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## Fertility and Labor Share of Child Care Service

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# Fertility and Labor Share of Child Care Service\*

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## Abstract

Our paper presents an examination of how preferences for rearing children affect fertility and income growth. As described in reports of the related literature, an aging society with an increase in life expectancy reduces fertility because the preference for children decreases relatively. However, in the model with the endogenous child care service price, a decrease in preference for children does not always reduce fertility because a decrease in the price of child care service raises fertility. Then, income growth can not decrease because fertility does not always increase. The subsidy for child care service increases both the share of using child care service and the labor share of the child care service sector. Then, the wage rate of the child care service sector rises, too.

**JEL Classification:** J13

**Keywords:** Fertility, Income growth, Two sector model

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# 1 Introduction

Our paper presents an examination of how preferences for rearing children affects fertility in the endogenous child care service price. The preference for rearing children is related to life expectancy. An increase in life expectancy reduces the preference for rearing children. Thereby, fertility decreases. This result was derived by Yakita (2001), van Groezen, Leers, and Meijdam (2008) and others as reported in the related literature. However, the relative decrease in the preference for rearing children with an increase in life expectancy raises the preference for consumption in the old period. Consequently, the saving rate increases. Then, the capital accumulation is facilitated. The capital stock per capita increases because of increased savings and decreased fertility.

The aim of these analyses is to examine how fertility is determined in an endogenous fertility model with a child care service sector. Our paper sets a two-sector model, with a final goods sector and a child care service sector, based on Yasuoka (2019).<sup>1</sup> With an endogenous price of child care services, our manuscript derives the result that an increased preference for children can not always raise fertility. Therefore, a decrease in the preference for rearing children as an increase in life expectancy can not always reduce fertility. This result differs from those in reports of the related literature in that a decrease in the preference for rearing children by an increase in life expectancy reduces the fertility.

Moreover, we verify the effects of a child care service subsidy on fertility. Because of the subsidy, both the share of using child care services and the labor share of the child care service sector increases. Then, the wage rate of the child care service sector rises. This result shows positive correlation between fertility and labor force participation. This result is consistent with data provided by Sleebos (2003).

Apps and Rees (2004) and Ferrero and Iza (2004) consider child care services and parental child care time. Thereby, they derive correlation between the fertility and female labor participation. These analyses derive a positive correlation. If one considers

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<sup>1</sup>Yasuoka (2019) sets a two-sector model: a final goods sector and an elderly care service sector. However, Yasuoka (2019) considers a small open economy without consideration of capital accumulation.

only parental child care, then negative correlation is obtainable, as shown by Galor and Weil (1996). By virtue of capital accumulation, the demand for child care services increases and the child care time decreases. These earlier reports describe that fertility is positively correlated with female labor force participation. Different from the results obtained from these reports of the literature, we derive how the subsidy for child care service affects the share of using child care services and the labor share of the child care service sector.

Van Groezen, Leers, and Meijdam (2003) show that the child allowance, as a subsidy for child care, raises fertility. However, if one considers capital accumulation, then the child allowance reduces fertility, as explained by Fanti and Gori (2009), because a decrease in capital accumulation reduces the wage income. Yasuoka and Miyake (2010) present the negative effect of subsidy policy on fertility because of an increase in child care services.

The remaining parts of paper are arranged to present our examination and the salient conclusions. Section 2 sets the model. Section 3 derives the equilibrium and presents an examination of how the preference for children affects fertility. Section 4 examines effects of a subsidy for child care service on fertility and income growth. The final section concludes the paper.

## 2 The Model

Individuals in households live in two periods: young and old. During the young period, the children work to obtain income. They consume goods in the old period. We assume the following log utility function:

$$u_t = \alpha \ln n_t + (1 - \alpha) \ln c_{t+1}, \quad 0 < \alpha < 1. \quad (1)$$

In that equation,  $n_t$  and  $c_{t+1}$  denote fertility (number of children) and consumption in the old period. In this model, child care of two types exists: self child care, representing parental time spent, and purchased child care services provided through the child care service market. If the individuals use the child care service, then the lifetime budget

constraint is shown as

$$\bar{w}_t = z_t n_t + \frac{c_{t+1}}{1 + r_{t+1}}. \quad (2)$$

In that equation,  $\bar{w}_t$  represents the wage income,  $z_t$  denotes the price of child care services, and  $r_{t+1}$  denotes the interest rate. Therefore, the parents use the child care services; they can provide a full time supply of labor.

