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On the excess entry theorem

in the presence of network effect-sensitive consumers

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## KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho Nishinomiya 662-8501, Japan On the excess entry theorem in the presence of network effect-sensitive consumers

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#### Abstract

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We reconsider the "excess entry theorem" in the case of a network product market. Heterogeneous consumers, who are sensitive to network effects, have passive expectations and Cournot oligopolistic competition prevails in the market. We demonstrate that if the network effect elasticity of network size in the equilibrium is sufficiently large, the number of firms under free entry is socially insufficient, compared with the second-best criteria. Otherwise, the socially excessive entry arises. Furthermore, we examine the case of responsive expectations and of network effect-insensitive consumers.

Keywords: excess entry theorem; network effect-sensitive consumers; a fulfilled expectation; Cournot oligopoly

JEL Classification: D42, L12, L15

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#### 1. Introduction

As in the review by Suzumura (2012), the "excess entry theorem" casts serious doubt on the conventional wisdom that the relative efficiency of resource allocation increases monotonically as the number of firms expands, that is, an increase in the number of firms promotes market competition and consequently improves social welfare. Since the seminal papers of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), there have many studies extending the theorem to various contexts: spatial competition, vertically related market structure, horizontally differentiated products markets, technology licensing, and network effects (see Matsumura and Okamura, 2006; Mukherjee and Mukherjee, 2008; Kagitani et al., 2016; Basak and Mukherjee, 2016; and Toshimitsu, 2020).

As digital technology progresses, we have observed the remarkable growth of information and communications technology (ICT) industries, e.g., telecommunications, Internet business, and social network services. Currently, many global companies are entering to the industries in newly industrializing countries such as China and Korea as well as in advanced countries such as the US and those in the EU. Thus, it is an important problem to examine how new entrants into such network product and service markets affect social efficiency. Toshimitsu (2020) introduced network effects into a standard quadratic utility function, a model that is closely related to ours, considered the social efficiency of a network product market, and demonstrated that if the degree of network effects is sufficiently large, the number of firms under free entry is socially insufficient, based on the second-best criteria. Toshimitsu (2020) assumed that homogeneous consumers have utility function with the same preferences regarding network effects. In particular, the marginal utility increase caused by an increase in network effects does not depend on the

consumer's preference (type), i.e., consumers are insensitive to network effects. However, in this paper, we assume that consumers who are sensitive to network effects (hereafter, CSNEs) exist in a network product market and have different preferences regarding network effects. This assumption is similar to that of Rohlfs (1974) and Lambertini and Orsini (2004), who consider direct network effects in a telecommunications industry. As shown below, the marginal utility increase caused by an increase in network effects depends on the consumer's type.

The purpose of the paper is to reconsider the "excess entry theorem" in the presence of CSNEs. In Section 2, we show that whether the number of firms under free entry is socially insufficient or excessive depends on the elasticity of network effects with respect to network size in the equilibrium. In Section 3, we also investigate the case of responsive expectations to confirm the result. Following Toshimitsu (2020), we examine the case of consumers who are insensitive to network effects (hereafter, CINEs). In addition, as future research problems, we consider the properties of network function and consumer expectations.

#### 2.The Model: The Presence of CSNEs

#### 2.1 An inverse demand function with multiplicatively added network effects

To analyze the excess entry theorem in the case of Cournot oligopolistic competition associated with network effects, we assume the presence of a direct network effect as already observed in ICT industries. In particular, we consider a linear market city where there is a continuum of consumers, indexed  $\theta \in [0,1]$ . To simplify, we assume that consumers are uniformly distributed with a density of one in the market, and the utility function (willingness-to-pay) of consumer  $\theta$ 

is given by:  $u(\theta) = N(X^e)\theta$ , where  $N(X^e)$  is an increasing network function of expected network sizes,  $X^e$ .<sup>[1](#page-4-0)</sup>

Given the price, a consumer purchases at most either one unit of the product or none. Hence, the surplus of consumer  $\theta$  is expressed as:  $v(\theta) = \max\{u(\theta) - P, 0\}$ . The index of the marginal consumer who has the same surplus from purchasing either one unit of the product or none is  $\hat{\theta} = \frac{P}{N(X^e)}$ .  $N(X)$  $\theta = \frac{1}{N(N^2)}$ . The quantity demanded of the network product in the market is given by:

 $X = 1 - \hat{\theta}$ ,  $X \in [0,1]$ . We obtain the following inverse demand function:

$$
P = N(X^{e})(1 - X), \quad X = \sum_{i=1}^{n} x_{i}, \tag{1}
$$

where  $x_i$  is the output of firm *i*. We assume that production costs are zero. For example, marginal costs of production in network product industries are either negligible or zero. Thus, firm *i*'s profit function is expressed as:

$$
\pi_i = Px_i - f = N(X^e)(1 - X)x_i - f,\tag{2}
$$

where *f* is a fixed entry cost.

