# Losses from competition policy in a general oligopolistic equilibrium<sup>\*</sup>

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We develop a general oligopolistic equilibrium model where free entry industries and restricted entry industries coexist, and examine the welfare effect of a competition policy, i.e. increasing the number of firms in the latter industries. We show an intriguing possibility that this policy can reduce welfare by raising the wage and promoting firm exit in the free entry industries.

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## 1 Introduction

It is one of the central questions in industrial organization whether a competition policy increases welfare.<sup>1)</sup> When one supposes the simplest situation in which identical firms compete in a single oligopolistic industry, the above policy raises welfare by eliminating inefficiency associated with market power. Is the same still valid even in more realistic situations? While there is a large literature that addresses this question, Lahiri and Ono (1988) demonstrate a striking result that if two sets of firms that

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We define a competition policy simply as increasing the number of firms. Chapters 14 and 16 in Belleflamme and Peitz (2015) provide a comprehensive account of competition policies.

differ in production cost are allowed, an increase in the number of inefficient firms can reduce welfare. More recently, Shimomura and Thisse (2012) construct a model in which oligopolistic and monopolistic competition coexists, and prove that increasing the number of oligopolistic firms unambiguously enhance welfare although it decreases the number of monopolistically competitive firms.

This paper also belongs to this strand of literature, but we reconsider the welfare effect of competition policy by applying a general oligopolistic equilibrium (GOLE) model first developed by Neary (2003, 2016).<sup>2)</sup> We assume a continuum of oligopolistic industries in which entry is impossible (the number of firms is exogenously given) in one set of industries and entry is free (the number of firms is endogenously determined) in the other set of industries. Then, we show that an increase in the number of firms in the former set of industries can worsen welfare.<sup>3)</sup>

As noted earlier, Neary (2003, 2016) is the first to develop the GOLE model. In particular, Neary (2003) assumes that all oligopolistic industries have the same number of firms, and shows that increasing it raises welfare increases unless all industries are identical. Using a GOLE model in which oligopolistic and perfectly competitive industries coexist, Crettez and Fagart (2009) find that the same is no longer valid because the oligopolistic industries may over-produce and the market distortion associated with this over-production is accelerated by the competition policy. In addition, Kamei (2014) shows that the competition policy reduces welfare

For the detailed account of the GOLE model, see Neary (2003, 2016) and Colacicco (2015).

<sup>3)</sup> Supposing a single industry that consists of oligopolistic and monopolistically competitive firms, Shimomura and Thisse (2012) find that welfare improves as the oligopolistic firms increase. The main difference between their and our models is whether the competition policy changes the factor price (wage). In Shimomura and Thisse (2012), the wage is always unity from the profit maximization condition of the perfectly competitive numeraire sector.

by allowing for economies of scale in the original Neary (2003) model. This is because the competition policy prevents firms from enjoying the gains from economies of scale by decreasing output per-firm. We also show that the competition policy can be welfare-reducing, but the underlying reason is quite different. In our model, the competition policy raises the wage and leads to crowding out of the firms in the free entry industries, thereby being welfare-reducing.

This paper is organized as follows. Section 2 presents a model, and Section 3 proves and discusses the main results. Section 4 concludes.

#### 2 Model

This section presents the model. There is a continuum of oligopolistic industries in a closed interval [0, 1] in which all goods are produced from labor only. We begin by considering the consumer behavior, and then turn to the firm behavior. The utility maximization problem of a representative consumer is given by

$$\max_{\{c_i\}} \quad \int_0^1 \ln c_i di \quad \text{subject to} \quad \int_0^1 p_i c_i di = I, \tag{1}$$

where  $c_i$  and  $p_i$  is a consumption and price of good *i*, respectively, and *I* is national income. Then, the first-order condition for utility maximization is  $1/c_i = \lambda p_i$ , where  $\lambda$  is a Lagrangean multiplier which represents marginal utility of income.<sup>4)</sup> A crucial assumption commonly made in the GOLE model is that all firms take  $\lambda$  as given in maximizing their profit. That is, oligopolistic firms are 'large' in their product market, but 'small' in the whole economy including the factor (labor) market. Given this assumption, we set  $\lambda = 1$  in what follows.

