Endogenous Fertility and Pension System*

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This paper presents consideration of two public pension systems having a Defined Contribution (DC) or a Defined Benefit (DB) structure and presents an examination of how the pension policy affects the fertility and income growth in an endogenous fertility model. DC benefits for older people change according to a budget under a constant contribution rate by younger people. However, DB entails a contribution rate that changes based on the maintenance of a balanced budget, providing constant benefits for older people. In both a small economy and a closed economy, the dynamics of the fertility is not brought about by DC, but by DB. However, the level of pension benefit has the same effect on fertility and income growth at both DC and DB at the steady state, not a transitional path.

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1 Introduction

This paper presents derivation of how fertility and income growth are determined under different pension systems. This paper presents consideration of pension systems of two types: Defined Contribution (DC) and

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Defined Benefit (DB). The pension system affects fertility through the household disposable income. The pension benefit increases the income during the old period. However, the contribution rate for pensions decreases the income received during the young period.

Actually, DC benefits for older people change according to a budget under a constant rate of contributions by younger people. Some related studies have examined endogenous fertility models in DC pensions: van Groezen, Leers, and Meijdam (2003), van Groezen and Meijdam (2008), Yasuoka and Goto (2011), and Yasuoka and Goto (2015). In addition to effects of DC pensions on fertility, Wigger (1999) shows how the contribution rate affects income growth.

By contrast, some researchers have examined fixed benefit pensions such as DB: Oshio and Yasuoka (2009) and Lin and Tian (2003). In a DB pension, the contribution rate continues increasing in a society with fewer children because the pension benefit must be a constant level in the DB pension. In a society with fewer children, the DB pension can not be maintained without a child allowance. In the DB pension setting, the effect of pension reform that is financed by a consumption tax on the income growth is examined.[1)]

The difference between DC and DB is the component that is changed to hold the balanced budget of a pension: the contribution rate or the replacement ratio. Whereas DB changes the contribution rate, DC changes the replacement ratio. If one considers public debt, then no balanced budget of a pension need be set. This model is presented by Ono (2003).

1) We notify how the children are considered in the endogenous fertility model: consumption goods and investment goods. In Nishimura and Zhang (1992), Zhang and Zhang (1998), Lin and Tian (2003) and others, the endogenous fertility model incorporates investment goods. By contrast, van Groezen, Leers, and Meijdam (2003), Fanti and Gori (2009), Oshio and Yasuoka (2009) and others set the model with consumption goods.

Considering the pension system in terms of uncertainty is important. Thøgerson (1998) sets a model with wage uncertainty and demonstrates that a public pension system is better than the private pension system. Borgmann (2005) sets a model with income and population growth uncertainty and demonstrates which pension system (DC or DB) is the more desirable in terms of social welfare.

The analyses put forward in this paper can derive the following results. In a DC pension system, no dynamics of fertility occurs. In contrast, in a DB pension system, dynamics of fertility occur. Therefore, if fertility is low, it continues to decrease. The pension benefit eventually becomes unsustainable in DB. This result is derived in a small open economy. However, even if one considers a closed economy, the result does not change. In the steady state, no difference exists between DC and DB in examining the effects of the pension on endogenous fertility and income growth.

This paper gives important policy implications. In Japan, the public pension was reformed at 2004. Before the reform, the pension was fundamentally DB: the pension benefit is fixed. However, because children have become fewer, the pension system has changed to DC with the contribution rate fixed. The analyses presented in this paper demonstrate that even if children decrease in the future, the pension system can be maintained.

The remainder of this paper is the following. Section 2 of this paper establishes the model. Section 3 derives the equilibrium in the small open economy. Section 4 derives the equilibrium in the closed economy. The final section presents results.

2 The Model

This model economy consists of a two-period (young and old) overlapping generations model. This model has three agents: households, firms, and a government.