We should notice consumer expectations that in general play an important role in a market with network effects. Based on the definition of Hurkens and López (2014), we examine the case of passive and responsive expectations: passive expectations imply that consumers first form expectations of network sizes and then firms compete in quantities, given the expected network sizes. Finally, consumers make optimal purchasing decisions, given their expectations. The

<span id="page-4-0"></span><sup>&</sup>lt;sup>1</sup> We assume that a network function and an expected network size are symmetrical:  $N_{\theta}(X_{\theta}^e) = N(X^e)$  for  $\theta \in [0,1]$ , where  $X_{\theta}^e = X^e$  is the expected network size of consumer θ. Furthermore, it holds that  $\frac{\partial u(\theta)}{\partial V(\theta)} = \theta > 0$ .  $(X^e)$ *u*  $N(X)$  $\frac{\partial u(\theta)}{\partial N(X^e)} = \theta > 0$ . That is, the larger is the value of  $\theta$ , the higher

decisions then lead to the determination of actual market shares and network sizes. Thus, in the equilibrium, the realized and expected network sizes are the same (see Katz and Shapiro, 1985). Responsive expectations imply that firms first compete in quantities and then consumers form expectations of network sizes. Finally, consumers make optimal purchasing decisions, given the quantities and their expectations. We will examine the case of responsive expectations in Section 3.

#### 2.2 A fulfilled Cournot equilibrium under passive expectations

Using equations (1) and (2), the first-order condition (FOC) of profit maximization of firm *i* is given by:

$$
\frac{\partial \pi_i}{\partial x_i} = P - N(X^e) x_i = N(X^e) (1 - 2x_i - X_{-i}) = 0,
$$
\n(3)

where 1, . *n*  $i = \sum_{i} x_{-i}$  $i=1, -i \neq i$  $X_{-i} = \sum x_i$  $=\sum_{-i=1,-i\neq i} x_{-i}$ . Furthermore, the second-order condition (SOC) and the cross effect are:

 $\frac{2\pi}{\sqrt{m_{i}^{2}}}$  =  $-2N(X^{e})$  < 0 *i*  $N(X)$ *x*  $\frac{\partial^2 \pi_i}{\partial x_i^2} = -2N(X^e)$ ∂ and  $\frac{2\pi}{2K_i} = -N(X^e) < 0.$  $i^{U \times 1} - i$  $N(X)$ *x X* π −  $\frac{\partial^2 \pi_i}{\partial x_i} = -N(X^e)$  $\partial x_i\partial$ The latter implies that strategic substitutes

arise.

 $\overline{a}$ 

At the symmetric fulfilled equilibrium, i.e.,  $X^e = X$  and  $x_i = x$ , based on the FOC, we derive the following individual and total outputs: $<sup>2</sup>$  $<sup>2</sup>$  $<sup>2</sup>$ </sup>

is the marginal utility of network effects.

<span id="page-5-0"></span> $2$  The equilibrium outputs depend only on the number of firms. This is because of the specification of the model, e.g., a stand-alone benefit does not exist and the marginal cost of production is zero. However, even relaxing the specification, the results do not significantly change. Even so, according to some values of stand-alone benefit and marginal cost, the corner solution, i.e.,  $X^p = 1$  and  $x^p = \frac{1}{n}$ , is possible. In this paper, we do not consider the corner solution.

$$
x^{p} = \frac{1}{n+1} \text{ and } X^{p} = \frac{n}{n+1},
$$
 (4)

where superscript *p* denotes the case of passive expectations. We derive the following effects of an increase in the number of firms on the outputs:

$$
\frac{dx^p}{dn} = -\frac{x^p}{n+1} \text{ and } \frac{dX^p}{dn} = x^p + n\frac{dx^p}{dn} = \frac{x^p}{n+1}.
$$
 (5)

Thus, it holds that  $\frac{dx^p}{dx^p} = -\frac{dX^p}{dx^p}$ .  $\frac{dx}{dn} = -\frac{dx}{dn}$ 

