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An inverted-U effect of patents on economic growth in an overlapping generations model

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An inverted-U effect of patents on economic growth in an overlapping generations model*

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Abstract

This paper analyzes how patent protection affects economic growth in a continuous-time overlapping generations model with lab-equipment type R&D-based growth. We show that increasing patent breadth may generate an inverted-U effect of patents on economic growth, an effect which is partly consistent with an empirically observed nonmonotonic relationship between patent protection and economic growth. This paper also shows that the combinations of heterogeneous households with finite lifetimes and the lab-equipment type R&D specification are relevant for deriving the inverted-U effect of patent protection on economic growth.

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*Any errors are our responsibility.

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1 Introduction

In recent decades, many countries have been strengthening their protection of intellectual property rights (IPR) by reforming their patent systems. The Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), which entered into effect in 1995, has intensified the policymakers' concerns for patent policies. For example, Park (2008) provides an index of patent rights on a scale of 0-5 (a larger number implies stronger protection) and shows that the strength of patent rights in the US increased from 0.74 in 1960 to 4.88 in 2005.[1]

In general, it is widely believed that stronger patent protection enables the patent holders to obtain greater rent from charging a higher price. In turn, it is likely that this process promotes innovation, thereby increasing productivity and economic growth.[2] However, recent empirical studies cannot verify a clear monotonically positive correlation between patent protection and economic growth (e.g., Gould and Gruben, 1996; Falvey et al., 2006; Qian, 2007; Lerner, 2009). Instead, these studies indicate the possibility of a nonmonotonic relationship between patent protection and economic growth. For example, Qian (2007) evaluates the effects of patent protection on pharmaceutical innovations for 26 countries that established pharmaceutical patent laws from 1978–2002 and finds that there appears to be an optimal level of intellectual property rights regulation above which further enhancement reduces innovative activities.[3]

In this paper, we analyze the effects of patent protection on economic growth in a continuous-time overlapping generations (OLG) model as in Blanchard (1985) with lab-equipment type R&D-based growth as in Rivera-Batiz and Romer (1991). We show that increasing patent breadth may generate an inverted-U effect of patents on economic growth, an effect which is partly consistent with an empirically observed nonmonotonic relationship between patent protection and economic growth.[4]

[1]Park (2008) examines five categories of patent rights (patent duration, coverage, enforcement mechanism, restrictions on patent scope, and membership in international treaties) and assigns a score from 0 to 1.

[2]This may not be the case if we consider sequential innovation. In this case, stronger patent protection may impede sequential innovation. See Chu et al. (2012a) for example.

[3]In addition, Table 2 in Gould and Gruden (1996) shows that the per capita income growth rate of countries with middle-level patent protection is lower than that of countries with the second-lowest level of patent protection.

[4]Aghion et al. (2005) find evidence of an inverted-U relationship between competition and innovation by using panel data. Since increasing patent breadth implies reduced competition in the current model, the finding of this paper is consistent with the evidence provided by Aghion et al. (2005).

Intuitively, broader patent breadth increases the proportion of income that goes to monopolistic profits and increases the equilibrium interest rate, leading to a conventional positive effect on economic growth through the enhancement of innovation activities (the “interest rate effect"). However, this scenario also has a potentially negative effect on economic growth by enlarging the growth-reducing effect of “generation-turnover", which arises in the overlapping generations framework because a fraction of older and therefore wealthier individuals die and they are replaced by poorer newborns with less accumulated wealth (the “generation-turnover effect"). It is known that the extent of the generation-turnover effect is determined by the equilibrium per capita asset-consumption ratio because this ratio captures the relative differences in aggregate per capita consumption in the economy and the consumption by newborns. The higher interest rate caused by the broader patent breadth motivates households to save more for their future consumption and increases the equilibrium per capita asset-consumption ratio. Under the lab-equipment type R&D specification, the value of innovations is determined by the price of final goods through a zero profit condition in the R&D sector and is independent of the patent breadth. Combinations of these two factors increase the equilibrium per capita asset-consumption ratio, enlarge the growth-reducing effects of “generation-turnover", and thus negatively affect economic growth.

We also show that our paper's inverted-U effect of patent breadth on economic growth depends upon our lab-equipment type R&D specification. If we follow Romer (1990) and consider the knowledge-driven R&D specification, where R&D activities require labor inputs, the growth-reducing “generation-turnover effect" disappears. Therefore, the broader patent breadth always increases economic growth through the growth-enhancing “interest rate effect". Intuitively, under the knowledge-driven R&D specification, R&D activities require labor inputs, and the value of innovations is positively related to the equilibrium wage rate through a zero profit condition in the R&D sector. The lower wage rate caused by the broader patent breadth decreases the value of innovations, decreases the equilibrium per capita asset-consumption ratio, and thus completely cancels out the growth-reducing effect of “generation-turnover". Therefore, under the knowledge-driven R&D specification, the broader patent breadth always increases economic growth through the growth-enhancing "interest rate effect". Our analyses show that the combinations of heterogeneous households with finite lifetime and the lab-equipment type R&D specification are relevant for deriving the inverted-U effect of patent protection on economic growth.[5]

This study relates most closely to the macroeconomic literature of patent policy and economic growth. The seminal study in this literature is that of Judd (1985), who finds that an infinite patent length maximizes economic growth. However, subsequent studies (e.g., O'Donoghue and Zweimuller, 2004; Furukawa, 2007; Horii and Iwaisako 2007; Chu et al., 2012 a,b) show that strengthening patent protection in various forms could generate a nonmonotonic effect on economic growth.[6] Iwaisako and Futagami (2013) also show that the contrasting effects of patent breadth on innovation and physical capital accumulation may generate an inverted-U effect of patents on economic growth.

However, most of these studies have focused on the economies of an infinitely living homogeneous household. Chou and Shy (1993) and Sorek (2011) are two exceptional studies that analyze the growth implications of patent policy in an OLG framework of finitely living households.[7] Using a two period OLG model of an expanding-variety growth, Chou and Shy (1993) show that, under one-period patent length, investment in new innovations is always higher than under infinite patent length, because young agents can buy no existing patents from old agents and must invest all their savings in new innovations.[8] In accordance with Chou and Shy (1993), Sorek (2011) develops a two period OLG model of quality-ladder growth and clarifies the parameter conditions under which the shorter patent length enhances economic growth. However, to the best of my knowledge, there are no existing studies that analyze the growth implications of patent policy in a continuous-time

[5]Similar to our study, Chu et al. (2012b) develops a R&D growth model with elastic labor supply and finds that increasing patent breadth may generate an inverted-U effect on innovation depending on whether the model features the knowledge-driven or lab-equipment driven innovation process.

[6]Examples include O'Donoghue and Zweimuller (2004) on leading breadth and patentability requirements, Furukawa (2007) and Horii and Iwaisako (2007) on patent protection against imitation, and Chu et al. (2012b) on patent breadth. See Chu et al. (2012a) for a more comprehensive literature review.

[7]Diwakar et al.(2018) also develop a two period OLG model of expanding-variety growth with physical capital accumulation. As with Iwaisako and Futagami (2013), the researchers show that the contrasting effects of patent breadth on innovation and physical capital accumulation may generate an inverted-U effect of patents on economic growth.

