t w_f l_{fN} + t_N p_N\right). \end{aligned} \tag{E15}$$

Using the definition of β, (54), (55), and (56), (E15) can be rewritten as

$$r_N = \frac{\beta}{\delta} + \frac{r\chi_m}{\alpha_{Nl}^m} + \frac{r\chi_f}{\alpha_{Nl}^f} + (1-\beta)\Lambda. \tag{E16}$$

The expression of the optimal child tax/subsidy takes the same form as (61), regardless of whether the income tax rates are differentiable or not.

Appendix F: The Derivations of (52) and (53)

Multiplying each term in (50) by $N_{t_N}^{-1}N_{t_m}$ and subtracting the resulting equation from (48) yields

$$\begin{aligned} t_m \quad : \quad 0 = & -w_m l_m + (1-t_m) w_m l_{mt_m} - (l_m + h_m)^{\phi} (l_{mt_m} + h_{mt_m}) \\ & + (2+\mu) N^{1-\sigma} s_m^{\sigma} h_m^{\sigma-1} h_{mt_m} - \lambda (w_m l_m + t_m w_m l_{mt_m}) \\ & + (1+\lambda) p_N N N_{t_N}^{-1} N_{t_m}. \end{aligned} \tag{F1}$$

Using (11) and the first equation in (A2), (F1) can be rewritten as (52), Similarly, we obtain (53).

Appendix G: The Proof of Proposition 4

(52) can be rewritten as

$$\begin{aligned} \frac{t_m}{1-t_m} = & -\left(\frac{1+\lambda}{\lambda}\right) \frac{l_m}{(1-t_m) l_{mt_m}} + \left(\frac{1}{\lambda}\right) \frac{(1+\mu) w_m h_{mt_m}}{w_m l_{mt_m}} \\ & + \left(\frac{1+\lambda}{\lambda}\right) \frac{p_N N N_{t_N}^{-1} N_{t_m}}{(1-t_m) w_m l_{mt_m}} \\ = & -\left(\frac{1+\lambda}{\lambda}\right) \frac{1}{\frac{(1-t_m) l_{mt_m}}{l_m}} + \left(\frac{1}{\lambda}\right) (1+\mu) \frac{(1-t_m) w_m h_m \frac{(1-t_m) h_{mt_m}}{h_m}}{(1-t_m) w_m l_m \frac{(1-t_m) l_{mt_m}}{l_m}} \\ & + \left(\frac{1+\lambda}{\lambda}\right) \frac{(1+t_N) p_N N \frac{(1-t_m) N_{t_m}}{N}}{(1-t_m) w_m l_m \frac{(1-t_m) l_{mt_m}}{l_m} \frac{(1+t_N) N_{t_N}}{N}}. \end{aligned} \tag{G1}$$

Using (54)–(56), (G1) can be rewritten as (57). Similarly, we obtain (58).

Appendix H: The Proof of Proposition 5

From (60) we obtain

$$\begin{aligned}\frac{t_N}{1+t_N} = & -\left(\frac{1+\lambda}{\lambda}\right)\frac{1}{\frac{(1+t_N)N_{t_N}}{N}} - \frac{t_m}{1+t_m}\frac{(1+t_m)w_m l_m}{(1+t_N)\,p_N N}\frac{N l_{mN}}{l_m} \\ & - \frac{t_f}{1+t_f}\frac{(1+t_f)w_f l_f}{(1+t_N)\,p_N N}\frac{N l_{fN}}{l_f} - \frac{1}{\lambda}\left[1-(2+\mu)(1-\rho)\right]\frac{c'}{(1+t_N)\,p_N} \\ & + \frac{1}{\lambda}\left[1-(2+\mu)\rho\right]\frac{(1-t_m)w_m l_m}{(1+t_N)\,p_N N}\frac{N l_{mN}}{l_m} \\ & + \frac{1}{\lambda}\left[1-(2+\mu)(1-\rho)\right]\frac{(1-t_f)w_f l_f}{(1+t_N)\,p_N N}\frac{N l_{fN}}{l_f} \\ & - \frac{1}{\lambda}\left\{1-(2+\mu)\left[(1-\rho)(1-\gamma)+\rho\gamma\right]\right\}.\end{aligned} \tag{H1}$$

Using (54)–(56) and (59), (H1) can be rewritten as (61).