#### 2.3 Is free entry socially excessive?

Before examining the excess entry theorem, we define the number of firms under free entry, given the zero profit condition, i.e.,  $\pi^p = 0$ , as follows:  $n^p \in \left\{ n \geq 2 \middle| P^p x^p - f = 0 \right\}$  where

$$
P^p = N(X^p)(1 - X^p) \text{ and } x^p = \frac{X^p}{n}.
$$

The social welfare function based on the second-best criteria is given by:

$$
Wp(n) = \int_0^{Xp} P(N(Xp), Z) dZ - nf,
$$
\n(6)

where  $P(N(X^p), Z) = N(X^p)(1 - Z)$  and  $X^p(n) = nx^p(n)$ . Given equation (6), the second-best number of firms is given by:

$$
\frac{\partial W^p}{\partial n} = \frac{\partial X^p}{\partial n} P^p + \int_0^{X^p} \frac{\partial N(X^p)}{\partial n} (1 - Z) dZ - f
$$
  
=  $\frac{\partial X^p}{\partial n} P^p + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} X^p \left( 1 - \frac{X^p}{2} \right) - f = 0.$  (7)

Thus, the socially second-best number of firms can be defined as:

$$
n^{*p} \in \left\{ n \geq 2 \Big| \frac{\partial X^p}{\partial n} P^p + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} X^p \left( 1 - \frac{X^p}{2} \right) - f = 0 \right\}.
$$

 We consider the excess entry theorem in the presence of CSNEs. Evaluating equation (7) at the number of firms under free entry, we obtain:

$$
\left. \frac{\partial W^p}{\partial n} \right|_{P^p x^p = f} = P^p \left( \frac{\partial X^p}{\partial n} - x^p \right) + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} X^p \left( 1 - \frac{X^p}{2} \right). \tag{8}
$$

In view of equation (8), following the terminology of Mankiw and Whinston (1986), the first term expresses a "business-stealing" effect of intense market competition, which reduces social welfare, and the second term expresses a "business-augmenting" effect by network effects that improve social welfare. Using the FOC, i.e.,  $P^p = N(X^p)x^p$ , and  $\frac{dx^p}{dx^p} = -\frac{dX^p}{dx^p}$ ,  $rac{dx}{dn} = -\frac{dx}{dn}$ , equation (8)

is rewritten as:

$$
\frac{\partial W^p}{\partial n}\Big|_{P^p x^p = f} = P^p n \frac{\partial x^p}{\partial n} + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} X^p \left(1 - \frac{X^p}{2}\right)
$$

$$
= -N(X^p) X^p \frac{\partial X^p}{\partial n} + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} X^p \left(1 - \frac{X^p}{2}\right)
$$

$$
= N(X^p) \frac{\partial X^p}{\partial n} \left\{-X^p + \frac{\partial N(X^p)}{\partial X^p} \frac{X^p}{N(X^p)} \left(1 - \frac{X^p}{2}\right)\right\}.
$$
(9)

Because  $N(X^p) \frac{\partial X}{\partial x} > 0$ ,  $N(X^p) \frac{\partial X^p}{\partial x^p}$ *n*  $\frac{\partial X^p}{\partial n} > 0$ , we derive the following relationship:

$$
\left. \frac{\partial W^p}{\partial n} \right|_{P^p x^p = f} > (\langle 0 \Leftrightarrow -X^p + \varepsilon(X^p) \left( 1 - \frac{X^p}{2} \right) > (\langle 0, 0, 10) \right)
$$
\n(10)

where  $\varepsilon(X^p) = \frac{\partial N(X^p)}{\partial X^p} \frac{X^p}{N(X^p)}$  $X^p = \frac{\partial N(X^p)}{\partial Y^p} \frac{X^p}{N(X^p)}$  $\mathcal{E}(X^r) \equiv \frac{1}{\partial X^p} \frac{1}{N(X)}$  $\equiv \frac{\partial N}{\partial \theta}$ denotes the network effect elasticity of network size in a free

entry equilibrium. On the right-side hand of equation (10),  $-X^p$  expresses the negative business-stealing effect and  $\varepsilon(X^p) \left(1 - \frac{\lambda}{2}\right)$  $\varepsilon(X^p) \left(1 - \frac{X^p}{2}\right)$  $\begin{pmatrix} 2 \end{pmatrix}$ is the positive business-augmenting effect. Thus,

