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Abstract

We consider the product line strategies of duopolistic firms, each of which can supply two vertically-differentiated products under nonnegative output constraints and expectations of their rival's product line reaction. Considering a game of firms with heterogeneous (homogeneous) unit costs for high- (low-) quality products, we derive the equilibria for the game and conduct comparative statics of the equilibria outcomes on the relative superiority of the high-quality product and relative cost efficiency. In two of the equilibria, we find that where the cost-inefficient firm supplies a high-quality good, social welfare can worsen as its unit cost decreases. We also characterize the result using the production substitution of differentiated goods within a firm and the high-quality good between firms. Further, by comparing social welfare in the first-best equilibria with those in the Cournot duopoly equilibria, we find that the social welfare of the market worsens in the multiproduct Cournot duopoly equilibria as the relative superiority of the high-quality good increases.

Keywords: Multiproduct firm; Duopoly; Production substitution; Vertical product differentiation;

JEL Classification Codes: D21, D43, L13, L15

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1 Introduction

Real-world economies often include oligopolistic competition in the one market segment in which firms supply multiple vertically-differentiated products. In the mobile phone market, for example, Apple supplies iPhone X to the first line segment and Samsung competes by supplying its Galaxy S9 to businesses. In the second line segment, Apple supplies iPhone 8 and Samsung responds with the Galaxy S8. However, despite obvious interest, there are few studies of oligopolistic competition in these markets in the economics literature.

Yet another example involves the diffusion line markets in fashion brands or the fast-fashion brands of clothing and apparel companies. A diffusion line is a secondary line of merchandise created by a high-end fashion house or designer that retails at lower prices, with many clothing and apparel companies adopting a diffusion strategy. For example, the fashion brand Armani supplies high-end customers with its products labelled as 'Armani' in its first line and low-end young customers with its products branded as 'Armani Exchange' in its second line. Likewise, Prada supplies high-end customers with its 'Prada' brand in the first line and low-end customers with its 'Miu Miu' brand in the second line. These clothing and apparel companies often supply vertically-differentiated multiproducts in their markets.

In the existing literature on vertical product differentiation, the quality of the goods firms produce is treated as an endogenous variable. For instance, in Bonanno (1986) and Motta (1993), firms initially choose the level of quality and then compete in a Cournot or Bertrand fashion in an oligopolistic market. Elsewhere, Shaked and Sutton (1987) consider a two-stage game model in which each of the horizontally- and vertically-differentiated multiproduct firms pays a fixed sunk cost for R&D or advertising expenditure to improve (the perceived) quality of its products in the first stage, and then chooses its respective prices in the second.

In terms of a horizontally-differentiated multiproduct model, Bental and Spiegel (1984) consider an optimal set of product varieties in a monopoly and analyze the relationship between the degree of differentiation between any two varieties and the variety price, or the cost of installing an additional variety. Shaked and Sutton (1990) consider a two-stage price game model in which each of the horizontally-differentiated multiproduct firms (potential entrants) selects in the first stage which product(s) it will produce, and then incurs a "sunk cost" for each product entered and chooses its respective prices in the second. They then *graphically* characterize the market structure at equilibria using two parameters to measure the expansion and competition effects.

However, none of these studies considers the case where firms sell multiple products differentiated in terms of quality (vertically) in the same market. The one closely related to the present study is Ellison (2005), who analyzes a market where each firm sells high- and low-end versions of the same product. Although each firm produces two differentiated goods, they are sold in different markets—each with different types of consumers.

In markets where firms supply multiple vertically-differentiated products, they sometimes compete with rivals that supply one or some vertically-differentiated products (i.e., the rival chooses a single product line) to the same market segment. For this reason, Kitamura and Shinkai (2015a) consider the competition between two firms where each can choose a product line of two vertically-differentiated products in the same market segment. However, few previous studies have addressed an oligopolistic market where multiproduct firms produce multiple goods differentiated in terms of quality (see, for example Johnson and Myatt 2003).[1]

According to Johnson and Myatt (2003), firms that sell multiple quality-differentiated products frequently change their product lines when a competitor enters the market. They then provide an explanation for the common strategies of using "fighting brands" and "pruning" product lines. In particular, Johnson and Myatt (2003) endogenize not only the level of quality of each good, but also the number of goods that each firm supplies to the market.

Unlike most extant studies, in our model here—and in Kitamura and Shinkai (2015a)—both the quality level and the number of differentiated goods that each firm supplies are exogenously given. We also do not explicitly consider the stage of product line choice with a fixed "sunk cost" as in Shaked and Sutton (1987, 1990). This setting seems appropriate to explore the relationship between the production substitution and the difference in the potential value of the goods according to consumers or the unit costs of the two goods. The results of the present paper results also relate to those of marketing studies on product segmentation and product distribution strategies. For instance, Calzada and Valletti (2012) consider a model of film distribution and consumption involving a film studio that can release two versions of the one film—one for theaters and the other for video (however, they do not consider oligopolistic competition between film studios). They show that the optimal strategy for the studio is to introduce versioning (the simultaneous release of a film with one version for theaters and another for video) if their goods are not close substitutes for each other.

In our earlier work (Kitamura and Shinkai 2015a), we considered a game that includes heterogeneous unit production costs between firms for high-quality products, but

[1] For the sake of simplicity, in this paper, we focus on a duopoly model.

homogeneous costs for low-quality products. We then described the firms' product line strategies using the relative quality of these products and the cost efficiency ratios of the firms for the high-quality good. We first derived equilibria by assuming that, in any equilibrium, each rival firm chooses positive outputs for both the high- and low-quality good. Consequently, these equilibria are included cases in which a firm chooses negative outputs for one of the goods for some parameter range (the relative quality ratio or cost inefficiency ratio for the high-quality good). We then retroactively excluded the ranges of parameters in equilibria that result in any negative outputs and graphically describe the firms' product line strategies based on the relative quality of the products and the cost efficiency ratios between the firms in the case of high-quality goods.

Nonetheless, in our earlier work (Kitamura and Shinkai 2015a), we did not describe the firms' equilibrium profits and equilibrium welfare, although we also established a result that indirectly supports that in Calzada and Valletti (2012). In their model, "versioning" and "sequencing" correspond to the simultaneous supply and sequential supply, respectively, of high- and low-quality goods, as in our model. In the case of sequential supply, the film studio supplies a high-quality film version to theaters and then launches a low-quality DVD version in the same market.

While our previous study (Kitamura and Shinkai 2015a) assumed that each rival firm chooses positive outputs for both goods in duopolistic competition, it is crucial that each firm considers its rivals' product line strategies when selecting its own strategy. In these cases, it is important that each firm picks its own product line strategies for multiple products, given their expectations of the rival's product line reactions. Therefore, in the present study, we consider the product line strategies of duopolistic firms where each supply two vertically-differentiated products under nonnegative output constraints, and with an expectation of their rival's product line reactions. This differentiates this study from our previous work (Kitamura and Shinkai 2015a) in many ways.

First, in this analysis, we *explicitly* examine the product line strategies of duopolistic firms supplying two vertically-differentiated products under a nonnegative output constraint and an expectation regarding the rival's product line reactions. We demonstrate that there are five nontrivial equilibria with positive outputs for one or both products and that both firms have positive profits in each equilibrium. In these equilibria, the ranges of the two ratio parameters for which positive equilibrium outputs exist for the two firms differ. We then graphically describe the firms' product line strategies in equilibrium, based on the relative quality of the products and each firm's relative cost efficiency for the high-quality good (Figure 1).

Second, at every one of the nontrivial equilibria, by comparing the equilibrium total outputs and profits of the two firms, we graphically describe the differences between the two firms' total outputs and profits in the five equilibria, based on the relative product quality and their relative cost efficiency ratios. We then conduct a comparative statics analysis on these two ratio parameters (Figure 2)[2].

Third, we also derive social welfare in every equilibrium. In addition, in a multiproduct Cournot duopoly, there exist two equilibria in which a reduction of the relative marginal cost inefficiency decreases social welfare. In both of these (which we derive in cases C and E), the result that we derive is similar to that in Lahiri and Ono (1988) for a single-product Cournot oligopoly. Lahiri and Ono (1988) show that a cost reduction in a firm with a sufficiently low market share decreases social welfare, but that a cost reduction in

[2]Professor John Sutton suggested this to us in his comment during our presentation of an earlier version of this paper (Shinkai and Kitamura 2015b) at the 2015 Annual Conference of the European Association for Research Industrial Economics in Munich, Germany. His comment and suggestion has much improved our analysis, and we therefore wish to express our gratitude.

any firm always increases social welfare if the market *share is the same* among all firms. We identify two equilibria in which a reduction of the relative marginal cost inefficiency decreases social welfare, when the equilibrium market share of the inefficient firm in the production of the high-quality good is low (cases C and E in equilibrium in Figures 1 and 3, respectively).

Through comparative statics based on the firms' relative cost inefficiency ratio for the high-quality good, we also find that a direct production substitution quantity between the high- and low-quality goods takes place within the inefficient firm. This is when production of the high-quality good exceeds an indirect production substitution quantity in the efficient firm through the strategic substitution between the cost-efficient and cost-inefficient firm for both goods. That is, we extend the analysis in Lahiri and Ono (1988) to a vertically-differentiated multiproduct duopoly.

However, the mechanism underpinning our result differs from that in Lahiri and Ono (1988). In sum, our result derives from the production substitution from the high- to the low-quality good *within* the cost-inefficient firm and the subsequent production substitution of good H *between* firms by means of a strategic substitute . This is because we consider a duopoly model in which each of the firms can produce two vertically - differentiated goods if it wishes. However, Lahiri and Ono's (1988) result is *only* caused by the production substitution *between* the cost-efficient and cost-inefficient firm, as the cost of the inefficient firm decreases.

Furthermore, we derive the first-best equilibria in which the social planner can select the quantities of all goods for maximizing social welfare under nonnegative constraints for the output of goods. Then, when we compare the social surpluses in the first-best equilibria with those in the Cournot duopoly equilibria derived earlier, we show that the social welfare of the market worsens in the multiproduct Cournot duopoly equilibria as the relative superiority of the high-quality good increases.

The rest of this paper is organized as follows. Section 2 presents our model. In Section 3, we derive the duopoly equilibria with two vertically-differentiated products in the same market under a nonnegative output constraint and an expectation with regard to the rival product line reactions. We then graphically describe the firms' product line strategies in equilibrium based on the relative quality of the differentiated products and the firms' relative cost efficiency of the high-quality good (Figure 1). By comparing the total profits of the two firms at every equilibrium, we graphically describe the differences between the firms' total outputs and the profits in the five equilibria based on the relative product quality and their relative cost efficiency ratios.