Appendix I: The Derivations of (66)

Substituting (26) for h_{iN} in (17) yields

$$\begin{aligned}0 = & -\left[(1-\rho)(1-\gamma)+\rho\gamma\right](1+t_N)p_N - (1-\rho)c'(N) \\ & + \left(\frac{1-\sigma}{\sigma}+1-\rho\right)\omega_m^{\frac{-\sigma}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}} + \left(\frac{1-\sigma}{\sigma}+\rho\right)\omega_f^{\frac{-\sigma}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}}.\end{aligned} \tag{I1}$$

Totally differentiating (I1) with respect to ρ and N, we obtain

$$0 = 2(0.5-\gamma)(1+t_N)p_N + c'(N) - (1-\rho)c''\frac{\partial N}{\partial \rho} + \left(\omega_f^{\frac{-\sigma}{1-\sigma}} s_f^{\frac{\sigma}{1-\sigma}} - \omega_m^{\frac{-\sigma}{1-\sigma}} s_m^{\frac{\sigma}{1-\sigma}}\right). \tag{I2}$$

Here, using (17), (55), and (59), we obtain the following result:

$$\frac{\chi_i}{\alpha_{Nl}^i} = \frac{\omega_i^{\frac{-\sigma}{1-\sigma}} s_i^{\frac{\sigma}{1-\sigma}}}{(1+t_N)\,p_N}, \quad i = m, f. \tag{I3}$$

Substituting (I3) into the fourth term in (I2) and dividing the resulting equation by $(1-\rho)c''$ yields (66).

Appendix J: The Derivation of (78)

We substitute (75) for h_c in (73) and consider the maximization of U. Allowing for (17), the first-order condition with respect to N is

$$\begin{aligned} 0 \;=\; \frac{\partial U}{\partial N} &= \rho(1-t_m)w_m l_{mN} + (1-\rho)(1-t_f)w_f l_{fN} - (1-\rho)c' \\ &\quad - \left[(1-\rho)(1-\gamma)+\rho\gamma\right](1+t_N)p_N \\ &\quad - \left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c h_{cN} \\ &\quad + N^{1-\sigma}\left(s_m^{\sigma}h_m^{\sigma-1}h_{mN} + s_f^{\sigma}h_f^{\sigma-1}h_{fN} + s_c^{\sigma}h_c^{\sigma-1}h_{cN}\right) \\ &\quad + (1-\sigma)N^{-\sigma}\left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} + \frac{(s_c h_c)^{\sigma}}{\sigma}\right]. \end{aligned} \tag{J1}$$

Using (11), (12), (17), (74), and (77), (J1) can be rewritten as

$$\begin{aligned} 0 \;=\; & \rho(1-t_m)w_m l_{mN} + (1-\rho)(1-t_f)w_f l_{fN} - (1-\rho)c' \\ & - \left[(1-\rho)(1-\gamma)+\rho\gamma\right](1+t_N)p_N - \left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c h_{cN} \\ & + (1-t_m)w_m h_{mN} + (1-t_f)w_f h_{fN} + \left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c h_{cN} \\ & + \left(\frac{1-\sigma}{\sigma}\right)\{(1-t_m)w_m h_{mN} + (1-t_f)w_f h_{fN} \\ & + \left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c h_{cN}\}. \end{aligned} \tag{J2}$$

Using (17), (J2) can be rewritten as (78).

Appendix K: The Proof of Proposition 6

By defining the Lagrange function as L and the Lagrange multiplier on the revenue constraint as λ, and by making use of (17), the first-order conditions of the government's social welfare maximization (85) subject to the revenue constraint (86) with respect to t_y, t_m, t_f, t_N, and t_c are given by

$$\begin{aligned} 0 \;=\; \frac{\partial L}{\partial t_y} &= -p_y y_m - (1+t_y)p_y y_m' - p_y y_f - (1+t_y)p_y y_f' \\ &\quad + y_m^{\varphi-1}y_m' + y_f^{\varphi-1}y_f' - \lambda[y_m + y_f + t_y(y_m' + y_f')]p_y, \end{aligned} \tag{K1}$$

