

Effects of Linguistic Compounding Rules and Rhythm on the Learning of the Multiplication Table in Japanese

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Abstract

The present paper argues that some languages are intrinsically more advantageous than others in arithmetic education, especially for children learning multiplication. This is because at an early stage of their education, they are expected to memorize the multiplication table as accurately as possible using language so that they can answer automatically what the product is when asked the factors, and because the way of processing numbers varies from language to language (Miura and Okamoto 2003). Thus, the method of memorizing the table, which is the key to success in becoming competent in basic numeracy, may vary depending on the language learners speak. In other words, linguistic differences can lead to differences in the amount of time required for them to memorize and recall the information in the table. Moreover, it can be assumed that such linguistic differences may also have an effect on the amount of time learners need for computation, and their degree of accuracy may also vary depending on language background (Miura, Okamoto, Kim, Steere, and Fayol 1993).

A way of chanting the times table without referring to meanings has been developed in Japan since it was brought in from China more than a thousand years ago (Ando 2002, Daily Yomiuri 2010). What makes this chanting easy is the variety of rhythmic patterns used, because they can help Japanese learners both memorize and recall the equals smoothly in combination with information on the phonemes in the sequence of each equation. To be more accurate, the application of multiple kinds of su-

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prasegmental units such as rhythm and accent can help strengthen the bond between sounds and meanings. In the Japanese multiplication method called *Kuku* (literally meaning ‘9 times 9’), several phonological rules used for making compounds have been adopted into the way the equations are read so that it becomes easy for learners to memorize them and makes their recall automatic and reflexive. Thus, it can be hypothesized that Japanese-speaking people need to go through phonological coding (i.e., changing letters into sounds in recognition: Baddeley 1966, Baddeley and Wilson 1992, Leinenger 2014) rigidly to recall a target equation for computation purposes regardless of its manner of reading, be it aloud or silently.

Moreover, the level of reliance on phonological coding may vary across individuals depending on their native language.

The goal of the present study is to examine the validity of the above assumption through an experiment with informants speaking four different native languages: English, German, Chinese, and Japanese. The validity of the aforementioned assumption was supported by the results.

I. Introduction

Memorizing all the equations in the multiplication table correctly is surely the key to success in becoming numerate, especially for the study of multiplication and division. In Japan, for example, second graders aged 7 to 8 start to learn multiplication relying on the times table in math class, and most of them master the table in around several months at the most, although two academic years are officially scheduled to learn it (Otaka 2002). On the other hand, this is not always the case in other countries because their languages are different from Japanese in terms of how to read the equations in the table. Therefore, the amount of time required to master the table may vary a great deal depending on the language the learners speak. In particular, as will be shown later, differences in phonology may be the most influential factor in acquiring a high ability at computation. This is because numerical figures, as well as letters, have to be recognized by the learners through the process of encoding them into sounds, and the process of encoding may vary depending on the language. Thus, it is possible to postulate that learners speaking some languages can remember the times table more efficiently than their counterparts speaking other languages due to linguistic specificities in phonology and syntax.

The purpose of the present paper is to examine the validity of the above hypothesis through an experiment with informants from different linguistic backgrounds, i.e., English, German, Chinese and Japanese. Which language of the four

will be most advantageous to memorizing and recalling the times table?

II. Understanding counting in the four languages

Let us begin with a linguistic analysis of Chinese numerical words used in counting. There are only 10 words needed, with which one can indicate numbers up to 99 as shown in Table 1. Neither allomorphs¹⁾ nor different versions of the numbers from 1 to 10 are used when numbers up to 99 are read in counting, which is unique and most economical compared to other languages. The number 21, for example, is composed of three words 2, 10, and 1, which are read in isolation from left to right: 2-10-1 (/èr-shi-yī/).

Table 1 Chinese numbers up to 99

1	2	3	4	5	6	7	8	9	10
yī	èr	sān	sì	wǔ	liù	qī	bā	jiǔ	shí
11	12	13	20	21	33	99			
shí yī	shí èr	shí sān	èr shí	èr shí yī	sān shí	jiǔ shí jiǔ			

Japanese counting also involves 10 numerical words to count to 99. These words were borrowed from Chinese in the 6th century (Ando 2002). In addition to these, there is another type of number words in Japanese. These are the native Japanese numbers, but they are not used for calculations except for *yon* ‘four’ and *nana* ‘seven’, which are shortened forms of *yottsu* and *nanatsu*, respectively. This is probably because the native words are mostly longer than the loaned ones by one mora (or syllable-timing unit) in length. Thus, from the point of view of the Principle of Economy, it makes sense that the shorter versions of numerical words have been given preference over longer ones for use in computation.

Table 2 Japanese numbers

1	2	3	4	5	6	7	8	9	10
ichi	ni	san	shi (yon)	go	roku	shichi (nana)	hachi	kyuu	juu
11	12	13	20	21	33	99			
juu.ichi	juu.ni	juu.san	ni.juu	ni.juu.ichi	sa.juu.san	kyuu.juu.kyuu			

C.f. Native Japanese numbers: 1 = *hitotsu*, 2 = *futatsu*, 3 = *mittsu*, 4 = *yottsu*, 5 = *itsutsu*, 6 = *muttsu*, 7 = *nanatsu*, 8 = *yattsu*, 9 = *kokonotsu*, 10 = *too*

1) Allomorphs are any of two or more actual representations of a morpheme, such as the plural endings /s/ (as in *cats*), /z/ (as in *bags*), and /ɪz/ (as in *bushes*) in English, for example.

