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Inequality and Institutional Quality in a Growth Model

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Inequality and Institutional Quality in a Growth Model*

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Abstract

A macroeconomic growth model of an occupational choice between being a diligent worker and being a criminal is developed. Imperfect protection of property rights renders agents heterogeneous in their holding wealth over time even though they are homogeneous in the initial period. The model is so tractable that one can explicitly derive the distribution of individual wealth and compute the Gini coefficient analytically in the stationary state. Using the Gini coefficient, we investigate how institutional quality affects inequality across agents in the economy. Our findings are as follows. In the case of a relatively higher capital share, as institutional quality improves, inequality widens in the early stage of development of institutional quality; in contrast, inequality shrinks once institutions have sufficiently matured. In the case of a lower capital share, inequality monotonically shrinks as institutional quality improves. Furthermore, we present government policies that reduce inequality and achieve the first-best outcome.

Keywords: Inequality, Institutional quality, Heterogeneous agents, Gini coefficient, Economic growth.

JEL Classification Numbers: D23, D63, O11, O41, O43.

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1 Introduction

The purpose of this study is to investigate how institutional quality affects inequality across individuals in the economy. According to the United Nations (2013), inequality in developing countries increased by 11 percent between 1990 and 2010. This report implies that in many developing countries, inequality across individuals remains persistent. In this paper, we attribute persistent inequality in developing countries to institutional quality. Figure 1 is a scatter plot that displays the relationship between institutional quality and inequality.[1] Although it does not provide information on the causality between them, we observe their presumably negative relationship in the figure.[2] We theoretically explore the mechanism behind this observation.

[Figure 1 around here]

It is widely known that institutional quality significantly impacts economic performance (e.g., North 1990; Knack and Keefer 1995; Hall and Jones 1999; Acemoglu, Johnson, and Robinson 2001, 2002; Rodrik, Subramanian, and Trebbi 2004). In an economy where institutional quality is so low that property rights are not secured and personal possessions are easily stolen by others (possibly including a corrupt government), individuals lack incentives to work diligently. In such an economy, people are frequently robbed of their earnings, and many of them do not hesitate to become criminals. If many people acquire their earnings by committing crimes rather than by working diligently, the production activities in the economy will be inefficiently implemented, being away from the production possibility frontier. Furthermore, in the circumstances where nobody including government

[1]The scatter plot in Figure 1 is based on 123 countries. The data on the Gini coefficient are collected from the dataset developed by Solt (2009, 2019). The index of "Legal System and Property Rights" in the Economic Freedom of the World in Gwartney, Lawson, Hall, and Murphy (2018) is used as a proxy of institutional quality. Both data are averaged over 2000-2017.

[2]Chong and Gradstein (2007) provide empirical evidence showing that inequality monotonically declines as institutional quality improves. Nevertheless, one may consider the possibility that the relationship between institutional quality and inequality is inverted U-shaped; Chong and Calderón (2000) empirically show that as institutional quality improves, inequality widens in poor countries but shrinks in rich countries.

officials rescues crime victims, the latter have no choice but to become poor; thus, inequality across individuals widens. We model these scenarios.

In our model, each agent lives for one period and bears one offspring, as in the model of Aghion and Bolton (1997). Then, each family line continues forever with two adjacent generations being linked by bequest. In the beginning of a given period, an agent receives an amount of wealth from the agent's parent as a bequest. There are two important characteristics to be explained in our model. The first is the whereabouts of the bequest, and the second is the agent's occupational choice. With regard to the bequest, there is no technology that can be used to store wealth until the end of the period, and thus, the agent must convert it into capital, which is used by a representative firm to produce general goods. However, wealth may be stolen by criminals in the beginning of the period. If wealth is stolen, the agent cannot earn interest income at the end of the period. With regard to the occupational choice, the agent chooses to be a diligent worker or a criminal in the beginning of the period. The presence of criminals is an essential factor that produces agents' heterogeneity in holding wealth, which is affected by institutional quality (measured by the probability of criminals being arrested). If the agent becomes a diligent worker, he or she will earn a wage income at the end of the period. If the agent becomes a criminal, he or she joins a criminal gang and is directed to steal wealth from others by the boss of the criminal gang. The gang converts the collected wealth into capital and earns interest income at the end of the period. The gang evenly distributes the interest income to all members. However, when committing a crime, any member may be arrested. Even a member that was arrested can nonetheless receive the same amount of rewards from the gang as do the other members.[3]

In equilibrium, the wage income is equal to a criminal's earnings because of the no-arbitrage condition between being a diligent worker and being a criminal. Such a condition

[3] As such, the criminal gang plays a role of insurance in our model. Although one can construct a model such that each member of the gang is rewarded as he or she works for the gang, the analysis becomes complicated, while the main result regarding inequality and institutional quality is unchanged.

determines the number of diligent workers, which is less than the population in the economy unless institutional quality is sufficiently high. The number of workers being smaller than the total population is a source of production inefficiency. In this case, the aggregate consumption is smaller and capital accumulation is limited relative to the case of employment of the entire population. We derive not only the law of motion of the aggregate capital but also the law of motion of individual wealth. Furthermore, we obtain the distribution of individual wealth and the Gini coefficient that measures wealth inequality in the stationary state. The tractable Gini coefficient enables us to investigate the relationship between institutional quality and inequality.

Our findings are as follows. If the capital share is relatively large, the effect of institutional quality on inequality is inverted U-shaped. In other words, in the early stage of development of institutional quality, inequality widens as institutional quality improves. However, once institutions have sufficiently matured, inequality shrinks with further improvement of institutional quality. If the capital share is small, the effect of institutional quality on inequality is monotonic. In other words, inequality monotonically declines as institutional quality improves. Furthermore, we identify government policies that reduce wealth inequality and achieve the first-best outcome even though there is no strong protection of property rights.

The remainder of this paper is organized as follows. In the following section, we discuss the related literature. In section 3, we develop a growth model of the occupational choice between being a diligent worker and being a criminal. In section 4, we obtain the dynamics of the aggregate capital in general equilibrium. In section 5, we derive the distribution of individual wealth analytically in the stationary state. In section 6, we compute the Gini coefficient that measures wealth inequality and investigate how institutional quality affects inequality. In section 7, government policies that reduce wealth inequality and achieve the first-best outcome are introduced. Section 8 concludes this paper.

2 Related literature

Over the past twenty years, many researchers have studied the interaction between inequality and economic growth. In the pioneering work of Galor and Zeira (1993), a model of agents' occupational choices between being unskilled workers and being skilled workers in a financially constrained economy was developed. They investigated the interaction between inequality within an economy and macroeconomic variables such as aggregate output and investment. By deriving multiple equilibria, the authors explained persistent income differences across countries. Subsequently, Banerjee and Newman (1993) also considered agents' occupational choices between being workers, self-employed, and entrepreneurs, and investigated the interplay between the occupational choices and the distribution of individual wealth. Additionally, Aghion and Bolton (1997) developed a model of economic growth and income inequality in a financially constrained economy and analyzed the trickle-down effect of capital accumulation. The important characteristic of all of the above studies is that the interaction between wealth distribution and macroeconomic outcomes is investigated. However, because of the nonlinearity of the dynamics of individual wealth, it is very difficult to obtain an explicit wealth distribution analytically from the above studies' models. Although, as in those studies, the dynamics of individual wealth is nonlinear in our model, an explicit wealth distribution can be obtained in our model. Furthermore, one can compute the Gini coefficient from the derived wealth distribution analytically, which can be used to investigate the relationship between institutional quality and inequality.

In addition to the above three studies, Galor and Moav (2000) develop a model in which income inequality arises both between the two groups of skilled and unskilled workers and within each group. Galor and Moav (2004) develop a unified theory of the effects of inequality on the process of development, in which physical capital accumulation is replaced by human capital accumulation as a growth engine.[4] In the model of Aghion, Howitt, and

[4] As in the model of Galor and Moav (2004), Asano (2012) also highlights the role of human capital accumulation and derives a nonmonotonic relationship between inequality and economic growth.

Violante (2002), it is demonstrated that the enlarged generality of new technologies leads to income inequality. Mendoza, Quadrini, and Ríos-Rull (2009) study the difference in the dynamic behaviors of wealth inequality in two countries characterized by different degrees of financial development after financial liberalization. Matsuyama (2002) develops a model of mass-consumption economies and demonstrates that a certain level of inequality is necessary for sustainable economic growth. There are also studies in the literature on political economy and economic growth that deal with inequality. Among others, Alesina and Rodrik (1994), Persson and Tabellini (1994), and Bertola (1993) show that inequality negatively affects economic growth in a politico-economic equilibrium.