$$
\begin{aligned}
0 \quad = \quad \frac{\partial L}{\partial t_m} &= -w_m l_m + (1-t_m) w_m l_{mt_m} + (1-t_m) w_m l_{mN} N_{t_m} \\
&- (l_m + h_m)^{\phi} (l_{mt_m} + h_{mt_m}) + (1-t_f) w_f l_{fN} N_{t_m} - c' N_{t_m} \\
&- (1+t_N) p_N N_{t_m} - (1+t_c) p_c h_{cN} N_{t_m} \\
&+ (2+\mu)(1-\sigma) N^{-\sigma} N_{t_m} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} + \frac{(s_c h_c)^{\sigma}}{\sigma} \right] \\
&+ (2+\mu) N^{1-\sigma} [s_m^{\sigma} h_m^{\sigma-1} (h_{mt_m} + h_{mN} N_{t_m}) \\
&+ s_f^{\sigma} h_f^{\sigma-1} h_{fN} N_{t_m} + s_c^{\sigma} h_c^{\sigma-1} h_{cN} N_{t_m}] \\
&- \lambda (w_m l_m + t_m w_m l_{mt_m} + t_m w_m l_{mN} N_{t_m} \\
&+ t_f w_f l_{fN} N_{t_m} + t_N p_N N_{t_m} + t_c p_c h_{cN} N_{t_m}),
\end{aligned} \tag{K2}
$$

$$
\begin{aligned}
0 \quad = \quad \frac{\partial L}{\partial t_f} &= (1-t_m) w_m l_{mN} N_{t_f} - w_f l_f + (1-t_f) w_f l_{ft_f} + (1-t_f) w_f l_{fN} N_{t_f} \\
&- (l_f + h_f)^{\phi} \left(l_{ft_f} + h_{ft_f}\right) - c' N_{t_f} - (1+t_N) p_N N_{t_f} - (1+t_c) p_c h_{cN} N_{t_f} \\
&+ (2+\mu)(1-\sigma) N^{-\sigma} N_{t_f} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} + \frac{(s_c h_c)^{\sigma}}{\sigma} \right] \\
&+ (2+\mu) N^{1-\sigma} [s_m^{\sigma} h_m^{\sigma-1} h_{mN} N_{t_f} + s_f^{\sigma} h_f^{\sigma-1} (h_{ft_f} + h_{fN} N_{t_f}) + s_c^{\sigma} h_c^{\sigma-1} h_{cN} N_{t_f}] \\
&- \lambda (t_m w_m l_{mN} N_{t_f} + w_f l_f + t_f w_f l_{ft_f} + t_f w_f l_{fN} N_{t_f} + t_N p_N N_{t_f} + t_c p_c h_{cN} N_{t_f}),
\end{aligned} \tag{K3}
$$

$$
\begin{aligned}
0 \quad = \quad \frac{\partial L}{\partial t_N} &= (1-t_m) w_m l_{mN} N_{t_N} + (1-t_f) w_f l_{fN} N_{t_N} - c' N_{t_N} \\
&- p_N N - (1+t_N) p_N N_{t_N} - (1+t_c) p_c h_{cN} N_{t_N} \\
&+ (2+\mu)(1-\sigma) N^{-\sigma} N_{t_N} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} + \frac{(s_c h_c)^{\sigma}}{\sigma} \right] \\
&+ (2+\mu) N^{1-\sigma} \left(s_m^{\sigma} h_m^{\sigma-1} h_{mN} N_{t_N} + s_f^{\sigma} h_f^{\sigma-1} h_{fN} N_{t_N} + s_c^{\sigma} h_c^{\sigma-1} h_{cN} N_{t_N} \right) \\
&- \lambda \left(t_m w_m l_{mN} N_{t_N} + t_f w_f l_{fN} N_{t_N} + p_N N + t_N p_N N_{t_N} + t_c p_c h_{cN} N_{t_N} \right),
\end{aligned} \tag{K4}
$$