In Section 3, we conduct comparative statics of these two ratio parameters (Figure 2), and in Section 4, we derive the social welfare for every equilibrium derived in Section 2. We then conduct comparative statics based on the relative product quality and the firms' relative cost inefficiency ratio for the high-quality good (Figure 3). In addition, we derive the first-best equilibrium, in which the social planner can choose the quantities of all goods for maximizing social welfare under nonnegative output constraints for the goods, and compare the social surplus in the first-best equilibria with those in the Cournot duopoly equilibria. Finally, we conclude the paper in Section 5.

2 The Model and the Game Equilibria

Suppose there are two firms ($i = 1, 2$) in a duopoly, each of which produces two goods (H and L), which differ in terms of quality. We assume a continuum of consumers, represented by a taste parameter, θ, which is uniformly distributed between 0 and r (> 0), with a density of one. We further assume that a consumer is of type $\theta \in [0, r]$, for

$r > 0$. The consumers' preferences are the standard Mussa and Rosen preferences. Thus, the utility (net benefit) of consumer θ who buys good α $(= H, L)$ from firm i $(= 1, 2)$ is given by

$$U_{i\alpha}(\theta) = V_{\alpha}\theta - p_{i\alpha} \qquad i =, 1, 2 \quad \alpha = H, L. \tag{1}$$

To maximize their own surplus, each consumer decides whether to buy nothing or one unit of good α from firm i.

Let V_H and V_L denote the quality of the high- and the low-quality goods, respectively. Then, the maximum amount that consumers are willing to pay for each good is assumed to be $V_H = \mu V_L = \mu > V_L = 1$. Thus, for simplicity, we normalize the quality of the low-quality good by setting $V_L = 1$ and assume that the quality of the high-quality good is μ times that of the low-quality good.

Note that the consumers' preferences and the utility of each consumer *never changes* when *the quality of both products change exogenously.*

Good α $(= H, L)$ is assumed to be homogeneous for all consumers. Suppose that there always exists a consumer $\underline{\theta}_{iL}, i = 1, 2$ who is indifferent between purchasing good L and purchasing nothing in a monopoly or a duopoly. For this consumer, $\underline{\theta}_{iL}$ satisfies

$$\begin{aligned} & U_{iL}(\underline{\theta}_L) = 0 \\ \Leftrightarrow \ & \underline{\theta}_{iL} = \frac{p_{iL}}{V_L} = p_{iL}, i = 1, 2. \end{aligned} \tag{2}$$

We can derive the demand for good H as $Q_H = r - \widehat{\theta}$, and that for good L as $Q_L = \widehat{\theta} - \underline{\theta}_{iL}$, as shown in Figure 1, where $Q_\alpha = q_{i\alpha} + q_{j\alpha}$, for $\alpha = H, L$ and $j = 1, 2$.

Without loss of generality, we set $r = 1$. Here, $\widehat{\theta}$, the threshold between the demand for H and that for L, is given by

$$\widehat{\theta} = (p_H - p_L)/(\mu - 1). \tag{3}$$

Then, as in Kitamura and Shinkai (2015a), we derive the following inverse demand functions:

$$\begin{cases} p_H = V_H(1 - Q_H) - Q_L = \mu(1 - Q_H) - Q_L \\ p_L = V_L - Q_H - Q_L = 1 - Q_H - Q_L, \end{cases} \tag{4}$$

where $Q_\alpha = q_{i\alpha} + q_{j\alpha}$ and p_α and $q_{i\alpha}$ denote the price of good α and firm i's output of good α, respectively, for $\alpha = H, L$ and $i, j = 1, 2$.

Moreover, suppose that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L = 0$, where $c_{i\alpha}$ is firm i's marginal and average cost for good α. This implies that a high-quality good incurs a higher cost of production than does a low-quality good. Here, without loss of generality, we assume $c_{2H} > c_{1H} = 1 > c_{iL} = 0$, which means that firm 1 is more efficient than firm 2. Under these assumptions, each firm's profit is defined in the following manner:

$$\pi_i = (p_H - c_{iH})q_{iH} + p_L q_{iL} \qquad i = 1, 2. \tag{5}$$

Firm $i(= 1, 2)$ chooses the outputs for H and L to maximize its profit function in Cournot fashion under nonnegative output constraints, provided that firm $j(\neq i)$ chooses *any given* product line strategy $\mathbf{s}_j \in \mathbf{S}_j \equiv \{(0, 0), (+, 0), (0, +), (+, +)\}$, where $(0, 0)$ implies $(q_{jH} = 0, q_{jL} = 0)$, $(+, 0)$ implies $(q_{jH} > 0, q_{jL} = 0)$, and so on. Thus, for any given

$\mathbf{s}_j \in \mathbf{S}_j$

$$\max_{q_{iH}, q_{iL}} \pi_i = \{\mu(1 - q_{iH} - q_{jH}) - q_{iL} - q_{jL} - c_{iH})q_{iH} + (1 - q_{iH} - q_{jH} - q_{iL} - q_{jL})q_{iL} \quad (6)$$
$$s.t. \;\; q_{iH} \geq 0, q_{iL} \geq 0, \; i \neq j, \; i, j = 1, 2.$$

The necessary and complementary conditions for this maximization problem are

$$\frac{\partial \pi_i}{\partial q_{iH}} \leq 0, \; \frac{\partial \pi_i}{\partial q_{iL}} \leq 0, \quad (7)$$

$$q_{iH} \cdot \frac{\partial \pi_i}{\partial q_{iH}} = q_{iL} \cdot \frac{\partial \pi_i}{\partial q_{iL}} = 0, \quad (8)$$

$$q_{iH} \geq 0, \; q_{iL} \geq 0, \;\; i = 1, 2. \quad (9)$$

Each firm chooses its product line strategy for the two vertically- differentiated products; that is, whether to produce positive (zero) quantities of product H and L, given the rival firm's product line strategy.

Note that each inequality $\partial \pi_i / \partial q_{i\alpha} \leq 0$ in (7) and the corresponding complementary slackness condition $q_{i\alpha} \cdot \partial \pi_i / \partial q_{i\alpha} = 0$ in (8) imply that if the marginal revenue of firm i for product $\alpha (= H, L)$ is below (the same as) its marginal cost, then firm i does not produce (does produce) a positive quantity of the product.

In the following, we present the equilibria of a Cournot duopoly game, in which each firm can choose its product line and outputs for the two vertically-differentiated goods. The firms operate under a nonnegative output constraint. After presenting the equilibrium, we describe the firms' product line strategies based on the products' relative quality and the firms' relative cost efficiency with respect to the high-quality good in equilibrium.

There are 15 cases to be solved based on each firm's product line strategies, given the firm's expectation of its rival's product line strategies, except for the trivial case in which neither firm produces H or L. After performing lengthy calculations and checking the nonnegative constraints for the outputs in each equilibrium, we find that 10 of the 15 cases have no equilibrium in the corresponding games. Owing to space limitations, we omit these calculations and the proofs of our results. Thus, we examine the following five cases:

- **Case A**: $\mathbf{s}_1 = (0, +),\ \mathbf{s}_2 = (0, +)$

  In this case, a duopoly market for the low-quality good is realized in equilibrium:

$$(q_{1H}^{*A}, q_{1L}^{*A}, q_{2H}^{*A}, q_{2L}^{*A}) = \left(0,\ \frac{1}{3},\ 0,\ \frac{1}{3}\right) \quad if\ 1 < \mu \leq 2, \tag{10}$$

  where the last inequality must hold because of the necessary condition. In Figure 2, area A corresponds to this case. The relative superiority of the high-quality product H, μ, is too small compared with the firms' relative cost efficiency of the high-quality good, c_{2H}. Therefore, neither firm produces product H; instead, each firm produces only the low-quality product L.

  From (4), (5), and (10), each firm's equilibrium price and profit are

$$(p_H^{*A}, p_L^{*A}) = \left(\frac{3\mu - 2}{3},\ \frac{1}{3}\right) \tag{11}$$

$$(\pi_1^{*A}, \pi_2^{*A}) = \left(\frac{1}{9}, \frac{1}{9}\right). \tag{12}$$

- **Case B**: $\mathbf{s}_1 = (+, 0),\ \mathbf{s}_2 = (0, +)$

In this case, each firm specializes in the product that is more cost-efficient for the firm. Thus, we obtain

$$(q_{1H}^{*B}, q_{1L}^{*B}, q_{2H}^{*B}, q_{2L}^{*B}) = \left(\frac{2\mu - 3}{4\mu - 1}, 0,\ 0,\ \frac{\mu + 1}{4\mu - 1}\right) \tag{13}$$

$$if\ \ 4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}), \tag{14}$$

where the last inequality must hold, given the necessary condition. In Figure 1, area B corresponds to this case. In area B, the relative cost inefficiency of the high-quality good of firm 2, c_{2H}, is relatively strong compared with μ, the relative quality superiority of the high-quality product H. From (4), (5), and (10), we obtain the corresponding equilibrium price and profit of each firm:

$$(p_H^{*B}, p_L^{*B}) = \left(\frac{(\mu + 1)(2\mu - 1)}{4\mu - 1}, \frac{\mu + 1}{4\mu - 1}\right) \tag{15}$$

$$(\pi_1^{*B}, \pi_2^{*B}) = \left(\frac{\mu (2\mu - 3)^2}{(4\mu - 1)^2}, \frac{(\mu + 1)^2}{(4\mu - 1)^2}\right). \tag{16}$$

We also find that $q_{1H}^{*B} - q_{2L}^{*B} = \frac{\mu - 4}{4\mu - 1} \geq 0 \Leftrightarrow q_{1H}^{*B} \geq q_{2L}^{*B}$ and that

$$\pi_1^{*B} - \pi_2^{*B} = \frac{1}{4\mu - 1}\left(\mu^2 - 3\mu + 1\right) > 0 \text{ for } \frac{1}{2}\sqrt{5} + \frac{3}{2} < 4 < \mu. \tag{17}$$

- **Case C**: $\mathbf{s}_1 = (+, 0),\ \mathbf{s}_2 = (+, +)$

In case C, firm 2 (which has a higher unit cost for the high-quality product H) produces both products, but firm 1, which is efficient in the production of product H, specializes in product H:

$$(q_{1H}^{*C}, q_{1L}^{*C}, q_{2H}^{*C}, q_{2L}^{*C}) = \left(\frac{\mu + c_{2H} - 2}{3\mu},\ 0,\ \frac{2\mu^2 + (1 - 4\mu)c_{2H} - 2}{6\mu(\mu - 1)},\ \frac{c_{2H}}{2(\mu - 1)} \right) \quad (18)$$

where

$$q_{1H}^{*C} > q_{2H}^{*C},\ q_{2L}^{*C} > 0 \text{ and } q_{2H}^{*C} \gtreqless q_{2L}^{*C} \Leftrightarrow \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) \lesseqgtr \mu, \quad (19)$$

and

$$\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu \Leftrightarrow q_{2H}^{*C} > 0 \quad (20)$$

hold. Furthermore, we obtain

$$c_{2H} \geq 2 \text{ and } \mu > 4. \quad (21)$$

For $q_{1H}^{*C} > 0$, the inequality $\mu > 2 - c_{2H}$ holds because $c_{2H} \geq 2$. In Figure 1, areas C.1 and C.2 correspond to this case. In area C.1, the quality superiority μ of the high-quality product is high compared with the relative cost inefficiency, c_{2H}, of the high-quality good for firm 2. Moving from area C.1 to area C.2, the relative quality superiority μ decreases and becomes small compared with the relative cost inefficiency, c_{2H}, of good H for firm 2. Hence, firm 2 substitutes the production of high-quality good H for that of low-quality good L. The corresponding equilibrium price and profit for each firm are

$$(p_H^{*C}, p_L^{*C}) = \left(\frac{\mu + c_{2H} + 1}{3}, \ \frac{2\mu - c_{2H} + 2}{6\mu} \right), \tag{22}$$

$$\begin{aligned} \pi_1^{*C} &= \frac{(\mu + c_{2H} - 2)^2}{9\mu}, \\ \pi_2^{*C} &= \frac{4\mu^3 - 4(4c_{2H} - 1)\mu^2 + 4(2c_{2H} - 1)(2c_{2H} + 1)\mu - (7c_{2H} - 2)(c_{2H} - 2)}{36\mu(\mu - 1)}. \end{aligned} \tag{23}$$

$$\Delta\pi^{*C} = \frac{1}{12\mu(\mu - 1)}(4(c_{2H} - 1)\mu(2\mu - c_{2H} - 3) + (c_{2H} + 2)(c_{2H} - 2)). \tag{24}$$

When $\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu, c_{2H} \geq 2$, we see that $\Delta\pi^{*C} = \pi_1^{*C} - \pi_2^{*C} > 0$.