If the individuals care for the children by themselves, then the budget constraint is

$$(1 - \phi n_t) \bar{w}_t = \frac{c_{t+1}}{1 + r_{t+1}}, 0 < \phi < 1. \quad (3)$$

In that equation,  $\phi$  is needed for the care time for a child. Because of the child care time  $\phi n_t$ , the parents supply labor time of  $1 - \phi n_t$ .

There exist firms of two types in this model economy: one produces final goods; the other firm produces child care services. The production function in the firm of the final goods is assumed as

$$Y_t = K_t^\theta (B_t L_t)^{1-\theta}, 0 < \theta < 1. \quad (4)$$

The output of final goods  $Y_t$  is produced by inputting the capital stock  $K_t$  and the effective labor  $L_t$ .  $B_t = b \frac{K_t}{L_t}$ , ( $b > 0$ ) shows the Romer (1986) and Grossman and Yanagawa (1993) type of externality. With competitive market, the effective wage rate  $w_t$  and the interest rate  $1 + r_t$  are given by the follows,

$$w_t = (1 - \theta) b^{1-\theta} \frac{K_t}{L_t}, \quad (5)$$

$$1 + r_t = \theta b^{1-\theta}. \quad (6)$$

The profit function  $\pi_t$  in the child care service is assumed as

$$\pi_t = z_t \rho L_t^c - w_t^c L_t^c, 0 < \rho. \quad (7)$$

Therein,  $L_t^c$  represents the labor input to child care services. Also,  $w_t^c$  denotes the wage rate in child care services. With a competitive market, the wage rate in the child care service sector is shown as presented below:

$$z_t = \frac{w_t^c}{\rho}. \quad (8)$$

As described in this paper, we assume that individuals have ability  $a$ . This ability  $a$  is assumed to differ among individuals; it is distributed in  $[0, \bar{a}]$ . This ability  $a$  shows labor productivity in production of the final goods. If individuals work in the final goods market, they can obtain  $\bar{w}_t = aw_t$ . If they work in the child care market, they can obtain  $\bar{w}_t = w_t^c$ , irrespective of  $a$ . This setting is based on the explanation presented by Meckl and Zink (2004).<sup>2</sup> Therefore, with high  $a$ , they work in the final goods sector. Otherwise, they work in the child care service sector.<sup>3</sup> The cut off ability  $\tilde{a}$  is given as

$$\tilde{a} = \frac{w_t^c}{w_t}. \quad (9)$$

Then, the labor shares of child care service sector and the final goods sector are shown respectively by  $\frac{\tilde{a}}{\bar{a}}$  and  $\frac{\bar{a}-\tilde{a}}{\bar{a}}$ .

If the workers in the final goods sector use child care services, their fertility is shown as<sup>4</sup>

$$n_t = \frac{\alpha aw_t}{z_t}. \quad (10)$$

If care time for children is given without child care service, then the fertility can be presented as<sup>5</sup>

$$n_t = \frac{\alpha aw_t}{\phi aw_t} = \frac{\alpha}{\phi}. \quad (11)$$

We define the ability  $a$  to hold  $\frac{\alpha aw_t}{z_t} = \frac{\alpha}{\phi}$  as  $\hat{a}$ .  $\hat{a}$  shows indifference between child care by the market service and that by one's own time. If individuals have ability  $a > \hat{a}$ , they use the child care service. Otherwise, they care for children with their own time. Therefore, the share of  $\frac{\bar{a}-\hat{a}}{\bar{a}}$  uses the child care service. Because of these equations  $\hat{a}, \tilde{a}$ , the following equation can be obtained:

$$\hat{a} = \frac{\tilde{a}}{\phi\rho}. \quad (12)$$

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<sup>2</sup>Meckl and Zink (2004) considers labor of two types: skilled and unskilled. Skilled laborers obtain  $a^2w_t$ . By contrast, unskilled laborers obtain  $aw_t$ . Substantially, this setting is the same as that used for the analyses presented herein.

<sup>3</sup>Productivity  $a$  shows no absolute ability of labor but a relative ability. Low  $a$  shows insufficient skill in the final goods sector, but high skill in the child goods sector.

<sup>4</sup>We can obtain fertility to maximize utility (1) subject to (2).