$$
\begin{aligned}
0 \quad = \quad \frac{\partial L}{\partial t_c} &= (1-t_m) w_m l_{mN} N_{t_c} + (1-t_f) w_f l_{fN} N_{t_c} - c' N_{t_c} \\
&- (1+t_N) p_N N_{t_c} - p_c h_c - (1+t_c) p_c h_{ct_c} - (1+t_c) p_c h_{cN} N_{t_c} \\
&+ (2+\mu)(1-\sigma) N^{-\sigma} N_{t_c} \left[\frac{(s_m h_m)^{\sigma}}{\sigma} + \frac{(s_f h_f)^{\sigma}}{\sigma} + \frac{(s_c h_c)^{\sigma}}{\sigma} \right] \\
&+ (2+\mu) N^{1-\sigma} [s_m^{\sigma} h_m^{\sigma-1} h_{mN} N_{t_c} \\
&+ s_f^{\sigma} h_f^{\sigma-1} h_{fN} N_{t_c} + s_c^{\sigma} h_c^{\sigma-1} (h_{ct_c} + h_{cN} N_{t_c})] \\
&- \lambda (t_m w_m l_{mN} N_{t_c} + t_f w_f l_{fN} N_{t_c} + t_N p_N N_{t_c} \\
&+ p_c h_c + t_c p_c h_{ct_c} + t_c p_c h_{cN} N_{t_c}).
\end{aligned} \tag{K5}
$$

Using (11), (12), and (K4), after some manipulation, (K2) and (K3) can be rewritten as

$$t_m : 0 = -(1+\lambda)w_m l_m - \lambda t_m w_m l_{mt_m} + (1+\mu)(1-t_m)w_m h_{mt_m} + (1+\lambda)p_N N N_{t_N}^{-1} N_{t_m},$$

$$t_f : 0 = -(1+\lambda)w_f l_f - \lambda t_f w_f l_{ft_f} + (1+\mu)(1-t_f)w_f h_{ft_f} + (1+\lambda)p_N N N_{t_N}^{-1} N_{t_f}.$$

These two equations are the same as (52) and (53), respectively. Using the process of Appendix G, we observe that the two equations lead to (89) and (90), respectively.

We next derive (92). Multiplying each term in (K4) by $N_{t_N}^{-1} N_{t_c}$ and subtracting the resulting equation from (K5), we obtain

$$\begin{aligned} 0 &= \beta p_N N N_{t_N}^{-1} N_{t_c} + (1-\beta)p_c h_c + (1-\beta)(1+t_c)p_c h_{ct_c} \\ &\quad - (1-\beta)(2+\mu)N^{1-\sigma} s_c^{\sigma} h_c^{\sigma-1} h_{ct_c} - p_c h_c - t_c p_c h_{ct_c}. \end{aligned}$$

Using (74), this can be rewritten as

$$\begin{aligned} \frac{t_c}{1+t_c} &= -\beta \frac{(1+t_N)p_N N}{(1+t_c)p_c h_c} \frac{-\frac{(1+t_c)N_{t_c}}{N}}{-\frac{(1+t_c)h_{ct_c}}{h_c}} \frac{1}{-\frac{(1+t_N)N_{t_N}}{N}} - (1-\beta)\frac{1}{-\frac{(1+t_c)h_{ct_c}}{h_c}} \qquad \text{(K6)} \\ &\quad + (1-\beta) - (1-\beta)(2+\mu)\left[(1-\rho)(1-\nu)+\rho\nu\right] + \frac{1}{-\frac{(1+t_c)h_{ct_c}}{h_c}}. \end{aligned}$$

Using (54) and (87), after some manipulations, this can be rewritten as (92).

Finally, we derive the optimal child tax/subsidy expression. By multiplying each term in (K4) by $N_{t_N}^{-1}$ and making use of (A2), (74), as well as the fact that $h_{iN}N = h_i$ $(i = m, f, c)$, after some manipulations, (K4) can be rewritten as

$$\begin{aligned} 0 &= (1-t_m)w_m l_{mN} + (1-t_f)w_f l_{fN} - c' - p_N N N_{t_N}^{-1} - (1+t_N)p_N \qquad \text{(K7)} \\ &\quad - (1+t_c)p_c h_{cN} + (2+\mu)\left(\frac{1}{\sigma}\right)\{(1-t_m)w_m h_{mN} + (1-t_f)w_f h_{fN} \\ &\quad + \left[(1-\rho)(1-\nu)+\rho\nu\right](1+t_c)p_c h_{cN}\} \\ &\quad - \lambda\left(t_m w_m l_{mN} + t_f w_f l_{fN} + p_N N N_{t_N}^{-1} + t_N p_N + t_c p_c h_{cN}\right). \end{aligned}$$