In contrast, English-speaking learners need 27 words to count to 99, as shown in Table 3, which is far more than in the previous two languages.

Table 3 English numbers

1	2	3	4	5	6	7	8	9	10
one	two	three	four	five	six	seven	eight	nine	ten
11	12	13	14	15	16	17	18	19	
eleven	twelve	thirteen	fourteen	fifteen	sixteen	seventeen	eighteen	nineteen	
20	30	40	50	60	70	80	90	99	
twenty	thirty	forty	fifty	sixty	seventy	eighty	ninety	ninety-nine	

It is interesting to note that numbers larger than 11 follow the decimal number system in terms of their readings in both Chinese and Japanese, whereas this is not the case in English. In Japanese and Chinese for example, 21 has the structure of two tens plus one (i.e., $2 \times 10 + 1$); /ni.juu.ichi/, /èr shí yī/, respectively, whereas in English it is twenty plus one (i.e., $20 + 1$). Note that 20 *twenty* in English is a single word. Thus, it can be claimed that structure of the decimal system is more analytical in the former two languages than in English, and this may help learners understand the system more readily (Miura and Okamoto 1999, Miura, Okamoto, Vlahovic-Stetic, Kim, and Han 1999).

III. Data on primary schoolers' numeracy performance across the world

A study was conducted internationally by the International Association for the Evaluation of Educational Achievement (IEA) in 2015 with the view of studying and comparing the arithmetic performances of 4th graders (aged 10-11) living in 49 countries. In Japan, 9,100 students participated in the study (TIMSS: Trends in International Mathematics and Science Study, 2015). According to the report, Japan ranked 5th, next to Singapore, Hong Kong, Korea, and Taiwan. Those five countries, which are all located in Eastern Asia, turned out to have significantly higher scores than other countries in numeracy. Note further that three of the top five are Chinese-speaking countries. In Singapore, English is in fact one of the four official languages, but 76% of Singaporeans are descendants of Chinese immigrants and speak Chinese (Mandarin Chinese or various dialects), whether as a native language or second language.

Judging from the fact that Chinese, Korean, and Japanese-speaking children do well in international comparisons of mathematical ability, the characteristics of their languages may have had positive effects on their efforts to memorize the times ta-

ble. In Japan, as referenced above, second graders start chanting the times table over and over, and they naturally learn it by heart without knowing or learning the patterns of multiplication. What is more, they master the whole times table in a comparatively short period of time (Otaka 2002). Incidentally, regarding math education in Korea, their own multiplication table has been developed as well since the educational device was imported from China. The Korean times table is as a system quite similar to that of Chinese (see Section II). The only difference is that Korean does not have a tonal system.

IV. Compounding rules in English and Japanese

It is useful to refer to compounding rules in these languages in this section because these are often used by people trying to memorize phrases efficiently as a kind of mnemonic.

Words can be combined to form various kinds of compounds such as compound nouns, compound verbs, and compound adjectives. These are very common and normally have two or more parts. The first part tells us what kind of object or person it is, or what its purpose is, i.e., the first part modifies the meaning of the second part by means of semantic restriction. Thus, the second part identifies or modifies the object or person in question. Compound nouns, for example, often have a meaning different or more specific than the two separate words. Therefore, compounding is an efficient way of building new words from a limited vocabulary. Two compound nouns in English are demonstrated in the following table.

Table 4 Compounds in English

First part: type or purpose	Second part: what or who	Compound noun
break	water	breakwater
boy	friend	boyfriend

Compound nouns can be written either as a single word, as two words with a hyphen, or as two words. There are no clear rules about this in English orthography.

E.g. *hanger-on*, *passer-by*, *washing machine*, *driving license*

Stress is important in pronunciation, as it distinguishes a compound noun and a phrase composed of an adjective and a noun. The stress usually falls on the first syllable in compounds in English.

E.g. *dáncing téacher* vs. *dáncing teacher*
white hóuse vs. *White House*

It is interesting to note from a different point of view that stress is used to define words in English. Words in English can be defined as a chunk composed of one or more syllables with one meaning having only one culmination in loudness (i.e., the primary stress).

4.1 Six ways of making compounds in Japanese

Various kinds of compounds are made in Japanese, too. However, as opposed to English, stress is not used at all, but other possible properties in this language are. Regarding ways of forming compounds,²⁾ Japanese outnumbers English. There are at least six ways as follows.

- (1) Pitch accent
- (2) Sequential voicing
- (3) Fortition of /h/ (/h/ → /p/)
- (4) Vowel alternation
- (5) Consonant epenthesis
- (6) Rhythmicity based on monomoraic syllables and the bimoraic foot

4.2. Pitch accent

Pitch is a phonetic factor related to the frequencies of vowel sounds. Japanese is a language with pitch accent based on two phonological units, High and Low, whereas English has stress accent. English uses pitch only for intonation. In Japanese, a culmination in pitch occurs only once within a phonological word, and this is so in the case of compounds, as shown below. The diacritics /ˊ/ and /ˋ/ indicate that words with the former have no accent (the pitch remains high or rising flat) and those with the latter are accented on the mora marked with it.