[8]Chou and Shy (1993) refers to this effect as the "crowding-out effect of the long duration of patents".

OLG framework.[9] Therefore, this paper contributes to the literature of patent policy and economic growth by highlighting novel interactions between the "generation-turnover effect" and the innovation process through which patent protection has an inverted-U effect on economic growth.

This study also relates to the literature of factor shares and economic growth (e.g., Bertola 1993, 1996; Bertola et al. 2006). Stronger patent protection increases the proportion of income that goes to monopolistic profits, which, in turn, reduces the proportion of income that goes to workers. In particular, this paper closely relates to that of Bertola (1996), who examines the effects of factor shares on economic growth in a simple AK type endogenous growth model. Bertola (1996) shows that the effect of factor shares on economic growth depends crucially on the assumptions of saving behaviors. In the standard infinite horizon optimizing model of balanced-growth, aggregate savings are positively related to private rates of return on investment or, for a given technology, the share of capital of production in aggregate income. Therefore, distributing income from labor to capital is beneficial for growth. However, in an overlapping generations model, higher rates of return on an older agent's wealth imply lower disposable income for young laborers with a high saving propensity. Consequently, distributing income from labor to capital may be harmful for growth under certain parameter conditions. Our paper's inverted-U effect of patent protection on economic growth in an OLG framework is partly indebted to this Bertola (1996) seminal contribution. However, by employing an R&D-based growth model, this paper focuses on the growth implications of patent policy that affect incentives for innovations more directly. Thus, this paper complements the analyses conducted by Bertola (1996). This paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the effect of patent breadth on economic growth under the lab-equipment type R&D specification. Section 4 briefly considers the knowledge-driven R&D case. Section 5 concludes the paper.

[9] Olivier (2000) constructed a continuous-time overlapping generations model with variety expansion type innovation to show that speculative bubbles may promote innovation and growth.

2 Model

To analyze the effect of patent protection on economic growth, we consider a continuous-time OLG model as in Blanchard (1985) with lab-equipment type R&D-based growth as in Rivera-Batiz and Romer (1991). The economy consists of age-specific heterogeneous households whose lifetime is finite. For production, three sectors exist: a final goods sector, an intermediate goods sector, and an R&D sector. In accordance with Goh and Olivier (2002), we introduce patent breadth into the model.

2.1 Basic Assumptions

The population of an economy consists of different cohorts that are distinguishable by their date of birth denoted as j. Each cohort j consists of a measure $L_j(v)$ of households at time $v \geq j$, where $j \in (-\infty, v)$ is the cohort index and $v \in (-\infty, \infty)$ is continuous calendar time. Each household encounters an age-independent instantaneous risk of death μ, which is assumed to be exogenous and constant, as in Blanchard (1985). Thus, the probability that a household born at time j survives until time $v \geq j$ is given by $\mathrm{e}^{-\mu(v-j)}$. Moreover, due to the law of large numbers, the value of μ also refers to the fraction of households dying at each instant.

At every instant of time, a new cohort is born. The birth rate of the economy is denoted by λ, which is assumed to be exogenous and constant, as in Buiter (1988). Hence, the size of the cohort born at time v is given by $\lambda L(v)$, where $L(v)$ is the size of the whole population in the economy at time v. Without loss of generality, we set $L(0)$ to L. Thus, population grows at the rate of $b = \lambda - \mu$, where $\lambda - \mu > 0$, and the size of the whole population in the economy at time v is given by $L(v) = L\mathrm{e}^{(\lambda-\mu)v} = L\mathrm{e}^{bv}$.

Since the size of the cohort born at time j is given by $\lambda L(j)$ and every household confronts an age-independent instantaneous risk of death μ, the size of the cohort j at time $v \geq j$ is given by $L_j(v) = \lambda L(j)\mathrm{e}^{-\mu(v-j)} = \lambda L\mathrm{e}^{(\lambda-\mu)j}\mathrm{e}^{-\mu(v-j)} = \lambda L\mathrm{e}^{\lambda j}\mathrm{e}^{-\mu v}$. Therefore, the size of the cohort j relative to the whole population at time v is given by

$$\frac{L_j(v)}{L(v)} = \frac{\lambda L e^{\lambda j} e^{-\mu v}}{L e^{(\lambda-\mu)v}} = \lambda e^{\lambda(j-v)}. \tag{1}$$

2.2 Households

The expected lifetime utility of the household born at time j (i.e., cohort j) is

$$U_j^E = \int_j^\infty \ln[c_j(v)] e^{-(\rho+\mu)(v-j)} dv, \tag{2}$$

where $c_j(v)$ is the consumption at time v for a household born at time j and $\rho > 0$ is the subjective time discount rate. As in Yaari (1965) and Blanchard (1985), we assume that every household insures themselves against the risk of dying with positive assets by using their savings to buy actuarial notes of a fair life insurance company. Therefore, under the assumption of the perfectly competitive annuity market, those who survive at time v receive the insurance premium μ as well as the interest rate $r(v)$. Consequently, the budget constraint of the household born at time j is given by

$$\dot{a}_j(v) = [r(v) + \mu] a_j(v) + w(v) - c_j(v), \tag{3}$$

where $a_j(v)$ is the asset holdings at time v for a household born at time j, and $w(v)$ is the wage rate at time v. Note that the newly born household receives no share of existing wealth; that is, $a_v(v) = 0$. Moreover, as in Grossman and Helpman (1991), all assets are held by the form of the shares of monopolistic firms.

The household maximizes (2) for the consumption subject to (3). We obtain the household's consumption Euler equation and the transversality condition as follows:

$$\frac{\dot{c}_j(v)}{c_j(v)} = r(v) - \rho, \tag{4}$$

$$\lim_{v\to\infty} a_j(v)\exp\left[-\int_j^v \{r(x)+\mu\}dx\right] = 0. \tag{5}$$

Integrating (3) with respect to v over $v \in [t, \infty)$, and using (4) and (5) yield consumption at time t for a household born at time j:

$$c_j(t) = (\mu+\rho)\left[a_j(t)+h(t)\right], \tag{6}$$

where

$$h(t) \equiv \int_t^\infty w(v)\exp\left[-\int_t^v \{r(x)+\mu\}dx\right]dv.$$

Here, $h(t)$ represents the human wealth (i.e., the present value of the expected future labor income). In addition, since $a_t(t) = 0$, the relation $c_t(t) = (\mu+\rho)h(t)$ holds.