While the abovementioned studies mainly focus on the relationship between inequality and economic growth, Chong and Gradstein (2007, 2019) and Gradstein (2007) study bidirectional causalities between institutional quality and inequality by developing a rent-seeking model. Our model is not a rent-seeking model. Although we consider only a one-way causality from institutional quality to inequality, the most important characteristic of our model is that criminals appear endogenously depending on the extent of protection of property rights against crimes. Then, our paper can be also related to the literature on the economics of crimes pioneered by Becker (1968) in that the presence of criminals leads to allocative and/or production inefficiencies. In the literature, Ehrlich (1973) investigates the interaction between crime activities and collective law enforcement. Chiu and Madden (1998) develop a model in which agents' income may be supplemented by burglary and demonstrate that the higher extent of income inequality leads to more burglaries. İmrohoroğlu, Merlo, and Rupert (2000, 2004) develop static and dynamic general equilibrium models to analyze the characteristics of criminal activities in the United States. Using an equilibrium search model, Burdett, Lagos, and Wright (2003) study the relationships between crime, unemployment, and inequality. Although criminal activities are introduced in our model as they are in the existing studies, our interest is in how the extent of protection of property rights affects the distribution of individual wealth.

3 Model

A closed economy consists of a continuum of agents with the total population normalized to one, a representative firm, and a representative criminal gang. The economy continues from period 0 to $+\infty$ in discrete time indexed by t. Each agent lives for one period and gives birth to one offspring; accordingly, each family line lasts forever. An agent in a certain period may become a diligent worker or a member of the criminal gang. Whereas all agents are homogeneous in the initial period, the heterogeneity of agents originates from the presence of criminals, which is a source of inequality across agents. In each period, by employing capital and workers, the representative firm produces general goods that can be used for consumption and bequest. Meanwhile, the boss of the criminal gang directs the members (criminals) to steal the possessions of others. The criminal gang collects the stolen wealth and rewards the members for their misdeeds.

3.1 Timing of events

Consider the timing of events in period t from the perspective of a certain agent denoted by $i \in \Omega$ (or interchangeably, that of family i), where Ω is a whole set of family genealogies. Each period is separated into two subperiods. As shown in Figure 2, there are two aspects to be considered: first, the whereabouts of a bequest, and second, the occupational choice. Note that production in period t occurs at the end of the period.

[Figure 2 around here]

3.1.1 Bequest

In the beginning of period t, agent i receives an amount k_{it} of wealth as a bequest from his or her parent (who died in period $t-1$). Because there is no technology for storing the bequest from the beginning to the end of the period, the bequest must be converted into capital by a one-for-one technology, which broadly includes physical and human capital.[5] At the end of period t, the capital that agent i has is lent out to the representative firm via a perfect capital market, and agent i earns interest income in the capital market in the same period. However, agent i may have the wealth stolen by criminals before it has been converted into capital.[6]

3.1.2 Occupational choice

In the beginning of period t, agent i chooses to be a worker or a member of the criminal gang. If agent i becomes a worker, he or she earns wage income w_t by supplying one unit of labor to the representative firm at the end of period t.

If agent i becomes a criminal, he or she steals a wealth from others in the beginning of period t. When committing a crime, agent i may be arrested with a certain probability $p \in [0, 1]$. The criminal gang collects the wealth that criminals steal and converts it into capital, which is lent out to the representative firm.[7] The criminal gang earns interest income from the firm, and afterwards evenly distributes the income to each member. Agent i obtains earnings $\tilde{w}_t$ at the end of period t even if he or she has actually been arrested. In other words, the agent's illegal earnings are guaranteed once he or she has become a member of the criminal gang.[8] The probability of being arrested, p, is assumed to be constant for all $t \geq 0$. One can regard p as reflecting, broadly, institutional quality related to the protection of property rights and the extent of legal enforceability in the economy. We assume that there is no free-rider problem in the criminal gang such that criminals pretend to be arrested without committing crimes. One should note that there is a possibility

[5]In other word, bequests can be used for investment projects to produce physical capital and/or for education to form human capital. In any case, physical capital and human capital are assumed to be perfect substitutes in this paper.

[6]One can consider various types of crimes agents face, including robbery, fraud, expropriation, embezzlement, etc.

[7]One can imagine that by laundering money, the criminal gang can access the capital market without being arrested.

[8]It is assumed that if the agent is arrested, the prison term is sufficiently short that the agent is released before his or her lifetime ends.

that any member of the criminal gang steals from another member because each member chooses his or her target randomly.

3.1.3 Consumption and bequest

Agents extract their utility exclusively from consumption and bequest that they leave to their offspring at the end of period t. Agent i's income, I_{it}, including wage income and interest income (which may be 0 because the agent may be robbed in the beginning of period t) is determined at the end of period t. Afterwards, the agent makes a decision as to the quantity he or she consumes and leaves a bequest.

3.2 Production

The representative firm produces general goods by applying a Cobb-Douglas production technology such that $y_t = Ak_t^{\alpha} l_t^{1-\alpha}$ where $\alpha \in (0,1)$ is the capital share, y_t is the total output, k_t is the total capital, l_t is the total labor, and A is the technology level of the production function. It is assumed that capital depreciates entirely in one period. Since the capital and labor markets are competitive, they are paid their marginal products:

$$w_t = (1-\alpha)Ak_t^{\alpha} l_t^{-\alpha} \tag{1}$$

$$r_t = \alpha A k_t^{\alpha-1} l_t^{1-\alpha}, \tag{2}$$

where r_t is a (gross) interest rate.

3.3 Criminal gang

Since l_t is the population of diligent workers, the population of criminals is equal to $1 - l_t$. A criminal in period t randomly chooses an agent as his or her target to steal the amount of wealth k_{it}. For simplicity, it is assumed that any criminal does not choose an agent chosen by another criminal and that one criminal robs one agent. As previously mentioned, there is a possibility that a criminal steals from another criminal. As such, $1-l_t$ also measures the probability that any one of agents is robbed by a criminal in period t. When committing a crime, a criminal is arrested with probability p. The criminal gang can collect the stolen wealth only from the criminals who are not arrested.

Because of the random choice of targets and because the probability that any given criminal is not arrested is equal to $1-p$, the law of large numbers implies that the total stolen wealth that the criminal gang obtains is equal to $(1-p)(1-l_t)\int_{i\in\Omega} k_{it}di$, where $\int_{i\in\Omega} k_{it}di$ is the average capital stock that agents have in period t, which is equal to the total capital stock used for production in equilibrium. Since the total amount of stolen wealth is lent out to the representative firm, the criminal gang earns interest income $(1-p)(1-l_t)r_t\int_{i\in\Omega} k_{it}di$, which is evenly paid to members for committing crimes. Then, the representative criminal gang solves the following maximization problem: $\max_{1-l_t}(1-p)(1-l_t)r_t\int_{i\in\Omega} k_{it}di-\tilde{w}_t(1-l_t)$. Because the black labor market of criminals is competitive, the zero-profit condition for the representative criminal gang yields

$$\tilde{w}_t = (1-p)r_t \int_{i\in\Omega} k_{it}di. \tag{3}$$

3.4 Agents

Agents choose their occupations in the beginning of a period and decide how much they consume and leave as bequest at the end of a period. Therefore, we solve their optimization problem backwards.

3.4.1 Utility maximization

Agent i has a utility function

$$u_{it} := c_{it}^{1-\sigma} k_{it+1}^{\sigma}, \tag{4}$$

where $\sigma \in (0, 1)$ is a weight parameter between consumption and bequest. At the end of period t, the agent faces a budget constraint such that

$$c_{it} + k_{it+1} \leq I_{it}. \tag{5}$$

Agent i maximizes Eq. (4) subject to Eq. (5). From the first-order condition, we obtain

$$c_{it} = (1 - \sigma)I_{it} \tag{6}$$

and

$$k_{it+1} = \sigma I_{it}. \tag{7}$$

Eqs. (4), (6), and (7) yield agent i's indirect utility as follows:

$$V_{it} := \max u_{it} = \sigma^{\sigma}(1 - \sigma)^{1-\sigma} I_{it}. \tag{8}$$

3.4.2 Income

An agent loses his or her interest income $r_t k_{it}$ with a certain probability because of criminals. Suppose that such a probability is $1 - q_t$. Then, the agent can retain income $r_t k_{it}$ at the end of period t with probability q_t and loses it with probability $1 - q_t$. It is straightforward to show that the probability that an agent is robbed and the robber is not arrested is equal to $(1 - l_t)(1 - p)$. Therefore, it follows that $1 - q_t = (1 - l_t)(1 - p)$, or, equivalently,

$$q_t = 1 - (1 - l_t)(1 - p). \tag{9}$$

Thus, agent i's income at the end of period t is represented as follows:

$$I_{it} = \begin{cases} \omega_{it} + r_t k_{it} & \text{with probability } q_t \\ \omega_{it} & \text{with probability } 1 - q_t, \end{cases} \tag{10}$$

where $\omega_{it} = w_t$ if agent i is a diligent worker and $\omega_{it} = \tilde{w}_t$ if he or she is a criminal. It is assumed that the wealth in the initial period is determined exogenously, is common across agents, and is given by $k_{i0} = k_0$. Since the total population of agents is consistently equal to one for all $t \geq 0$, according to the law of large numbers, q_t also measures the population of agents who can retain interest income at the end of period t.

