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How Can a Central Bank Exit Quantitative Easing Without Rapidly Shrinking its Balance Sheet?

Atsushi Tanaka
(School of Economics, Kwansei Gakuin University)

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KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

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Abstract

This study constructs a simple dynamic optimization model of a central bank and examines its optimal behavior after exiting quantitative easing using interest-bearing liabilities instead of selling assets and rapidly shrinking its balance sheet. With high interest payments on liabilities, the bank may be forced to expand the monetary base to maintain its solvency, which leads to higher inflation. The model shows when the bank faces such a situation and derives the optimal paths of the monetary base supply and liabilities to deal with this. The study applies the model to the Bank of Japan and examines how the bank can exit quantitative easing.

Keywords: central bank, monetary base, inflation, quantitative easing, exit strategy, solvency.

JEL Codes: E52, E58

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† School of Economics, Kwansei Gakuin University

Address: 1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan

E-mail: atanaka@kwansei.ac.jp

1. Introduction

This study constructs a simple dynamic optimization model of a central bank and examines its optimal behavior after exiting quantitative easing using interest-bearing liabilities instead of selling assets and rapidly shrinking its balance sheet. With high interest payments, the bank may be forced to expand the monetary base to maintain its solvency. The model shows when the bank faces such a situation and how it can optimally deal with this.

Many central banks in industrialized countries introduced the unconventional monetary easing policy after the Lehman shock, and one of the main measures of the policy is quantitative easing, whereby a central bank purchases a substantial amount of assets to supply reserves, and thus expands its balance sheet. When the economy recovers, the bank must exit quantitative easing. It must absorb the piled up reserves to avoid inflation.

One of the exit strategies to do so is to shrink the balance sheet rapidly by selling assets. However, this strategy is not feasible because interest rates rise and asset prices fall at and after the exit, which imposes a large capital loss on the bank. If it waits for asset redemption, the bank can avoid the capital loss, but the balance sheet shrinks only slowly. Thus, a feasible exit strategy is to absorb a large part of the reserves with other liabilities such as using reverse repos or paying high interest on the excessive part of reserves while waiting for asset redemption, as suggested by Bernanke (2009). Such an exit strategy can absorb a large amount of reserves without rapidly shrinking the balance sheet, but it imposes high interest payments on the bank.

The heavy interest burden deteriorates the central bank's financial health. An unsound central bank may cause a large expansion of the monetary base and thus high inflation, as some developing countries have experienced.

The financial health of the central bank, especially the role of its capital, have been examined and among the early literature are Leone (1994), Stella (1997), Ueda (2003), Dalton and Diziobek (2005), Ize (2005), and Milton and Sinclair (2011)[1]. The influence of the financial health on monetary policy is discussed by Stella (1997).

[1] See Hall and Reis (2015), Appendix A for an extensive survey of the literature in this field.

Empirical evidence of a negative relationship between central bank financial health and monetary policy performance is presented by Klüh and Stella (2008) and Adler et al. (2012).

A large loss deteriorates a central bank's financial health. The bank can restore its solvency by receiving fiscal support from the government. However, if this option is not available, the bank has no choice but to expand the monetary base to maintain solvency. By introducing a central bank's intertemporal budget constraint, Hall and Reis (2015) theoretically examine when the bank becomes insolvent, and Del Negro and Sims (2015) present the simulation result regarding the Fed's solvency when it follows the Taylor rule after an exit from quantitative easing. Berriel and Bhattarai (2009) and Berriel and Mendes (2015) examine a central bank's behavior to avoid insolvency. The former study uses a model that includes the bank's capital as well as inflation and output in its objective function, and the latter presents a simulation of a central bank's behavior by setting a lower bound for its capital. Benigno and Nisticò (2015) investigate how a central bank can restore solvency; they show the condition to restore solvency and present some numerical examples of time paths of a central bank's capital, monetary base supply, and other variables.

This study explicitly examines a dynamic optimizing behavior of a central bank and derives optimal time paths of its monetary base supply and other variables when it bears high interest payments for exiting quantitative easing. The study focuses on the case of no fiscal support from the government[2], since a central bank does not generally become insolvent with fiscal support so long as the government remains solvent[3].