when the former effect is larger (smaller) than the latter, social welfare decreases (increases) with

an increase in the number of firms, compared with the second-best criteria. Equation (10) is also rewritten as:

$$
\left. \frac{\partial W^p}{\partial n} \right|_{P^p x^p = f} > \left( \langle < \right) 0 \Leftrightarrow \varepsilon(X^p) > \left( \langle < \right) \frac{X^p}{1 - \frac{X^p}{2}} = \frac{2n^p}{n^p + 1} \tag{11}
$$

where  $1 < \frac{2n^p}{n} < 2$ . 1 *p p n n*  $\lt \frac{2n}{n}$ + We summarize the above result in the following proposition.

#### *Proposition*

*If the network effect elasticity of network size in a free entry equilibrium is sufficiently large (small), the number of firms under free entry is insufficient (excessive), based on the second-best criteria.*

#### 3. Discussion

In this section, we reexamine the proposition in the case of CSNEs with passive expectations, with respect to the following; (i) the case of responsive expectations; and (ii) the case of consumers who are insensitive to network effects (CINEs)

#### 3.1 Responsive expectations case

Under responsive expectations, with regard to the perceived inverse demand function with network effects, it holds that  $X^e = X$ . Thus, equation (1) is revised as:

$$
P = N(X)(1 - X), \quad X = \sum_{i=1}^{n} x_i.
$$
 (12)

In view of equation (12), we derive the following first-order property of the inverse demand function.

$$
\frac{\partial P(X)}{\partial X} = \frac{\partial N(X)}{\partial X} (1 - X) - N(X) = N(X) \frac{1 - X}{X} \left\{ \varepsilon(X) - \frac{X}{1 - X} \right\}
$$

where  $\varepsilon(X) = \frac{\partial N(X)}{\partial X} \frac{X}{N(X)}$  $\mathcal{E}(X) \equiv \frac{\partial X}{\partial X} \frac{X}{N(X)}$  $=\frac{\partial N}{\partial \theta}$ is a network effect elasticity of total output (network size) in the

case of responsive expectations. We assume that  $\frac{\partial P(X)}{\partial X} < 0 \Leftrightarrow \frac{X}{1-X} > \varepsilon(X)$ .  $\frac{\partial P(X)}{\partial X}$  < 0  $\Leftrightarrow$   $\frac{X}{1-X}$  >  $\varepsilon$  $\partial X$  1-The price

elasticity of total output is given by:  $-\frac{\partial P(X)}{\partial X} \frac{X}{P(X)} = \frac{X}{1-X} - \varepsilon(X) > 0.$  $-\frac{\partial P(X)}{\partial X}\frac{X}{P(X)} = \frac{X}{1-X} - \varepsilon(X) > 0$ . Furthermore, with

respect to the second-order property, we have 2  $\frac{P(X)}{\partial X^2} = \frac{\partial N(X)}{\partial X} \frac{1-X}{X} \bigg\{ E(X) - \frac{2X}{1-X} \bigg\}$  $\frac{\partial^2 P(X)}{\partial X^2} = \frac{\partial N(X)}{\partial X} \frac{1-X}{X} \left\{ E(X) - \frac{2X}{1-X} \right\}$  where

$$
E(X) = \frac{\partial^2 N(X)}{\partial X^2} \frac{X}{\frac{\partial N(X)}{\partial X}}.
$$

The profit function of firm *i* is given by:  $\pi_i = Px_i - f = N(X)(1-X)x_i - f$ . The FOC is

$$
\frac{\partial \pi_i}{\partial x_i} = P + \frac{\partial P}{\partial X} x_i = N(X) \left( 1 - 2x_i - X_{-i} \right) + \frac{\partial N}{\partial X} \left( 1 - X \right) x_i = 0,\tag{13}
$$

We assume that the following SOC and cross effect hold:

$$
\frac{\partial^2 \pi_i}{\partial x_i^2} = 2 \frac{\partial P}{\partial X} + \frac{\partial^2 P}{\partial X^2} x_i
$$
  