<sup>5</sup>We can obtain fertility to maximize utility (1) subject to (3). The fertility of child care service workers can be given as (11).

We assume  $\phi\rho < 1$  for simplicity. Then, we can obtain  $\hat{a} > \tilde{a}$ . Therefore, individuals with ability  $[0, \tilde{a}]$  work in the child care service sector and care for their children using their own time. Individuals who have ability  $[\tilde{a}, \hat{a}]$  work in the final goods sector and care for their children using their own time. Individuals who have ability  $[\hat{a}, \bar{a}]$  work in the final goods sector and use the child care services for their children.

### 3 Equilibrium

This section presents derivation of the equilibrium of the model economy in this paper. The equilibrium can be given by the child care service market and the capital market. First, we consider the child care market equilibrium. Demand for child care service is given as  $\int_{\hat{a}}^{\bar{a}} \frac{\alpha aw_t}{z_t} \frac{1}{\bar{a}} da$ . The supply for child care services is given as (9). Then, because of these two equations, the following equation can be shown as the child care service market equilibrium condition:

$$\frac{\alpha(\bar{a}^2 - \hat{a}^2)}{2\phi\hat{a}} = \rho\tilde{a}. \quad (13)$$

With (12) and (13), we can obtain  $\hat{a}$  and  $\tilde{a}$  as

$$\hat{a} = \bar{a} \sqrt{\frac{\alpha}{2\phi^2\rho^2 + \alpha}}, \quad (14)$$

$$\tilde{a} = \phi\rho\bar{a} \sqrt{\frac{\alpha}{2\phi^2\rho^2 + \alpha}}. \quad (15)$$

After obtaining  $\frac{d\hat{a}}{d\alpha} > 0$ ,  $\frac{d\hat{a}}{d\phi} < 0$ ,  $\frac{d\hat{a}}{d\rho} < 0$  and  $\frac{d\tilde{a}}{d\alpha} > 0$ ,  $\frac{d\tilde{a}}{d\phi} > 0$ ,  $\frac{d\tilde{a}}{d\rho} > 0$ , one can consider a decrease in the preference for children  $\alpha$  as representing the case of an aging society with fewer children. A decrease in  $\alpha$  decreases  $\hat{a}$ : the share of using child care services increases because the number of children at each household decreases.

The average number of children or fertility  $n^a$  is given as

$$\begin{aligned} n^a &= \int_{\hat{a}}^{\bar{a}} \frac{\alpha aw_t}{z_t} \frac{1}{\bar{a}} da + \int_0^{\hat{a}} \frac{\alpha}{\phi} \frac{1}{\bar{a}} da \\ &= \frac{\alpha(\bar{a}^2 + \hat{a}^2)}{2\phi\bar{a}\hat{a}}. \end{aligned} \quad (16)$$

The sign of  $\frac{dn^a}{d\alpha}$  is ambiguous because

$$\frac{dn^a}{d\alpha} = \frac{n^a}{\alpha} - \frac{\alpha \left( \frac{\bar{a}^2}{\hat{a}^2} - 1 \right)}{2\phi\bar{a}} \frac{d\hat{a}}{d\alpha}. \quad (17)$$

The sign of  $\frac{dn^a}{d\alpha}$  is determined depending on the level of  $n^a$ . Large  $n^a$  brings about  $\frac{dn^a}{d\alpha} > 0$ . However, with small  $n^a$ , we can obtain  $\frac{dn^a}{d\alpha} < 0$ : even if the preference for children increases, the fertility can not always increase because of the child care service market. As shown by (10) and (11), an increase in  $\alpha$  raises fertility. However, with an increase in the demand for child care service, the price of child care service rises, which reduces the demand for child care service. Then, this reduces the fertility. The former positive effect and the latter negative effect on fertility co-exist. Therefore, the effect on fertility is ambiguous. The following proposition can be established.

**Proposition 1** An increase in the preference for children  $\alpha$  can not always raise fertility. An increase in the preference for children  $\alpha$  raises  $\hat{a}$ .

If we consider the model economy without child care service or a one-sector model, then the fertility can always be raised by virtue of an increase in  $\alpha$ . However, by considering child care services as a two-sector model, an increase in  $\alpha$  does not always raise fertility.