3.4.3 Optimal occupation

From Eqs. (8) and (10), we obtain agent i's expected indirect utility as follows:

$$E(V_{it}) = \sigma^{\sigma}(1-\sigma)^{1-\sigma}(\omega_{it} + q_t r_t k_{it}). \tag{11}$$

Agent i chooses his or her occupation to maximizes the expected indirect utility. Since Eq. (11) holds for any agent $i \in \Omega$ and $E(V_{it})$ increases with ω_{it}, Remark 1 below immediately follows from Eq. (11).

Remark 1. *Consider any agent $i \in \Omega$. If $w_t > \tilde{w}_t$, agent i becomes a worker, and if $w_t < \tilde{w}_t$, agent i becomes a criminal. If $w_t = \tilde{w}_t$, agent i is indifferent to the occupational choice of being a worker and being a criminal.*

4 Equilibrium

Given the common initial individual wealth, $k_{i0} = k_0$, a competitive equilibrium is expressed by sequences of interest rate $\{r_t\}$, workers' wage rate $\{w_t\}$, criminals' earnings $\{\tilde{w}_t\}$, allocation $\{(c_{it}, k_{it+1})\}$ and $\{k_t, l_t\}$, and agents' occupational choices for all $t \geq 0$ so that the optimization conditions of all agents, the representative firm, and the representative criminal gang hold and the markets in general goods, capital, and labor all clear.

4.1 Capital market

As previously explained, the bequest received from a parent is converted into capital by a one-for-one technology. Then, the capital market clearing condition is given by

$$k_t = \int_{i\in\Omega} k_{it} di. \tag{12}$$

By applying Eq. (12), we can obtain the aggregate income, $\int_{i\in\Omega} I_{it} di$, as shown in Lemma 1 below.

Lemma 1. *The aggregate income, $\int_{i\in\Omega} I_{it} di$, is given by*

$$\int_{i\in\Omega} I_{it} di = A k_t^{\alpha} l_t^{1-\alpha}. \tag{13}$$

Proof. See the Appendix.

Eq. (13) is consistent with national accounting because all of general goods produced in period t are consistently distributed to agents as their incomes.

4.2 Labor market

According to Remark 1, if $w_t > \tilde{w}_t$, any agent becomes a worker, and it holds that $l_t = 1$. Since $w_t = (1-\alpha)A k_t^{\alpha} l_t^{-\alpha}$ decreases with l_t and $\tilde{w}_t = (1-p)\alpha A k_t^{\alpha} l_t^{1-\alpha}$ increases with l_t, if $(1-\alpha)A k_t^{\alpha} > (1-p)\alpha A k_t^{\alpha} \Longleftrightarrow p > (2\alpha-1)/\alpha$, all agents become workers. Otherwise, some agents may choose to be criminals. However, if $w_t < \tilde{w}_t$ in period t, no agents become diligent workers. In this case, the marginal product of labor in production becomes infinite, and the wage rate increases. Therefore, for workers and criminals to coexist in the economy for all $t \geq 0$, it must hold that $w_t = \tilde{w}_t \Longleftrightarrow l_t = (1-\alpha)/[\alpha(1-p)]$ where if $p = (2\alpha-1)/\alpha$, it holds that $l_t = 1$. Formally, the following lemma is obtained.

Lemma 2. *The population of diligent workers in equilibrium is given by*

$$l_t = \begin{cases} 1 & \text{if } p \geq \frac{2\alpha-1}{\alpha} \\ \frac{1}{1-p}\left(\frac{1-\alpha}{\alpha}\right) & \text{if } 0 \leq p < \frac{2\alpha-1}{\alpha}. \end{cases} \tag{14}$$

Proof. From the discussion immediately preceding Lemma 2, it is obvious that if $p \geq (2\alpha - 1)/\alpha$, it holds that $l_t = 1$. If $0 \leq p < (2\alpha - 1)/\alpha$, it must hold that $w_t = \tilde{w}_t \iff (1 - p)l_t = (1 - \alpha)/\alpha$ for the economy to be feasible for all $t \geq 0$. The last equality yields the second equality of Eq. (14). □

The intuition behind Lemma 2 is as follows. As capital share α increases, the population of workers decreases. This is because as α increases, the interest income that the criminal gang can earn becomes large relative to the probability of being arrested and the formal wage rate. Then, the number of criminals increases. In contrast, as the probability of being arrested increases, agents are more likely to choose not to be criminals, and thus, the population of workers increases. If p is greater than or equal to a threshold, given by $(2\alpha - 1)/\alpha$, no agents become criminals.

By applying Eq. (14) to Eq. (9), q_t (i.e., the probability of not being a crime victim) is computed as follows:

$$q_t = \begin{cases} 1 & \text{if } p > \frac{2\alpha-1}{\alpha} \\ p + \frac{1-\alpha}{\alpha} & \text{if } p \leq \frac{2\alpha-1}{\alpha}. \end{cases} \tag{15}$$

It is clear that q_t increases with p. However, q_t decreases as the capital share increases because the population of criminals increases.

4.3 General goods market

We aggregate Eq. (7) across all agents to obtain

$$\int_{i\in\Omega} k_{it+1} di = \sigma \int_{i\in\Omega} I_{it} di. \tag{16}$$

By applying Eqs. (12) and (13), we can rewrite Eq. (16) as follows:

$$k_{t+1} = \sigma A k_t^{\alpha} l_t^{1-\alpha}. \tag{17}$$

4.4 Capital accumulation

The labor input given by Eq. (14) is intensively rewritten as $l_t = \min\{1, (1-\alpha)/[\alpha(1-p)]\} =: l(p)$, where as p increases from 0 to $(2\alpha-1)/\alpha$, $l(p)$ increases from $(1-\alpha)/\alpha$ to 1. Substituting $l_t = l(p)$ into Eq. (17) yields

$$k_{t+1} = l(p)^{1-\alpha} \sigma A k_t^{\alpha}. \tag{18}$$

Figure 3 presents a phase diagram for the dynamic behavior of capital given by Eq. (18). It is straightforward to obtain the steady-state capital as follows:

$$\bar{k} := (\sigma A)^{\frac{1}{1-\alpha}} l(p). \tag{19}$$

As Figure 3 shows, in the process of economic development, the aggregate capital monotonically increases and converges to the steady state when the initial aggregate capital stock is low. One notes that if p is greater than the threshold value, $(2\alpha-1)/\alpha$, it holds that $l(p) = 1$ and capital is accumulated successfully even though the probability of being arrested is not equal to 1 and the institutional quality is not the highest. If $p < (2\alpha-1)/\alpha$, $l(p)$ increases with p, which implies that capital accumulation is enhanced as institutional quality improves.

[Figure 3 around here]

Thus far, we have investigated the aggregate capital. In the next section, we investigate the wealth distribution in the stationary state. To do so, we obtain Proposition 1 below regarding the wage and interest rates in the steady state.

Proposition 1. *The wage and interest rates in the steady state of Eq. (18) are given by*

$$\bar{w} := (1-\alpha)A^{\frac{1}{1-\alpha}}\sigma^{\frac{\alpha}{1-\alpha}} \tag{20}$$

$$\bar{r} := \alpha\sigma^{-1}, \tag{21}$$

respectively.

Proof. The claims follow from Eqs. (1), (2), and (19). □

One notes from Eqs. (20) and (21) that both wage and interest rates are independent of p.

5 Stationary state and wealth distribution

In this section, we derive the dynamics of individual wealth (bequests) and analytically obtain the wealth distribution in the stationary state. The stationary state is defined as a state in which the aggregate capital is equal to the steady-state value given by Eq. (19) and individual wealth, $\{k_{it}\}$, follows a time-invariant distribution. The idea clearly underlying this definition is that even though macroeconomic variables such as the aggregate capital, the aggregate consumption, and the total output become constant in the steady state, each family's wealth may vary over the time-invariant stationary distribution.

We investigate the relationship between wealth inequality and institutional quality by deriving the Gini coefficient from the wealth distribution. In the following, we replace the term "agent i" with "family i" because we consider the dynamics of wealth of family genealogies. To focus on a meaningful situation in which there are criminals in equilibrium, we impose the following parameter assumption throughout the following analysis.[9]

[9] Assumption 1 implies that $\alpha > 1/2$. Mankiw, Romer, and Weil (1992) estimate the physical and human capital shares. According to the researchers' estimation, the sum of the capital shares is approximately 0.51–0.59.