- E.g. 1. yamaˊ + hototoˋgisu → yama.hototoˋgisu
 'mountain' 'quail' 'mountain quail'
2. hana.uriˊ + musumeˋ → hana.uri.muˋsume
 'flower-selling' 'girl' 'girl who sells flowers'
3. Ryu.u.gaˋ.ku.se.e + Kyo.o.giˋ.ka.i → Ryu.u.ga.ku.se.e.kyo.o.giˋ.ka.i
 'Student from abroad' 'conference'

For example, the first compound of *yama.hototoˋgisu* 'mountain quail' is composed of two words (or morphemes) with different pitch patterns, but they are united by taking a new pitch pattern having one culmination in pitch. That is, the third syllable

2) There are various kinds of compounds in Japanese as well, such as compound nouns (e.g., *yuki-yama* 'snow mountain'), compound verbs (e.g., *nage-dasu* 'discard'), compound adjectives (e.g., *hoso-nagai* 'lanky'), and compound adverbs (e.g., *hito-mazu* 'for the time being').

ble from the end (the antepenultimate) is accented.

A similar phenomenon to this unification of different pitch patterns exemplified above can be observed also from the readings of equations set in the times table. The sonogram of figure 1 is of the utterance of the equation $6 \times 7 = 42$ (*ro`ku* \times *shichi`* = *shi`juu ni`*) for example recorded by a Japanese-speaking informant.

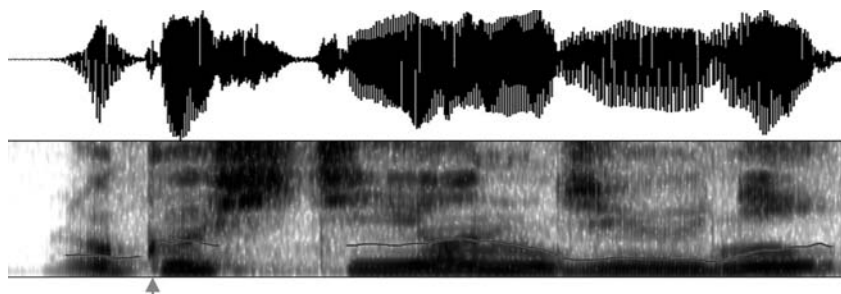


Figure 1 Pitch movement in the utterance of the equation *roku shichi shi.juu ni* ($6 \times 7 = 42$) in Japanese

Note that pitch (shown in a blue line) rises on the second mora (*ku*) and maintains its highness until the third mora from the end. If the numerical words *6 ro`ku* and *7 shichi`* are pronounced in isolation, their pitch patterns will not be the same as those used in the equation here (Otaka 2002).

4.3 Sequential voicing

Rendaku ‘sequential voicing’ occurs on the word-initial voiceless consonant of the second word when a compound is made, except when the second element already contains a voiced consonant (Lyman’s law). Compounds made through this phonological rule are exemplified below.

- E.g.
- | | | | | | |
|----|----------|---|---------|---|------------------|
| 1. | [take] | + | [sao] | → | [takezao] |
| | ‘bamboo’ | | ‘pole’ | | ‘bamboo pole’ |
| 2. | [hito] | + | [tsuma] | → | [hitozuma] |
| | ‘person’ | | ‘wife’ | | ‘someone’s wife’ |
| 3. | [hana] | + | [tʃi] | → | [hanaʒi] |
| | ‘nose’ | | ‘blood’ | | ‘nose bleeding’ |
| 4. | [buta] | + | [ʃiru] | → | [butaʒiru] |
| | ‘pig’ | | ‘soup’ | | ‘pork soup’ |
| 5. | [hon] | + | [tana] | → | [hondana] |
| | ‘book’ | | ‘shelf’ | | ‘bookshelf’ |
| 6. | [ko] | + | [taiko] | → | [kodaiko] |
| | ‘small’ | | ‘drum’ | | ‘small drum’ |

Rendaku is a powerful rule that applied to compounding in Japanese. However, it does not always occur when two words are combined to make a compound, even though the second word has a voiceless consonant word-initially (e.g., yaki + tori → yaki.tori ‘grilled chicken’ as opposed to yama + tori → yama.dori ‘mountain quail’). No one has ever successfully explained why *Rendaku* sometimes applies, but at other times does not.

4.4 Fortition of /h/

Word-initial /h/ ([h, ϕ , ζ]³⁾ of the second element is realized as the voiceless bilabial plosive *p* when preceded by a word ending with a nasal sound, or a high vowel *i* or *u*.⁴⁾ In the latter case, the word-final syllable composed of a voiceless consonant plus a high vowel is replaced by *p*, resulting in the geminate *pp*. As we shall see below, this phonological rule is also involved in reading some equations of the times table, in that the left hand sides of the equations (multiplicand + multiplier) are treated as if they were compound nouns.