2.3 Aggregation

Recalling the fact that the size of the cohort j relative to the whole population at time v is given by (1), we can define the aggregate per capita asset holdings at time t, $a(t)$ and the aggregate per capita consumption at time t, $c(t)$, as follows:

$$a(t) \equiv \int_{-\infty}^t a_j(t)\frac{L_j(t)}{L(t)}dj = \int_{-\infty}^t a_j(t)\lambda e^{\lambda(j-t)}dj, \tag{7}$$

$$c(t) \equiv \int_{-\infty}^t c_j(t)\frac{L_j(t)}{L(t)}dj = \int_{-\infty}^t c_j(t)\lambda e^{\lambda(j-t)}dj. \tag{8}$$

Substituting (6) into (8) and using (7), we obtain

$$c(t) = (\mu+\rho)[a(t)+h(t)]. \tag{9}$$

Moreover, differentiating (8) with respect to t and using (9), as shown in Appendix A, we can write the dynamics of $c(t)$ as:

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - \lambda(\mu + \rho)\frac{a(t)}{c(t)}, \tag{10}$$

where we have that $(\mu + \rho)\frac{a(t)}{c(t)} = \frac{c(t) - c_t(t)}{c(t)}$. The third term on the right side of (10) reports the difference in aggregate per capita consumptions in an economy and consumptions by newborns. Note that the aggregate per capita consumptions in an economy $c(t)$ is always higher than the consumptions of newborns $c_t(t)$ because newborns have no accumulated assets. Consequently, the aggregate per capita consumption expenditure growth in (10) will always be lower than the individual consumption growth in (4) because a fraction of μ of older and therefore wealthier individuals die, and they are replaced by poorer newborns. Since the latter can afford less consumption than the former, the turnover of generations slows the aggregate consumption expenditure growth compared to individual consumption expenditure growth.

2.4 Final goods sector

Final goods $Y(t)$ are produced by competitive firms.

$$Y(t) = A \cdot L_y(t)^{1-\alpha} \int_0^{N(t)} x_i(t)^{\alpha} di, \tag{11}$$

where $A > 0$ is a productivity parameter, $L_y(t)$ is production labor, $x_i(t)$ is the intermediate good i, and $N(t)$ is the number of intermediate goods. Given the price of the intermediate goods $p_i(t)$ and wage rate $w(t)$, the profit maximization yields

$$w(t) = (1 - \alpha)\frac{Y(t)}{L_y(t)}, \tag{12}$$

$$p_i(t) = \alpha A L_y(t)^{1-\alpha} x_i(t)^{\alpha-1}. \tag{13}$$

As explained below, labor is used only for final goods production. Thus, the labor market clearing condition becomes $L_y(t) = L(t)$.

2.5 Intermediate goods sector

There is a continuum of intermediate goods $i \in [0, N(t)]$. One unit of intermediate goods is produced with a units of final good inputs. A single firm holding the patent monopolistically supplies each intermediate good i. The profit function of each intermediate good firm is $\pi_i(t) = [p_i(t) - a]x_i(t)$. The familiar unconstrained profit-maximizing price is $p_i(t) = \frac{a}{\alpha}$. Here, we follow Goh and Olivier (2002) to introduce patent breadth $\beta > 1$ as a policy variable such that $p_i(t) = \max\left\{\beta, \frac{1}{\alpha}\right\} a$.[10] We focus on the interesting case in which $\beta \in (1, \frac{1}{\alpha})$. Consequently, a broader patent breadth β enables the monopolistic firms to charge a higher markup capturing Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".[11] Substituting $p_i(t) = p(t) = \beta a$ into (13) and $\pi_i(t) = [p_i(t) - a]x_i(t)$ shows that relations $x_i(t) = x(t)$ and $\pi_i(t) = \pi(t)$ hold for all $i \in [0, N_t]$. Therefore, henceforth, we can omit the index i. Under these specifications, the profit of each intermediate good firm satisfies

$$\pi(t) = \frac{\beta - 1}{\beta} p(t)x(t) = \frac{\beta - 1}{\beta} \frac{\alpha Y(t)}{N(t)}, \tag{14}$$

where the second equality follows from (13), $L_y(t) = L(t)$ and $Y(t) = AL(t)^{1-\alpha}N(t)x(t)^{\alpha}$. Thus, substituting $p(t) = \beta a$ into (14) yields

$$x(t) = \frac{\alpha}{\beta a} \frac{Y(t)}{N(t)}. \tag{15}$$

Hence, using (15), we can rewrite $Y(t) = AL(t)^{1-\alpha}N(t)x(t)^{\alpha}$ as follows:

$$Y(t) = \tilde{A}(\beta)L(t)N(t), \tag{16}$$

[10] Generally, governments control the degree of patent protection through patent length and breadth. In this paper, for simplicity, we assume that the patent length is fixed and infinite and that governments control the degree of patent protection using only patent breadth.

[11] Specifically, we assume that the broader the government makes patent breadth, the more difficult it is to produce imitative goods. We specify the unit cost of producing imitative goods as βa. Each firm that produces an intermediate good charges a price such that producers of imitative goods cannot earn positive profits, as follows: $p_i(t) = \beta a$.

where

$$\tilde{A}(\beta) \equiv A^{1/1-\alpha}(\alpha/\beta a)^{\alpha/1-\alpha} > 0.$$

Since $\frac{\beta \tilde{A}'(\beta)}{\tilde{A}(\beta)} = -\frac{\alpha}{1-\alpha} < 0$, we can see that a broader patent breadth negatively affects the volume of final goods production through its distortional effects on factor inputs allocations.

2.6 R&D Sector

Denote $V_i(t)$ as the value of the patent on variety $i \in [0, N_t]$. $\pi_i(t) = \pi(t)$ from (14) implies that $V_i(t) = V(t)$ for all $i \in [0, N(t)]$. If households possess one unit of stock in the time interval dt, they can obtain a profit of $\pi(t)$ and a capital gain or loss of $\dot{V}(t)$. Alternatively, households can invest $V(t)$ units of funds in the risk-free asset. Therefore, in equilibrium, the no-arbitrage condition for $V(t)$ is

$$r(t)V(t) = \pi(t) + \dot{V}(t). \tag{17}$$

Competitive entrepreneurs employ R&D inputs for innovation. In accordance with Rivera-Batiz and Romer (1991), we consider the lab equipment type R&D specification. Devoting $\eta(t)$ units of the final good, R&D firms can invent one unit of intermediate goods. We assume that the R&D cost $\eta(t)$ is given by $\eta L(t)$, which expresses the dilution effect that removes the scale effect, as in Laincz and Peretto (2006). Given the value of the patent on variety $V(t)$, the zero profit condition yields

$$V(t) = \eta(t) = \eta L(t). \tag{18}$$

With noting that $\dot{V}(t)/V(t) = \dot{L}(t)/L(t) = b$ from (18), combining (14), (16), (17) and (18) yield

$$r(t) = \frac{\alpha}{\eta}\Omega(\beta) + b \equiv r(\beta), \tag{19}$$

where $\Omega(\beta) \equiv \frac{\beta-1}{\beta}\tilde{A}(\beta)$. Since $\Omega'(\beta) = \frac{1}{\beta}\left[\frac{1-\alpha\beta}{(\beta-1)(1-\alpha)}\right]\Omega(\beta) > 0$ and $r'(\beta) = \frac{\alpha}{\eta}\Omega'(\beta) > 0$, a broader patent breadth positively affects the equilibrium interest rate.