Assumption 1. *The probability of being arrested satisfies the following inequality:*

$$0 \leq p < \frac{2\alpha - 1}{\alpha}.$$

5.1 Derivation of stationary wealth distribution

Under Assumption 1, Eqs. (7), (10), and $w_t = \tilde{w}_t$ yield the law of motion of family i's wealth as follows:

$$k_{it+1} = \begin{cases} \sigma \left(w_t + r_t k_{it} \right) & \text{with probability } q \\ \sigma w_t & \text{with probability } 1 - q, \end{cases} \tag{22}$$

where $q := p + (1 - \alpha)/\alpha$ from Eq. (15). As discussed in section 4, the economy with any initial wealth, $k_{i0} = k_0$, monotonically converges to the steady state. In this process, both w_t and r_t also converge to their steady-state values given by Eqs. (20) and (21). Therefore, from Eq. (22), the law of motion of family i's wealth in the stationary state becomes

$$k_{it+1} = \begin{cases} \sigma \bar{w} + \alpha k_{it} & \text{with probability } q \\ \sigma \bar{w} & \text{with probability } 1 - q, \end{cases} \tag{23}$$

where we have applied Eq. (21) to the first equation of (22). If q were equal to 1, the law of motion of individual wealth would become $k_{it+1} = \sigma \bar{w} + \alpha k_{it}$, and since $\alpha \in (1/2, 1)$, k_{it} would converge to $\sigma \bar{w}/(1 - \alpha)$ for sufficiently large t. However, once family i has been robbed in period t, it cannot earn interest income, and its wealth in period $t + 1$ becomes $k_{it+1} = \sigma \bar{w}$. Figure 4 shows the dynamic behavior of individual wealth in the stationary state.

[Figure 4 around here]

5.1.1 Wealth distribution

To obtain the support of the distribution of individual wealth in the stationary state, we derive Lemma 3 below.

Lemma 3. *Consider any family* $i \in \Omega$*. Then, under Assumption 1, family* i *becomes a crime victim in some period* $t \geq 0$ *almost surely.*

Proof. The probability that family i is robbed at least once from period 0 to period t is given by $1 - q^{t+1}$. The claim follows from the fact that $\lim_{t \to \infty}(1 - q^{t+1}) = 1$. □

Lemma 3 implies that any family is robbed with probability 1 in the long run. Since the law of motion of family i's wealth follows Eq. (22), if family i with k_{it} is robbed in period t, the wealth that family i has in period $t+1$ is given by $k_{it+1} = \sigma w_t$. This outcome yields the support of the wealth distribution in the stationary state in Lemma 4 below.

Lemma 4. *Under Assumption 1, the support of the distribution of individual wealth in the stationary state is given by* $\{k^{(j)}\}_{j=0}^{\infty}$*, where* $k^{(j)} = \sigma \bar{w} \sum_{s=0}^{j} \alpha^s$.

Proof. See the Appendix.

We define $P^{(j)}$ as the population of families that have individual wealth $k^{(j)}$ in the stationary state. Then, we can obtain $P^{(j)}$ explicitly in Lemma 5 below.

Lemma 5. *Under Assumption 1, it holds that* $P^{(j)} = q^j(1-q)$ *for* $j = 0, 1, \cdots, \infty$.

Proof. See the Appendix.

Now, we can construct the distribution of individual wealth in the stationary state, which is denoted by $\{(k^{(j)}, P^{(j)})\}_{j=0}^{\infty}$.

Proposition 2. *Suppose that Assumption 1 holds. Then, the distribution of individual wealth in the stationary state is given by*

$$\{(k^{(j)}, P^{(j)})\}_{j=0}^{\infty} = \left\{ \left(\sigma \bar{w} \sum_{s=0}^{j} \alpha^s, q^j(1-q) \right) \right\}_{j=0}^{\infty}. \tag{24}$$

Proof. The claim immediately follows from Lemmata 2 and 3. □

Based on Proposition 2, the average and variance of the distribution of individual wealth are computed as

$$(\sigma A)^{\frac{1}{1-\alpha}} l(p)$$

and

$$\frac{\alpha^2 \bar{k}^2 q(1-q)}{1-\alpha^2 q},$$

respectively. One notes that the average is equal to the steady-state capital stock, which is consistent with the capital market clearing condition. The variance depends upon the average capital stock, $\bar{k}$, which becomes greater as the average capital stock increases, which means that the variance is not an appropriate measure of inequality.

6 Wealth inequality

In this section, we investigate the relationship between institutional quality and wealth inequality. The explicit derivation of the wealth distribution in the previous section enables us to derive the Gini coefficient for it.

6.1 Gini coefficient

Using the stationary wealth distribution given by Eq. (24), it is straightforward to construct the Gini coefficient.

Proposition 3. *Under Assumption 1, the Gini coefficient regarding the weal distribution of Eq. (24) is given by*

$$g(q) := \frac{\alpha q(1-q)}{1-\alpha^2 q}, \tag{25}$$

where $q \in [(1-\alpha)/\alpha, 1)$.

Proof. See the Appendix.

Because q has a one-to-one linear relationship with p that reflects institutional quality, q also reflects institutional quality. In the following, we investigate the relationship between institutional quality and wealth inequality. As p increases from 0 to $(2\alpha - 1)/\alpha$, q also increases from $(1-\alpha)/\alpha$ to 1. As shown in Proposition 4 below, whether the Gini coefficient monotonically decreases or is inverted U-shaped as institutional quality improves depends upon the capital share.

Proposition 4. *Suppose that Assumption 1 holds. Then, the following hold.*

- *If* $1/2 < \alpha \leq (-1 + \sqrt{5})/2$, $g(q)$ *monotonically decreases with* $q \in [(1 - \alpha)/\alpha, 1)$.
- *If* $\alpha > (-1 + \sqrt{5})/2$, $g(q)$ *increases with* $q \in [(1 - \alpha)/\alpha, (1 - \sqrt{1 - \alpha})/\alpha)$ *and* $g(q)$ *decreases with* $q \in [(1 - \sqrt{1 - \alpha})/\alpha, 1)$.

Proof. See the Appendix.

Proposition 4 implies that if the capital share is relatively large, the Gini coefficient is inverted U-shaped with respect to q. At the early stage of the development of institutional quality, inequality across agents widens as institutional quality improves, whereas inequality starts to shrink once institutional quality has attained a certain threshold level. This feature may be called the institutional Kuznets curve. The inverted U-shaped Gini coefficient is consistent with empirical evidence presented by Chong and Calderón (2000), who show that institutional quality is positively linked with inequality in poor countries and negatively linked with inequality in rich countries. In contrast, if the capital share is relatively small, the Gini coefficient monotonically decreases with q, which implies that inequality shrinks as institutional quality improves. The monotonic decrease in inequality with improving institutional quality is consistent with empirical evidence presented by Chong and Gradstein (2007). Figure 5 presents charts for two examples. In panel A with $\alpha = 1/1.2$, the inverted U-shaped Gini coefficient pattern is observed, in which the Gini coefficient is maximized when $q = 0.71$. In panel B with $\alpha = 1/1.8$, a monotonic decrease in the Gini coefficient is observed.

[Figure 5 around here]

We can intuitively understand the possibility of the inverted U-shaped Gini coefficient by looking at Figure 4. Once a family has become a crime victim, the wealth that the family has declines to $\sigma \bar{w}$. Therefore, if the probability of being safe, q, gets smaller, the wealth distribution is more skewed towards lower wealth, and then inequality shrinks as many families become poor. In contrast, if q gets greater, the wealth distribution is more skewed towards higher wealth, and inequality shrinks as many families become richer. However, if q takes an intermediate value, the individual wealth is thoroughly distributed. In this case, the families are more heterogeneous in holding wealths, and thus, inequality becomes high.

7 Policy analysis

In this section, we investigate whether government policy can reduce wealth inequality and attain the first-best outcome. It is assumed that the government is endowed with three policy instruments. The first is taxation of (or subsidization of) diligent workers' wage income, the second is taxation of (or subsidization of) all agents' capital income (regardless of whether the agents are diligent workers or criminals), and the third is lump-sum taxation of (or subsidization of) all agents. In this case, the law of motion of individual wealth becomes

$$k_{it+1} = \begin{cases} \sigma \left(\omega_{it} + (1-\tau^k) r_t k_{it} - \tau_t^f \right) & \text{with probability} \quad q_t \\ \sigma \left(\omega_{it} - \tau_t^f \right) & \text{with probability} \quad 1 - q_t. \end{cases} \tag{26}$$

In Eq. (26), $\tau^k \in (-\infty, 1]$ is the tax rate on capital income if $\tau^k > 0$ and the subsidy rate if $\tau^k < 0$, τ_t^f is the lump-sum tax if $\tau_t^f > 0$ and the lump-sum transfer if $\tau_t^f < 0$, and $\omega_{it} = \tilde{w}_t = (1-\tau^k)(1-p)\alpha A k_t^{\alpha} l_t^{1-\alpha}$ if agent i is a criminal and $\omega_{it} = (1-\tau^w)w_t$ if that agent is a diligent worker, where $\tau^w \in (-\infty, 1]$ is the tax rate on wage income if $\tau^w > 0$

and the subsidy rate if $\tau^w < 0$. In what follows, we focus on two cases. The first is the case in which $\tau := \tau^k = \tau^w > 0$ and $\tau_t^f < 0$ and the second is the case in which $\tau^k = 0$.