2.1 Households

Each household lives in two periods, young and old, and supplies labor during the young period to gain income. This model economy assumes that some child-care service is necessary to rear children. The lifetime budget constraint is given as

$$z_t n_t + c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = (1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}. \tag{1}$$

Therein, n_t represents the number of children; z_t denotes the price for caring for a child as child care service.[2] In addition, c_{1t} and c_{2t+1} respectively signify consumption in young and old periods. Here, w_t shows the wage rate. Interest rate $1+r_{t+1}$ represents the return to savings. Younger people face contribution rate τ_t for the pension system. Older people receive pension benefit p_{t+1}.

A household's utility function is assumed as

$$u_t = \alpha \ln n_t + \beta \ln c_{1t} + (1-\alpha-\beta)\ln c_{2t+1}, \quad 0<\alpha<1,\ 0<\beta<1,\ \alpha+\beta<1. \tag{2}$$

This function form is generally used in the endogenous fertility model.[3]

Under budget constraint (1) in the model of child care service, households decide the allocations of c_{1t}, c_{2t+1}, and n_t to maximize their utility as

$$c_{1t} = \beta\left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{3}$$

$$c_{2t+1} = (1-\alpha-\beta)(1+r_{t+1})\left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}\right), \tag{4}$$

$$n_t = \frac{\alpha\left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}\right)}{z_t}. \tag{5}$$

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2) This paper assumes the child care provided as the goods and service. On the other hand, some papers assume the child care time to have the children as Galor and Weil (1996), de la Croix and Doepke (2003) and others.

3) This utility form is used by van Groezen, Leers, and Meijdam (2003) and others. This is the conventional form in an endogenous fertility model with consumption goods.

2.2 Firms

A representative firm produces final good Y_t with constant returns to scale or a neoclassical product function, as shown by

$$Y_t = F(K_t, A_t L_t),\ F_K > 0,\ F_L > 0,\ F_{KK} < 0,\ F_{LL} < 0. \tag{6}$$

The firm inputs capital stock K_t and labor L_t. Also, A_t denotes labor productivity; A_t is assumed as set to unity. With a perfectly competitive market, wage rate w_t and interest rate r_t are shown as

$$w_t = f(k_t) - f'(k_t)k_t, \tag{7}$$

$$1 + r_t = f'(k_t). \tag{8}$$

In those equations, $k_t \equiv \frac{K_t}{L_t}$ and $f(k_t) \equiv \frac{Y_t}{L_t}$. The capital stock depreciates fully in one period.

This model includes a child care service sector. Based on Yasuoka and Miyake (2010), we assume $Y_t^c = \rho L_t^c$ as the child care service production function ($0 < \rho$). Here, Y_t^c and L_t^c respectively denote the outputs of child care service and the labor input for child care service sector. Assuming free labor mobility between the final goods sector and child care service sector, the profit function π_t is

$$\pi_t = z_t \rho L_t^c - w_t L_t^c. \tag{9}$$

Then, profit maximization derives the price of child care service as

$$z_t = \frac{w_t}{\rho}. \tag{10}$$

2.3 Government

The government supplies the policy of pay-as-you-go pensions. We consider pension systems of two types: DC and DB.

Defined Contribution: DC This pension system fixes the contribution rate for younger people ($\tau_t = \bar{\tau}$) and determines the benefit level for older people, which depends on the intergenerational population ratio, to hold a balanced budget. Considering the balanced budget, the budget constraint can be expressed as

$$N_t p_{t+1} = \bar{\tau} N_{t+1} w_{t+1} \ \leftrightarrow \ p_{t+1} = \bar{\tau} n_t w_{t+1}. \tag{11}$$

In that equation, N_t and N_{t+1} respectively denote the population size of older people in $t+1$ period (younger people in t period) and that of the younger people in the $t+1$ period. The intergenerational population ratio is given as $n_t = \frac{N_{t+1}}{N_t}$. Large n_t, which represents the intergenerational population ratio, increases the benefit for older people.

Defined Benefit: DB This pension system fixes the benefit level for older people ($p_{t+1} = \bar{p} w_{t+1}$) and determines the contribution rate for younger people. Considering a balanced budget, the budget constraint is

$$N_t \bar{p} w_{t+1} = \tau_{t+1} N_{t+1} w_{t+1} \ \leftrightarrow \ \tau_{t+1} = \frac{\bar{p}}{n_t}. \tag{12}$$

With a large population of younger people, the contribution rate is low.