- E.g.
- | | | | | | | |
|----|---------|---|---------------|---|------------|-----------------------|
| 1. | ichi | + | hai | → | ip.pai | ‘a cup of ~’ |
| | ‘one’ | | ‘cup’ | | | |
| 2. | satsu | + | ϕ uuukee | → | sap.puukee | ‘scene with no taste’ |
| | ‘kill’ | | ‘scene’ | | | |
| 3. | kan | + | hai | → | kam.pai | ‘cheers!’ |
| | ‘dry’ | | ‘glass’ | | | |
| 4. | on | + | ϕ u | → | om.pu | ‘music notes’ |
| | ‘sound’ | | ‘symbol’ | | | |

4.5. Vowel alternation

When some compounds are formed, a word-final vowel of the first element may change into different one as exemplified below. In many cases, the mid front vowel *e* becomes the low vowel *a*, but this is not always the case. There are also other types of vowel alternation in the making of compounds in Japanese as follows.

E.g. /e/→/a/:

- | | | | | | | |
|----|---------|---|---------|---|----------|---------------|
| 1. | sake | + | ya | → | saka.ya | ‘liquor shop’ |
| | ‘wine’ | | ‘shop’ | | | |
| 2. | koe | + | iro | → | kowa.iro | ‘voice color’ |
| | ‘voice’ | | ‘color’ | | | |

3) In Japanese, /h/ has 3 allophones [h, ϕ , ζ]. It is realized as [ϕ] before /u/, [ζ] before /i/, and [h] before /a/, /e/, or /o/.

4) /u/ is an unrounded high back vowel in Japanese, but this is often replaced by /u/ for convenience’ sake. /u/ is used hereafter in this paper.

/i/→/o/:

3. **ki** + kage → **kokage** ‘shade’
 ‘tree’ ‘shadow’

4. **hi** + kage → **hokage** ‘light’
 ‘fire’ ‘shadow’

/i/→/e/:

5. /**ki**/ + hai → **kehai** ‘hint’
 ‘air’ ‘distribute’

/o/→/a/:

6. /**siro**/ + ko → **sirako** ‘soft roe’
 ‘white’ ‘baby’

4.6. Consonant epenthesis

There are two types of compounding by means of gemination. One is called *Renjoo* in Japanese, which used to be prevalent in Old Japanese during the period between the end of the Heian Era and the following Muromachi Era (10th~14th c.: Yamaguchi et al. 1997). When a word ending with a coda consonant called a ‘mora phoneme’ (/n/, /m/), or /t/ is compounded with a word beginning with a vowel or a glide /j, w/, one of the consonants /n/, /m/, or /t/ is epenthesisized or geminated between the two words as the onset of the second word.

E.g. Double consonant 1: **Renjoo**

1. **ten** + oo → **tennoo** ‘emperor’ 天皇
 ‘heaven’ ‘king’
2. **han** + oo → **hannoo** ‘reaction’ 反応
 ‘return’ ‘reply’
3. **sam** + i → **sammi** ‘third rank’ 三位
 ‘three’ ‘position’
4. **set** + in → **settin** ‘toilet’ 雪隠
 ‘snow’ ‘hide’

The other type of double consonant occurs when a first element ending with a syllable composed of a voiceless consonant and a high vowel is compounded with a second factor beginning with a voiceless plosive, whereupon the final syllable of the first element is replaced by a voiceless consonant identical to the word-initial consonant of the second element. This type of sound change is well-known as *Sokuonbin* (‘sound change with a double consonant’) in the development of verbal forms in Old Japanese (e.g., /kahi/ (*te*-form of the verb *kahu* ‘buy’) + /te/ → /kat.te/).

E.g. Double consonant 2: **Sokuonbin**

3. **Ketsuu** + kon → **kekkon** ‘marriage’ 結婚

- | | | | | | |
|----|--------------|---|-----------|---|---------------------|
| | ‘tie’ | | ‘wedding’ | | |
| 4. | sak <u>u</u> | + | ka | → | sakka ‘writer’ 作家 |
| | ‘make’ | | ‘person’ | | |
| 5. | bet <u>s</u> | + | soo | → | bessoo ‘cottage’ 別荘 |
| | ‘another’ | | ‘house’ | | |

4.7. Rhythmicity based on monomoraic syllables and the bimoraic foot

Although Japanese is a mora-counted language, the bimoraic foot composed of two moras also functions as a rhythmic unit in Japanese (Otaka 2006). What can make up a bimoraic foot are any syllable of the shape CV followed by the mora phoneme *n* (e.g., *pan* ‘bread’), any syllable of the shape CVV (e.g., *too* ‘ten’), or any syllable (CVC) ending with part of a geminate (e.g., *kek-kaku* ‘pneumonia’). However, the sequence of two CV moras are sometimes treated as a pseudo-bimoraic foot if they are sandwiched between two bimoraic sequences, as exemplified below.

- E.g. *nen-gara-nen-juu* ‘all the year round’ \Leftarrow *nen-ga-nen-juu*
enu -eti-kee ‘NHK’ \Leftarrow *enu -eiti-kee*
iti-nii-san-sii-goo-roku ‘1, 2, 3, 4, 5, 6’ \Leftarrow *iti-ni-san-si-go-roku*

In the first example above, the original form of the adverb was *nen.ga.nen.juu*, but it has taken the form *nen.gara.nen.juu* in modern Japanese. That is, the nonsense syllable *ra* appears between the particle *ga* and the morpheme *nenjuu* ‘through the year’ in order to make the whole sequence fit a bimoraic rhythm. Note that as every morpheme involved in this adverb is composed of two moras, it sounds rhythmical. Similarly, in the second example, the second morpheme H of NHK ‘Japan Broadcasting Company’ is normally pronounced /e.i.ti/ with three moras in isolation, but it gets shortened by one mora to /e.ti/ in this compound because it is sandwiched between two bimoraic sequences. In the third example, the numerical numbers 2 *ni*, 4 *si*, and 5 *go* are all monomoraic, but become bimoraic when used in counting together with other bimoraic numbers such as *iti* ‘1’, *san* ‘3’, and *roku* ‘6’.