Next, we examine the effects of a preference for children  $\alpha$  on income growth. Defining  $k_t = \frac{K_t}{N_t}$  ( $N_t$  denotes the population size of younger people) as the capital stock per capita and  $L_t = \left(\frac{\bar{a}-\hat{a}}{\bar{a}} + \frac{(\hat{a}-\bar{a})(1-\phi n)}{\bar{a}}\right) N_t$  as effective labor, the capital market equilibrium is given as <sup>6</sup>

$$\begin{aligned} K_{t+1} &= N_t(1-\alpha) \left( \int_{\bar{a}}^{\bar{a}} a w_t \frac{1}{\bar{a}} da + \int_0^{\bar{a}} w_t^c \frac{1}{\bar{a}} da \right) \\ &= \frac{1-\alpha}{\bar{a}} \frac{\bar{a}^2 + \hat{a}^2}{2} w_t. \end{aligned} \quad (18)$$

Then, the income growth rate  $1+g \equiv \frac{k_{t+1}}{k_t}$  is given as

$$\frac{k_{t+1}}{k_t} = \frac{(1-\alpha)(1-\theta)b^{1-\theta}(\bar{a}^2 + \rho^2\phi^2\hat{a}^2)}{2n^a(\bar{a} - (1 - (1 - \rho\phi)(1 - \alpha))\hat{a})}. \quad (19)$$

As shown by (19), an increase in the preference for children  $\alpha$  reduces the income growth rate directly because saving for consumption in the old period decreases. However, an

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<sup>6</sup> $L_t$  denotes the labor input for final goods sector. Therefore, workers of the child care service sector are not included.

increase in  $\alpha$  raises  $\hat{a}$ . This effect raises the income growth rate. Moreover, an increase in  $\alpha$  reduces  $L_t$ , which positively affects income growth. If the average fertility  $n^a$  increases, then this effect on the income growth rate is negative because of the dilutive effect on the capital stock. Therefore, the increase in effects on income growth is ambiguous because these effects on income growth exist. Then, the following proposition can be established.

**Proposition 2** An increase in  $\alpha$  can not reduce income growth rate because of ambiguous effects on average fertility and positive effects on  $\hat{a}$ .

As shown by reports of the related literature, an increase in life expectancy, which entails a decrease in the preference for children, raises the income growth rate because saving increases and fertility decreases. However, if we regard the price of child care service as a two-sector model, the income growth rate can not always increase because fertility can decrease and  $\hat{a}$  increases. An increase in  $\hat{a}$  increases  $\tilde{a}$ , as shown by (12). The wage income of the child care service workers increases and saving increases. Therefore, these counter-effects exist. The effect of an increase in preference for children on income growth is ambiguous.

## 4 Subsidy for Child Care Services

This section presents examination of the effects of a subsidy for child care services on fertility. Defining  $\epsilon$  and  $\tau$  respectively as the subsidy rate of child care service and the income tax rate, household budget constraints (2) and (3) are changed as shown below:

$$(1 - \tau)\bar{w}_t = (1 - \epsilon)z_t n_t + \frac{c_{t+1}}{1 + r_{t+1}}, \quad (20)$$

$$(1 - \phi n_t)(1 - \tau)\bar{w}_t = \frac{c_{t+1}}{1 + r_{t+1}}. \quad (21)$$

The government budget constraint is given as

$$\epsilon \int_{\hat{a}}^{\bar{a}} z_t n_t \frac{1}{\bar{a}} da = \tau \left( \int_{\hat{a}}^{\bar{a}} a w_t \frac{1}{\bar{a}} da + \int_{\tilde{a}}^{\hat{a}} (1 - \phi n) a w_t \frac{1}{\bar{a}} da + \int_0^{\tilde{a}} (1 - \phi n) w_t^c \frac{1}{\bar{a}} da \right). \quad (22)$$

In this case,  $\tilde{a}$  are given as (9). Also,  $\hat{a}$  is shown as

$$\hat{a} = \frac{(1-\epsilon)\tilde{a}}{(1-\tau)\phi\rho}. \quad (23)$$

By total differentiation of (22) and (23) with respect to  $\epsilon$ ,  $\tau$ ,  $\hat{a}$ , and  $\tilde{a}$  at the approximation of  $\epsilon = 0$  and  $\tau = 0$ , one can obtain the following equation:

$$d\epsilon = \frac{\bar{a}^2 - \alpha\hat{a}^2 + (1-\alpha)\tilde{a}^2}{\alpha(\bar{a}^2 - \hat{a}^2)}d\tau, \quad (24)$$

$$d\hat{a} = \hat{a}d\tau - \frac{\tilde{a}}{\phi\rho}d\epsilon + \frac{1}{\phi\rho}d\tilde{a}. \quad (25)$$