Such an optimal behavior is theoretically examined by Tanaka (2019). However, its model examines the bank's net assets instead of handling the assets and liabilities separately. As Bhattarai et al. (2015) and Berriel and Mendes (2015) demonstrate, it is important for a central bank to have less liquid assets in quantitative easing. This study

[2] The Bank of Japan Act does not provide any rule regarding the government's fiscal support, and, as highlighted by Reis (2015, p.3), the fiscal support for the Federal Reserves and the European Central Bank is not clear.

[3] Previous literature such as Reis (2015), Hall and Reis (2015), Benigno and Nisticò (2015), and Berentsen et al. (2016) compare several rules of fund transfers to and from the government.

assumes that the assets purchased by a central bank before an exit from quantitative easing are much less liquid than the liabilities. The assets bear only low returns, and the bank cannot sell them but can only shrink their volume slowly, while the liabilities used to absorb excessive funds at the exit impose high interest payments on the bank. This situation may make the bank insolvent, and the model in the current study reveals the bank's optimal paths of the liabilities and the monetary base to restore solvency.

This rest of this paper is organized as follows. Before considering the case in which a central bank shrinks its balance sheet, section 2 examines a simple case in which the balance sheet is constant. The model is constructed, and a central bank's optimal behavior is derived and graphically illustrated. Section 3 modifies the model so that the balance sheet slowly shrinks through redemption. The bank's optimal behavior is derived, and it is applied to the case of the Bank of Japan.

2. Model with the Balance Sheet Constant

2.1. Model Setting

This section considers the case in which a central bank does not shrink its balance sheet after an exit from quantitative easing. In Table 1, it is shown that before an exit from quantitative easing, a central bank has built up the asset holdings A to supply a substantial amount of monetary base H. The assets bear only a low interest rate r_A since they are purchased during quantitative easing. At the exit, the bank absorbs a large amount of the monetary base via some liabilities L. The liabilities can be the those obtained by any fund-absorbing operation such as reverse repos or the special reserves paying interests. In either case, the central bank needs to pay a high interest rate r_L, which is higher than r_A, on the liabilities so that the private banks have an incentive to switch some funds from H to L instead of using them for credit creation[4].

Insert Table 1 around here.

[4] In this model, L can be negative. In some cases, as shown later, L is positive at the exit, but the central bank chooses to reduce L gradually to make it negative. This means that the bank stops fund-absorbing operation, and that it starts purchasing new assets that bear r_L.

The model in this study starts at the exit, $t = 0$. At $t = 0, \cdots, \infty$, the central bank controls H and thus changes L, while A is assumed to be constant in this section. The central bank's profit π is defined as

$$\pi = r_A A - r_L L - C, \tag{1}$$

where C is the bank's operating cost, which is exogenous and assumed to be constant. In this study, the bank has no transfer to and from the government[5]. Changes in balance sheet variables satisfy,

$$\dot{L} = -\dot{H} - \dot{K} = -\dot{H} - r_A A + r_L L + C, \tag{2}$$

where $\dot{L} = dL/dt$, $\dot{H} = dH/dt$, $\dot{K} = dK/dt = \pi$.

The central bank's objective is to stabilize the inflation. The price level is determined by the following simple equation

$$P = \alpha_0 + \alpha_1 H + \alpha_2 \eta, \tag{3}$$

where P is the price level, η is a vector of variables affecting P. The inflation $\dot{P} = dP/dt$ is,

$$\dot{P} = \alpha_1 \dot{H} + \alpha_2 \dot{\eta}, \tag{4}$$

where $\dot{\eta} = d\eta/dt$. Since the current study focuses on the central bank's behavior and its influence on inflation, any change in η is not considered here for simplicity, and therefore,

$$\dot{P} = \alpha_1 \dot{H}. \tag{4'}$$

The central bank exits quantitative easing at $t = 0$, and it attempts to make the inflation $\dot{P}$ near the target $\dot{P}^*$ by controlling $\dot{H}$ at $t = 0, \cdots, \infty$. It minimizes the following quadratic loss function;

$$\min_{\dot{H}} \int_0^\infty exp(-\delta t) \left\{ \frac{1}{2} \left(\dot{P} - \dot{P}^* \right)^2 \right\} dt, \tag{5}$$

where δ is a discount factor, $\delta < r_L$. Putting equations (2), (4'), and (5) together we have,

$$\min_{\dot{H}} \int_0^\infty exp(-\delta t) \left\{ \frac{1}{2} \left(\alpha_1 \dot{H} - \alpha_1 \dot{H}^* \right)^2 \right\} dt \tag{5'}$$