The reason such rhythmicity as observed above occurs is probably because if phrases display some kind of rhythmic regularity in addition to Double Articulation, they can be more easily memorized and recalled in recognition. According to the French linguist André Martinet (1960), the stream of speech can be divided into meaningful signs (morphemes) and further subdivided into meaningless elements (phonemes). This property that all natural languages share is called Double Articulation or the Duality of Patterning. Note that if rhythmicity is added to this intrinsic property, the result is something we might call Triple Articulation, which can help make phrases both memorable and sustainable in long-term memory. That is why

rhythmicity often applies when compounds are formed. Judging from the fact that fixed phrases such as proverbs and verses often involve rhythmic regularities including rhyming in English also, this property of rhythm may be universal across languages.

V. Japanese ways of memorizing the multiplication table

The equations shown in Table 5 are examples from *Kuku*,⁵⁾ which are traditionally read in some extraordinary ways. This implies that Japanese learners have made it a practice to memorize the equations somewhat differently from the standardized pronunciation of Japanese numerals. Note that various phonological processes referred to in the previous section are fully involved in the table.

Table 5 Japanese times table

$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 5 = 5$	$1 \times 9 = 9$
in ichi ga iti	in ni ga ni	in go ga go	in kuu ga kuu
$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$
ni ichi ga ni	ni nin ga si	ni san ga roku	ni si ga hati
$3 \times 3 = 9$	$3 \times 6 = 18$	$3 \times 8 = 24$	$4 \times 8 = 32$
sa zan ga kuu	sabuu roku juuu hati	san pa ni juuu si	si wa san juu ni
$5 \times 4 = 20$	$5 \times 8 = 40$	$5 \times 9 = 45$	$6 \times 9 = 54$
go si ni juuu	go ha yon juuu	gok kuu yon juuu go	rok kuu go juuu si
$7 \times 8 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$	$9 \times 8 = 72$
siti ha go juuu roku	hap pa roku juu shi	hak kuu siti juuu ni	kuu ha siti juuu ni

First, the number 1, *iti*, located in the first place of the left member (multiplier) of the nine equations ($1 \times 1 = 1 \sim 1 \times 9 = 9$) has been changed to *in*, while this is not the case for the one in the second place of the left member (e.g., $2 \times 1 = 2$). This alteration between *ti* and *n* cannot be observed in the compounds in the previous section, but only in verbal declensions. The latter type of sound change is called *Hatsuonbin* ‘sound change with a coda /n/’ in Japanese grammar (e.g., /*tobi*/ + /*te*/ → /*ton.de*/: *te*-form of the verb *tobu* ‘fly’). Therefore, it can be claimed that

5) In old days, Japanese people started with the largest numbers (9×9) and moved down to the smallest (1×1) when learning the table, but today they recite *Kuku* in the opposite way. The product of 9×9 , *ku-ku*, was the first to be chanted in this method, which is why the times table is called *Kuku* in Japanese.

the use of *Hatsuonbin* has been extended to the readings of some equations as a kind of mnemonic, resulting in a new allomorph *in*. In fact, this process has applied to CV syllables of many compound nouns, especially when the C is a nasal consonant (*n*, *m*) followed by a high vowel (*i*, *u*), although this process does not obligatorily apply when forming compounds.

E.g., *kanzashi* ‘comb’ ← *kami* ‘hair’ + *sasi* ‘insertion’
tondabayashi ‘city name in the Kansai area’ ← *tom***i** + *ta* + *hayasi*
akindo ‘merchant’ ← *aki* ‘dealing’ + *hi***t***o* ‘person’

Nasal insertion occurs in the reading of the equation of $2 \times 2 = 4$ (*ni nin ga si*) as well without causing any replacement. This process probably applies to avoid the repetition of the syllable *ni* ‘two’, which would violate the Obligatory Contour Principle (see below).

The subject marker *ga* is used only in equations whose right members (products) are smaller than 10. This is because numbers larger than ten involve more moras than those smaller than 10 overall; thus one does not use *ga*, worth one mora, so as to keep the utterance as short as possible.⁶⁾

Next, sequential voicing is observable in the Japanese times table. For example, the number 3 is pronounced in four different ways in the chart. When it is the multiplier of the equation $3 \times 3 = 9$, it is pronounced *zan* instead of the standard *san*. The reason why the repetition of *san* is avoided is the same as in the cases of $2 \times 2 = 4$ (*ni nin ga si*) and $1 \times 1 = 1$ (*in ichi ga iti*) above. On the other hand, in the case of $3 \times 6 = 18$, the initial 3 becomes *sabuu*, which is another allomorph of *san*. Thus, the number 3 is pronounced alternatively as *sa*, *san*, *zan*, and *sabuu*.