=  $2N(X) \left\{ \frac{\partial N(X)}{\partial X} \frac{X}{N(X)} \frac{1-X}{X} - 1 \right\} + \frac{\partial N(X)}{\partial X} \left\{ \frac{\partial^2 N(X)}{\partial X^2} \frac{X}{\frac{\partial N(X)}{\partial X}} \frac{1-X}{X} - 2 \right\} x_i$   
=  $-\frac{1-X}{X} \left[ 2N(X) \left\{ \frac{X}{1-X} - \varepsilon(X) \right\} + \frac{\partial N(X)}{\partial X} \left\{ \frac{2X}{1-X} - \varepsilon(X) \right\} x_i \right] < 0$ 

and

$$
\frac{\partial^2 \pi_i}{\partial x_i \partial X_{-i}} = \frac{\partial P}{\partial X} + \frac{\partial^2 P}{\partial X^2} x_i
$$
  
=  $-\frac{1-X}{X} \left[ N(X) \left\{ \frac{X}{1-X} - \varepsilon(X) \right\} + \frac{\partial N(X)}{\partial X} \left\{ \frac{2X}{1-X} - \mathcal{E}(X) \right\} x_i \right] < 0.$ 

Thus, because the cross effect is negative, we have the following inequation:

$$
N(X)\left\{\frac{X}{1-X} - \varepsilon(X)\right\} + \frac{\partial N(X)}{\partial X}\left\{\frac{2X}{1-X} - \mathcal{E}(X)\right\} x_i > 0.
$$
 (14)

In the symmetric equilibrium, i.e.,  $x_i = x$ , from the FOC, we derive as follows:

$$
N(X^r)\left\{1-\left(1+\frac{1}{n}\right)X^r\right\}+\frac{\partial N(X^r)}{\partial X}\left(1-X^r\right)\frac{X_r}{n}=0.\tag{15}
$$

where  $X^r = nx^r$  and superscript *r* denote the case of responsive expectations. Taking the first-order property of the inverse demand function and equation (14), we obtain the effects of an increase in the number of firms on the total and individual outputs in the equilibrium as follows:

$$
\frac{dX^r}{dn} = \left(\frac{1}{n}\right)\frac{\Gamma}{\Delta} > 0 \quad \text{and} \quad n\frac{dx^r}{dn} = \frac{dX^r}{dn} - \frac{X^r}{n} = -\frac{\Gamma}{\Delta} < 0,
$$

where  $\Gamma = N(X^r) \left\{ \frac{1}{1 - X^r} - \varepsilon(X^r) \right\} \left\{ \frac{1}{n} > 0 \right\}$  $\int_{r}^r$  *X<sup>r</sup>*  $\int_{r}^r$  *x<sup>r</sup>*  $\int_{r}^r$  $N(X^r)$   $\left\{ \frac{X^r}{1 - Y^r} - \varepsilon(X^r) \right\} \frac{X^r}{Y^r}$  $\Gamma \equiv N(X^r) \left\{ \frac{X^r}{1 - X^r} - \varepsilon(X^r) \right\} \frac{X^r}{n} >$  $\left\{\frac{1}{1-X^r}-\varepsilon(X^r)\right\}\frac{1}{n}>0$  and

$$
\Delta = \left(1 + \frac{1}{n}\right)N(X^r)\left\{\frac{X^r}{1 - X^r} - \varepsilon(X^r)\right\} + \frac{\partial N(X^r)}{\partial X^r}\left\{\frac{2X^r}{1 - X^r} - \mathcal{E}(X^r)\right\}\frac{X^r}{n} > 0.
$$

Before examining the excess entry theorem, we define the number of firms under free entry, given the zero profit condition, i.e.,  $\pi^r = 0$ , as follows:  $n^r \in \{ n \ge 2 | P^r x^r - f = 0 \}$  where

$$
P^r = N(X^r)(1 - X^r) \text{ and } x^r = \frac{X^r}{n}.
$$

The social welfare function based on the second-best criteria is given by:

$$
W^{r}(n) = \int_{0}^{X^{r}} P(N(Z), Z) dZ - nf,
$$
\n(16)

where  $P(N(Z), Z) = N(Z)(1-Z)$  and  $X^{r}(n) = nx^{r}(n)$ . The second-best number of firms is given by:

$$
\frac{\partial W^r}{\partial n} = \frac{\partial X^r}{\partial n} P^p - f = 0.
$$
\n(17)

Thus, the socially second-best number of firms is defined as:  $n^{*r} \in \left\{ n \geq 2 \middle| \frac{\partial X^r}{\partial x^r} - f = 0 \right\}.$  $\in \left\{ n \geq 2 \Big| \frac{\partial X^r}{\partial n} P^r - f = 0 \right\}$ 