2.7 Market clearing conditions

The final goods are used for household consumption, the intermediate goods production, and R&D investments. Thus, the final goods market clearing condition becomes

$$Y(t) = c(t)L(t) + ax(t)N(t) + \eta L(t)\dot{N}(t). \tag{20}$$

In addition, the asset market clearing condition is given as

$$a(t)L(t) = N(t)V(t). \tag{21}$$

3 Patent policy and economic growth

3.1 Equilibrium dynamics

The dynamic system of the economy for a given patent breadth β is illustrated by the following equations:

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\eta}\left[\frac{\beta-\alpha}{\beta}\tilde{A}(\beta) - \tilde{c}(t)\right], \tag{22}$$

$$\frac{\dot{c}(t)}{c(t)} = r(\beta) - \rho - \lambda(\mu+\rho)\frac{\eta}{\tilde{c}(t)}, \tag{23}$$

where $\tilde{c}(t) \equiv c(t)/N(t)$. Note that (22) is obtained from (15), (16) and (20); (23) is obtained from (10), (18), (19) and (21), respectively.

Summarizing equations (22) and (23), we can obtain the following differential equation of $\tilde{c}(t)$:

$$\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = -\frac{(1-\alpha)\tilde{A}(\beta)}{\eta} + b - \rho - \lambda(\mu+\rho)\frac{\eta}{\tilde{c}(t)} + \frac{\tilde{c}(t)}{\eta} \equiv \Gamma(\tilde{c}(t);\beta), \tag{24}$$

where $\Gamma_{\tilde{c}}(\tilde{c};\beta) \equiv \partial\Gamma(\tilde{c};\beta)/\partial\tilde{c} = \frac{1}{\eta} + \lambda(\mu+\rho)\frac{\eta}{\tilde{c}^2} > 0$, $\Gamma_{\tilde{c}\tilde{c}}(\tilde{c};\beta) \equiv \partial\Gamma_{\tilde{c}}(\tilde{c};\beta)/\partial\tilde{c} = -\lambda(\mu+\rho)\frac{\eta}{\tilde{c}^3} < 0$, $\lim_{\tilde{c}\to 0}\Gamma(\tilde{c};\beta) = -\infty$, $\lim_{\tilde{c}\to\infty}\Gamma(\tilde{c};\beta) = \infty$ and $\Gamma_\beta(\tilde{c};\beta) \equiv \partial\Gamma(\tilde{c};\beta)/\partial\beta = -\frac{1-\alpha}{\eta}\tilde{A}'(\beta) > 0$. Figure 1 describes the dynamic properties of $\tilde{c}(t)$ in (24) showing that there exists a unique steady state

$\tilde{c}^*(\beta)$ that is unstable (i.e., $\tilde{c}^*(\beta) \equiv \{\tilde{c} \mid \Gamma(\tilde{c};\beta) = 0\}$). This finding implies that the forward looking variable $\tilde{c}(t)$ must jump to $\tilde{c}^*(\beta)$ at the initial date. Otherwise, the monotonic dynamics of $\tilde{c}(t)$ would lead to either 0 or ∞, which contradicts the equilibrium conditions.[12] From (24), we can derive $\tilde{c}^*(\beta)$ explicitly as

$$\tilde{c}^*(\beta) = \frac{\eta}{2}\left\langle \rho + \frac{(1-\alpha)\tilde{A}(\beta)}{\eta} - b + \left\{\left[\rho + \frac{(1-\alpha)\tilde{A}(\beta)}{\eta} - b\right]^2 + 4\lambda(\mu+\rho)\right\}^{\frac{1}{2}}\right\rangle, \tag{25}$$

where $\tilde{c}^*_\beta(\beta) \equiv \partial \tilde{c}^*(\beta)/\partial\beta$ and

$$\frac{\tilde{c}^*_\beta(\beta)\beta}{\tilde{c}^*(\beta)} = -\frac{\alpha}{\eta}\frac{\tilde{A}(\beta)}{\left\{\left[\rho + \frac{(1-\alpha)\tilde{A}(\beta)}{\eta} - b\right]^2 + 4\lambda(\mu+\rho)\right\}^{\frac{1}{2}}} < 0. \tag{26}$$

Since $\tilde{c}^*_\beta(\beta) < 0$, we can see that the broader patent breadth negatively affects the equilibrium consumption-number of intermediate goods ratio.

In the steady state equilibrium, since the per capita output $y(t)$ is given by $y(t) \equiv \frac{Y(t)}{L(t)} = \tilde{A}(\beta)N(t)$ from (16), $c(t)$, $N(t)$ and $y(t)$ grows at the same balanced-growth rate γ. From (23), the balanced-growth rate γ is determined by the following equation:

$$\gamma = r(\beta) - \rho - \lambda(\mu+\rho)\frac{\eta}{\tilde{c}^*(\beta)} \equiv \gamma(\beta). \tag{27}$$

3.2 Effects of patent breadth on economic growth

In this subsection, we examine the effects of patent breadth on the balanced-growth rate γ. By differentiating (27) with respect to β, we obtain

$$\frac{\partial\gamma(\beta)}{\partial\beta} = \frac{\alpha}{\eta}\Omega'(\beta) + \lambda(\mu+\rho)\eta\left(\frac{1}{\tilde{c}^*(\beta)}\right)^2 \tilde{c}^*_\beta(\beta). \tag{28}$$

[12]First, $\tilde{c}(t) = 0$ leads to $c_j(t) = 0$ and violates the first order conditions. Second, $\tilde{c}(t) = \infty$ violates the resource constraints of final goods given by (21).

By substituting (26) and $\Omega'(\beta) = \frac{1}{\beta}\left[\frac{1-\alpha\beta}{(\beta-1)(1-\alpha)}\right]\Omega(\beta)$ into (28) and rearranging them, we obtain the following expression:

$$\beta\frac{\partial\gamma(\beta)}{\partial\beta} = \frac{\alpha}{\eta}\tilde{A}(\beta)\Phi(\beta), \tag{29}$$

where

$$\Phi(\beta) \equiv \frac{1-\alpha\beta}{(1-\alpha)\beta} - \lambda(\mu+\rho)\frac{\eta}{\tilde{c}^*(\beta)}\frac{1}{\left\{\left[\rho + \frac{(1-\alpha)\tilde{A}(\beta)}{\eta} - b\right]^2 + 4\lambda(\mu+\rho)\right\}^{\frac{1}{2}}}.$$