The change of *h* into *p* can be observed from $3 \times 8 = 24$ (*san pa nijuu si*) because the preceding 3 (*san*) ends with a nasal. On the other hand, the 8 (*hati*) changes also into one of its allomorphs *wa* as in $4 \times 8 = 32$ (*si wa san.juu ni*). Thus, the number *hati* can be said to have two more allomorphs (*pa* and *wa*).

Another type of process called *double consonants* can be seen from some equations as well as from compound nouns.

E.g. $6 \times 9 = :$ /roku/ + /ku/ → /rokku/
 $8 \times 8 = :$ /hati/ + /ha/ (/hati/) → /happa/
 $8 \times 9 = :$ /hati/ + /ku/ → /hakku/

Finally, the function of rhythm should be considered as well. Based on the

6) Regarding the origin of this tradition, Ando (2002) argues that it was originally adopted to meet the need of abacus users in the Edo era, who wanted to distinguish between numbers less than 10 and those greater than 10 when using an abacus for calculation.

analyses of the readings of the equations in Table 5, it appears that the use of the bimoraic foot as a rhythmic chunk has been extended to the readings of the equations so that each equation can have its own specific rhythmic pattern. Table 9 shows the rhythmic patterns of nine example equations. Each bimoraic foot is indicated with parentheses, and the moras are demarcated with dots. In most cases, a chunk of the bimoraic foot falls also on an entire morpheme, as seen from $9 \times 1 = 9$: *kuu.(i.ti).ga.kuu* (1-2-1-1). However, this is not the case for such an equation as $4 \times 2 = 8$: *si.(ni.ga).(ha.ti.)* (1-2-2). Since equations are semantically and syntactically divided into two parts, the left and right hand sides, the subject-marking particle *ga* can be combined with a preceding monosyllabic morpheme to make a rhythmic chunk. In contrast, as seen from the previous case of $9 \times 1 = 9$ (*kuu.(i.ti).ga.kuu*: 1-2-1-1), *ga* does not get chunked with the following syllable, although it is a monosyllabic number. This is because the particle *ga* functions to demarcate the left and right hand sides of an equation and belongs to the former syntactically.

Let us observe the variety of patterns in the equations in Table 6. Out of the nine examples, only the pattern /2-2-2-2/ is used twice, for $9 \times 4 = 36$ and $9 \times 9 = 81$. The rest all have different patterns from each other. This variety in rhythm may help prevent confusion in equations involving the same number, 9 in this case.

Table 6 Rhythmic patterns based on monomoraic syllables and bimoraic foot

$9 \times 1 = 9$ <i>kuu.(i.ti).ga.kuu</i> 1-2-1-1	$9 \times 2 = 18$ <i>(kuu.ni).(juu.uu).(ha.ti)</i> 2-2-2	$9 \times 3 = 27$ <i>kuu.(sa.n).ni.(juu.uu).(si.ti)</i> 1-2-1-2-2
$9 \times 4 = 36$ <i>(kuu.si).(sa.n).(juu.uu).(ro.kuu)</i> 2-2-2-2	$9 \times 5 = 45$ <i>(kuu.go).si.(juu.uu).go</i> 2-1-2-1	$9 \times 6 = 54$ <i>kuu.(ro.kuu).go.(juu.uu).si</i> 1-2-1-2-1
$9 \times 7 = 63$ <i>kuu.(si.ti).(ro.kuu).(juu.uu).(san)</i> 1-2-2-2-2	$9 \times 8 = 72$ <i>(kuu.ha).(si.ti).(juu.uu).ni</i> 2-2-2-1	$9 \times 9 = 81$ <i>(kuu.kuu).(ha.ti).(juu.uu).(i.ti)</i> 2-2-2-2

In summary, it can be argued that most of the processes used in compound formation also apply to the readings of equations of the multiplication table in Japanese. They are based on either of two major linguistic principles, the Principle of Economy (PE) and the Obligatory Contour Principle (OCP). The former states that speakers of any language tend to minimize their efforts when using their languages, whereas the latter states that two identical/similar sounds are likely to be avoided in use.⁷⁾ Due to the application of PE, shorter versions of possible allomorphs are likely to be adopted in the readings of many equations.

7) For example, tongue twisters are known to be difficult to say fast because they violate the OCP. ↗

- E.g. 3: $san \rightarrow sa$
 4: $si, yon \rightarrow si$
 8: $hati \rightarrow ha, pa, wa$
 9: $kyuuu \rightarrow ku$

On the other hand, the application of OCP allomorphs such as *zan* from *san* in $3 \times 3 = 9$ (*sa zan ga ku*) and *pa* from *hati* in $8 \times 8 = 64$ (*hap pa roku.j uuu si*) are likely to occur. The latter case might appear to violate the principle because of the two identical consonants in *pp*. However, this is not the case because the first element of a geminate consonants except for the nasal geminate *nn* is phonologically a mere pause in Japanese phonology, which is identical to a rest in music (e.g., /λ/ if one mora is regarded as /J/) from the point of view of rhythmic function.

VI. English ways of reading the multiplication table

In English, there are several ways to read one equation with no phonological processes applying, as shown below. That is, there are no traditionally developed or modified ways of reading the equations that can mnemonically help learners to memorize and recall them. This means that English-speaking people may need more time to recite the times table than their Japanese counterparts. In other words, the way of reading the equations in English does not much function as a chant. With regards to German, the way of reading the equations is almost the same as English because of their close linguistic relationship. For example, the equation $2 \times 4 = 8$ (or $2 \cdot 4 = 8$) is read *Zwei mal vier ist acht*, which is exactly the same in essence as English shown below.