3 Equilibrium in Small Open Economy

Next we present derivation of the equilibrium for a small open economy. The interest rate is given by an exogenous interest rate r; the wage rate w is also decided exogenously. We explain the equilibrium of DC and BC.

3.1 DC Case

Considering (5), (10), and (11), fertility n_t can be obtained as

$$n_t = \frac{\alpha(1-\bar{\tau})}{\frac{1}{\rho} - \frac{\alpha\bar{\tau}}{1+r}}. \tag{13}$$

Here, $\frac{1}{\rho} > \frac{\alpha\bar{\tau}}{1+r}$ should be held to be positive n_t. There is no dynamics of n_t.

3.2 DB Case

Considering (5), (10), and (12), we can obtain fertility n_t as

$$n_t = \alpha\rho\left(1 + \frac{\bar{p}}{1+r} - \frac{\bar{p}}{n_{t-1}}\right). \tag{14}$$

For given n_{t-1}, fertility in t period n_t is derived. The fertility dynamics of (14) can be portrayed as the figure below.

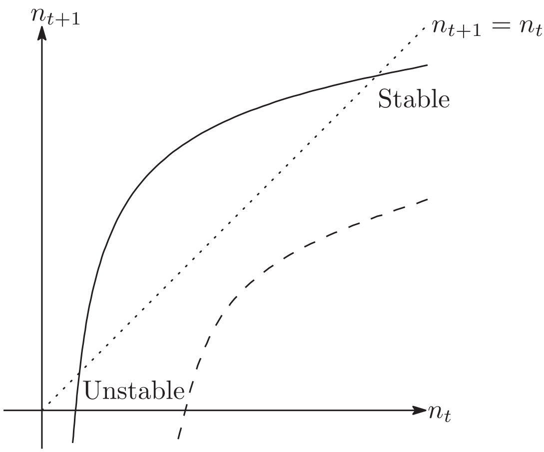

Fig. 1: Dynamics of n_t.

The solid line has two steady state equilibria: one for the stable steady state equilibrium and the other for the unstable one. The dashed line has no steady state. With $\alpha\rho\left(1+\frac{\bar{p}}{1+r}\right)^2 - 4\bar{p} \geq 0$, one can obtain the steady state equilibrium.[4)]

4) Assuming steady state $n = n_t = n_{t+1}$, one can obtain $n^2 - \alpha\rho\left(1+\frac{\bar{p}}{1+r}\right)n + \alpha\rho\bar{p} = 0$. Then, $n = \frac{\alpha\rho\left(1+\frac{\bar{p}}{1+r}\right) \pm \sqrt{\alpha^2\rho^2\left(1+\frac{\bar{p}}{1+r}\right)^2 - 4\alpha\rho\bar{p}}}{2}$ is obtainable if $\alpha^2\rho^2\left(1+\frac{\bar{p}}{1+r}\right)^2 - 4\alpha\rho\bar{p} > 0$.

4 Equilibrium in Closed Economy

This section presents derivation of the fertility dynamics as the equilibrium in the closed economy. In a closed economy, capital accumulation is considered. Capital accumulation is found by the capital market equilibrium. The capital market equilibrium is shown as

$$n_t k_{t+1} = (1-\tau_t)w_t - c_{1t} - z_t n_t. \tag{15}$$

For this section, we assume $A_t = a\frac{K_t}{L_t}$ as the externality of capital accumulation, similarly to Romer (1986), Grossman and Yanagawa (1993) and others. Then, the wage rate (7) and the interest rate (8) are given as shown below:

$$w_t = (af(a) - f'(a))k_t, \tag{16}$$

$$1 + r_t = f'(a). \tag{17}$$

In this section, $f(a) = \frac{Y_t}{A_t L_t}$ and $k_t = \frac{K_t}{L_t}$ are defined.