Note that $\Phi(\beta)$ satisfies the following properties: $\frac{\partial\Phi(\beta)}{\partial\beta} < 0$, $\lim_{\beta\to 1}\Phi(\beta) = \frac{\left[\rho+\frac{(1-\alpha)\tilde{A}(1)}{\eta}-b\right]\tilde{c}^*(1)+\lambda(\mu+\rho)\eta}{\left[\rho+\frac{(1-\alpha)\tilde{A}(1)}{\eta}-b\right]\tilde{c}^*(1)+2\lambda(\mu+\rho)\eta} > 0$, and $\lim_{\beta\to\frac{1}{\alpha}}\Phi(\beta) = -\lambda(\mu+\rho)\frac{\eta}{\tilde{c}^*(\frac{1}{\alpha})}\frac{1}{\left\{\left[\rho+\frac{(1-\alpha)\tilde{A}(\frac{1}{\alpha})}{\eta}-b\right]^2+4\lambda(\mu+\rho)\right\}^{\frac{1}{2}}} < 0$. Because $sign[\frac{\partial\gamma(\beta)}{\partial\beta}] = sign[\Phi(\beta)]$ from (29), the above properties of $\Phi(\beta)$ show that there exists a unique $\beta^{op} \in (1, \frac{1}{\alpha})$ such that $\frac{\partial\gamma(\beta^{op})}{\partial\beta} = 0$, $\frac{\partial\gamma(\beta)}{\partial\beta} > 0$ $\forall\beta \in [1, \beta^{op})$ and $\frac{\partial\gamma(\beta)}{\partial\beta} < 0$ $\forall\beta \in (\beta^{op}, 1/\alpha]$. Therefore, as depicted in Figure 2, suppose that the parameter conditions ensure that the relation $\gamma(\beta^{op}) > 0$ holds; we can demonstrate that there exists a unique $\beta^{op} \in (\beta_{min}, \beta_{max})$ such that $\gamma(\beta^{op}) > \gamma(\beta)$ $\forall\beta \in [\beta_{min}, \beta_{max}]$, $\frac{\partial\gamma(\beta)}{\partial\beta} > 0$ $\forall\beta \in [\beta_{min}, \beta^{op})$, and $\frac{\partial\gamma(\beta)}{\partial\beta} < 0$ $\forall\beta \in (\beta^{op}, \beta_{max}]$ where $\beta_{min} = \max\{1, \beta_l\}$, $\beta_{max} = \min\{\beta_u, 1/\alpha\}$, $\beta_l, \beta_u \equiv \{\beta \mid \gamma(\beta) = 0\}$ and $\beta_u > \beta_l$.[13] Figure 2 depicts the case where the relations $\beta_l > 1$ and $\beta_u > \frac{1}{\alpha}$ hold.[14] These results are summarized in the following proposition.

Proposition 1 *Suppose that the parameter conditions ensure that the relation $\gamma(\beta^{op}) > 0$ holds, and an inverted-U relationship exists between patent breadth and the balanced-growth rate in a continuous-time OLG model with a lab-equipment type R&D growth.*

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Note that the conditions $\gamma(\beta^{op}) > 0$ are necessary for our economy to have parameter regions to ensure a positive balanced-growth rate. From (27), patent breadth β affects the balanced-growth rate γ via the following two effects: the "interest rate effect" (the first term of the right side of

[13]From (27), the parameter conditions for $\gamma(\beta^{op}) > 0$ are given by $r(\beta^{op}) - \rho - \lambda(\mu+\rho)\frac{\eta}{\tilde{c}^*(\beta^{op})} > 0$.

[14]Of course, other cases (i.e., $\beta_l > 1$ and $\beta_u < \frac{1}{\alpha}$; $\beta_l < 1$ and $\beta_u > \frac{1}{\alpha}$; $\beta_l < 1$ and $\beta_u < \frac{1}{\alpha}$) might hold under certain parameter configurations.

(27)) and the “generation-turnover effect" (the third term of the right side of (27)). Let us first explain the “interest rate effect". From (14), the broader patent breadth increases the share of income that is attributed to monopolistic profits. Since the value of innovations (monopolistic firms) V is determined by the price of final goods through a zero profit condition in the R&D sector (i.e., equation (18)) and is independent of the patent breadth under the lab-equipment type R&D specification, the increase in monopoly profits caused by the broader patent breadth increases the equilibrium interest rate, as shown in (19), and thus increases the balanced-growth rate. Therefore, the “interest rate effect" positively affects the balanced-growth rate.

Let us next explain the “generation-turnover effect". From (10) and (27), the generation-turnover term under the lab-equipment type R&D specification is given by $\lambda(\mu+\rho)\frac{a(t)}{c(t)} = \lambda(\mu+\rho)\frac{\eta}{\tilde{c}^*(\beta)}$. Since $\tilde{c}^*_\beta(\beta) < 0$ from (26), the broader patent breadth increases the equilibrium per capita asset-consumption ratio $\frac{a(t)}{c(t)}$, and thus decreases the balanced-growth rate γ. Therefore, the “generation-turnover effect" negatively affects the balanced-growth rate. Intuitively, from (4), the rise in interest rate caused by the broader patent breadth motivates households to save more for their future consumptions (increases the consumption growth rate of each household), which positively affects the equilibrium per capita asset-consumption ratio. In addition, under the lab-equipment type R&D specification, the value of innovations (monopolistic firms) V is determined by (18) and is independent of the patent breadth. Combinations of these two factors increase the equilibrium per capita asset-consumption ratio, enlarges the growth-reducing effects of generation-turnover, and thus negatively affects the balanced growth rate.

As the patent breadth enlarges, the positive interest rate effect decreases due to the diminishing marginal effects of patent breadth on monopoly profits, whereas the negative generation turn-over effect increases due to the gradual increase in the equilibrium per capita asset-consumption ratio. Therefore, an inverted-U relationship exists between the patent breadth and the balanced-growth rate under the lab-equipment type R&D specification.

4 The knowledge-driven R&D specification

Before concluding this paper, we should note that our paper's inverted-U effect of patent breadth on economic growth depends upon our lab-equipment type R&D specification. To clarify this point, we consider an alternative R&D specification. In accordance with Romer (1990), we consider the knowledge-driven R&D specification, where R&D activities require labor inputs. However, to avoid lexicographic explanations, we relegate the detailed analysis of the knowledge-driven R&D case to the Appendix B. Appendix B shows that, under the knowledge-driven R&D specification, the broader patent breadth always increases economic growth through the growth-enhancing "interest rate effect". The growth-reducing "generation-turnover effect" disappears in the knowledge-driven R&D case. Intuitively, as in the lab-equipment R&D case, the higher interest rate caused by the broader patent breadth motivates households to save more for their future consumption, increases the equilibrium per capita asset-consumption ratio, and thus enlarges the growth-reducing effect of generation-turnover. However, under the knowledge-driven R&D specification, R&D activities require labor inputs, and the value of innovations is positively related to the equilibrium wage rate through a zero profit condition in the R&D sector. The lower wage rate caused by the broader patent breadth decreases the value of innovations, decreases the equilibrium per capita asset-consumption ratio, and thus completely cancels out the abovementioned growth-reducing effect of generation-turnover. Therefore, the growth-reducing "generation-turnover effect" disappears, and the broader patent breadth always increases economic growth through the growth-enhancing "interest rate effect". The analyses in Appendix B clarify that the combinations of heterogeneous households with finite lifetimes and the lab-equipment type R&D specification are relevant for deriving the inverted-U effect of patent protection on economic growth.

5 Conclusion

This paper analyzed how patent protection affects economic growth in a continuous-time OLG model with a lab-equipment type R&D-based growth. We showed that increasing patent breadth may generate an inverted-U effect of patents on economic growth, which is partly consistent with an empirically observed nonmonotone relationship between patent protection and economic growth. We also showed that the combinations of heterogeneous households with a finite lifetime and a lab-equipment type R&D specification are relevant for deriving the inverted-U effect of patent protection on economic growth.