[5] Many central banks have the rule to make remittance to the government when the profit is positive. Such remittance can be easily introduced into the model in this study, but it does not make any crucial change in the result; the remittance is therefore omitted for simplicity.

$$\text{s.t.}\,\dot{L} = -\dot{H} - r_A A + r_L L + C, \tag{2}$$

$$L = L_0 \text{ at } t = 0. \tag{6}$$

where $\dot{H}^*$ is the target increase in the monetary base that is consistent with the inflation target $\dot{P}^*$, $\dot{H}^* = \dot{P}^*/\alpha_1$. Equation (6) is the initial condition where L_0 is the amount of funds that the central bank must absorb to end quantitative easing. Given δ, r_A, r_L, A, C, $\dot{H}^*$, and L_0, the central bank sets $\dot{H}$ for all $t \geq 0$. $\dot{H}$ is a control variable, and L is a state variable.

2.2. Optimal Behavior

The current value Hamiltonian $\mathcal{H}$ is

$$\mathcal{H} = \frac{1}{2}\left(\alpha_1 \dot{H} - \alpha_1 \dot{H}^*\right)^2 + m\left(-\dot{H} - r_A A + r_L L + C\right), \tag{7}$$

where m is the current value multiplier. The first-order conditions are as follows.

$$\partial\mathcal{H}/\partial\dot{H} = {\alpha_1}^2\left(\dot{H} - \dot{H}^*\right) - m = 0, \tag{8 − 1}$$

$$\dot{m} = \delta m - \partial\mathcal{H}/\partial L = (\delta - r_L)m, \tag{8 − 2}$$

where $\dot{m} = dm/dt$. Equations (8-1), (8-2), and (2) reduce to the next differential equations

$$\dot{L} = r_L L - \dot{H} - r_A A + C, \tag{9 − 1}$$

$$\ddot{H} = (\delta - r)\left(\dot{H} - \dot{H}^*\right), \tag{9 − 2}$$

where $\ddot{H} = d^2H/dt^2$. A steady state $\left(\dot{H}_S, L_S\right)$ is

$$\dot{H}_S = \dot{H}^*, L_S = \frac{\dot{H}^* + r_A A - C}{r_L}. \tag{10 − 1}$$

The steady state of inflation is

$$\dot{P}_S = \dot{P}^* = \alpha_1 \dot{H}^*. \tag{10 − 2}$$

Figure 1 illustrates a phase diagram of the variables $\dot{H}$ and L. The $\dot{L} = 0$ locus is derived from equation (9-1) and its slope is $1/r_L$. L is increasing in the area to the left of the $\dot{L} = 0$ and is decreasing in the area to its right. The $\ddot{H} = 0$ locus is derived from equation (9-2) and it is vertical at $\dot{H}^*$. $\dot{H}$ is decreasing if $\dot{H} > \dot{H}^*$ and is increasing if $\dot{H} < \dot{H}^*$ since $\delta < r_L$. The steady state $\left(\dot{H}_S, L_S\right)$ is a saddle point. As discussed by Reis (2015), a central bank needs to satisfy the following no-Ponzi game condition

$$\lim_{t\to\infty} exp(-r_L t)L \leq 0, \tag{11}$$

to stay solvent.

Insert Figure 1 around here.

From equations (9-1) and (9-2), a second-order differential equation of L is derived

$$\ddot{L} - \delta\dot{L} + r_L(\delta - r_L)L = (\delta - r_L)\big(C - r_A A - \dot{H}^*\big), \tag{12}$$

where $\ddot{L} = d^2L/dt^2$. Solving differential equation (12) derives the optimal paths of L.

$$L = k_1 exp(r_L t) + k_2 exp\{(\delta - r_L)t\} + L_S. \tag{13}$$

As $1 > r_L > \delta > 0$ is assumed, only the path with $k_1 = 0$ can reach L_S.