The continuum of industries consists of a set of industries  $i \in \left[0, \widetilde{i}\right]$  in

Note that λ is an endogenous variable that is determined in the whole general equilibrium system.

which entry is restricted (the number of firms is exogenously given) and a set of industries  $j \in [\tilde{i}, 1]$  in which entry is free so that the number of firms is endogenously given by the zero-profit condition. Denoting by m > 1 and  $n_j > 1$  the number of firms in the former and latter industries, the inverse demand function is given by  $p_i = 1/(\sum_{k=1}^m x_{ik})$  and  $p_j =$  $1/(\sum_{l=1}^{n_j} x_{jl})$ , where  $x_{ik}$  and  $x_{jl}$  are the output of a representative firm in each set of industries.<sup>5)</sup> The labor input coefficient  $\alpha_i$  differs across industries, and fixed labor f > 0 must be employed in the free entry industries. Summarizing these assumptions, the profit of a representative firm in each set of industries is defined as follows.

$$\pi_i \equiv p_i x_{ik} - w \alpha_i x_{ik}, \quad \pi_j \equiv p_j x_{jl} - w \alpha_j x_{jl} - w f,$$

where  $\pi_i$  and  $\pi_j$  are the profit, and w is the wage. Assuming Cournot competition and a symmetry within each industry, the output per firm and price in industry i are<sup>6)</sup>

$$x_i = \frac{m-1}{m^2 w \alpha_i}, \quad p_i = \frac{m w \alpha_i}{m-1}.$$
 (2)

Similarly, the output per firm and price in industry j are

$$x_j = \frac{n_j - 1}{n_j^2 w \alpha_j}, \quad p_j = \frac{n_j w \alpha_j}{n_j - 1}.$$
(3)

Substituting (3) into the zero-profit condition  $\pi_j = 0$  and solving for  $n_j$ , the number of firms in the free entry industries is computed as

$$n_j = \frac{1}{\sqrt{wf}},\tag{4}$$

which turns out to be common for all  $j \in \left[\tilde{i}, 1\right]$ .

Thus far, we have assumed that the wage is given to all firms. We now endogenize it, and describe the full general equilibrium by introducing the

<sup>5)</sup> We assume that the number of firms *m* is common for all the restricted entry industries since the analysis becomes too complicated if it is different.

<sup>6)</sup> Subscripts k and l are dropped hereafter because all firms within each industry are identical.

labor market-clearing condition:

$$L = \int_0^{\widetilde{i}} m\alpha_i x_i di + \int_{\widetilde{i}}^1 n \left(\alpha_j x_j + f\right) dj = \frac{m - \widetilde{i}}{mw},$$

where L > 0 is the labor endowment, and the last equality uses Eqs. (2), (3) and (4). Solving this equation for w, the equilibrium wage is explicitly obtained as

$$w = \frac{m - i}{mL}.$$
(5)

Substituting (5) into (2), (3) and (4), the outputs, prices and the number of firms in the free entry industries are derived, and the model is closed. Eq. (5) immediately leads to:

**Lemma.** A competition policy in the form of increasing m raises the equilibrium wage.

In order to understand the intuition behind this result, let us note that an increase in m has the following two impacts. An increase in m has a positive effect on aggregate labor demand, and hence the wage will increase. On the other hand, an increase in m leads some firms in the free entry sectors to exit, which serves to decrease aggregate labor demand and put downward pressure on the wage. As a result, the competition policy defined above has both a positive and negative impact on the equilibrium wage, but it ends up raising the wage because the former effect dominates the latter effect. This effect of the competition policy on the equilibrium wage will be useful in the subsequent arguments.

#### **3** Welfare effects

Based on the model in the last section, we address the welfare effect of the competition policy. Substituting the demand functions  $c_i = 1/p_i$  and  $c_j = 1/p_j$  into the direct utility function in (1), indirect utility or welfare

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W is expressed by

$$W = -\int_0^{\widetilde{i}} \ln p_i - \int_{\widetilde{i}}^1 \ln p_j dj.$$
(6)

In order to find how m affects  $p_i$  and  $p_j$  in (6), we substitute (2), (3) and (4) into the inverse demand functions  $p_i = 1/(mx_i)$  and  $p_j = 1/(n_jx_j)$ . Then, these prices become a function of m as follows.

$$p_i = \frac{\left(m - \widetilde{i}\right)\alpha_i}{(m-1)L}, \quad p_j = \frac{w\alpha_j}{1 - \sqrt{wf}},\tag{7}$$

where w in  $p_j$  is given by (5). Differentiation of these prices with respect to m leads to

$$\frac{dp_i}{dm} = -\frac{\left(1-\widetilde{i}\right)\alpha_i}{(m-1)^2L} < 0, \quad \frac{dp_j}{dm} = \frac{\partial p_j}{\partial w} \cdot \frac{dw}{dm} = \frac{\widetilde{i}\alpha_j\left(1-\frac{\sqrt{wf}}{2}\right)}{m^2L\left(1-\sqrt{wf}\right)^2} > 0,$$

that is, the competition policy lowers  $p_i$  and raises  $p_j$ . Relating this effect on prices to Eq. (6), it is generally ambiguous whether the competition policy enhances welfare.