7.1 Case 1: $\tau = \tau^k = \tau^w > 0$ and $\tau_t^f < 0$

In case 1, the government implements a lump-sum transfer policy whereby the government imposes taxes on diligent workers' wage income and all agents' interest income and redistributes the tax revenue to all agents (regardless of whether they are diligent workers or criminals) at the end of each period in a lump-sum manner.

Because wage income after tax is given by $(1-\tau)w_t = (1-\tau)(1-\alpha)Ak_t^{\alpha}l_t^{-\alpha}$ and the earnings of criminals are given by $\tilde{w}_t = (1-\tau)(1-p)\alpha Ak_t^{\alpha}l_t^{1-\alpha}$, the no-arbitrage condition between being a diligent worker and being a criminal becomes

$$w_t = (1-\alpha)Ak_t^{\alpha}l_t^{-\alpha} = (1-p)\alpha Ak_t^{\alpha}l_t^{1-\alpha}, \tag{27}$$

which is exactly the same as in the previous section. Therefore, Lemma 2 still holds under the current policy, and thus, the probability of not being a crime victim is given by Eq. (15). Because of these outcomes together with a constant weight parameter, σ, that describes the decision between consumption and bequest, Eqs. (18)-(21) still hold under the current policy.

Assuming a balanced government budget, we obtain the government's budget constraint as $\tau(w_t l_t + r_t k_t) + \tau_t^f = 0$. From Eqs. (2), (9), and (27), it follows that $r_t k_t = (1-l_t)w_t + r_r k_t q_t$. Then, $\tau_t^f = -\tau(w_t l_t + r_t k_t)$ can be rewritten as

$$\tau_t^f = -\tau(w_t + r_t k_t q_t). \tag{28}$$

Then, in the stationary state the no-arbitrage condition allows Eq. (26) to be rewritten as follows:

$$k_{it+1} = \begin{cases} \sigma[(1-\tau)\bar{w} - \bar{\tau}^f] + (1-\tau)\alpha k_{it} & \text{with probability } \; q \\ \sigma[(1-\tau)\bar{w} - \bar{\tau}^f] & \text{with probability } \; 1-q, \end{cases} \tag{29}$$

where $\bar{\tau}^f = -\tau(\bar{w} + \bar{r}\bar{k}q)$ from Eq. (28). The same procedure as in section 5 yields the distribution of individual wealth in the stationary state as follows:

$$\{(k^{(j)}, P^{(j)})\}_{j=0}^{\infty} = \left\{ \left(\sigma[(1-\tau)\bar{w} - \bar{\tau}^f] \sum_{s=0}^{j} [(1-\tau)\alpha]^s, q^j(1-q) \right) \right\}_{j=0}^{\infty}. \tag{30}$$

Furthermore, the Gini coefficient is computed as follows:

$$g_L(q,\tau) := \frac{(1-\tau)\alpha(1-q)q}{1-(1-\tau)^2\alpha^2 q}. \tag{31}$$

Proposition 5. *Suppose that Assumption 1 holds. Suppose also that the government runs a balanced budget with $\tau = \tau^k = \tau^w > 0$ and $\tau_t^f < 0$. Then, the Gini coefficient of the wealth distribution decreases with tax rate, τ; i.e., the lump-sum transfer policy reduces wealth inequality in the economy.*

Proof. The claim follows from the fact that $\partial g_L(q,\tau)/\partial\tau < 0$. □

It is observed that although wealth inequality is reduced by the lump-sum transfer policy under Assumption 1, this policy cannot attain the first-best outcome because it still holds that $q < 1$.

7.2 Case 2: $\tau^k = 0$

In case 2, the government implements a wage subsidization policy if $\tau^w < 0$ and a wage taxation policy if $\tau^w > 0$. Assuming the balanced government budget, we obtain the government's budget constraint as follows:

$$\tau^w w_t l_t + \tau_t^f = 0. \tag{32}$$

Because wage income after the policy is given by $(1-\tau^w)w_t = (1-\tau^w)(1-\alpha)Ak_t^\alpha l_t^{-\alpha}$ and the earnings of criminals are given by $\tilde{w}_t = (1-p)\alpha Ak_t^\alpha l_t^{1-\alpha}$, the no-arbitrage condition between being a diligent worker and being a criminal becomes

$$(1-\tau^w)(1-\alpha)Ak_t^\alpha l_t^{-\alpha} = (1-p)\alpha Ak_t^\alpha l_t^{1-\alpha} = \tilde{w}_t. \tag{33}$$

Based on Eq. (33), the same procedure as in section 4 yields the population of diligent workers in equilibrium as follows:

$$l_t = \min\{1, (1-\tau^w)(1-\alpha)/[(1-p)\alpha]\} =: l(p, \tau^w). \tag{34}$$

Proposition 6. *Suppose that Assumption1 holds. Suppose also that the government runs a balanced budget with $\tau^k = 0$ and $\tau^w = -\alpha(1-p)/(1-\alpha)+1$. Then, all agents become diligent workers.*

Proof. From Eq. (34), if $\tau^w = -\alpha(1-p)/(1-\alpha)+1$, it follows that $l(p,\tau^w) = 1$. □

In contrast to case 1, this policy affects the population of workers in equilibrium. Accordingly, by setting $\tau^w = -\alpha(1-p)/(1-\alpha)+1$, the government can achieve the first-best outcome, and all agents become diligent workers, correcting the production inefficiency; additionally, wealth inequality is perfectly eliminated. One should note that under Assumption 1, it holds that $\tau^w = -\alpha(1-p)/(1-\alpha)+1 < 0$; i.e., the government cannot implement wage taxation policy to achieve the first-best outcome.

8 Conclusion

How does institutional quality affect inequality? To address this question, we develop a macroeconomic growth model with an occupational choice. Our model demonstrates that if the capital share is relatively large, inequality widens as institutional quality improves in the early stage of development of institutional quality. However, once institutions have sufficiently matured, inequality declines as institutional quality improves. If the capital share is small, inequality monotonically shrinks as institutional quality improves. Our model can describe developing countries where people live in circumstances without peace and order. The policy implication of our model is that whereas policymakers should aim to improve institutional quality to reduce inequality, there may be a possibility that the government's taxation and subsidies by themselves can resolve production inefficiency and reduce inequality even though there is no strong protection of property rights.

Our model can be extended to a financially constrained economy. Whether the results obtained in this paper still hold in such an economy is unclear. This question is left for future research.

Appendix

Proof of Lemma 1

Suppose that Γ_t is a set of agents who earn interest income at the end of period t without being robbed. Then, it follows that

$$\begin{aligned}\int_{i\in\Omega} I_{it}di &= \int_{i\in\Gamma_t}(\omega_{it}+r_tk_{it})di + \int_{i\in\Omega\setminus\Gamma_t}\omega_{it}di \\ &= \int_{i\in\Omega}\omega_{it}di + r_t\int_{i\in\Gamma_t}k_{it}di. \qquad \text{(A.1)}\end{aligned}$$

Define Λ_t as a set of agents who are diligent workers in period t. Then, the first term of the right-hand side of Eq. (A.1) becomes $\int_{i\in\Omega}\omega_{it}di = \int_{i\in\Lambda_t} w_t di + \int_{i\in\Omega\setminus\Lambda_t}\tilde{w}_t di$. Furthermore, since q_t measures the population of agents in Γ_t and since criminals choose targets randomly, the second term of the right-hand side of Eq. (A.1) becomes $r_t\int_{i\in\Gamma_t} k_{it}di = r_t q_t \int_{i\in\Omega} k_{it}di = r_t q_t k_t$, where Eq. (12) has been used to obtain the last equality. Then, using Eqs. (1), (2), (3), (9), (12), and (A.1), we compute $\int_{i\in\Omega} I_{it}di$ as follows:

$$\begin{aligned}\int_{i\in\Omega} I_{it}di &= w_t l_t + \tilde{w}_t(1-l_t) + [1-(1-l_t)(1-p)]r_t k_t \\ &= w_t l_t + (1-l_t)(1-p)r_t k_t + [1-(1-l_t)(1-p)]r_t k_t \\ &= A k_t^{\alpha} l_t^{1-\alpha}.\end{aligned}$$