With the initial condition (6), the optimal paths of L and $\dot{H}$ are derived as follows:

$$L = (L_0 - L_S)exp\{(\delta - r_L)t\} + L_S, \tag{14 - 1}$$

$$\dot{H} = (2r_L - \delta)(L_0 - L_S)exp\{(\delta - r_L)t\} + \dot{H}^*. \tag{14 - 2}$$

The resulting inflation path is derived by substituting (14-2) into (4'). The convergence locus shown by equations (14-1) and (14-2) is illustrated in Figure 1. Its slope is $1/(2r_L - \delta)$, which is smaller than the slope of $\dot{L} = 0$ locus.

2.3. Policy Implications

(a) Case of $L_0 > L_S$

This subsection examines the optimal path of the monetary base. At the exit, a central bank holds the liabilities L_0. Suppose L_0 is substantially large so that $L_0 > L_S$. Such L_0 is shown as L_0^1 in Figure 1. The bank can choose any value of $\dot{H}$ on the horizontal dotted line at $L = L_0^1$. If the bank sets $\dot{H} = \dot{H}^*$, then L starts moving upward, and condition (11) does not hold. To avoid an increase in L, the bank must increase $\dot{H}$ to reach the $\dot{L} = 0$ locus, but it is not optimal either. The bank's optimal behavior is to increase $\dot{H}$ further to the convergence locus. The no-Ponzi game condition is binding, and only (11), which has an equal sign, holds. It is now the transversality condition for the model, and the solution is shown by equations (14-1) and (14-2). The bank's optimal solution only gradually moves along the convergence locus toward the steady state point

$(\dot{H}_S, L_S)$[6].

In the case of $L_0 > L_S$, the no-Ponzi game condition (11) is binding, and the central bank cannot stay solvent unless it accelerates the monetary base increases. The optimal increase in the monetary base is larger than the increase only to prevent any expansion in L; it should be large enough to shrink L. The path of the resulting inflation is derived from by (14-2) and (4'). It shows the bank cannot avoid considerable inflation after the exit, and the inflation only gradually falls toward the inflation target $\dot{P}^*$.

Higher r_L deteriorates the situation. It makes the convergence locus flatter as its slope is $1/(2r_L - \delta)$, and it makes L^S smaller; both lead to larger $\dot{H}$.

(b) Case of $L_0 \leq L_S$

Suppose the central bank need not have a large amount of liabilities, $L_0 \leq L_S$ at the exit. Such L_0 is shown as L_0^2 in Figure 1 If the bank sets $\dot{H} = \dot{H}^*$ for all $t \geq 0$, the minimal value of loss function (5') is achieved. Equation (9-1) at $t = 0$ is

$$\dot{L} = r_L L_0^2 - \dot{H}^* - r_A A + C \leq r_L L_S - \dot{H}^* - r_A A + C = 0$$

as equation (10-1) holds. L never increases throughout $t \geq 0$, and condition (11) is not binding. Therefore, in contrast to the above case (a), the bank can always set $\dot{H}$ at the target $\dot{H}^*$ and thus the inflation $\dot{P}$ at the target $\dot{P}^*$.

3. Model with Shrinking the Balance Sheet Slowly

3.1 Model Setting

Although a central bank does not sell the assets to avoid any capital loss at and after an exit from quantitative easing, it can reduce the asset holdings by redemption. In this section, the above model is modified for a central bank to shrink the asset holdings at rate ρ. The rate is assumed to be given to the central bank because the redemption schedule is predetermined.

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6 When the bank reaches the steady state point, *L* remains unchanged, and *A* is constant, while *H* continues to expand by $\dot{H}_S$. This is because π is negative by the same amount, which continuously decreases *K* by the same amount. Negative *K* is allowed since it is not in the no-Ponzi game condition. If positive *K* is preferred, the bank can easily achieve it by accelerating $\dot{H}$ slightly over the convergence locus. It puts the bank on a path for *L* to diverge downward. Negative *L* makes π and *K* positive sooner or later.