To seek more, let us substitute (7) into (6), and differentiate the resulting expression with respect to m. Then, the welfare effect of increasing m is derived as follows.

$$\begin{split} \frac{dW}{dm} &= -\int_{0}^{\widetilde{i}} \frac{1}{p_{i}} \cdot \frac{dp_{i}}{dm} di - \int_{\widetilde{i}}^{1} \frac{1}{p_{j}} \cdot \frac{dp_{j}}{dm} dj \\ &= \frac{\widetilde{i}\left(1-\widetilde{i}\right)}{(m-1)\left(m-\widetilde{i}\right)} - \frac{\widetilde{i}\left(1-\widetilde{i}\right)\left(2-\sqrt{wf}\right)}{2\left(1-\sqrt{wf}\right)m\left(m-\widetilde{i}\right)} \\ &= \frac{\widetilde{i}\left(1-\widetilde{i}\right)\Delta}{2m(m-1)\left(m-\widetilde{i}\right)\left(1-\sqrt{wf}\right)}, \end{split}$$
  
where  $\Delta \equiv 2 - (m+1)\sqrt{wf} = 2 - \sqrt{\frac{(m+1)^{2}\left(m-\widetilde{i}\right)f}{mL}}$ 

It follows from this result that the welfare effect of the competition policy depends on the sign of  $\Delta$ . More specifically, we establish:

**Proposition.** A competition policy in the form of increasing m reduces welfare if

$$(m+1)^2 \left(m-\tilde{i}\right) f - 4mL > 0.$$
(8)

*Proof.* The competition policy is welfare-reducing (dW/dm < 0) if  $\Delta < 0$ , which is rewritten as

$$2 < \sqrt{\frac{(m+1)^2 \left(m - \tilde{i}\right) f}{mL}}.$$

Rearranging this inequality, we arrive at the proposition. ||

It is a little difficult to know why the competition policy is detrimental under condition (8). To see this, it is helpful to rewrite (8) as a condition on m. Noting that inequality (8) is rewritten as

$$fm^{3} + f\left(2 - \widetilde{i}\right)m^{2} + \left[f\left(1 - 2\widetilde{i}\right) - 4L\right]m - \widetilde{i}f > 0,$$

we can say that the competition policy is welfare-reducing if m is sufficiently large. This finding is intuitively explained as follows. When m is sufficiently large, the pro-competitive effect of increasing m is small since the oligopolistic equilibrium is sufficiently close to the competitive equilibrium. Therefore, the marginal pro-competitive effect of increasing m is quite small. In contrast, the decrease in  $n_j$  led by the competition policy has a negative welfare effect, and plays a primary role in the overall welfare effect. In other words, under condition (8), the negative welfare effect arising from crowding out in the free entry industries is overweighs the positive welfare effect from the pro-competitive effect. Consequently, the competition policy decreases welfare.

**Remark.** The above result rests on the assumption of logarithmic utility. If, instead, quadratic utility is used as in Neary (2003, 2016), nothing clear is obtained.

### 4 Conclusion

Applying Neary's (2003, 2016) GOLE model to the situation where restricted entry industries and free entry industries coexist, we have shown a possibility that a competition policy, i.e. an increase in the number of firms in the restricted entry sectors, raises the equilibrium wage and forces the firms to exit in the free entry sectors, thereby reducing welfare.

Despite that the above result may be useful in competition policy-making, it is admittedly based on many simplifying assumptions. First, we ignore product differentiation by straightforwardly following Neary (2003, 2016). If products are differentiated within each industry and/or across industries, our results are expected to be modified. Second, it is interesting to endogenize  $\tilde{i}$  that divides the two sets of industries. In this extended model, a competition policy affects welfare not only through the wage but also through this threshold of industries. It is our future research agenda to enrich the analysis by noting these limitations of this paper.

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