Evaluating equation (17) at the number of firms under free entry, we obtain:

$$
\left. \frac{\partial W^r}{\partial n} \right|_{p^r x^r = f} = \frac{\partial X^r}{\partial n} P^r - P^r x^r
$$
\n
$$
= P^r \left( \frac{\partial X^r}{\partial n} - \frac{X^r}{n} \right) = P^r \frac{\partial x^r}{\partial n} n < 0. \tag{18}
$$

Therefore, irrespective of network effects, the number of firms under free entry is socially excessive, compared with the second-best number of firms. In the case of responsive expectations, there is no positive business-augmenting effect caused by network effects because, given the inversed demand included with network effects, firms decide their output. Thus, in the equilibrium, there is only a negative business-stealing effect, similar to the case of Cournot oligopolistic competition in a homogeneous product market.

#### 3.2 CINEs with passive expectations

In the previous sections, assuming the utility function of CSNEs, we have derived the inverse demand function with multiplicative network effects, i.e., equations (1) and (12). In this section, we consider the utility function of consumers, who are insensitive to network effects (CINEs). In particular, all consumers have identical preferences for network effects, i.e.,  $u(\theta) = \theta + N(X^e)$ . Thus, the surplus of consumer  $\theta$  having passive expectations is given by:

 $v(\theta) = \max{\lbrace \theta + N(X^e) - P, 0 \rbrace}$ . We obtain the following inverse demand function with additive separable network effects.

$$
P = 1 - X + N(Xe), \quad X = \sum_{i=1}^{n} x_i,
$$
\n(19)

where we assume  $-1 + \frac{\partial N(X^e)}{\partial X} < 0$ . *e*  $N(X)$ *X*  $-1+\frac{\partial N(X^e)}{\partial X^e}$  < 0. The profit function of firm *i* is expressed as:

$$
\pi_i = Px_i - f = \left\{1 - X + N\left(X^e\right)\right\} x_i - f.
$$
 The FOC is  $\frac{\partial \pi_i}{\partial x_i} = P + \frac{\partial P}{\partial X} x_i = 1 - 2x_i - X_{-i} + N(X^e) = 0.$ 

Based on the FOC and by the same procedure as in the previous sections, we have the following equation determining total output in the symmetric fulfilled Cournot equilibrium under passive expectations.

$$
1 - \left(1 + \frac{1}{n}\right)X^p + N(X^p) = 0.
$$
\n(20)

Thus, the effects of an increase in the number of firms on the total and individual outputs are as follows:

$$
\frac{dX^p}{dn} = \frac{\left(\frac{1}{n}\right)X^r}{1 + \frac{1}{n} - \frac{\partial N(X^p)}{\partial X^p}} > 0 \quad \text{and} \quad n\frac{dx^p}{dn} = -\frac{\frac{X^p}{n}\left(1 - \frac{\partial N(X^p)}{\partial X^p}\right)}{1 + \frac{1}{n} - \frac{\partial N(X^p)}{\partial X^p}} < 0. \tag{21}
$$

The social welfare function is represented as:  $W^p(n) = \int_0^{X^p} P(N(X^p), Z) dZ - nf$ , where  $P(N(X^p), Z) = 1 - Z + N(X^p)$  and  $X^p(n) = nx^p(n)$ .

Thus, using the FOC of the social welfare function, we have the following socially second-best

number of firms: 
$$
n^{\#p} \in \left\{ n \ge 2 \middle| \frac{\partial W^p(n)}{\partial n} = \frac{\partial X^p}{\partial n} P^p + \int_0^{X^p} \frac{\partial N(X^p)}{\partial n} dZ - f = 0 \right\}.
$$

Evaluating the FOC of the social welfare function at the number of firms under free entry, we

obtain:

$$
\left. \frac{\partial W^p}{\partial n} \right|_{P^p x^p = f} = \frac{\partial X^p}{\partial n} P^p + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} \int_0^{X^p} dZ - P^p x^p
$$
\n
$$
= P^p \left( \frac{\partial X^p}{\partial n} - x^p \right) + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} X^p.
$$
\n(22)