- **Case D**: $\mathbf{s}_1 = (+, +), \ \mathbf{s}_2 = (0, +)$

In this case, and in contrast to case C, firm 1 is efficient in producing product H and supplies both products. However, the inefficient firm 2 specializes in product L.

$$(q_{1H}^{*D}, q_{1L}^{*D}, q_{2H}^{*D}, q_{2L}^{*D}) = \left(\frac{\mu - 2}{2(\mu - 1)}, \frac{4 - \mu}{6(\mu - 1)}, \ 0, \ \frac{1}{3} \right) \ \text{ if } \ 2 < \mu < 4 \ \textit{and} \ \mu \leq 2c_{2H}. \tag{25}$$

where the last inequalities must hold given both the positive output condition and the necessary condition.

In addition, we have $q_{1L}^{*D} \gtreqless q_{1H}^{*D} \Leftrightarrow \mu \lesseqgtr 5/2$. Areas D.1 and D.2 correspond to this case. The relative superiority, μ, of the high-quality good is relatively small compared with the relative cost inefficiency, c_{2H}, of the high-quality good for firm 2, especially in case D.2.

From (4), (5), and (10), the corresponding equilibrium price and profit for each firm are

$$(p_H^{*D}, p_L^{*D}) = \left(\frac{3\mu + 2}{6}, \frac{1}{3}\right) \tag{26}$$

$$(\pi_1^{*D}, \pi_2^{*D}) = \left(\frac{9\mu^2 - 32\mu + 32}{36(\mu - 1)}, \frac{1}{9}\right). \tag{27}$$

Here,

$$\pi_1^{*D} - \pi_2^{*D} = \frac{1}{4(\mu - 1)}(\mu - 2)^2 > 0, \text{for } \mu \leq 2c_{2H},\ 2 < \mu < 4. \tag{28}$$

- **Case E**: $\mathbf{s}_1 = (+, +),\ \mathbf{s}_2 = (+, +)$

In case E, both firms produce both products,

$$(q_{1H}^{*E}, q_{1L}^{*E}, q_{2H}^{*E}, q_{2L}^{*E}) = \left(\frac{\mu + c_{2H} - 3}{3(\mu - 1)}, \frac{2 - c_{2H}}{3(\mu - 1)}, \frac{\mu - 2c_{2H}}{3(\mu - 1)}, \frac{2c_{2H} - 1}{3(\mu - 1)}\right) \tag{29}$$
$$if\ 1 < c_{2H} < 2 \text{ and } 3 - c_{2H} < \mu,$$

where the last inequalities must hold because of both the positive output and the necessary condition. In addition, we obtain

$$q_{1H}^{*E} \gtreqless q_{1L}^{*E} \Leftrightarrow \mu \gtreqless 5 - 2c_{2H},\ q_{2H}^{*E} \gtreqless q_{1L}^{*E} \text{ and } q_{2L}^{*E} \gtreqless q_{1H}^{*E} \Leftrightarrow \mu \lesseqgtr c_{2H} + 2.$$

Furthermore, we show that

$$q_{2H}^{*E} \gtreqless q_{2L}^{*E} \Leftrightarrow \mu \gtreqless 4c_{2H} - 1.$$

In this case, c_{2H} is very small compared with μ. In Figure 1, this case corresponds to areas E.1, E.2, E.3, and E.4. Moving from area E.4 to E.1, the relatively inefficient firm 2 reduces its output of high-quality product H.

Then, we obtain the corresponding equilibrium price and profit of each firm from (4), (5), and (10):

$$(p_H^{*E}, p_L^{*E}) = \left(\frac{\mu + c_{2H} + 1}{3}, \frac{1}{3} \right) \tag{30}$$

$$\begin{aligned} \pi_1^{*E} &= \frac{1}{9(\mu - 1)} \left(\mu^2 + (2c_{2H} - 5)\,\mu + (c_{2H} - 2)(c_{2H} - 4) \right), \\ \pi_2^{*E} &= \frac{1}{9(\mu - 1)} \left(\mu^2 - (4c_{2H} - 1)\,\mu + 4c_{2H}^2 - 1 \right). \end{aligned} \tag{31}$$

$$\begin{aligned} \text{For } 1 &< c_{2H} < 2, \mu \geq 2c_{2H} > \frac{1}{2}(c_{2H} + 3), \\ \pi_1^{*E} - \pi_2^{*E} &= \frac{1}{3\,(\mu - 1)} \,(c_{2H} - 1)\,(2\mu - c_{2H} - 3) > 0. \end{aligned} \tag{32}$$

By combining these five cases, we obtain the following proposition. Further, we show the product line strategy of the duopoly game under the rival's nonnegative output belief in the c_{2H}-μ plane in Figure 1.

[Insert Figure 1 here]

Proposition 1 *In the duopoly equilibrium of the game, given the rival's expectation of a nonnegative quantity, the following inequalities hold for the outputs of the high- and low-quality goods for each firm:*

$$0 < q_{2H}^{*E} < q_{1H}^{*E} \leq q_{1L}^{*E} < q_{2L}^{*E}$$
$$\text{for } (c_{2H}, \mu) \in \{(c_{2H}, \mu) \in R^{2++} \mid \mu > 2c_{2H}, \mu \leq 5 - 2c_{2H} \text{ and } 1 < c_{2H} < \frac{5}{4}\} \text{ (E.1)},$$

$$0 < q_{2H}^{*E} < q_{1L}^{*E} < q_{1H}^{*E} < q_{2L}^{*E} \text{ for } (c_{2H}, \mu) \in$$
$$\{(c_{2H}, \mu) \in R^{2++} \mid \mu > 2c_{2H}, \mu > 5 - 2c_{2H}, \mu < c_{2H} + 2 \text{ and } 1 < c_{2H} < 2\} \text{ (E.2)},$$

$$0 < q_{1L}^{*E} \leq q_{2H}^{*E} < q_{2L}^{*E} < q_{1H}^{*E} \text{ for } (c_{2H}, \mu) \in$$
$$\{(c_{2H}, \mu) \in R^{2++} \mid \mu \geq c_{2H} + 2, \mu < 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \text{ (E.3)},$$

$$0 < q_{1L}^{*E} < q_{2L}^{*E} \leq q_{2H}^{*E} < q_{1H}^{*E} \text{ for } (c_{2H}, \mu) \in$$
$$\{(c_{2H}, \mu) \in R^{2++} \mid \mu \geq 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \text{ (E.4)}.$$

$$
\begin{aligned}
q_{1L}^{*C} &= 0 < q_{2L}^{*C} < q_{2H}^{*C} < q_{1H}^{*C} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \mu > \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) > 4, c_{2H} \geq 2\} \text{ (C.1)}, \\
q_{1L}^{*C} &= 0 < q_{2H}^{*C} \leq q_{2L}^{*C} < q_{1H}^{*C} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) > \mu \geq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) > 4 \\
, c_{2H} &\geq 2\} \text{ (C.2)}.
\end{aligned}
$$

$$
\begin{aligned}
q_{1H}^{*B} &\geq q_{2L}^{*B} > q_{1L}^{*B} = q_{2H}^{*B} = 0 \\
\text{for } (c_{2H}, \mu) &\in \{(c_{2H}, \mu) \in R^{2++} \mid 4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}), \mu \geq \frac{5}{2} \text{ (B)}.
\end{aligned}
$$

$$
\begin{aligned}
q_{2L}^{*D} &= \frac{1}{3} > q_{1H}^{*D} > q_{1L}^{*D} > q_{2H}^{*D} = 0 \text{ when } \frac{5}{2} < \mu < 4, \ \mu \leq 2c_{2H} \text{ (D.1)}, \\
q_{2L}^{*D} &= \frac{1}{3} > q_{1L}^{*D} \geq q_{1H}^{*D} > q_{2H}^{*D} = 0 \text{ when } 1 < \mu \leq \frac{5}{2}, \mu \leq 2c_{2H}, \text{(D.2)}.
\end{aligned}
$$

$$
q_{1H}^{*A} = q_{2H}^{*A} = 0 < q_{1L}^{*A} = q_{2L}^{*A} = \frac{1}{3} \text{ when } 1 < \mu \leq 2 \text{ (A)}.
$$

where A, B, C.1, C.2, D.1, D.2, E.1, E.2, E.3, and E.4 indicate the area in the c_{2H}*-*μ *plane in Figure 1.*

Note that each equilibrium output presented in Proposition 1 is that of a duopoly game, given the firms' expectations about their rival's nonnegative output(s).

The result presented in Proposition 1 leads firms to infer *correctly* the quality superiority and relative cost-efficiency ratios *ex post* by observing the output strategies in equilibrium. Note that we assume $c_{2H} > c_{1H} = 1$ and $V_H = \mu V_L = \mu > V_L = 1$. Thus, the horizontal and vertical axes in Figure 1 show the relative cost ratio c_{2H} and the quality ratio μ, respectively. At any point (c_{2H}, μ) in areas E.1, E.2, E.3, and E.4 in Figure 1, the relative cost ratio c_{2H} is between one and two. Thus, the difference between the unit costs of the two firms is small. The equilibrium in case E corresponds to these areas. In areas E.1, E.2, and E.3, the relative superiority of the high-quality good μ is not very high. Thus, both firms are likely to supply high- and low-quality goods.