This is our desired conclusion. □

Proof of Lemma 4

According to Lemma 3, any family becomes a crime victim almost surely. Suppose that family i with k_{it} is robbed in period $t \geq 0$. Then, the wealth that family i has in period $t+1$ is given by $k_{it+1} = \sigma w_t =: k_{t+1}^{(0)}$. Looking forward from period $t+1$ onward, we note that Eq. (22) implies that $k_{it+2} = \sigma w_{t+1} + \sigma^2 r_{t+1} w_t =: k_{t+2}^{(1)}$ with probability q and $k_{it+2} = \sigma w_{t+1} =: k_{t+2}^{(0)}$ with probability $1-q$, $\cdots$, $k_{it+j+1} = \sigma w_{t+j} + \sigma^2 r_{t+j} w_{t+j-1} + \cdots + \sigma^{j+1}\left(\Pi_{s=1}^{j} r_{t+s}\right) w_t =: k_{t+j+1}^{(j)}$ with probability q^j and $k_{it+j+1} = \sigma w_{t+j} =: k_{t+j+1}^{(0)}$ with probability $q^{j-1}(1-q)$. Since it holds that $r_{t+s} \to \bar{r}$, $w_{t+s} \to \bar{w}$, and $\sigma r_{t+s} \to \alpha$ for $s = 1, \cdots j$ as $t \to \infty$, it follows that $k^{(0)} := \lim_{t\to\infty} k_{t+j+1}^{(0)} = \sigma\bar{w}$ for $j = 0, 1, \cdots, \infty$ and $k^{(j)} := \lim_{t\to\infty} k_{t+j+1}^{(j)} = \sigma\bar{w}\sum_{s=0}^{j}\alpha^s$ for $j = 1, 2, \cdots, \infty$. Therefore, for sufficiently large t, family i's wealth takes one of the values in $\{k^{(j)}\}_{j=0}^{\infty}$. Because this outcome holds for all families in Ω, the support of the distribution of individual wealth in the stationary state is given by $\{k^{(j)}\}_{j=0}^{\infty}$, where $k^{(j)} = \sigma\bar{w}\sum_{s=0}^{j}\alpha^s$. □

Proof of Lemma 5

Suppose that $P_t^{(j)}$ is the population of families that have individual wealth, $k_t^{(j)}$, in period $t \geq 1$ (see the proof of Lemma 4 for the definition of $k_t^{(j)}$). Eq. (22) and the law of large numbers imply that the population of families that become crime victims in period t and have individual wealth, $k_{t+1}^{(0)}$, in period $t+1$ is equal to $P_{t+1}^{(0)} = 1-q$. Again, Eq. (22) and the law of large numbers yield $P_{t+2}^{(1)} = qP_{t+1}^{(0)} = q(1-q)$, $P_{t+3}^{(2)} = qP_{t+2}^{(1)} = q^2(1-q)$, $\cdots$, $P_{t+j+1}^{(j)} = qP_{t+j}^{(j-1)} = q^j(1-q)$, $\cdots$. Or equivalently, we obtain $P_{t+j+1}^{(j)} = q^j(1-q)$ for $j = 0, 1, \cdots, \infty$. It follows from the last equation that $P^{(j)} := \lim_{t\to\infty} P_{t+j+1}^{(j)} = q^j(1-q)$ for $j = 0, 1, \cdots, \infty$. Conversely, we can verify that $\sum_{j=0}^{\infty} P^{(j)} = \sum_{j=0}^{\infty} q^j(1-q) = 1$. This completes the proof. $\square$

Proof of Proposition 3

By definition, the Gini coefficient in the stationary state is given as follows:

$$g(q) = 1 - \sum_{j=0}^{\infty}[G(k^{(j)}) - G(k^{(j-1)})][B^{(j)} + B^{(j-1)}], \tag{A.2}$$

where $G(k^{(j)})$ is the cumulative distribution function of $k^{(j)}$, and $B^{(j)}$ is the cumulative proportion of wealth relative to the total wealth, which is given by $B^{(j)} := \sum_{\ell=0}^{j}(1-q)q^{\ell}k^{(\ell)}/\bar{k}$ according to Eq. (24). In Eq. (A.2), it is assumed for convenience that $k^{(-1)} = B^{(-1)} = 0$. Using Eq. (24) allows $B^{(j)} + B^{(j-1)}$ to be computed as

$$B^{(j)} + B^{(j-1)} = \left(\frac{1-q}{1-\alpha}\right)\left(\frac{\sigma\bar{w}}{\bar{k}}\right)\left[2\left(\frac{1-q^j}{1-q} - \frac{\alpha(1-(\alpha q)^j)}{1-\alpha q}\right) + (1-\alpha^{j+1})q^j\right]. \tag{A.3}$$

From Eq. (24), it follows that

$$G(k^{(j)}) - G(k^{(j-1)}) = (1-q)q^j. \tag{A.4}$$

Additionally, from Eqs. (14), (19), and (20), it follows that

$$\frac{\sigma \bar{w}}{\bar{k}} = 1 - \alpha q. \tag{A.5}$$

Eqs. (A.3)-(A.5) allow $[G(k^{(j)}) - G(k^{(j-1)})][B^{(j)} + B^{(j-1)}]$ to be computed as follows:

$$\begin{aligned} &[G(k^{(j)}) - G(k^{(j-1)})][B^{(j)} + B^{(j-1)}] \\ &= \frac{(1-q)^2(1-\alpha q)}{1-\alpha}\left[\frac{2(1-\alpha)}{(1-q)(1-\alpha q)}q^j - \frac{1+q}{1-q}(q^2)^j + \frac{\alpha+\alpha^2 q}{1-\alpha q}(\alpha q^2)^j\right]. \end{aligned} \tag{A.6}$$

Eq. (A.6) yields

$$\sum_{j=0}^{\infty}[G(k^{(j)}) - G(k^{(j-1)})][B^{(j)} + B^{(j-1)}] = 2 - \frac{1-\alpha q}{1-\alpha} + \frac{\alpha(1+\alpha q)(1-q)^2}{(1-\alpha)(1-\alpha q^2)}. \tag{A.7}$$

Eqs. (A.2) and (A.7) lead to our desired conclusion. □

Proof of Proposition 4

From Eq. (25), it follows that

$$g'(q) = \alpha^2\left(q - \frac{1-\sqrt{1-\alpha}}{\alpha}\right)\left(q - \frac{1+\sqrt{1-\alpha}}{\alpha}\right)\Big/(1-\alpha q^2)^2. \tag{A.8}$$

It holds that $g'(0) > g'((1-\sqrt{1-\alpha})/\alpha) = 0 > g'(1)$. Therefore, if $(1-\alpha)/\alpha \geq (1-\sqrt{1-\alpha})/\alpha$ and $\alpha > 1/2 \iff 1/2 < \alpha \leq (-1+\sqrt{5})/2$, it follows that $g'(q) < 0$ in $((1-\alpha)/\alpha, 1)$, and thus, $g(q)$ monotonically decreases with $q \in [(1-\alpha)/\alpha, 1)$. If $(1-\alpha)/\alpha < (1-\sqrt{1-\alpha})/\alpha$ and $\alpha > 1/2 \iff \alpha > (-1+\sqrt{5})/2$, it follows that $g'(q) > 0$ in $[(1-\alpha)/\alpha, (1-\sqrt{1-\alpha})/\alpha)$ and $g'(q) < 0$ in $((1-\sqrt{1-\alpha})/\alpha, 1)$. Therefore, $g(q)$ increases with $q \in [(1-\alpha)/\alpha, (1-\sqrt{1-\alpha})/\alpha)$, and $g(q)$ decreases with $q \in [(1-\sqrt{1-\alpha})/\alpha, 1)$.
□