The first (second) term of equation (22) is a business-stealing (-augmenting) effect. Taking the FOC of profit maximization of individual firms, i.e.,  $P^p = x^p$ , and the effects on the total and individual outputs, i.e., equation (21), equation (22) can be rewritten as:

$$
\frac{\partial W^p}{\partial n}\Big|_{P^p x^p = f} = X^p \frac{\partial x^p}{\partial n} + \frac{\partial N(X^p)}{\partial X^p} \frac{\partial X^p}{\partial n} X^p
$$
\n
$$
= X^p \frac{X^p}{1 + \frac{1}{n} - \frac{\partial N(X^p)}{\partial X^p}} \left\{ 2 \frac{\partial N(X^p)}{\partial X^p} - 1 \right\} > (<0).
$$
\n(23)

Thus, we obtain the following relationship:

$$
\left. \frac{\partial W^p}{\partial n} \right|_{P^p x^p = f} > \left( \langle < \right) 0 \Leftrightarrow \frac{\partial N(X^p)}{\partial X^p} > \left( \langle < \right) \frac{1}{2}.\tag{24}
$$

In the case of CINEs, if the degree of marginal network effects is larger (smaller) than a half, the number of firms under free entry is insufficient (excessive), based on the second-best criteria. For example, if we assume a linear network function, i.e.,  $N(X^e) = \beta X^e$  and  $0 < \beta < 1$ , it

holds that 
$$
\frac{\partial W^p}{\partial n}\Big|_{P^p x^p = f} > (\langle 0 \Leftrightarrow \beta > (\langle 0 \rangle \frac{1}{2})
$$
. See Proposition 1 in Toshimits (2020). However, if

assuming the case of CSNEs, the network effect elasticity of network size is unity, i.e.,  $\varepsilon(X^p) = 1$ , and, in view of equation (11), the number of firms under free entry is socially excessive, irrespective of the degree of network effects.

Furthermore, with respect to the case of CINEs having responsive expectations, we can derive the same results as in Section 3.1, i.e., the socially excess entry arises, irrespective of the degree of network effects.

#### 3.3 Further problems

#### (1) Network function

We have respectively considered the cases of CSNEs and CINEs having passive expectations, based on a general form of network function. As a result, whether the number of firms under free entry is socially insufficient or excessive depends on the nature of the network function. However, if the network effect is a linear function of network size, socially excessive entry always arises in the case of CSNEs while socially insufficient entry can arise in the case of CINEs. Thus, we need to investigate the properties of the network function.

Furthermore, assuming the cases of CSNEs and CINEs in a linear city market, we have derived the inverse demand function with multiplicative and additive network functions, respectively. Thus, we should consider the relationship between utility and network functions.<sup>[3](#page-14-0)</sup>

#### (2) Symmetric assumptions

We have assumed heterogeneous consumers in a linear market, i.e.,  $\theta \in [0,1]$ . This assumption is similar to that of Katz and Shapiro (1985). In this case, it may be natural to assume that consumers have different preferences regarding network effects and various expectations of network sizes. However, we assume a symmetric network function and the same expected network size, i.e.,  $N_{\theta}(X^e) = N(X^e)$  for  $\theta \in [0,1]$ , where  $X^e_{\theta} = X^e$  is the expected network

<span id="page-14-0"></span> $3$  Swann (2002) considered the functional form of the relationship between utility and the size of a network and found that the functional form can be linear, but only under strong conditions. Furthermore, he explored the conditions under which an individual utility is a linear function of

size of consumer  $\theta$ . Relaxing this assumption, we should examine not only the excess entry theorem, but also the properties of a fulfilled Cournot oligopolistic equilibrium itself.

#### 4. Conclusion

 $\overline{a}$ 

We have reconsidered the excess entry theorem in the presence of network effect-sensitive consumers having passive expectations in a network product market, where a Cournot oligopolistic competition prevails.

We have demonstrated as follows. In the case of passive expectations, the stronger is the strength of network effects, the larger is the business-augmenting effect over the business-stealing effect. Hence, the number of firms under free entry is socially insufficient. In other words, from the viewpoint of social welfare, a government should promote new entry to the market. With the responsive expectations, whether consumers are sensitive or insensitive to network effects, the number of firms under free entry is socially excessive. That is, taking into account that the expected network sizes of consumers are the same as the announced level of total outputs, firms thus perceive the inverse demand functions as well as the expected network sizes and determine their actual outputs. Thus, an increase in the number of firms promotes a negative business-stealing effect, but not a business-augmenting effect. In this case, a government should restrict the number of new entries.

The results depend on the types of consumer expectations, i.e., passive or responsive. We need to explore both theoretically and empirically how consumers form network sizes and in which products and services markets this occurs.

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