The child care service market equilibrium (13) is modified as the following form in the case of a subsidy:

$$\frac{\alpha w_t(1-\tau)\bar{a}^2 - \hat{a}^2}{(1-\epsilon)z_t} = \rho\tilde{a}. \quad (26)$$

Considering  $\frac{(1-\tau)w_t}{(1-\epsilon)z_t} = \frac{1}{\tilde{a}\phi}$  and total differentiation of this equation with respect to  $\hat{a}$ ,  $\tilde{a}$ , we can obtain

$$d\hat{a} = -\frac{\rho\phi\hat{a}}{\alpha\hat{a} + \rho\phi\tilde{a}}d\tilde{a}. \quad (27)$$

With (24), (25) and (27), we can obtain  $\frac{d\hat{a}}{d\tau}$  as shown below:

$$\frac{d\hat{a}}{d\tau} = \frac{\hat{a} - \frac{\tilde{a}(\bar{a}^2 - \alpha\hat{a}^2 + (1-\alpha)\tilde{a}^2)}{\rho\phi\alpha(\bar{a}^2 - \hat{a}^2)}}{1 + \frac{\alpha\hat{a} + \rho\phi\tilde{a}}{\rho^2\phi^2\tilde{a}}}. \quad (28)$$

If the numerator is positive, i.e., if the decrease effect of subsidy on  $\hat{a}$  is large,  $\frac{d\hat{a}}{d\tau}$  is negative. The share of individuals who use the child care service increases. The first term in the numerator shows the taxation effect. Taxation reduces demand for child care services;  $\hat{a}$  increases. The second term represents the subsidy effect. The subsidy raises the demand for child care services and  $\hat{a}$  decreases. If the subsidy effect is greater than the taxation effect, one can obtain  $\frac{d\hat{a}}{d\tau} < 0$ . From (27), the labor share of the child care services market increases.

In the case of subsidy, the average fertility (16) is modified to the following form:

$$n^a = \int_{\hat{a}}^{\bar{a}} \frac{\alpha(1-\tau)aw_t}{(1-\epsilon)z_t} \frac{1}{\bar{a}} da + \int_0^{\hat{a}} \frac{\alpha}{\phi} \frac{1}{\bar{a}} da = \frac{\rho\phi\tilde{a} + \alpha\hat{a}}{\phi\bar{a}}. \quad (29)$$

Here,  $\frac{dn^a}{d\tau}$  can be presented as shown below:

$$\frac{dn^a}{d\tau} = -\frac{\rho\tilde{a}}{\tilde{a}\hat{a}} \frac{d\hat{a}}{d\tau}. \quad (30)$$

With  $\frac{d\hat{a}}{d\tau} < 0$ , average fertility increases. Then, the following proposition can be established.

**Proposition 3** A subsidy for child care services can raise the share of the individuals who use child care services. If  $\hat{a}$  decreases because of an increase in the share of using child care service, then the average fertility can increase.

The subsidy facilitates the use of child care services. An increase in  $\alpha$  has a direct positive effect on fertility. However, the subsidy reduces  $\hat{a}$ , which differs from the result of an increase in  $\alpha$  by which  $\hat{a}$  rises. A decrease in  $\hat{a}$  shows an increase in  $\tilde{a}$ . An increase in  $\tilde{a}$  shows the share of working in the child care service sector. The wage rate of the child care sector increases, as shown by (9) and (27).

## 5 Conclusions

Our paper presents an examination of how the preference for children affects fertility and income growth. Because of endogenous child care service pricing, an increase in preference for children can not always raise the fertility caused by an increase in the price of child care service. This result demonstrates that even if the preference for children decreases, fertility does not always decrease. This result can not be derived in the related reports of the literature that an increase in life expectancy invariably reduces fertility. Therefore, income growth can not always decrease by an increase in the preference for children because fertility can not always increase. The dilutive effect on the capital stock does not occur.

Moreover, the subsidy for child care service increases the share of people using child care services. By virtue of the subsidy policy, the fertility increases. The share of using

the child care service sector increases. This result brings about an increase in the labor force participation rate.

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