However, as the quality ratio μ becomes sufficiently high and the relative cost ratio c_{2H} becomes sufficiently low in area E.4, the inefficient firm 2 produces far more of the high-quality good, which has a higher cost than that of the low-quality good (with no production cost). Naturally, the efficient firm 1 produces more of the high-quality good H than of the low-quality good L because its production cost for H is lower than that of the rival firm; its marginal revenue from good H is also high because its quality superiority μ is very high. From this illustration, we identify a substitution of production from the low- to the high-quality good in both firms as the point (c_{2H}, μ) moves from area E.1 to areas E.2, E.3, and E.4 in Figure 1. Note that this substitution is stronger for the efficient than for the inefficient firm.

This result is consistent with that of Calzada and Valletti (2012), where the optimal strategy for a film studio is to introduce versioning if their goods are not close substitutes. Thus, when the quality of the high-quality good H is large compared with that of good L, we can conclude that they are not close substitutes. Then, the result in the above proposition confirms that it would be better for both firms to supply both goods in the market; that is, to obey the "versioning strategy" of Calzada and Valletti (2012).

At any point (c_{2H}, μ) in areas C.1 and C.2, the relative superiority μ is large compared with the relative cost ratio c_{2H}. Thus, the margin of the efficient firm 1 for the high-quality good H, $p_H^{*C} - 1$ is very high, and the firm substitutes the production of good L by that of good H. In other words, the efficient firm 1 specializes in good H, with its relatively large margin compared with that for the low-quality good L (that is p_L^{*C}) . Moving from area C.1 to C.2, the inefficient firm 2 loses its incentive to supply the high-quality good more because c_{2H} increases but μ decreases. Thus, it reduces its output of the high-quality good and increases its output of the low-quality good. The equilibrium in case C corresponds to these areas.

In area B, the relative superiority μ is at a moderate level, but is smaller than those in areas C.1 and C.2, and the relative cost ratio c_{2H} is larger than those in areas C.1 and C.2. Hence, firm 2, with its inefficient production technology for the high-quality good, stops producing good H and specializes in the low-quality good L. Two monopoly markets appear in this case. The equilibrium in case B corresponds to this area. As the relative superiority μ decreases from the point (c_{2H}, μ) in area D.1 to that in area D.2, firm 1 with efficient production technology for the high-quality good reduces its output of good H and increases its output of the low-quality good, thus substituting production of the high-quality good with that of the low-quality good. As the relative superiority μ decreases further in the equilibrium in case A (area A), firm 1 ceases to produce the high-quality good H and specializes in the low-quality good. Consequently, the market in the equilibrium becomes a duopoly of the low-quality good.

Next, we provide a lemma on the equilibrium profits of the firms for the five nontrivial equilibria.

Lemma 1 *In the duopoly equilibrium of the game under the expectation of the rival* *firm's nonnegative quantities, the following equality and inequalities hold for the profits of each firm:*

In case A, the equilibrium profits of both firms are identical: $\pi_1^{*A} = \pi_2^{*A}$. *In the equilibria for cases B, C, D, and E, firm 1—which is efficient in producing the high-quality good—earns more than firm 2—which has inefficient technology for producing the high-quality good. Thus,* $\pi_1^{*k} > \pi_2^{*k}$, $k = B, C, D,$ *and* E.

Note that the firm that is cost-efficient in producing the high-quality product earns more than the inefficient firm does at all equilibria except that of case A.

In case A, taking into account the results of Proposition 1 and Lemma 1, we find that the relative superiority μ of the high-quality good is too small compared with the unit costs for H. Thus, both firms specialize in good L, and the market for good L becomes a Cournot duopoly. Hence, the two firms' equilibrium profits are identical.

3 Comparative statics of the difference in total equilibrium outputs and profits based on μ and c_{2H}

In this section, we conduct a comparative statics analysis of differences in total outputs and profits on the two ratios μ and c_{2H} in the five equilibria.

Note that μ stands for the relative superiority of the high-quality good, $V_H/V_L = \mu > 1 = V_L$. Let $\Delta\mu$ (Δc_{2H}) denote the variation of μ (c_{2H}). $\Delta\mu >$ ($<$) 0 implies that the firms' product innovation of the high-quality good succeeds in improving (fails to improve) the relative superiority of the high-quality good. $\Delta c_{2H} >$ ($<$) 0 implies that the process innovation on the high-quality good of the efficient firm 1 (the inefficient firm 2) succeeds in enhancing the relative cost efficiency of the high-quality good.

First, we investigate how the change of the relative superiority of the high-quality good μ or the relative cost ratio c_{2H} affects the difference in the total output between the cost-efficient firm 1 and the inefficient firm 2 at equilibrium $k(=A, B, C, D, E)$.

We denote by $Q_i^{*k} = q_{iL}^{*k} + q_{iH}^{*k}$ and $\Delta Q_{12}^{*k} \equiv Q_1^{*k} - Q_2^{*k}$, the total output of firm $i(=1,2)$ and the difference in total output of the high-quality good H between the efficient firm 1 and the inefficient firm 2, respectively, at equilibrium $k(=A, B, C, D, E)$.

From (10), (13), (14), (18), (21), (25), and (29), we see that

$$Q_1^{*A} = Q_2^{*A} = \frac{1}{3}, \Delta Q_{12}^{*A} = 0, \tag{33}$$

$$Q_1^{*B} = \frac{2\mu - 3}{4\mu - 1}, Q_2^{*B} = \frac{\mu + 1}{4\mu - 1}, \Delta Q_{12}^{*B} = \frac{\mu - 4}{4\mu - 1} > 0, \tag{34}$$

$$Q_1^{*C} = \frac{1}{3\mu}\left(\mu + c_{2H} - 2\right), Q_2^{*C} = \frac{1}{6\mu}\left(2\mu - c_{2H} + 2\right), \Delta Q_{12}^{*C} = \frac{1}{2\mu}\left(c_{2H} - 2\right) > 0, \tag{35}$$

$$Q_1^{*D} = Q_2^{*D} = \frac{1}{3}, \Delta Q_{12}^{*D} = 0 \tag{36}$$

and

$$Q_1^{*E} = Q_2^{*E} = \frac{1}{3}, \Delta Q_{12}^{*E} = 0. \tag{37}$$

From (33), we see that $\frac{\partial}{\partial \mu}\Delta Q_{12}^{*A} = \frac{\partial}{\partial c_{2H}}\Delta Q_{12}^{*A} = 0$. From (34), we can easily show that

$\frac{d}{d\mu}\Delta Q_{12}^{*B} > 0$ and $\frac{\partial}{\partial c_{2H}}\Delta Q_{12}^{*B} = 0$.

From (35), we see that $\frac{\partial}{\partial\mu}\Delta Q_{12}^{*C} \leq 0$ for $c_{2H} \geq 2$ and $\frac{1}{2}(2c_{2H}+\sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu$. From (36), we have $\frac{d}{d\mu}\Delta Q_{12}^{*D} = 0$, $\frac{d}{dc_{2H}}\Delta Q_{12}^{*D} = 0$ for $2 < \mu < 4$, since $Q_{12}^{*D} = 0$. From (37), we obviously see that $\Delta Q_{12}^{*E} = 0$, $\frac{\partial}{\partial\mu}\Delta Q_{12}^{*E} = 0$ and $\frac{\partial}{\partial c_{2H}}\Delta Q_{12}^{*E} = 0$.

Next, we investigate the effects of the difference in the profit of the cost efficient firm 1 and the inefficient firm 2 for the high-quality good H at every equilibrium when the relative superiority of the high-quality good μ and the relative cost ratio c_{2H} of the high-quality good *change*.

We denote by $\Delta\pi^{*k} \equiv \pi_1^{*k} - \pi_2^{*k}$ the difference in the profit of the efficient firm 1 for the high-quality good H and the inefficient firm 2 at equilibrium $k(= A, B, C, D, E)$.

Because $\Delta\pi^{*A} = 0$, we obviously see that $\frac{\partial}{\partial\mu}\Delta\pi^{*A} = \frac{\partial}{\partial c_{2H}}\Delta\pi^{*A} = 0$. From (17), we have $\frac{d}{d\mu}\Delta\pi^{*B} = \frac{d}{d\mu}(\frac{1}{4\mu-1}(\mu^2 - 3\mu + 1)) > 0$ if $1 \leq \mu$ and $\frac{\partial}{\partial c_{2H}}\Delta\pi^{*B} = 0$. From (24), we can show that $\frac{\partial}{\theta\mu}\Delta\pi^{*C} > 0$ for $2 < c_{2H}$,then$\frac{\partial}{\theta\mu}\Delta\pi^{*C} = \frac{1}{12\mu^2(\mu-1)^2}(c_{2H}^2 - 4)(4\mu^2 - 2\mu - 1) > 0$, and $\frac{\partial}{\partial c_{2H}}\Delta\pi^{*C} = \frac{1}{6\mu(\mu-1)}(4\mu^2 - 4(c_{2H}+1)\mu + c_{2H}) > 0$ when $\mu > \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) > 4$.

From (28),$\Delta\pi^{*D}$ is a function of only μ and we can show that $\frac{d}{d\mu}\Delta\pi^{*D} > 0 \Leftrightarrow \mu > 2$ because $2 < \mu < 4$. We have $\frac{d}{dc_{2H}}\Delta\pi^{*D} = 0$. From (32), we can see that $\frac{\partial}{\partial\mu}\Delta\pi^{*E} > 0$ and $\frac{\partial}{\partial c_{2H}}\Delta\pi^{*E} > 0$ for $\mu > c_{2H} + 1$, $1 < c_{2H} < 2$.

In summary, we can obtain the next proposition on the effect of changing ΔQ_{12}^{*k} and $\Delta\pi^{*k}, k = A, B, C, D, E$, when the relative superiority of the high-quality good μ and the relative cost ratio c_{2H} of high-quality good H change. The proof of the lemma is provided in the appendix.

Lemma 2 *At equilibrium A, changing the relative superiority of the high-quality good μ and the relative cost ratio of high-quality good c_{2H} has no effect on* ΔQ_{12}^{*A} and $\Delta\pi^{*A}$.

At equilibria B and D, ΔQ_{12}^{*B}, $\Delta\pi^{*B}$ and $\Delta\pi^{*D}$ *increase as the relative superiority of the high-quality good* μ *increases but* ΔQ_{12}^{*D} *does not change. There is no effect on* ΔQ_{12}^{*B}, ΔQ_{12}^{*D}, $\Delta\pi^{*B}$ and $\Delta\pi^{*D}$ *when the relative cost ratio of the high-quality good* c_{2H} *changes. At equilibrium C ,* ΔQ_{12}^{*C} *nonincreases, but* $\Delta\pi^{*C}$ *increases as the relative superiority of the high-quality good* μ *increases. Both* ΔQ_{12}^{*C} *and* $\Delta\pi^{*C}$ *increase as* c_{2H} *increases. At equilibrium E, changing the relative superiority of the high-quality good* μ *and the relative cost ratio of the high-quality good* c_{2H} *has no effect on* ΔQ_{12}^{*E}*, but* $\Delta\pi^{*E}$ *increases as the relative superiority of the high-quality good* μ *or the relative cost ratio of high-quality good* c_{2H} *increases.*

Figure 2 graphically summarizes the results in Lemma 2.