E.g. $2 \times 4 = 8$:

1. Two times four equals (is, is equal to, makes) eight.
2. Two fours are eight.
3. Twice four is eight.
4. Two multiplied by 4 is eight.

$12 \times 12 = 144$:

1. Twelve times twelve is equal to one hundred forty-four.
2. Twelve times twelve is equal to one forty-four.

↘ According to Goto (2016), children can easily recite multiplication tables when it comes to one figure multiplied by the same figure, while it takes longer for them to say tables containing the figures 4, 7, or 8. This is because these similar-sounding figures are hard for them to say: 4 (shi), 7 (shichi), 8 (hachi).

In evaluating these, it appears that the most effective way to memorize and recall the times table in English is the second one above because it is the shortest. However, even this method may still be confusing to English speakers because there is nothing except for the difference in phonemes that can be used to distinguish equations whose numerical words are composed of the same number of syllables: e.g., *Two twos are four, Three twos are six, Four twos are eight*, etc.

VII. Chinese ways of reading the multiplication table

Since Chinese is typologically an isolating language with neither declensions nor affixation,⁸⁾ unlike English, the equations of the table are just read word by word (i.e., syllable by syllable) as arranged in order. As shown below, the way of reading the equation of $3 \times 9 = 27$ for example is 3, 9, 2, 10, 7 (*sān.jiǔ.èr.shí.qī*). Each number is simply read in isolation from left to right. *Twenty*, which has the meaning of two tens (10 multiplied by 2), is expressed by simply conjoining the two numbers 2 and 10. However, tonal patterns characteristic of each word (monosyllable) seem to help the learners memorize the equations just like the melodies of songs can help learners remember the lyrics in music. So (2015) found experimentally that Chinese speakers can guess one potential answer just by listening to the tones of the left member of a given equation. In his experiment, he asked ten Chinese-speaking informants to choose the correct answer out of four options for the product of an equation after only listening to the tones of its left hand side. It turned out that all the informants determined the right answer with 100 percent accuracy.

E.g. $3 \times 9 = 27$	三	九	二	十	七	
	sān	jiǔ	èr	shí	qī	
	(1)	(3)	(4)	(2)	(1)	⇐ tonal patterns
$4 \times 7 = 28$	四	七	二	十	八	
	sì	qī	èr	shí	bā	
	(4)	(1)	(4)	(2)	(1)	⇐ tonal patterns
$8 \times 9 = 72$	八	九	七	十	二	
	ā	jiǔ	qī	shí	èr	
	(1)	(3)	(1)	(4)	(2)	⇐ tonal patterns

8) Precisely speaking, there is marginal suffixation in Mandarin, especially in the plural marker *men* for pronouns.

VIII. Experiment

8.1. Goal

From the arguments made above, it can be assumed that numbers can be more easily processed in some languages than others. That is, the difficulty learners experience in the process of memorizing the times table may vary depending on the characteristics of a given language, because the level of reliance on phonological coding differs depending on the language they speak. Phonological coding is, once again, the recognition process in which readers recode written, orthographic information into a sound-based code during silent reading.

We assume that German and English-speaking students are likely to have greater difficulty than Japanese and Chinese-speaking students in both memorizing and recalling the times table.

The goal of the present experiment is to confirm the validity of this assumption and to determine which language is most advantageous to mastering the table in terms of efficiency, accuracy, and sustainability. To this end, we need to explore which language group relies the most on phonological coding in order to recall the equalities, because the more phonological information one relies on to recall a target phrase, the easier it is, just like when trying to recall fixed phrases or chants. It is a fact that such phonological information as rhyming and rhythmicity can also help people memorize a target phrase effectively,⁹⁾ which likewise can be recalled only through phonological coding without accessing its meaning. Therefore, it is meaningful to examine how much one relies on phonological coding when trying to recall a target phrase. The level of the reliance may vary depending on the language one speaks.

8.2. Informants

A total of 23 informants in their early 20s studying at a Japanese university in 2016 participated individually in this experiment. The numbers of informants belonging to each language group are shown below with the number of male and female subjects. There were six English-speaking informants, two British and the rest American. The number of male and female subjects was equal in both the German and English-speaking groups, whereas the Chinese and Japanese-speaking groups showed a gender imbalance.

English speaking: 6 (3 females and 3 males)

German speaking: 4 (2 females and 2 males)

9) You may have had an experience in which part of the lyrics of a song which you had completely forgotten was suddenly recalled with the help of the melody falling on the part.