Multiplying each term in (78) by $(2+\mu)$, subtracting the resulting equation from (K7), and making use of (17), we obtain

$$\begin{aligned}0 \;=\;& [1-(2+\mu)(1-\rho)]\,c' + (1+\lambda)p_N N N_{t_N}^{-1} \\ &+ \{1-(2+\mu)\,[(1-\rho)(1-\nu)+\rho\nu]\}\,(1+t_c)p_c h_{cN} \\ &+ \lambda\,(t_m w_m l_{mN} + t_f w_f l_{fN} + t_N p_N + t_c p_c h_{cN}) \\ &+ \{1-(2+\mu)\,[(1-\rho)(1-\gamma)+\rho\gamma]\}\,(1+t_N)p_N \\ &- [1-(2+\mu)\rho](1-t_m)w_m l_{mN} - [1-(2+\mu)(1-\rho)](1-t_f)w_f l_{fN},\end{aligned} \tag{K8}$$

which yields

$$\begin{aligned}\frac{t_N}{1+t_N} \;=\;& \left(-\frac{1}{\lambda}\right)[1-(2+\mu)(1-\rho)]\,\frac{c'}{(1+t_N)p_N} + \left(\frac{1+\lambda}{\lambda}\right)\frac{1}{-\frac{(1+t_N)N_{t_N}}{N}} \\ &+ \left(-\frac{1}{\lambda}\right)\{1-(2+\mu)\,[(1-\rho)(1-\nu)+\rho\nu]\}\,\frac{\frac{Nh_{cN}}{h_c}}{\frac{(1+t_N)p_N N}{(1+t_c)p_c h_c}} \\ &+ \left(\frac{t_m}{1-t_m}\right)\frac{-\frac{Nl_{mN}}{l_m}}{\frac{(1+t_N)p_N N}{(1-t_m)w_m l_m}} + \left(\frac{t_f}{1-t_f}\right)\frac{-\frac{Nl_{fN}}{l_f}}{\frac{(1+t_N)p_N N}{(1-t_f)w_f}} \\ &- \left(\frac{t_c}{1+t_c}\right)\frac{\frac{Nh_{cN}}{h_c}}{\frac{(1+t_N)p_N N}{(1+t_c)p_c h_c}} + \left(-\frac{1}{\lambda}\right)\{1-(2+\mu)\,[(1-\rho)(1-\gamma)+\rho\gamma]\} \\ &+ \left(-\frac{1}{\lambda}\right)[1-(2+\mu)\rho]\frac{-\frac{Nl_{mN}}{l_m}}{\frac{(1+t_N)p_N N}{(1-t_m)w_m l_m}} + \left(-\frac{1}{\lambda}\right)[1-(2+\mu)(1-\rho)]\frac{-\frac{Nl_{fN}}{l_f}}{\frac{(1+t_N)p_N N}{(1-t_f)w_f l_f}}.\end{aligned} \tag{K9}$$

Using the definition of β, (54), (55), (56), (59), and (87), (K9) can be rewritten as (91).

Appendix L: The Proof of Proposition 7

First, we derive the following relationship:

$$1-\alpha_{Nh_c}\frac{\xi}{\delta} = 1 - \frac{(1+t_N)p_N N}{(1+t_c)p_c h_c}\frac{\frac{(1+t_c)N_{t_c}}{N}}{\frac{(1+t_N)N_{t_N}}{N}} = 1 - \frac{p_N N N_{t_N}^{-1}}{p_c h_c N_{t_c}^{-1}}. \tag{L1}$$

(82) yields

$$p_N N N_{t_N}^{-1} = -\frac{(1-\rho)Nc''}{(1-\rho)(1-\gamma)+\rho\gamma}, \tag{L2}$$

and (75) and (83) yield

$$p_c h_c N_{t_c}^{-1} = -\frac{(1-\rho)Nc''}{(1-\rho)(1-v)+\rho v}. \tag{L3}$$

Applying (L2) and (L3) to (L1), we obtain

$$1 - \alpha_{Nh_c}\frac{\xi}{\delta} = \frac{(1-2\rho)(v-\gamma)}{(1-\rho)(1-\gamma)+\rho\gamma}. \tag{L4}$$

This shows that if $\nu = \gamma$, then $1 - \alpha_{Nh_c}\frac{\xi}{\delta} = 0$ and hence, the first term in (92) vanishes. Thus, we obtain (95). Moreover, substituting (95) for r_c in (91), the fourth term in (91) offsets the second term in (93) and thus, we obtain (94).

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