[Insert Figure 2 here]

In equilibrium A, the relative superiority of the high-quality good μ is too low, so both firms supply the same output, consisting of only low-quality good L, to the market, irrespective of the relative cost ratio of the high-quality good c_{2H}. Hence, any change of μ and c_{2H} in area A has no effect on the difference in the total output and profit between firms, ΔQ_{12}^{*A} and $\Delta\pi^{*A}$.

In equilibrium D, the relative superiority of the high-quality good μ is relatively low, but the relative cost ratio of the high-quality good c_{2H} is not as low. The inefficient firm 2 stops producing the high-quality good and specializes in supplying the low-quality good L, but the efficient firm 1 supplies high-quality good H to the market, as well as low-quality good L. Hence, $\Delta\mu > 0$ in the area D brings about production substitution from the low-quality good L to the high-quality-good H in firm 1's product line, but there is no change in firm 2's product line. However, the total output Q_1^{*D} of firm 1 for both goods L and H does not change because they offset each other by perfect production substitution from good L to good H within the efficient firm 1 as μ increases. Consequently, if the relative superiority of the high-quality good μ increases, then only $\Delta\pi^{*D}$ increases but ΔQ_{12}^{*D} remains zero. However, Δc_{2H} has no effect on ΔQ_{12}^{*D} and $\Delta\pi^{*D}$ because the inefficient firm 2 never produces the high-quality good H in equilibrium D.

In equilibrium B, μ, the relative superiority of the high-quality good is higher and the relative cost inefficiency c_{2H} is not as high as that in equilibrium D. Hence, firm 1 stops producing the low-quality good L and specializes in producing the high-quality good H. By contrast, the inefficient firm 2 continues to specialize in producing good L because its relative cost inefficiency of good H compared with that of firm 1 is strong in this area. However, when μ is sufficiently high, firm 1 stops supplying good L and increases its output of good H, so firm 2 increases its output of good L. The increment in the former surpasses that of the latter so that $\Delta Q_{12}^{*B}(> 0)$ is increasing in μ. In addition, the difference in the profits of the two firms, $\Delta\pi^{*B}$also increases as μ increases because the markup of the high-quality good H is larger than that of the low-quality good.

In equilibrium C, μ, the relative superiority of the high-quality good is much higher, but the relative cost inefficiency c_{2H} is not higher than that in equilibrium B. Therefore, the efficient firm 1 keeps specializing in supplying good H, but the inefficient firm 2 supplies the high-quality good H, as well as good L, because the margin of good H is large enough for the inefficient firm 2 to produce good H. The difference in the firms' total output, ΔQ_{12}^{*C}, nonincreases (increases) as μ (c_{2H}) increases because the decrease in the output of good H (the total output of firm 1) by firm 1 outweighs the increase of the resultant total output of firm 2 through the production substitution from good L (H) to good H (L) in firm 2. As a consequence, the difference in the firms' total market shares ΔQ_{12}^{*C} shrinks. We can easily confirm these reason by consider reaction functions in case

C:

$$
\begin{aligned}
q_{1H} &= \frac{1}{2} - \frac{1}{2}q_{2H} - \frac{1}{2\mu}q_{2L} \\
q_{2H} &= \frac{1}{2} - \frac{1}{2}\frac{c_{2H}}{\mu} - \frac{1}{2}q_{1H} - \frac{1}{\mu}q_{2L} \\
q_{2L} &= \frac{1}{2} - \frac{1}{2}q_{1H} - q_{2H}.
\end{aligned}
$$

To pay attention to the slope of these reaction functions, an increase in μ leads to the smaller effect of q_{2L} on both $(q_{1H},\ q_{2H})$ and directly leads to expansion q_{2H} because of the decrease in cost-quality ratio c_{2H}/μ. As a result, an increase in μ causes to large increase q_{2H} and to decrease q_{1H}, so that ΔQ_{12}^{*C} decreases.

Proposition 2 *In a multiproduct duopoly which supplies high-quality goods by both firms and low-quality goods by only the inefficient firm, an increase in the relative superiority of the high-quality good shrinks the difference in the firms' market shares.*

The difference in the firms' profits $\Delta\pi^{*C}$ is increasing (decreasing) in μ (c_{2H}) because the increase of μ (c_{2H}) expands (shrinks) the markups from good H of both firms.

In equilibrium E, the relative cost inefficiency c_{2H} is very low. Hence, firm 1 begins to supply low-quality good L to the market, as well as good H. In this equilibrium, $\Delta Q_{12}^{*E} = 0$ because the production substitution quantities from one good to another offset each other as μ (c_{2H}) increases. Note that in equilibrium E, $Q_1^{*E} = Q_2^{*E} = 1/3$, from (29). This implies that both good H and good L are perfectly substituted in each firm, so changing μ or c_{2H} causes *a direct production substitution* between good H and good L *within* each firm, and *subsequently* does *an indirect production substitution of each good between the cost-efficient firm 1 and the cost-inefficient firm 2.* The former direct effect, however, works more intensely than the latter. Consequently, the difference in the firms' profit $\Delta\pi^{*E}$ as μ (c_{2H}) increases because the increase of both μ and c_{2H} enhances the production substitution from good L to good H in both firms, and efficient firm 1's markup on good H is larger than that of inefficient firm 2.

4 Welfare Analysis

4.1 The Social Welfare for the Cournot Duopoly Equilibria

In this section, we first define social welfare. We then present the social welfare in the equilibria derived in the preceding section and compare the equilibrium social welfare for the five cases. We define social welfare W^{*k}, for $k = A, B, C, D$, and E, and the social surplus as the sum of the consumer surplus CS^{*k} and the producer surplus PS^{*k}:

$$W^{*k} = CS^{*k} + PS^{*k}, k = A, B, C, D \text{ and } E.$$

We define CS^{*k} and PS^{*k} as

$$\begin{aligned} CS^{*k} &\equiv \int_{\underline{\theta}^{*k}}^{\widehat{\theta}^{*k}} (\theta - p_L^{*k}) d\theta + \int_{\hat{\theta}^{*k}}^{1} (\mu\theta - p_H^{*k}) d\theta \\ &= \frac{1}{2}\left[\mu + (1-\mu)(\widehat{\theta}^{*k})^2 - (\underline{\theta}^{*k})^2\right] - p_L^{*k}(\widehat{\theta}^{*k} - \underline{\theta}^{*k}) - p_H^{*k}(1 - \widehat{\theta}^{*k}) \end{aligned} \tag{38}$$

and

$$\begin{aligned} PS^{*k} &\equiv \pi_1^{*k} + \pi_2^{*k} \\ &= (p_H^{*k} - c_{1H}^{*k}) q_{1H}^{*k} + p_{1L}^{*k} Q_{1L}^{*k} + (p_H^{*k} - c_{2H}^{*k}) q_{2H}^{*k} + p_{2L}^{*k} Q_{2L}^{*k}. \end{aligned} \tag{39}$$

Then, from (38) and (39), the social surplus is defined as

$$W^{*k}(\hat{\theta}^{*k}) \equiv \int_{\underline{\theta}^{*k}}^{\hat{\theta}^{*k}} \theta d\theta + \int_{\hat{\theta}^{*k}}^{1} \mu\theta d\theta - c_{iH}q_{iH}^{*k} - c_{jH}^{*k}q_{jH}^{*k} \tag{40}$$
$$= -\frac{\mu-1}{2}\left(\hat{\theta}^{*k}\right)^2 + \frac{\mu}{2} - \frac{1}{2}(\underline{\theta}^{*k})^2 - c_{iH}q_{iH}^{*k} - c_{jH}^{*k}q_{jH}^{*k}, i,j = 1,2, j \neq i.$$

For case A, from (3), (2), (39), (12), (38), and (40), we have

$$\widehat{\theta}^{*A} = 1, \underline{\theta}^{*A} = p_L^{*A} = \frac{1}{3}, CS^{*A} = \int_{\frac{1}{3}}^{1} \left(\theta - \frac{1}{3}\right) d\theta = \frac{2}{9}$$

and

$$W^{*A}(\hat{\theta}^{*A}) = PS^{*A} + CS^{*A} = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}. \tag{41}$$

For case B, from (3), (2), (39), (16), (38), and (40), we obtain

$$\widehat{\theta}^{*B}(\mu) = \frac{2(\mu+1)}{4\mu-1}, \underline{\theta}^{*B}(\mu) = P_L^{*B}(\mu) = \frac{\mu+1}{4\mu-1}, CS^{*B} = \frac{1}{2\left(4\mu-1\right)^2}\left(4\mu^3 - 7\mu^2 + 9\mu - 5\right)$$

and

$$W^{*B}(\mu) = W^{*B}(\hat{\theta}^{*B}(\mu)) = \frac{1}{2\left(4\mu-1\right)^2}\left(12\mu^3 - 29\mu^2 + 31\mu - 3\right). \tag{42}$$

For case C, from (3), (2), (39), (23), (38), and (40), we have

$$\widehat{\theta}^{*C}(\mu, c_{2H}) = \frac{1}{6\mu(\mu-1)}\left(2\mu^2 + 2c_{2H}\mu + c_{2H} - 2\right), \underline{\theta}_L^{*C} = P_L^{*C} = \frac{1}{6\mu}\left(2\mu - c_{2H} + 2\right),$$

$$\begin{aligned} CS^{*C}(\mu, c_{2H}) &\equiv CS^{*C}(\widehat{\theta}^{*C}(\mu, c_{2H})) \\ &= \frac{1}{72\mu\left(\mu-1\right)}\left(16\mu^3 - 16(c_{2H}+2)\mu^2 + 4\left(c_{2H}+5\right)\left(c_{2H}+1\right)\mu + \left(5c_{2H}+2\right)\left(c_{2H}-2\right)\right) \end{aligned}$$

and

$$\begin{aligned} W^{*C}(\mu, c_{2H}) &\equiv W^{*C}(\hat{\theta}^{*C}(\mu, c_{2H})) \\ &= \frac{1}{72\mu\left(\mu-1\right)}(32\mu^3 - 32(c_{2H}+2)\mu^2 + 4(11c_{2H}^2 - 6c_{2H} + 19)\mu \\ &\quad - \left(17c_{2H} - 22\right)\left(c_{2H} - 2\right)). \end{aligned} \tag{43}$$

For case D, from (3), (2), (39), (27), (38), and (40), we have

$$\begin{aligned} \widehat{\theta}^{*D}(\mu) &= \frac{1}{2}\frac{\mu}{\mu-1}, \underline{\theta}_L^{*D} = P_L^{*D} = \frac{1}{3}, CS^{*D} = \frac{1}{72\left(\mu-1\right)}\left(9\mu^2 - 20\mu + 20\right), \\ W^{*D}(\hat{\theta}^{*D}(\mu)) &= \frac{1}{72\left(\mu-1\right)}\left(27\mu^2 - 76\mu + 76\right). \end{aligned} \tag{44}$$

For case E, from (3), (2), (39), (31), (38), and (40), we have

$$
\begin{aligned}
\widehat{\theta}^{*E}(\mu, c_{2H}) &= \frac{1}{3\mu - 3}\left(\mu + c_{2H}\right), \underline{\theta}^{*E} = p_L^{*E} = \frac{1}{3}, \\
CS^{*E}(\mu, c_{2H}) &\equiv CS^{*E}(\widehat{\theta}^{*E}(\mu, c_{2H})) \\
&= \frac{1}{18\left(\mu - 1\right)}\left(4\mu^2 - 4(c_{2H}+2)\mu + \left(c_{2H}+5\right)\left(c_{2H}+1\right)\right), \\
W^{*E}(\mu, c_{2H}) &\equiv W^{*E}(\hat{\theta}^{*E}(\mu, c_{2H})) \\
&= \frac{1}{18\left(\mu - 1\right)}\left(8\mu^2 - 8(c_{2H}+2)\mu + (11c_{2H}^2 - 6c_{2H} + 19)\right). \qquad (45)
\end{aligned}
$$

In observing the nonnegativity conditions for the equilibrium outputs of these four equilibria, (10), (25), (14), and (20), we can ensure that the value of the upper bound of the condition in case A is exactly the same as that of the lower bound in case D, that of the upper bound in case D is also the same as that of the lower bound in case B, and so on.