Chinese speaking: 7 (6 females and 1 male)

Japanese speaking: 6 (2 females and 4 males)

8.3. Procedures

All the informants were asked one by one to answer 64 questions on the equations of the times table. The questions were all written on a sheet of paper and the informants were asked to fill in their answers in the blanks provided by pencil. Example questions are as follows.

$$\begin{array}{l} 2 \times 3 = [\quad], 3 \times 6 = [\quad], 4 \times 5 = [\quad], 5 \times 9 = [\quad], \\ 6 \times 4 = [\quad], 7 \times 4 = [\quad], 8 \times 6 = [\quad], 9 \times 8 = [\quad], \text{ etc.} \end{array}$$

They were asked to answer as fast as possible twice in a quiet room. First, they filled in the blanks silently, then did the same the second time while saying “a-a-a-a-a . . .” aloud throughout the activity so as to interfere with their phonological coding to some extent. The time they needed to complete a task was measured with a stopwatch, and subsequently the differences in time between the two tests were measured and recorded. It was assumed that all the informants would perform the second test faster due to possible learning effects: The more one performs a certain task, the faster its performance.

The number of mistakes they made while answering the questions was also considered in the analyses as a measure of accuracy. It was also assumed that the more accurate their answers, the less likely they were to have computed the answer. In other words, they just recited the right hand sides of the equations in question without referring to their meanings.

IX. Results

The results for the mean time in seconds required for the informants to answer the 64 questions in the two kinds of tests are shown in Table 7. The English-speaking informants improved in the second test by 10 seconds. German-speaking informants took more time overall than English-speaking informants, and improved by 9 seconds on the second trial. In contrast, Chinese and Japanese-speaking informants took much less time than English and German-speaking informants, and took more time to complete the second trial than the first trial unlike the latter group. That is, no learning effect was observed for the former group of the two languages.

Table 7 The time required to complete 64 equations

Language (No. of informants)	Average time on trial #1 (second)	Average time on trial #2 (second)	Difference (second)
English ($n = 6$)	212.2	202.3	-10
German ($n = 4$)	257.1	248.2	-9
Chinese ($n = 7$)	91.7	100.8	9.1
Japanese ($n = 6$)	75.5	79.5	4

Table 8 presents the numbers of mistakes the informants made out of 64 questions by language. The results indicate that Chinese and Japanese-speaking informants performed both tests more accurately than their German and English-speaking counterparts. The former gave very few wrong answers, 0.6 to 1.4 on average out of 64 regardless of the test type, whereas the latter made 3 to 6.1 mistakes.

Table 8 The number of mistakes made during calculation

Language	No. of mistakes on trial#1 out of 64	No. of mistakes on trial#2 out of 64
English ($n = 6$)	4	6.1
German ($n = 4$)	3	3.7
Chinese ($n = 7$)	1	0.7
Japanese ($n = 6$)	0.6	1.4

X. Discussion

From the results shown in the previous section, it appears that English and German-speaking informants need more time to answer questions on the times table than their Chinese and Japanese-speaking counterparts. Moreover, the former are likely to make more mistakes than the latter in recalling the tables. These facts indicate that German and English-speaking people cannot automatically recite all the tables and thus sometimes need to calculate the answers, whereas Chinese and Japanese-speaking people are good at reciting the tables without calculation. This is why Chinese and Japanese-speaking people can quickly tell the product of a given equation upon hearing/reading its left hand side. In the process of this recitation, the phonological coding helps them to accurately connect the left and right hand sides of a given equation.

Saying “aaaaa . . .” while reciting the tables must have hindered Chinese and Japanese-speaking informants from giving correct answers more than it did their English and German-speaking counterparts. This implies that Chinese and Japanese-

speaking people may rely more on sounds than their English and German-speaking counterparts when recalling the equalities in the multiplying table. In other words, the times tables in Chinese and Japanese are stored in the vocabulary as fixed phrases or chants involving multiple phonological properties as well. That is why phonological coding plays a more important role for them in recalling the table than for their German and English-speaking counterparts.

A learning effect was more rigidly observed in the English and German groups than their Chinese and Japanese counterparts, because the former groups took less time to complete the same task the second time due to the positive effect of the first test. This is normal, because they more or less have to engage in calculation. In contrast, this was not the case for the Chinese and Japanese groups. They took significantly more time on the second test because their phonological coding was presumably hindered by the sounds they kept making during the test, i.e., they could not sufficiently concentrate on recalling the tables the second time.

XI. Conclusion

In conclusion, it can be argued that English and German-speaking people must more or less engage in calculation when trying to recite the times table, whereas Chinese and Japanese-speaking people can recite it quickly with the help of phonological knowledge as if recalling fixed phrases or chants without referring to their meanings. What helps the Japanese memorize and recall the tables with less effort are various rules of phonology such as rhythmicity, rhyming, tonal patterns, compounding rules, and brevity based on both the PE and the OCP. The Japanese way of reciting the times table called *Kuku* involves a variety of rhythmic patterns composed of bimoraic feet and single moras, following the OCP. As referred to earlier, applying the OCP in the readings of the table facilitates the process of memorizing and recalling the tables in recognition. In Chinese multiplication, on the other hand, the fixed tonal patterns for each equality of the table can help Chinese-speaking people to strengthen the bond between the phonemic sequence and its meaning effectively, which is why they can guess the product of a given equation upon listening to or reading its left hand side in a question. The learned melody of some phrases can help them recite it without activating its meaning.

Kuku in Japanese is the times table traditionally and linguistically developed based on specific mnemonics in which the readings of many of the equations follow the PE and the OCP as much as possible, so that memorizing and recalling them become efficient and automatic. What is more, once *Kuku* has been acquired by the learners, it is unlikely to fade easily thanks to the strengthened bond between the phonology and semantics of each equation.

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