Appendix A: Derivation of (10)

By differentiating (8) with respect to t, we obtain

$$\dot{c}(t) = \lambda c_t(t) - \lambda c(t) + \int_{-\infty}^{t} \dot{c}_j(t)\lambda e^{\lambda(j-t)} dj. \tag{30}$$

Substituting (4) into (30) and using (8) yield

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - \lambda \frac{c(t) - c_t(t)}{c(t)}. \tag{31}$$

Using (9) and $c_t(t) = (\mu + \rho)h(t)$ from (6), (31) can be rewritten as (10).

Appendix B: knowledge-driven R&D case

In this section, we examine the knowledge-driven R&D case. Contrary to the lab-equipment R&D case, we show that the broader patent breadth always increases the balanced-growth rate.

Minor changes

The structures of the households, the final goods sector, and the intermediate goods sector are the same as the lab-equipment R&D case. In accordance with Romer (1990), we consider the knowledge-driven R&D specification where R&D activities require labor inputs. To reflect this change, equation (16) is rewritten as

$$Y(t) = \tilde{A}(\beta) L_y(t) N(t). \tag{32}$$

In addition, from (12) and (32), we obtain

$$w(t) = (1 - \alpha)\frac{Y(t)}{L_y(t)} = (1 - \alpha)\tilde{A}(\beta)N(t). \tag{33}$$

R&D Sector

Competitive entrepreneurs employ R&D labor inputs for innovation. Devoting $\eta(t)$ units of labor inputs, R&D firms can invent one unit of intermediate goods. As in Laincz and Peretto (2006), we assume that the R&D cost $\eta(t)$ is given by $\frac{\eta L(t)}{N(t)}$. Given the value of the patent on variety $V(t)$, the zero profit condition yields

$$V(t) = w(t)\frac{\eta L(t)}{N(t)} = \eta(1-\alpha)\tilde{A}(\beta)L(t). \tag{34}$$

where the second equality follows from (33). The no-arbitrage condition is the same as the lab-equipment R&D case (i.e., equation (17)). Thus, noting that $\dot{V}(t)/V(t) = \dot{L}(t)/L(t) = b$ from (34), combining (14), (17), (32), and (34) yield

$$r(t) = \frac{1}{\eta}\frac{\beta-1}{\beta}\frac{\alpha}{1-\alpha}\frac{L_y(t)}{L(t)} + b. \tag{35}$$

Market clearing conditions

Labor is used for final goods production and R&D investments. Thus, the labor market clearing condition becomes

$$L(t) = L_y(t) + \frac{\eta L(t)}{N(t)}\dot{N}(t). \tag{36}$$

The final goods are used for consumption and intermediate goods production. The final goods market clearing condition is given as

$$Y(t) = c(t)L(t) + ax(t)N(t). \tag{37}$$

The asset market clearing condition is the same as the lab-equipment R&D case (i.e., equation (21)).

Equilibrium Dynamics

By substituting (15) and (32) into (37) and rearranging them, we obtain

$$\frac{L_y(t)}{L(t)} = \frac{\beta}{\beta-\alpha}\frac{\tilde{c}(t)}{\tilde{A}(\beta)}, \tag{38}$$

where $\tilde{c}(t) \equiv c(t)/N(t)$. Using (38), (35) is rewritten as

$$r(t) = \frac{1}{\eta}\frac{\alpha}{1-\alpha}\frac{\beta-1}{\beta-\alpha}\frac{\tilde{c}(t)}{\tilde{A}(\beta)} + b. \tag{39}$$

Thus, by substituting (38) into (36), we can express the dynamics of $N(t)$ as follows:

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\eta}\left[1 - \frac{\beta}{\beta-\alpha}\frac{\tilde{c}(t)}{\tilde{A}(\beta)}\right]. \tag{40}$$

In addition, combining (10), (21), (34) and (39) yields

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho - \lambda(\mu+\rho)\frac{\eta(1-\alpha)\tilde{A}(\beta)}{\tilde{c}(t)}. \tag{41}$$

Summarizing equations (40) and (41), we can obtain the following differential equation of $\tilde{c}(t)$:

$$\frac{\dot{\tilde{c}}(t)}{\tilde{c}(t)} = \frac{1}{\eta}\frac{1}{1-\alpha}\frac{\tilde{c}(t)}{\tilde{A}(\beta)} - \lambda(\mu+\rho)\frac{\eta(1-\alpha)\tilde{A}(\beta)}{\tilde{c}(t)} - \frac{1}{\eta} + b - \rho \equiv \Gamma^K(\tilde{c}(t);\beta), \tag{42}$$

where $\Gamma^K_{\tilde{c}}(\tilde{c};\beta) \equiv \partial\Gamma^K(\tilde{c};\beta)/\partial\tilde{c} = \frac{1}{\eta}\frac{1}{1-\alpha}\frac{1}{\tilde{A}(\beta)} + \lambda(\mu+\rho)\frac{\eta(1-\alpha)\tilde{A}(\beta)}{\tilde{c}^2} > 0$, $\Gamma^K_{\tilde{c}\tilde{c}}(\tilde{c};\beta) \equiv \partial\Gamma^K_{\tilde{c}}(\tilde{c};\beta)/\partial\tilde{c} = -2\lambda(\mu+\rho)\frac{\eta(1-\alpha)\tilde{A}(\beta)}{\tilde{c}^3} < 0$, $\lim_{\tilde{c}\to 0}\Gamma^K(\tilde{c};\beta) = -\infty$, $\lim_{\tilde{c}\to\infty}\Gamma^K(\tilde{c};\beta) = \infty$ and $\Gamma^K_{\beta}(\tilde{c};\beta) \equiv \partial\Gamma^K(\tilde{c};\beta)/\partial\beta = -\tilde{A}'(\beta)\left[\frac{\tilde{c}}{\eta(1-\alpha)}\left(\frac{1}{\tilde{A}(\beta)}\right)^2 + \frac{\lambda(\mu+\rho)\eta(1-\alpha)}{\tilde{c}}\right] > 0$. As in the lab-equipment R&D case, there exists a unique steady state $\tilde{c}^{K*}(\beta)$ that is unstable (i.e., $\tilde{c}^{K*}(\beta) \equiv \{\tilde{c} \mid \Gamma^K(\tilde{c};\beta) = 0\}$). This finding implies that the forward looking variable $\tilde{c}(t)$ must jump to $\tilde{c}^{K*}(\beta)$ at the initial date. From (42), we can derive $\tilde{c}^{K*}(\beta)$ explicitly as

$$\tilde{c}^{K*}(\beta) = \kappa\eta(1-\alpha)\tilde{A}(\beta), \tag{43}$$

where $\kappa \equiv \frac{\frac{1}{\eta}+\rho-b+\left\{\left(\frac{1}{\eta}+\rho-b\right)^2+4\lambda(\mu+\rho)\right\}^{\frac{1}{2}}}{2}$. Since $\tilde{c}_\beta^{K*}(\beta) = \kappa\eta(1-\alpha)\tilde{A}'(\beta) < 0$, we can see that the broader patent breadth negatively affects the equilibrium consumption-number of the intermediate goods ratio. In the steady state equilibrium, from (32), (38) and (43), the per capita output $y(t)$ is given by $y(t) = \frac{\beta}{\beta-\alpha}\tilde{c}^{K*}(\beta)N(t)$. Therefore, $c(t)$, $N(t)$ and $y(t)$ grow at the same balanced-growth rate γ^K. In addition, from (39) and (43), the equilibrium interest rate is given by

$$r(t) = \frac{\alpha(\beta-1)}{\beta-\alpha}\kappa + b \equiv r^K(\beta). \tag{44}$$