Hence, we obtain the following proposition on the equilibrium welfare in the five cases. The proof is in the appendix.

Lemma 3

$\frac{d}{d\mu}W^{*D}(\mu) > 0$ for $2 < \mu < 4$ in case D. $\frac{d}{d\mu}W^{*B}(\mu) > 0$ for $4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4})$ in case B. $\frac{\partial}{\partial\mu}W^{*C}(\mu, c_{2H}) > 0$ for $\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu$, $2 \leq c_{2H}$; $\frac{\partial}{\partial c_{2H}}W^{*C}(\mu, c_{2H}) > 0$ for $c_{2H} \geq 2$, $\frac{1}{2}(2 + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) \leq \mu < \frac{11}{8}c_{2H} + \frac{1}{8}\sqrt{121c_{2H}^2 - 134c_{2H} + 121} - \frac{3}{8}$; $\frac{\partial}{\partial c_{2H}}W^{*C}(\mu, c_{2H}) \leq 0$ for $c_{2H} \geq 2$, $\frac{11}{8}c_{2H} + \frac{1}{8}\sqrt{121c_{2H}^2 - 134c_{2H} + 121} - \frac{3}{8} \leq \mu$ in case C. $\frac{\partial}{\partial\mu}W^{*E}(\mu, c_{2H}) > 0$ for $1 < c_{2H} < 2$, $\mu > 2$, $c_{2H} > 2$. $\frac{\partial}{\partial c_{2H}}W^{*E}(\mu, c_{2H}) > 0$ for $\frac{1}{4}(11c_{2H} - 3) > \mu > 2c_{2H}$, $\frac{\partial}{\partial c_{2H}}W^{*E}(\mu, c_{2H}) \leq 0$ for $2c_{2H} < \frac{1}{4}(11c_{2H} - 3) \leq \mu$ in case E.

Figure 3 graphically summarizes the results in Lemma 3.

[Insert Figure 3 here]

An increase in the relative superiority of the high-quality good μ leads to an increase in the equilibrium output of good H in the market. This increases both: (i) the sum of consumers' willingness-to-pay (positive effect on welfare), $\partial\left(\int_{\underline{\theta}^{*k}}^{\hat{\theta}^{*k}}\theta d\theta+\int_{\hat{\theta}^{*k}}^{1}\mu\theta d\theta\right)/\partial\mu$; and (ii) the firms' relative production costs (negative effect on welfare), $-c_{iH}(\partial q_{iH}^{*k}/\partial\mu)-c_{jH}^{*k}(\partial q_{jH}^{*k}/\partial\mu)$, from (40). Lemma 3 implies that the direct effect (i) is stronger than the indirect effect (ii), so that the increase in μ improves social welfare.

Furthermore, in two cases, C and E, the marginal cost of good H for firm 2 impacts the social welfare because firm 2 produces good H in these cases. Lemma 3 means that with these combinations of the product line, social welfare is convex in c_{2H}. Namely, in Figure 3, the light blue dottd line stands for $\mu=\frac{11}{8}c_{2H}+\frac{1}{8}\sqrt{121c_{2H}^2-134c_{2H}+121}-\frac{3}{8}$ in case C and the green dotted line indicates $\mu=\frac{1}{4}(11c_{2H}-3)$ in case E, respectively.

The social welfare in this model consists of: (a) the sum of consumers' willingness-to-pay, (b) the production cost of good H for firm 1 ($c_{1H}=1$), and (c) the production cost of good H for firm 2 ($c_{2H}>1=c_{1H}$).

- (a) The sum of consumers' willingness-to-pay (indirect effect)

$$\frac{\partial\left(\int_{\underline{\theta}^{*C}}^{\hat{\theta}^{*C}}\theta d\theta+\int_{\hat{\theta}^{*C}}^{1}\mu\theta d\theta\right)}{\partial c_{2H}}=\frac{4-22\mu^2+18\mu^3-c_{2H}(5+4\mu)}{36\mu(\mu-1)},$$

$$\frac{\partial\left(\int_{\underline{\theta}^{*E}}^{\hat{\theta}^{*E}}\theta d\theta+\int_{\hat{\theta}^{*E}}^{1}\mu\theta d\theta\right)}{\partial c_{2H}}=\frac{-(\mu+c_{2H})}{9(\mu-1)}<0.$$

In case C, this positively (negatively) affects the social welfare if c_{2H} is sufficiently small (large), whereas it always negatively affects the social welfare in case E.

- (b) The production cost of good H for firm 1 (indirect effect)

$$\frac{\partial(-q_{1H}^{*C})}{\partial c_{2H}} = \frac{-1}{3(\mu-1)} < 0, \quad \frac{\partial(-q_{1H}^{*E})}{\partial c_{2H}} = \frac{-1}{3\mu} < 0.$$

These negatively affect the social welfare.

- (c) The production cost of good H for firm 2 (direct effect)

$$\frac{\partial(-c_{2H}q_{2H}^{*C})}{\partial c_{2H}} = \frac{1-\mu^2+c_{2H}(4\mu-1)}{3\mu(\mu-1)}, \quad \frac{\partial(-c_{2H}q_{2H}^{*E})}{\partial c_{2H}} = \frac{4c_{2H}-\mu}{3(\mu-1)}.$$

In both cases, this negatively (positively) affects the social welfare if c_{2H} is sufficiently small (large).

This implies that, in both cases (C and E), the direct effect of c_{2H} (the third factors) is stronger than any other effects; that is, the social welfare $W^{*C}(\mu, c_{2H})$ and $W^{*E}(\mu, c_{2H})$ decrease (increase) in c_{2H} when c_{2H} is small (large) enough from Lemma 3. Thus, the social welfare in these cases is a convex function in c_{2H}.[3] For these two cases, we have the following proposition:

Proposition 3 *In the multiproduct Cournot duopoly equilibria C and E, a reduction in the relative cost inefficiency can decrease social welfare. This may occur even when both firms have an equal market share of the total output of the two goods, but the inefficient firm in production has a lesser market share of the high-quality good H than the efficient firm.*

[3]The result of case E corresponds to Proposition 5 in Kitamura and Shinkai (2015b).

In Figure 3, in the equilibrium of both cases, social welfare has the tendency to decrease (increase) when c_{2H} decreases, if μ is sufficiently small (large) in case E or if c_{2H} is sufficiently large (small) in case C. The reason is that in case E, a decrease in the relatively high (low)-cost good H for firm 2 makes the inefficient firm 2 substitute production from good L to good H *within* firm 2. It then causes the efficient firm 1's production substitution from good H to good L *between* firm 1 and firm 2, and that makes social welfare worse. In case C, when c_{2H} is sufficiently large, social welfare becomes worse as c_{2H} falls because of the production substitution between the two firms for the high-quality good; that is, the relatively inefficient firm 2 sells more good H and efficient firm 1 sells less H.

Lahiri and Ono (1988) show that, in a single-product Cournot oligopoly, reducing the marginal cost of a minor firm with a sufficiently low share can decrease welfare by the production substitution. Moreover, they find that if all the firms have an equal market share, then a cost reduction in any firm improves social welfare. In case C, from proposition 2, ΔQ_{12}^{C} increases when μ is small or c_{2H} is large, so that the market share of inefficient firm 2 decreases as c_{2H} decreases. In this case, whereas Proposition 3 states the same result as Lahiri and Ono (1988), the cause of our result differs from theirs. Our result is caused by both a stronger production substitution from good L to good H within firm *2* and, subsequently, a weaker production substitution of good H between the cost-efficient and cost-inefficient firm by means of a strategic substitute, from (18) as large c_{2H} decreases.

However, Lahiri and Ono's (1988) result is only caused by the production substitution between the cost-efficient and cost-inefficient firm as the inefficient firm's cost decreases. Meanwhile, in case E, $\Delta Q_{12}^{E} = 0$ means that the two firms have the same market share for the total output of the two goods, but the inefficient firm 2 in production of the high-quality good has less market share for high-quality good H than the efficient firm 1 does. In this case, Proposition 3 in our model states that a cost reduction can decrease social welfare, even though both firms have the same market share for total outputs; that is, $Q_1^{*E} = Q_2^{*E} = 1/3$, from (37). This implies that both goods H and L are perfectly substituted in each firm, so the changing μ or c_{2H} causes not only production substitution between good H and good L within each firm, but also the production substitution of each good between the two firms.

We account for the mechanism of how reducing the relative marginal cost inefficiency decreases social welfare. In the equilibrium that we derive , both goods H and L are perfectly substituted in each firm. Therefore, changing μ or c_{2H} causes not only production substitution between good H and good L within each firm, but also the production substitution of each good between the two firms. This occurs because we consider a model in which each firm can afford to produce and supply vertically-differentiated multiproducts. However, Lahiri and Ono's (1988) result is caused only by the production outputs of each good between the cost-effective and cost-ineffective firm because they consider a model in which firms can supply a homogeneous single product, but firms have asymmetric production costs.