The central bank's balance sheet before and at the exit is the same as in Table 1. The asset holdings are A_0 at $t = 0$, and, after the exit, the bank shrinks them slowly at the rate ρ to zero at $t \to \infty$.

$$A = A_0 exp(-\rho t). \tag{15}$$

Table 2 shows the balance sheet at $t \to \infty$. L can be positive or negative. Negative L means that the bank clears the liabilities and starts purchasing new assets that bear r_L[7].

Insert Table 2 around here.

From the balance sheet constraint and equation (1) we have

$$\dot{L} = \dot{A} - \dot{H} - \dot{K} = -\rho A - \dot{H} - r_A A + r_L L + C$$
$$= r_L L - \dot{H} - (\rho + r_A) A_0 exp(-\rho t) + C. \tag{16}$$

Thus, the model in this section is as follows.

$$\min_{\dot{H}} \int_0^{\infty} exp(-\rho t) \left\{ \frac{1}{2} \left(\alpha_1 \dot{H} - \alpha_1 \dot{H}^* \right)^2 \right\} dt \tag{5'}$$

$$\text{s.t.} \dot{L} = r_L L - \dot{H} - (\rho + r_A) A_0 exp(-\rho t) + C, \tag{16}$$

$$L = L_0 \text{ at } t = 0. \tag{6}$$

Given $\delta, r_A, r_L, A_0, \rho, C, L_0,$ and H^*, the central bank sets $\dot{H}$ for all $t \geq 0$.

3.2. Optimal Behavior

The current value Hamiltonian $\mathcal{H}$ is

$$\mathcal{H} = \frac{1}{2} \left(\alpha_1 \dot{H} - \alpha_1 \dot{H}^* \right)^2 + m \{ r_L L - \dot{H} - (\rho + r_A) A_0 exp(-\rho t) + C \}, \tag{17}$$

where *m* is the current value multiplier. The first-order conditions are as follows.

$$\partial \mathcal{H} / \partial \dot{H} = \alpha_1^2 \left(\dot{H} - \dot{H}^* \right) - m = 0, \tag{18-1}$$

$$\dot{m} = \delta m - \partial \mathcal{H} / \partial L = (\delta - r_L) m. \tag{18-2}$$

Equations (18-1), (18-2), and (16) reduce to the next differential equations

$$\dot{L} = r_L L - \dot{H} - (\rho + r_A) A_0 exp(-\rho t) + C, \tag{19-1}$$

$$\ddot{H} = (\delta - r) \left(\dot{H} - \dot{H}^* \right). \tag{19-2}$$

[7] It is more realistic for a central bank to take a path of L to diverge negatively. See footnote 6.

From these equations, a second-order differential equation of L is derived;

$$\ddot{L} - \delta\dot{L} + r_L(\delta - r_L)L = (\rho + r_A)(\delta - r_L + \rho)A_0 exp(-\rho t) + (\delta - r_L)\left(\dot{H}^* - C\right). \quad (20)$$

Solving the differential equation (20) derives the optimal path of L.

$$L = (L_0 - \beta - \gamma)exp\{(\delta - r_L)t\} + \beta exp(-\rho t) + \gamma, \quad (21-1)$$

$$\text{where } \beta = \frac{\rho + r_A}{\rho + r_L}A_0, \gamma = \frac{\dot{H}^* - C}{r_L}. \quad (22)$$

From equations (21-1) and (16), the optimal path of $\dot{H}$ is

$$\dot{H} = (2r_L - \delta)(L_0 - \beta - \gamma)exp\{(\delta - r_L)t\} + \dot{H}^*. \quad (21-2)$$

3.3. Policy Implications

(a) Cases of $L_0 > \beta + \gamma$ and $L_0 \leq \beta + \gamma$

To see when the no-Ponzi game condition (11) is satisfied in this model, multiply both sides of equation (16) by $exp(-r_L t)$ and sum them from $t = 0$ to ∞ to obtain

$$\int_0^\infty exp(-r_L t)\left(\dot{L} - r_L L\right) dt$$

$$= -\int_0^\infty exp(-r_L t)\dot{H} dt$$

$$- \int_0^\infty exp[-(\rho + r_L)t](\rho + r_A)A_0 dt + \int_0^\infty exp(-r_L t)C dt.$$