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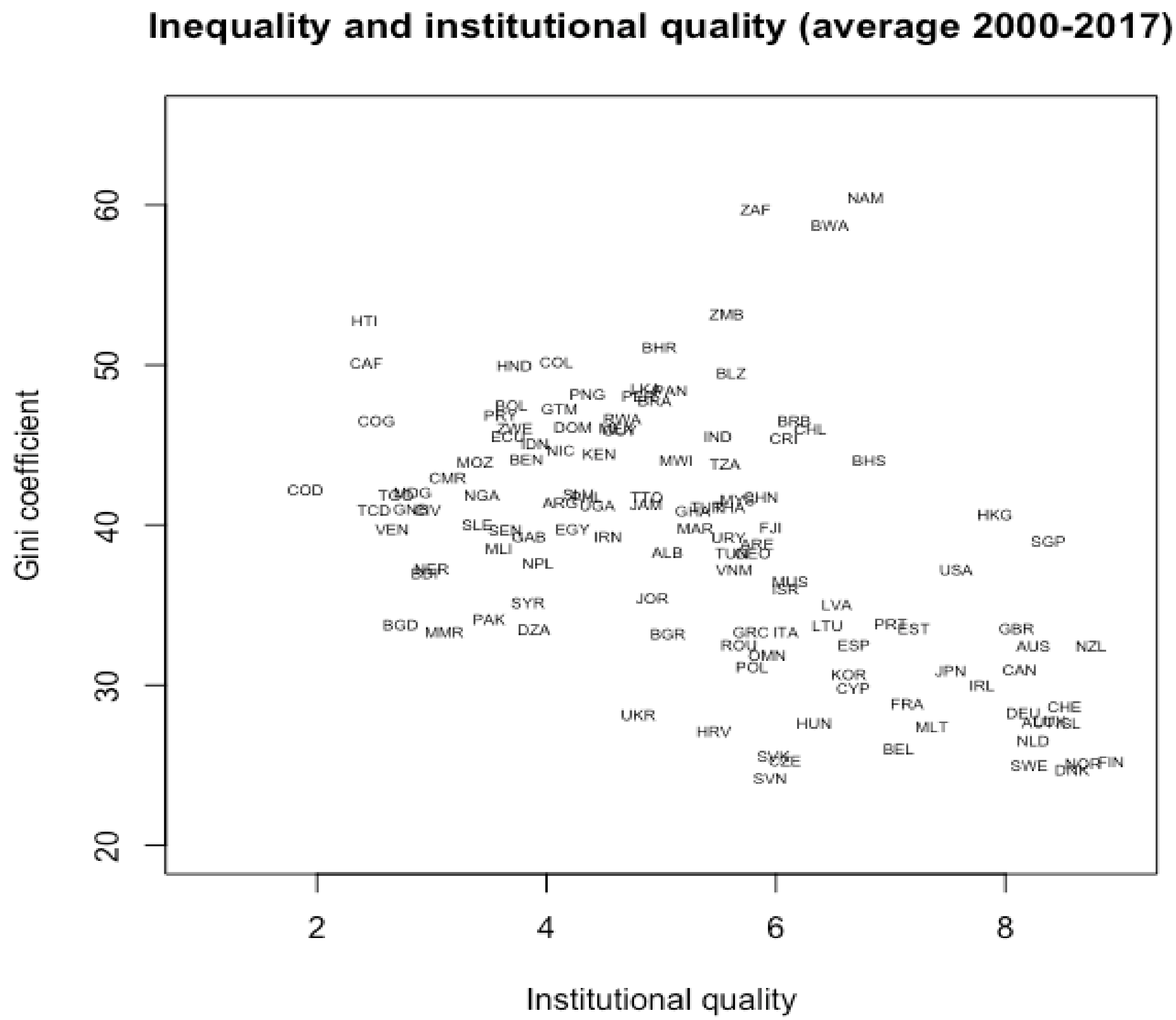


Figure 1. Institutional quality vs. Gini coefficient

Notes. This scatter plot is based on 123 countries. The data on the Gini coefficient are collected from the dataset developed by Solt (2009, 2019). The index of "Legal System and Property Rights" produced by Gwartney et al. (2018) is used for a proxy of institutional quality.

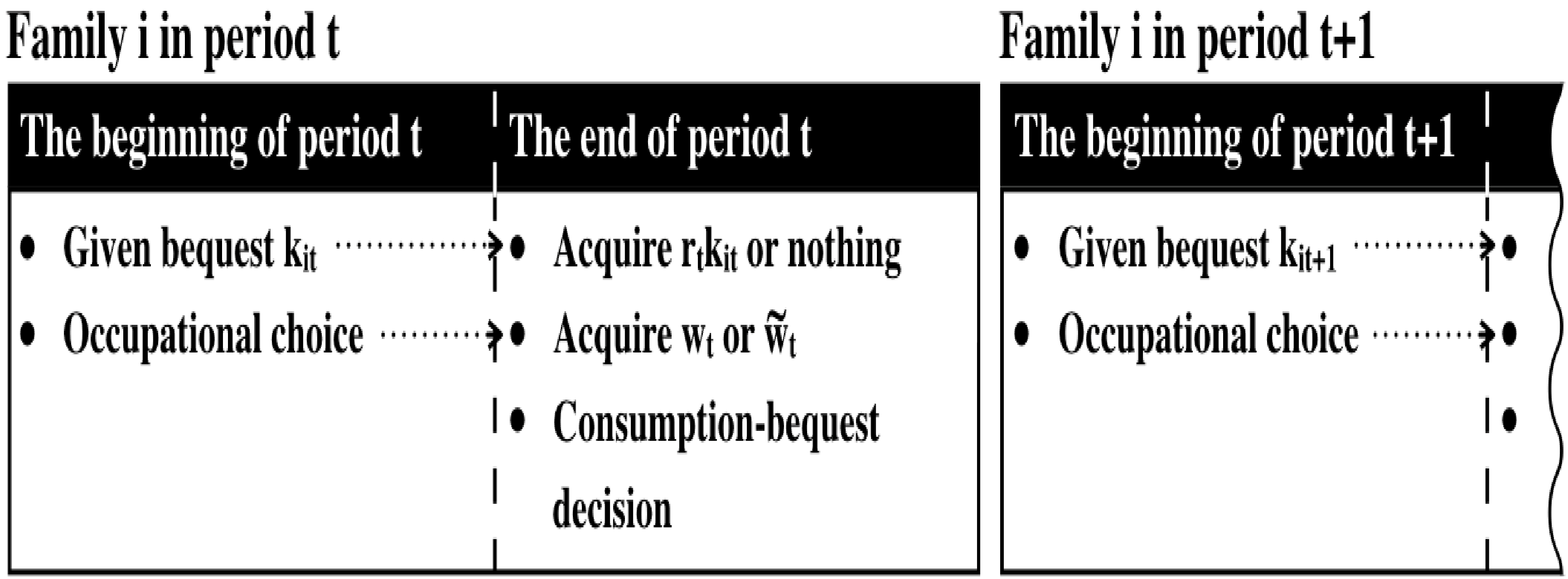


Figure 2. The timing of events

Notes. The bequest received from parent is converted into capital and agent i (family i) acquires an interest income at the end of each period. However, the interest income may be stolen by criminals. In each period, agent i (family i) makes a decision about a occupational choice. If he becomes a diligent worker, he earns a wage income, w_t, at the end of each period and if he becomes a criminal, he acquires a reward, $\widetilde{w}_t$, for his misdoings. At the end of each period, agent i (family i) makes a decision about how much he consumes and leaves bequest to his offspring.