4.1 DC Case

With (5), (10), (11), (15), (16), and (17), we can obtain the following capital accumulation equation and the fertility in DC as

$$\frac{k_{t+1}}{k_t} = \left(\frac{1-\bar{\tau}}{n_t} - \frac{\alpha+\beta}{\alpha\rho}\right)(af(a) - f'(a)), \tag{18}$$

$$n_t = \frac{\alpha\rho(1-\bar{\tau})}{1 - \frac{\alpha\rho\bar{\tau}\frac{k_{t+1}}{k_t}}{f'(a)}}. \tag{19}$$

Defining $1+g = \frac{k_{t+1}}{k_t}$, the following equations can be obtained.

$$1+g = \frac{\frac{(1-\alpha-\beta)(af(a)-f'(a))}{\alpha\rho}}{1+\frac{\bar{\tau}(af(a)-f'(a))}{f'(a)}}, \tag{20}$$

$$n = \frac{\alpha\rho(1-\bar{\tau})}{1-\frac{\bar{\tau}(1-\alpha-\beta)(af(a)-f'(a))}{f'(a)+\bar{\tau}(af(a)-f'(a))}}. \tag{21}$$

An increase in contribution rate $\bar{\tau}$ reduces the income growth rate because the pension reduces the incentive to save for the old period. However, the fertility can be pulled up by the pension. This effect is shown by $\frac{\bar{\tau}(1-\alpha-\beta)(af(a)-f'(a))}{f'(a)+\bar{\tau}(af(a)-f'(a))}$ in the denominator.

4.2 DB Case

With (5), (10), (12), (15), (16), and (17), we can obtain the following income growth and the fertility in the DB case,

$$1+g_t=\left(\frac{1-\frac{\bar{p}}{n_{t-1}}}{n_t}-\frac{\alpha+\beta}{\alpha\rho}\right)(af(a)-f'(a)), \tag{22}$$

$$n_t=\alpha\rho\left(1-\frac{\bar{p}}{n_{t-1}}+\frac{\bar{p}(af(a)-f'(a))}{f'(a)}\left(\frac{1-\frac{\bar{p}}{n_{t-1}}}{n_t}-\frac{\alpha+\beta}{\alpha\rho}\right)\right). \tag{23}$$

With (23), n_t depends on n_{t-1} as

$$n_t=\frac{\left(1-\frac{\bar{p}}{n_{t-1}}\right)\alpha\rho-\frac{\bar{p}(af(a)-f'(a))(\alpha+\beta)}{f'(a)}+\sqrt{D}}{2}, \tag{24}$$

where $D=\left(\left(1-\frac{\bar{p}}{n_{t-1}}\right)\alpha\rho-\frac{\bar{p}(af(a)-f'(a))(\alpha+\beta)}{f'(a)}\right)^2+\frac{4\alpha\rho\bar{p}(af(a)-f'(a))}{f'(a)}\left(1-\frac{\bar{p}}{n_{t-1}}\right)$.

The dynamics of n_t is the same with Fig. 1. Therefore, we can obtain a stable and unstable steady state.

If fertility continues decreasing, the income growth rate decreases too because of the high contribution rate. Similarly to the DC case, an increase in the pension level decreases the income growth rate. For an increased pension level, the effect on fertility is ambiguous.

Then, the following proposition can be established.

Proposition If the pension is provided as DB, then fertility and income growth show dynamics. Such is not true in a DC case. With low initial fertility, both income growth and the fertility continue to decrease.

This proposition presents an important policy implication. In Japan, the population size of the working generation continues to decrease. With a DB pension, both the income growth rate and the fertility rate are expected to be reduced in the future. Therefore, a change from DB to DC is necessary. By virtue of this reform, the income growth rate and fertility can be maintained at a constant level.

5 Conclusions

This paper sets the endogenous fertility model with a pay-as-you-go pension and examines how the pension system affects fertility. If the household can use child care services, then the DC pension gives no dynamics of fertility. This result is obtainable in the model not only of a small open economy but also of a closed economy.

By contrast, the DB pension brings about dynamics of fertility. With low fertility, the DB pension negatively influences fertility over time. Income growth continues decreasing, too. This paper demonstrates that a pension reform that fixes the contribution rate of a pension should be considered: The burden for younger people is not expected to be increased. However, older people are adversely influenced by the pension reform.

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