This becomes the following equation.

$$\lim_{t\to\infty} exp(-r_L t)\, L = L_0 - \beta - \gamma - \int_0^\infty exp(-r_L t)\left(\dot{H} - \dot{H}^*\right)dt.$$

If $L_0 \leq \beta + \gamma$, then the central bank achieves the target $\dot{H} = \dot{H}^*$ for all *t*. Condition (11) is always satisfied, and is therefore not binding. If $L_0 > \beta + \gamma$, the bank must make $\dot{H}$ larger than the target $\dot{H}^*$ to satisfy the no-Ponzi game condition. It is binding and the bank must follow the optimal paths (21-1) and (21-2) to stay solvent. The bank must accelerate the monetary base increase, causing inflation that is higher than the target $\dot{P} > \dot{P}^*$.

A change in r_L, ρ, or $\dot{H}^*$ has the following effects on the central bank.

$$\partial(\beta + \gamma)/\partial r_L < 0, \partial(\beta + \gamma)/\partial\rho > 0, \partial(\beta + \gamma)/\partial\dot{H}^* > 0,$$

$$\partial\dot{H}/\partial r_L \gtreqless 0, \partial\dot{H}/\partial\rho < 0, \partial\dot{H}/\partial\dot{H}^* > 0.$$

An increase in r_L, a decrease in ρ, or a decrease in $\dot{H}^*$ pushes the bank toward $L_0 >$

$\beta + \gamma$. The sign of $\partial \dot{H} / \partial r_L$ depends on *t*; it is positive at $t = 0$, but it later turns to negative. With higher r_L, the bank shrinks more L immediately after the exit to avoid the heavy interest burden, which forces the bank to accelerate the monetary base increase $\dot{H}$ early rather than later. Higher ρ allows the bank to make the monetary base increase slower.

(b) Case of the Bank of Japan

The model in this section is applied to the case of the Bank of Japan (BOJ). The BOJ has been expanding its balance sheet for quantitative easing. Suppose the bank exits the quantitative easing in April 2019 and we investigate how the bank can manage the situation.

The upper part of Table 3 shows the figures at $t = 0$. In March 2019, which is the end of fiscal year 2018, the BOJ has 511 trillion yen of assets and 502 trillion yen of monetary base. Since the cash and required reserves amount to 118 trillion yen, it is assumed that the BOJ absorbs 502-118=384 trillion yen with some interest-bearing liabilities at the exit in April. The figures $r_A = 0.43\%$ and $C = 0.198$ trillion yen are from the fiscal year 2018, and δ is assumed to be lower than r_L by 1%.

Insert Table 3 around here.

The lower part of Table 3 presents some simulations. In case (1), it is assumed that r_L is 2.5%[8], and that the BOJ targets 0% inflation, $\dot{P}^* = \alpha_1 \dot{H}^* = 0$. The majority of the assets held by the BOJ are Japanese Government Bonds (JGBs), and the average maturity of those held by the bank is between 7 and 8 years. In case (1), the BOJ stops purchasing the JGBs and the asset holdings are assumed to decrease at $\rho = 1/7 = 14.3\%$ due to the redemption. Then, $L_0 - \beta - \gamma = -55.97 < 0$, and the no-Ponzi game condition (11) is therefore not binding; the BOJ can hit the target $\dot{P}^* = \alpha_1 \dot{H}^* = 0$. Case (2) shows if r_L rises to 5.1%, $L_0 - \beta - \gamma$ becomes positive, and the BOJ starts having

[8] Iwata and the Japan Center for Economic Research (2014) assume the interest rate on a part of the excess reserves after exiting quantitative easing in Japan to be 2.5% or 3.0% in its simulation. In their simulation, Fujiki and Tomura (2017) assume the short-term interest rate to be 2.75% in the long run.

difficulty in hitting the target.