4.2 The Social Welfare for the First Best Equilibria

In this subsection, we would like to consider the first-best equilibrium in which the social planner can choose the quantities of all goods for maximizing the social welfare defined by (3), (4), and (40) under nonnegative constraints of outputs of goods:

$$\max_{q_{1H},q_{2H},q_{1L},q_{2L}} W(q_{1H},q_{2H},q_{1L},q_{2L}) = \frac{1}{2}[-\mu(1-q_{1H}-q_{2H})^2+\mu+2(1-q_{1H}-q_{2H})(q_{1L}+q_{2L}) -(q_{1L}+q_{2L})^2] - q_{1H} - c_{2H}q_{2H}$$

$$\textit{subject to } \; q_{1H},q_{2H},q_{1L},q_{2L} \geq 0.$$

Then, we can easily calculate and identify the following six cases that satisfy the Kuhn–Tucker condition for the above maximization problem:

- **Case 1**: $\mathbf{s}_1 = (0,+),\ \mathbf{s}_2 = (0,0)$ if $1 < \mu \leq 2$.

  Then, we have $(q_{1H}^{**F1}, q_{1L}^{**F1}, q_{2H}^{**F1}, q_{2L}^{**F1}) = (0,1,0,0)$.

- **Case 2**: $\mathbf{s}_1 = (0,0),\ \mathbf{s}_2 = (0,+)$ if $1 < \mu \leq 2$.

  Then, we have $(q_{1H}^{**F2}, q_{1L}^{**F2}, q_{2H}^{**F2}, q_{2L}^{**F2}) = (0,0,0,1)$.

- **Case 3**: $\mathbf{s}_1 = (0,+),\ \mathbf{s}_2 = (0,+)$ if $1 < \mu \leq 2$.

  Then, we have $(q_{1H}^{**F3}, q_{1L}^{**F3}, q_{2H}^{**F3}, q_{2L}^{**F3}) = (0, q_{1L}^{**F3}, 0, q_{2L}^{**F3})$ and

  $q_{1L}^{**F3} + q_{2L}^{**F3} = 1$.

- **Case 4**: $\mathbf{s}_1 = (+,+),\ \mathbf{s}_2 = (0,0)$ if $2 < \mu$.

  Then, we have $(q_{1H}^{**F4}, q_{1L}^{**F4}, q_{2H}^{**F4}, q_{2L}^{**F4}) = ((\mu-2)/(\mu-1), 1/(\mu-1), 0, 0)$.

- **Case 5**: $\mathbf{s}_1 = (+,0),\ \mathbf{s}_2 = (0,+)$ if $2 < \mu$.

  Then, we have $(q_{1H}^{**F5}, q_{1L}^{**F5}, q_{2H}^{**F5}, q_{2L}^{**F5}) = ((\mu-2)/(\mu-1), 0, 0, 1/(\mu-1))$.

- **Case 6**: $\mathbf{s}_1 = (+,+),\ \mathbf{s}_2 = (0,+)$ if $2 < \mu$.

  Then, we have $(q_{1H}^{**F6}, q_{1L}^{**F6}, q_{2H}^{**F6}, q_{2L}^{**F6}) = ((\mu-2)/(\mu-1), q_{1L}^{**F6}, 0, q_{2L}^{**F6})$ and

  $q_{1L}^{**F6} + q_{2L}^{**F6} = 1/(\mu-1)$.

According to the above result, in our model, the social planner does not let the inefficient firm produce the high-quality good. The resultant outcomes in cases 1 and 2 above are a monopoly for good L of an efficient firm 1 and an inefficient firm 2, respectively. The outcome in case 3 is a duopoly for good L. The outcomes in cases 4 and 5 are a monopoly for both goods and a monopoly specializing in good H for cost-efficient firm 1, and a monopoly specializing in good L for cost-inefficient firm 2, respectively. The outcome in case 6 is a monopoly for good H for cost efficient firm 1 in good H and a duopoly in good L. We can easily find that the social surpluses are all the same in the first-best equilibria for cases 1, 2, and 3 , thus we have

$$W^{**Fi}(q_{1H}^{**Fi}, q_{1L}^{**Fi}, q_{2H}^{**Fi}, q_{2L}^{**Fi}) = \frac{1}{2}, i = 1, 2, 3. \tag{46}$$

We can also derive the social surpluses in cases 4, 5, and 6,

$$W^{**Fi}(q_{1H}^{**Fi}, q_{1L}^{**Fi}, q_{2H}^{**Fi}, q_{2L}^{**Fi}) = \frac{1}{2(\mu - 1)}(\mu^2 - 3\mu + 3), i = 4, 5, 6. \tag{47}$$

Comparing the resultant product lines for both goods with positive production outputs for both firms in the first-best equilibrium with those in the Cournot competition equilibrium derived in Section 2, we see that case 3, the case 5 equilibrium, and case 6 in the first-best equilibria correspond to case A, the case B equilibrium, and case D in the Cournot competition equilibria, respectively. Examining the properties of the differences between the social surplus in all cases given by (46) and (47) in the first-best equilibrium and those corresponding to all five cases in the Cournot duopoly equilibrium, we conclude the following.[4]

Proposition 4 *In the multiproduct Cournot duopoly equilibria B, C, D, and E, an*

[4] For the proof, see Appendix.

increase in the relative superiority of the high-quality good brings about greater market inefficiency.

If $\mu < 2$, which corresponds to the case A in the Cournot duopoly equilibrium, then only the low-quality good is supplied in both the first-best equilibrium and the Cournot duopoly equilibrium, so that the change of μ does not affect social welfare. If $\mu \geq 2$, then in the first-best equilibrium, the social planner can make the efficient firm produce the high-quality good and both firms produce the low-quality good optimally when μ increases. In the Cournot duopoly, however, such optimal allocation is not realized because of the competition between these firms. Specifically, in cases B and D, the increase in μ leads to the efficient firm producing more high-quality goods, but this is underproduction from a social perspective: $\partial Q_H^{**Fi}/\partial\mu > \partial Q_H^{*B}/\partial\mu$ and $\partial Q_H^{**Fi}/\partial\mu > \partial Q_H^{*D}/\partial\mu$. Therefore, the increase in μ brings about greater market inefficiency in cases B and D in the Cournot duopoly equilibrium. In cases C and E, the inefficient firm also produces the high-quality good, so that the increase in μ induces the inefficient firm to produce more of the high-quality good, which increases the market inefficiency.

5 Conclusion

In this study, we consider a duopoly game with two vertically-differentiated products under nonnegative output constraints and an expectation with regard to the rival's product line strategies. We derive an equilibrium for the game and describe the firms' product line strategies and their realized profits in each equilibrium, based on the quality superiority of the goods and the relative cost efficiency.

We also show that the cost-efficient firm producing the high-quality good earns more than the inefficient firm, except in the special case where the relative superiority of the high-quality good μ is too small compared with the unit cost of the high-quality good H. In this case, both firms specialize in good L, and the market for good L becomes a Cournot duopoly. Thus, both firms' profits are the same. We also show that social welfare increases as the relative superiority of the high-quality good μ increases in all five cases.

However, in cases C and E, where the inefficient firm 2 produces good H, social welfare is convex in the relative cost inefficiency of good H, c_{2H}. Thus, in a multiproduct Cournot duopoly, a reduction in the relative marginal cost inefficiency decreases social welfare, whereas Lahiri and Ono (1988) obtain a similar result in a single-product Cournot oligopoly. We also illustrate that the mechanism for our result differs from that of Lahiri and Ono (1988). Our result is caused by both a stronger production substitution from good L to good H within firm 2 and, subsequently, a weaker production substitution of good H between the cost-efficient and cost-inefficient firms by means of a strategic substitute, from (18) as the large c_{2H} decreases (in equilibria C and E). However, the result of Lahiri and Ono (1988) is only caused by the production substitution between the cost-efficient and cost-inefficient firms as the cost of the inefficient firm decreases.

In addition, we clarify how the mechanism by which our result is caused is different from that for the result in Lahiri and Ono (1988). In equilibria C and E in our study, both goods H and L are perfectly substituted in each firm, so changing μ or c_{2H} causes not only the production substitution between good H and good L within each firm, but also the production substitution of each good between the two firms. This occurs because we consider a model in which each firm can afford to produce and supply vertically-differentiated multiproducts. By contrast, the result in Lahiri and Ono (1988) is caused only by the production substitution of the goods between the cost-effective firm and the cost-ineffective firm because they only consider a model in which firms supply a homogeneous single product, but the firms have asymmetric production costs.

Finally, we derive six first-best equilibria in which the social planner can choose the quantities of all goods for maximizing social welfare under nonnegative constraints of the outputs of goods. By comparing the social welfare in the first-best equilibria with those in the Cournot duopoly equilibria, we show that an increase in the relative superiority of the high-quality good makes the social welfare of the market worse in the multiproduct Cournot duopoly equilibria B, C, D, and E.