Therefore, by substituting (43) and (44) into (41), the balanced-growth rate γ^K is determined by the following equation:

$$\gamma^K = r^K(\beta) - \rho - \lambda(\mu+\rho)\frac{1}{\kappa} \equiv \gamma^K(\beta). \tag{45}$$

Effects of patent breadth on economic growth

In this subsection, we examine the effects of patent breadth on the balanced-growth rate γ^K. By differentiating (45) with respect to β, we obtain

$$\frac{\partial\gamma^K(\beta)}{\partial\beta} = \frac{\partial r^K(\beta)}{\partial\beta} = \frac{\alpha(1-\alpha)}{(\beta-\alpha)^2}\kappa > 0, \;\; \forall\beta \in \left(1, \frac{1}{\alpha}\right]. \tag{46}$$

These results indicate that the broader patent breadth always increases the balanced-growth rate. In addition, the conditions $\gamma^K(\frac{1}{\alpha}) > 0$ are necessary for our economy to have at least parameter regions, which ensure a positive balanced-growth rate.[15]

From (45), as in the lab-equipment R&D case, patent breadth β affects the balanced-growth rate γ^K via the following two effects: the "interest rate effect" (the first term of the right side of (45)) and the "generation-turnover effect" (the third term of the right side of (45)), while the parameter β is included only in the first term.

Let us first explain the "interest rate effect". From (14) and (35), as in the lab-equipment R&D

[15]From (45), the parameter conditions for $\gamma^K(\frac{1}{\alpha}) > 0$ are given by $r(\frac{1}{\alpha}) - \rho - \lambda(\mu+\rho)\frac{1}{\kappa} > 0$.

case, the broader patent breadth increases the share of income that is attributed to monopolistic profits, which directly increases the equilibrium interest rate (i.e., “direct effect"). In addition, from (33), the broader patent breadth negatively affects the equilibrium wage rate. Under the knowledge-driven R&D specification, R&D activities require labor inputs, and the value of innovations (monopolistic firms) V is positively related to the equilibrium wage rate through a zero profit condition in the R&D sector (i.e., equation (34)). Thus, the lower wage rate caused by the broader patent breadth leads to the lower equilibrium value of innovations (monopolistic firms). Although, the lower value of innovation (monopolistic firms) exerts both positive and negative effects on the equilibrium interest rate through general equilibrium effects, it eventually leads to a higher equilibrium interest rate (i.e., “asset price effect"). This finding implies that the “asset price effect" also increases the equilibrium interest rate. Due to these “direct effect" and “asset price effect", the broader patent breadth increases the equilibrium interest rate, as shown in (44), and thus increases the balanced-growth rate. Therefore, the “interest rate effect" positively affects the balanced-growth rate.

Let us next explain the “generation-turnover effect". From (10) and (45), the generation-turnover term under the knowledge-driven R&D specification is given by $\lambda(\mu+\rho)\frac{a(t)}{c(t)} = \lambda(\mu+\rho)\frac{1}{\kappa}$. This finding implies that the patent breadth has no effect on the equilibrium asset-consumption ratio $\frac{a(t)}{c(t)}$. Intuitively, as in the lab-equipment R&D case, the higher interest rate caused by the broader patent breadth motivates households to save more for their future consumptions, increases the equilibrium per capita asset-consumption ratio, and thus enlarges the growth-reducing effect of generation-turnover. However, under the knowledge-driven R&D specification, the value of innovations (monopolistic firms) V is determined by (34) and is positively related to the equilibrium wage rate. The lower wage rate caused by the broader patent breadth decreases the value of innovations (monopolistic firms), decreases the equilibrium per capita asset-consumption ratio, and thereby completely cancels out the abovementioned growth-reducing effect of generation-turnover. Therefore, the growth-reducing “generation-turnover effect" disappears in the knowledge-driven R&D case, and the broader patent breadth always increases economic growth through the growth-enhancing “interest rate effect". The above analyses clarify that the combinations of heterogeneous households with a finite lifetime and the lab-equipment type R&D specification are relevant for deriving the inverted-U effect of patent protection on economic growth.

References

[1] Aghion, P., Bloom, N., Blundell, R., Griffith, R., Howitt, P. (2005) “Competition and innovation: an inverted-U relationship", *Quarterly Journal of Economics*, Vol. 120(2), pp. 701–728.

[2] Bertola, G. (1993) “Factor shares and savings in endogenous growth", *American Economic Review*, Vol. 83(5), pp. 1184–1198.

[3] Bertola, G. (1996) “Factor shares in OLG models of growth", *European Economic Review*, Vol. 40(8), pp. 1541–1560.

[4] Bertola, G., Foellmi, R., Zweimüller, J. (2006) *Income distribution in macroeconomic models*: Princeton University Press.

[5] Blanchard, O. J. (1985) “Debt, deficits, and finite horizons”, *Journal of Political Economy*, Vol. 93(2), pp. 223–247.

[6] Buiter, W. (1988) “Death, birth, productivity growth and debt neutrality", *Economic Journal*, Vol. 98, issue 391, pp. 279–293.

[7] Chou, C., Shy, O. (1993) “The crowding-out effects of long duration of patents", *RAND Journal of Economics*, Vol. 24(2), pp.304–312.

[8] Chu, A.C., Cozzi, G., Galli, S. (2012a) “Does intellectual monopoly stimulate or stifle innovation?", *European Economic Review*, Vol.5 (4), pp.727–746.

[9] Chu, A.C., Shiyuan, P., Minjuan, S. (2012b) “When does elastic labor supply cause an inverted-U effect of patents on innovation?", *Economics Letters*, Vol. 117(1), pp.211–213.

[10] Diwakar, B., Sorek, G., Stern, M. (2017) “Patents and growth in OLG Economy with physical capital," *Auburn Economics Working Paper Series* auwp2017-06, Department of Economics, Auburn University

[11] Falvey, R., Foster. N. and Greenaway, D. (2006), “Intellectual property rights and economic growth", *Review of Development Economics*, Vol.10(4), pp.700–719.

[12] Furukawa, Y. (2007) “The protection of intellectual property rights and endogenous growth: Is stronger always better?", *Journal of Economic Dynamics and Control*, Vol. 31(11), pp. 3644-3670.

[13] Gilbert, R., Shapiro, C. (1990) “Optimal patent length and breadth", *RAND Journal of Economics*, Vol.21(1), pp.106-112.