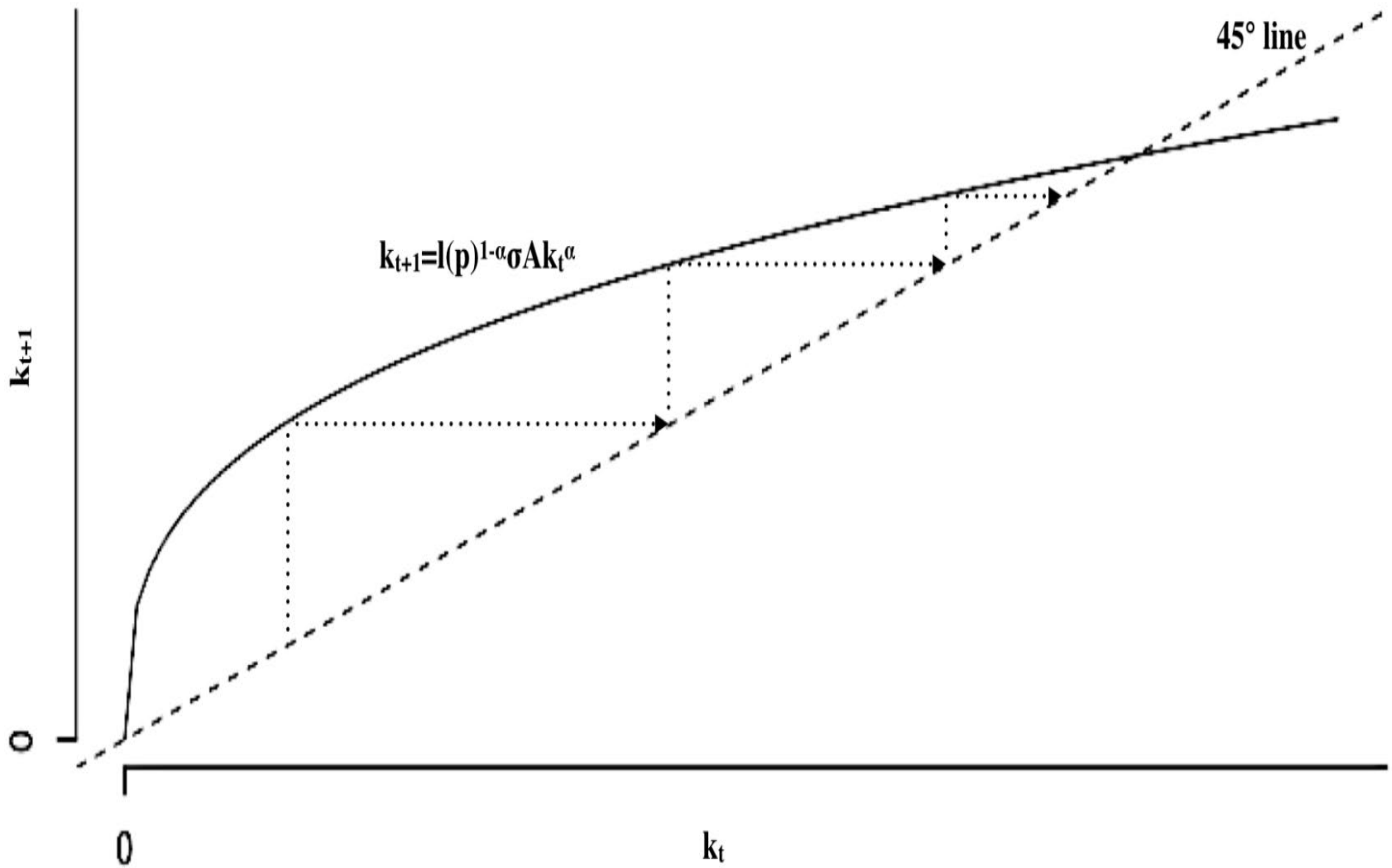


Figure 3. Phase diagram of aggregate capital

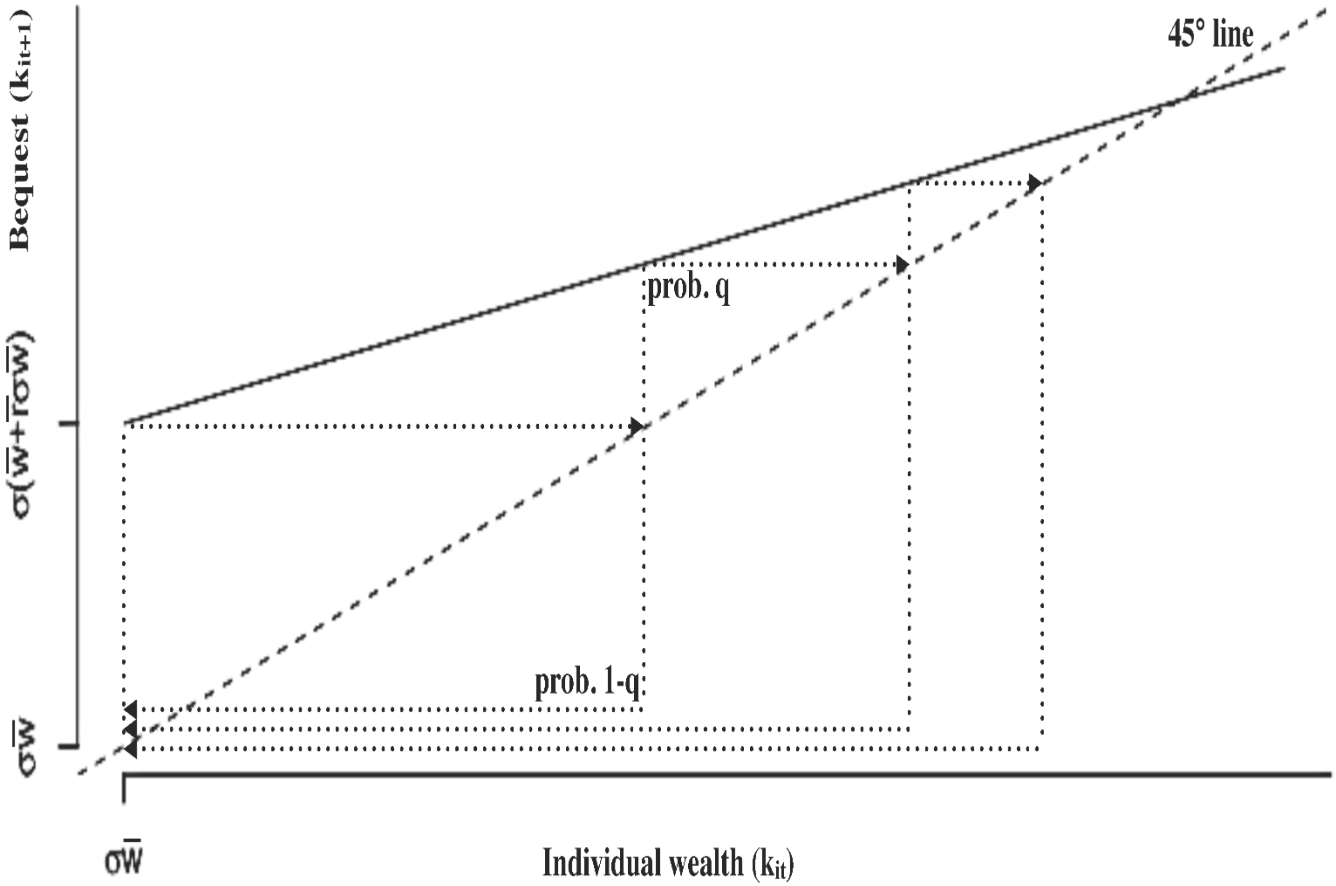
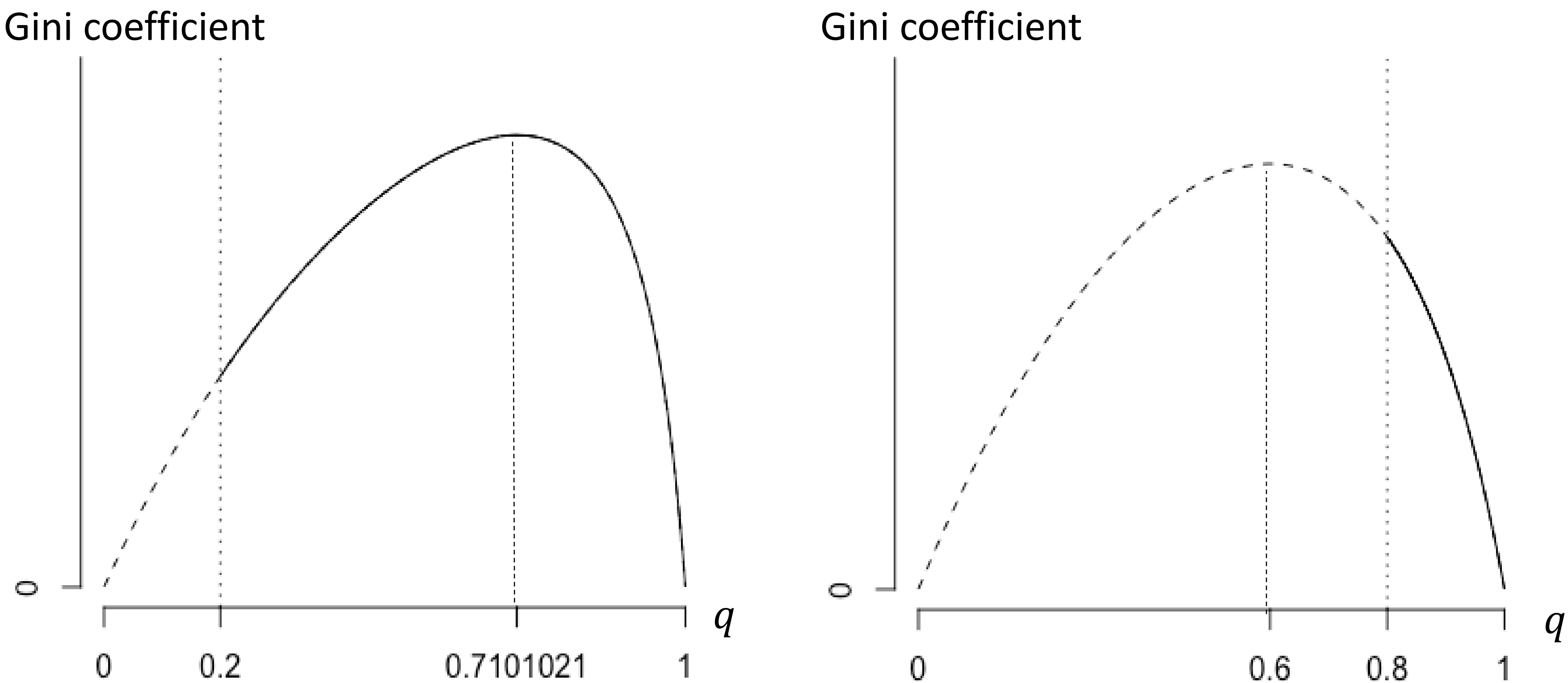


Figure 4. Phase diagram of individual wealth in the stationary state

Notes. In each period, the individual wealth is stolen with probability $1-q$, whereas it evolves with probability q.



Panel A: $\alpha = \frac{1}{1.2}$

Panel B: $\alpha = \frac{1}{1.8}$

Figure 5. The relationship between institutional quality and the Gini coefficient