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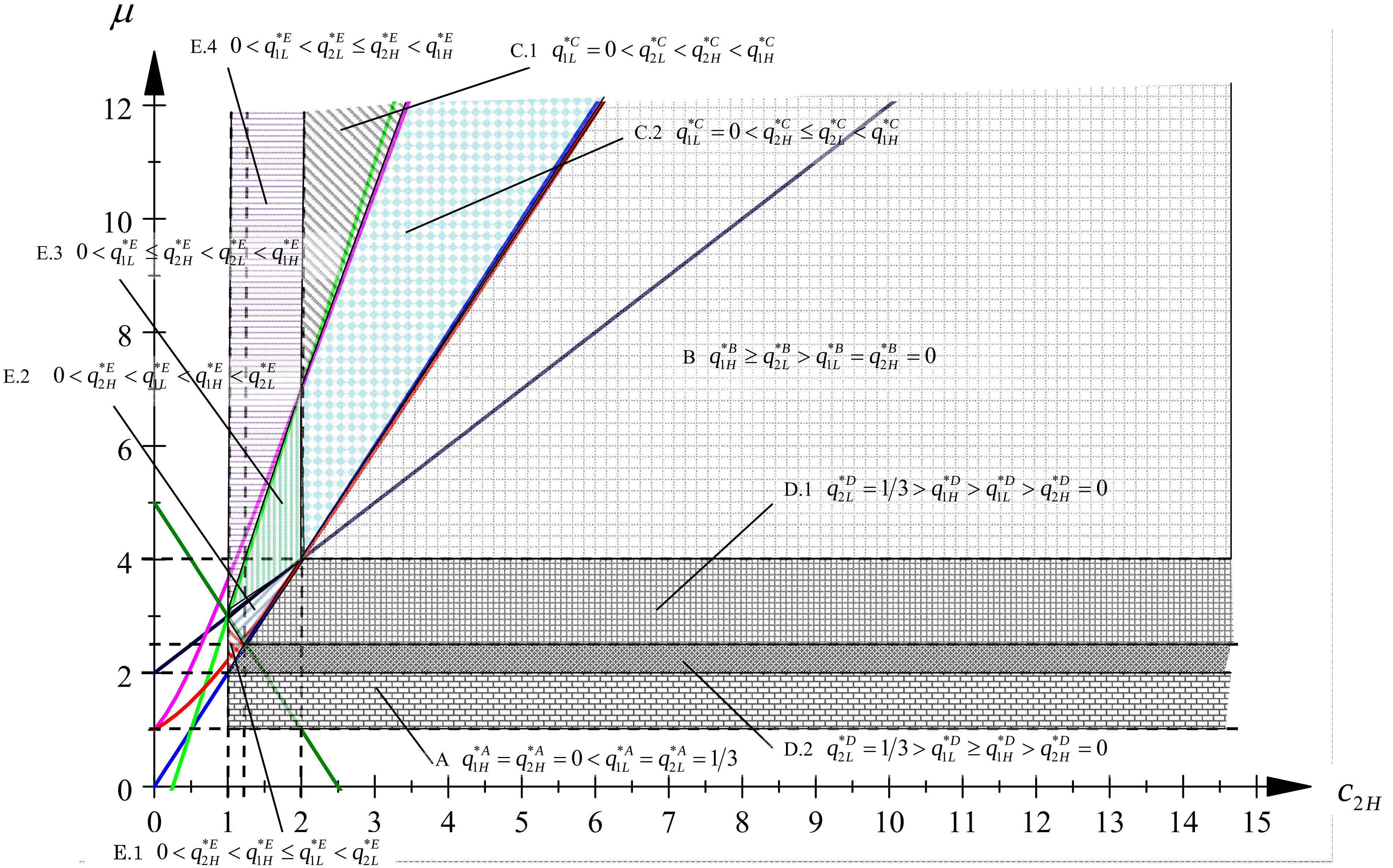
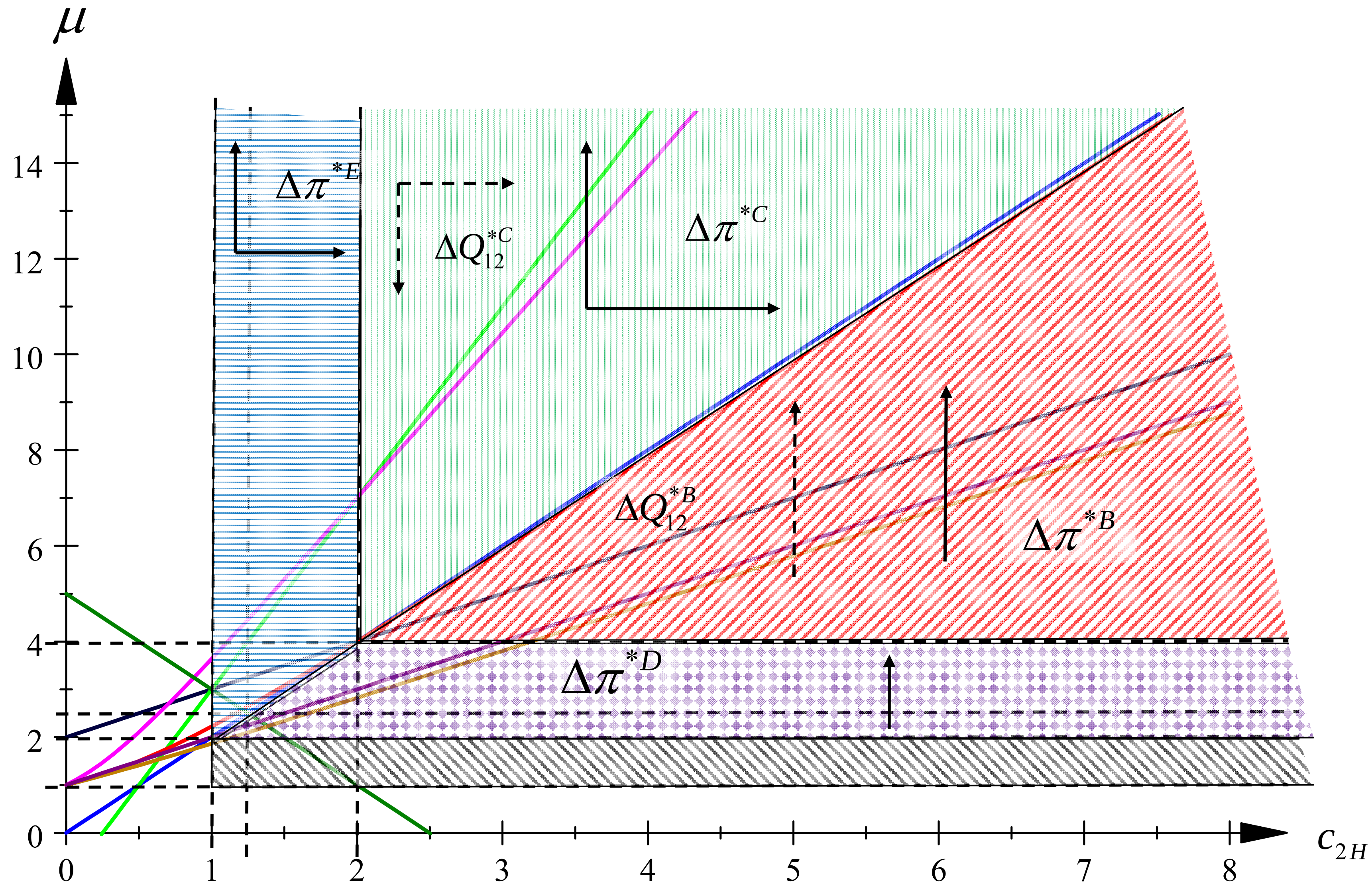


Figure 1 Equilibrium Output Strategies in $c_{2H} - \mu$ Plane



$\Delta\pi^{*k} \equiv \pi_1^{*k} - \pi_2^{*k}, k = A, B, C, D, E.$ $\Delta Q_{12}^{*k} \equiv Q_1^{*k} - Q_2^{*k}, k(=A, B, C, D, E)$ $\Delta\pi^{*A} = 0$ $\Delta Q_{12}^{*E} = 0, \Delta Q_{12}^{*C} > 0, \Delta Q_{12}^{*A} = 0$

$\uparrow$ ($\downarrow$) implies that $\Delta\pi^{*k}$ is increasing (decreasing) in μ. - - - → (← - - -) implies that ΔQ_{12}^{*k} increasing (decreasing) in c_{2H} , $k = B, C, D, E$.

Figure 2 Comparative Statics of ΔQ_{12}^{*k} and $\Delta\pi^{*k}$ w.r.t. μ and c_{2H}

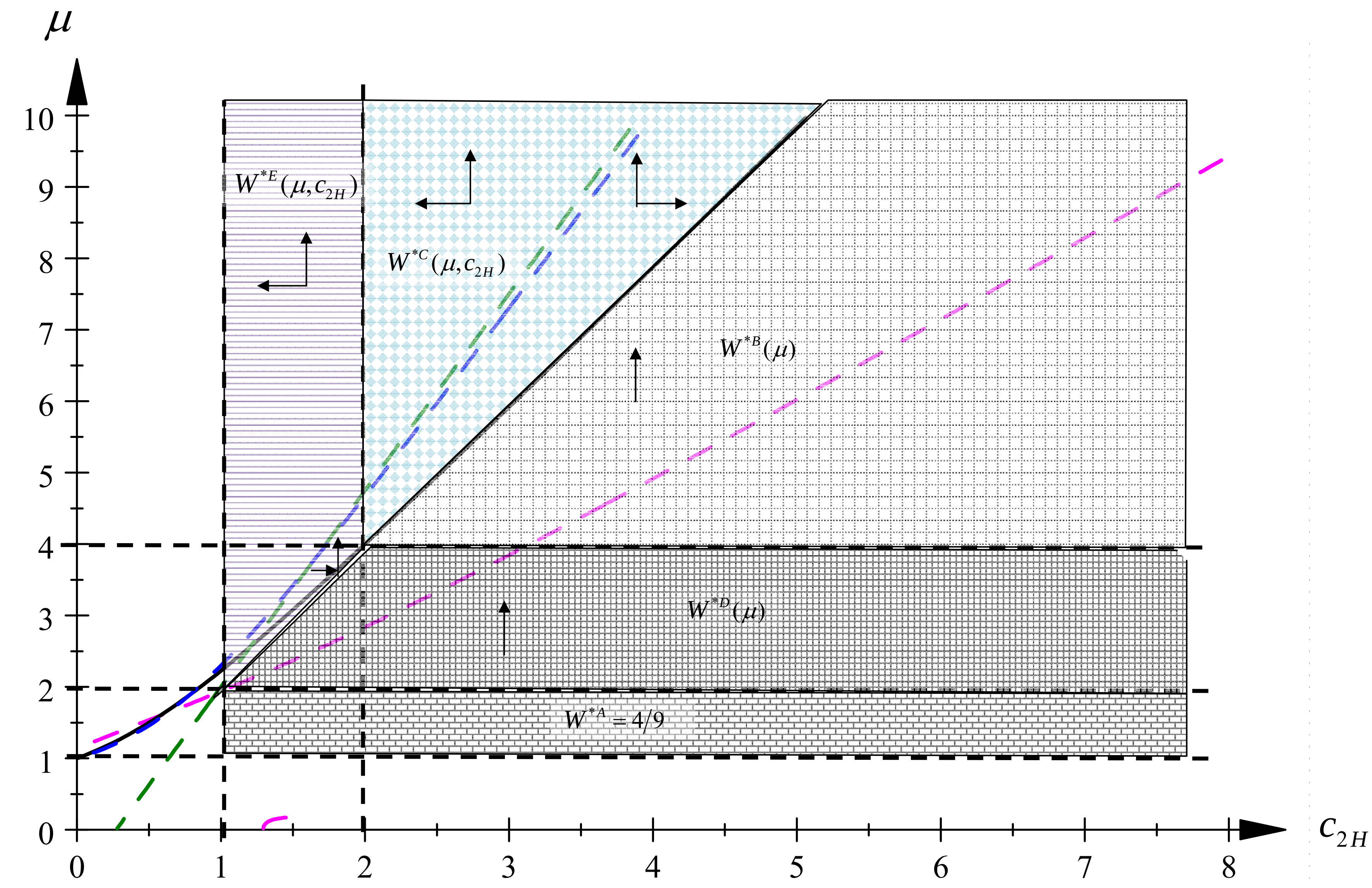


Figure 3 Welfare Comparison and Comparative Statics on μ and c_{2H}

$\mu = \frac{1}{2}(2c + \sqrt{4c^2 - 2c + 4})$ Black $\quad \mu = \frac{1}{4}(11c - 3)$ Green $\quad \mu = \frac{11}{8}c + \frac{1}{8}\sqrt{121c^2 - 134c + 121} - \frac{3}{8}$ Light Blue

$2(\mu - 1)\frac{14(\mu-1)+\sqrt{-18\mu-152\mu^2-272\mu^3+352\mu^4+9}}{-34\mu+44\mu^2+17} - c = 0$ Magenta ↑ implies that $W^{*k}(\mu)$ or $W^{*k}(\mu, c_{2H})$ increasing in μ. → implies that $W^{*k}(\mu)$ or $W^{*k}(\mu, c_{2H})$ increasing in c_{2H}. ← implies that $W^{*k}(\mu)$ or $W^{*k}(\mu, c_{2H})$ decreasing in c_{2H}.