[14] Goh, A.-T., Olivier, J. (2002) “Optimal patent protection in a two-sector economy”, *International Economic Review*, Vol. 43(4), pp. 1191–1214.

[15] Gould, D. M., Gruben, W.C. (1996) “The role of intellectual property rights in economic growth", *Journal of Development Economics*, Vol. 48(2), pp.323-350.

[16] Grossman, G. M., Helpman, E. (1991) *Innovation and growth in the global economy*. Cambridge, MA: The MIT Press.

[17] Horii, R., Iwaisako, T. (2007) “Economic growth with imperfect protection of intellectual property rights", *Journal of Economics*,Vol. 90(1), pp.45–85.

[18] Iwaisako,T., Futagami, K. (2013) “Patent protection, capital accumulation, and economic growth", *Economic Theory*, Vol. 52(2), pp. 631–668.

[19] Judd, K. L. (1985) “On the Performance of Patents", *Econometrica*, Vol. 53(3), pp. 567–585.

[20] Laincz, C., Peretto, P. (2006) “Scale effects in endogenous growth theory: an error of aggregation not specification", *Journal of Economic Growth*, Vol.11(3), pp. 263–288,

[21] Lerner, J. (2009) “The empirical impact of intellectual property rights on innovation: puzzles and clues", *American Economic Review*, Vol.99(2), pp. 343–48.

[22] O'Donoghue, T., Zweimüller, J. (2004) "Patents in a model of endogenous growth", *Journal of Economic Growth*, Vol.9(1), pp. 81–123.

[23] Olivier, J. (2000) "Growth-enhancing bubbles", *International Economic Review*, Vol. 41(1), pp. 133–151.

[24] Park, W. (2008) "International patent protection: 1960-2005", *Research Policy*, Vol. 37(4), pp. 761–766.

[25] Qian, Yi. (2007) "Do national patent laws stimulate domestic innovation in a global patenting environment? a cross-country analysis of pharmaceutical patent protection", 1978-2002, *Review of Economics and Statistics*, Vol. 89(3), pp. 436–453.

[26] Rivera-Batiz, L., Romer, P. (1991) "Economic integration and endogenous growth", *Quarterly Journal of Economics*, Vol. 106(2), pp. 531–555.

[27] Romer, P. (1990) "Endogenous technological progress", *Journal of Political Economy*, Vol. 98(5), pp. S71–S102.

[28] Sorek, G. (2011) "Patents and quality growth in OLG economy", *Journal of Macroeconomics*,Vol. 33(4), pp. 690–699.

[29] Yaari, M. E. (1965) "Uncertain lifetime, life insurance, and the theory of the consumer", *Review of Economic Studies*, Vol. 32(2), pp. 137–150.

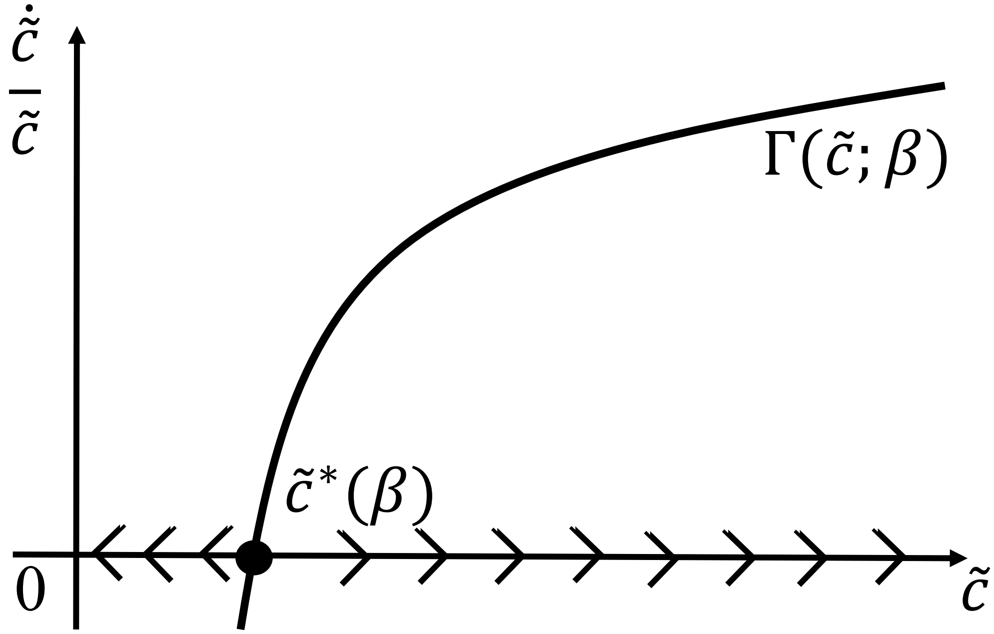

Figure 1: Phase diagram

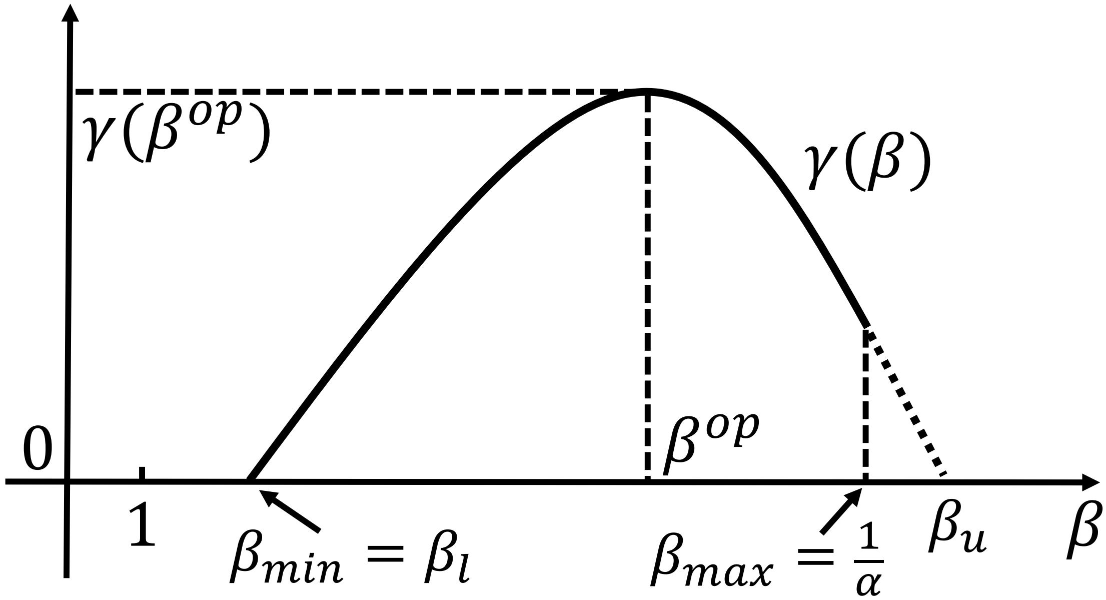

Figure 2: Determination of β^{op} when $1 < \beta_l$ and $\frac{1}{\alpha} < \beta_u$