It may not be appropriate to assume that the BOJ stops purchasing the JGBs completely at and after the exit. In fiscal year 2018, the government issued 47 trillion yen of JGBs, excluding refunding bonds, and the BOJ's holdings of JGBs increased by 22 trillion yen. At the end of fiscal year 2018, the total outstanding of JGBs is 977 trillion yen, 470 trillion yen of which is held by the BOJ. In such a situation, reducing the BOJ's holdings by $477/7 = 67$ trillion yen per year may lead to JGB market turmoil.

As the extreme case opposite to no purchase, suppose the BOJ purchases some JGBs to keep the asset holdings constant as is assumed in section 2. Case (3) in Table 3 shows that $L_0 - \beta - \gamma$ is positive, and that the monetary base should expand by 10.63 trillion yen at $t = 0$, which corresponds to a $\dot{H}_0/H_0 = 10.63/118 = 9.03\%$ increase. If the BOJ targets a $2\% \times H_0 = 2.35$ trillion yen increase, then case (4) shows that the monetary base should expand by 7.34 trillion yen, which corresponds to a 6.23% increase. Thus, the BOJ's success in exiting quantitative easing in April 2019 crucially depends on the fiscal deficit and JGB market condition. The more JGBs the BOJ must purchase for market stability, the more likely the bank has no option but to accelerate the monetary base increase and thus inflation.

4. Conclusion

This study examined a central bank's optimal behavior after an exit from quantitative easing when the bank cannot shrink its balance sheet rapidly. It constructed a dynamic optimization model of a central bank and derived the optimal paths of its monetary base supply and liabilities. The study investigated the model with the balance sheet constant and then modified the model to allow the balance sheet to shrink slowly. The analysis reveals the followings findings.

The condition for a central bank to face the solvency problem depends on various variables including the interest rates, the rate of asset redemption, asset holdings, the size of funds to be absorbed at an exit, the targets of monetary base increase and inflation, and the bank's operating cost.

When it does not face the solvency problem, a central bank can always hit the targets of monetary base increase and inflation. When it does, the bank is forced to accelerate the monetary base increase, which should be large enough to shrink the liabilities used for exiting quantitative easing, and thus to cause higher inflation. The situation deteriorates with higher interest on the liabilities or slower asset redemption.

The model is applied to the BOJ. If the BOJ exits quantitative easing in April 2019, then the success of the exit crucially depends on the fiscal deficit and Japanese Government Bond market condition. If it is allowed to stop the JGB purchase completely, the BOJ does not face the solvency problem. However, if it needs to purchase some JGBs to slow the decrease in JGB holdings, the bank has no option but to accelerate the monetary base increase substantially, which leads to higher inflation.

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Table 1. Central Bank's Balance Sheet Before and At the Exit

Before the exit, $t < 0$

Assets	Liabilities
Assets (A)	Monetary Base (H)
	Capital (K)

At the exit, $t = 0$

Assets	Liabilities
Assets (A)	Monetary Base (H)
	Liabilities (L)
	Capital (K)

Table 2. Central Bank's Balance Sheet After the Exit

After the exit, $t \to \infty$

Assets	Liabilities
New Assets (-L)	Monetary Base (H)
	Capital (K)

Table 3. Some Simulations of the Bank of Japan's Exit

At the exit, $t = 0$

Assets	Liabilities
$A_0 = 511$[a]	$H_0 = 118$
	$L_0 = 384$
	$K_0 = 9$

$r_A = 0.43\%$[b], $C = 0.198$, $\delta = r_L - 1\%$

Case	r_L	$\dot{H}^*$	ρ	$L_0 - \beta - \gamma$	$\dot{H}_0$
(1)	2.5%	0	14.3%	-55.97	0.00
(2)	5.1%	0	14.3%	0.03	0.00
(3)	2.5%	0	0.0%	303.74	10.63
(4)	2.5%	2.35	0.0%	209.61	7.34

Notes: Trillion yen unless otherwise stated.

[a] A_0 is equal to the total assets minus the liabilities other than H_0 and L_0.

[b] r_A is equal to the sum of the BOJ's profit and C divided by A_0.

Source: The BOJ's figures are from its Financial Statement and Annual Review.

Figure 1. Phase Diagram of $\dot{H}$ and L