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Abstract

We consider product line strategies of duopolistic firms supplying two vertically differentiated products with non-negativity output constraint and its expectation on rival's product line reaction. We consider a game in which there exists a heterogeneous unit production costs in high quality goods but is homogeneous in low quality product between firms. We derive equilibria for the game and characterize graphically firms' product line strategies and the realized profits of both firms through quality superiority and relative cost efficiency ratios. We also show that the efficient cost firm earns more than the inefficient firm except for the special case where both firms specialize in low quality good. We also illustrate that firms can correctly conjecture the *ex ante* relationship between the quality superiority of both goods and the relative cost efficiency ratios of firms on high quality good *ex post* in equilibrium.

Keywords: Multi-product firm; Duopoly; Substitution of Production between products; Vertical product differentiation

JEL Classification Codes: D21, D43, L13, L15

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1 Introduction

In real economy, there exits oligopolistic competition in the same segment market in which firms supply vertically differentiated multi-product. In such a market competition, it is important for each firm to choose its own product line strategies on multi-products under its expectation on its rival's product line reaction. In Kitamura and Shinkai (2015) published in Econ. Lett.), we characterize graphically firms' product line strategies through the quality superiority and the relative cost efficiency of high quality goods ratios between firms, when each rival firm chooses positive outputs in both a high and low quality goods.

In our previous work, we don't explore equilibrium profits of firms. In real oligopolistic market, there are some cases in which its rival firm faces with its rival which supplies each one of vertically differentiated two products (that is, the rival chooses a single product line) in a same segment market. Thus, it is crucial for each firm to consider which product line strategies the rival firm chooses, when it chooses its own best product line strategies. Furthermore, in our previous study, we analyze the case in which there exists cost heterogeneity in high quality product but with homogenous cost for low quality product.

In this paper, we consider product line strategies of duopolistic firms supplying two vertically differentiated products with non-negativity output constraint and its expectation on rival's product line reaction. We consider a game in which there is a heterogeneous in unit production cost of high quality good but is homogeneous in low quality product between firms. We show that there exist five nontrivial equilibria in which positive outputs for any products of the game. We characterize graphically firms' product line strategies through the quality superiority and the cost efficiency of high quality goods between firms in the equilibrium. Furthermore, we also compare the equilibrium profits of firms for the amounts and graphically characterize the relationship of profits of both firms in the equilibrium through quality superiority and relative cost efficiency ratios.

2 The Model

Suppose there are two firms, $i = 1, 2$, and each produce two goods (H and L) that differ in terms of quality, where 1 and 2 imply firms 1 and 2 in the duopoly case, respectively. We assume there is a continuum of consumers characterized by a taste parameter, θ, which is uniformly distributed between 0 and r (> 0), with density 1. We further assume that a consumer of type $\theta \in [0, r]$,for $r > 0$. Preferences are standard a la Mussa–Rosen. Thus, the utility (net benefit) of consumer θ who buys good α $(= H, L)$ from firm i $(= 1, 2)$ is given by

$$U_{i\alpha}(\theta) = V_\alpha \theta - p_{i\alpha} \qquad i =, 1, 2 \quad \alpha = H, L. \tag{1}$$

Each consumer decides to buy either nothing or one unit of good α from firm i to maximize his/her surplus.

Let V_H and V_L denote the quality level of the two goods. Then, the maximum amount that consumers are willing to pay for each good is assumed $V_H = \mu V_L = \mu > V_L = 1$. Thus, for simplicity, we normalize the quality of the low-quality good as $V_L = 1$ and we assume the quality of the high-quality good is μ-fold that of the low-quality good. Good α $(= H, L)$ is assumed homogeneous for any consumer.

Then, in the same way as in Kitamura and Shinkai (2015), we can derive the following inverse demand functions:

$$
\begin{cases}
p_H = V_H(r - Q_H) - Q_L = \mu(r - Q_H) - Q_L \\
p_L = V_L - Q_H - Q_L = 1 - Q_H - Q_L,
\end{cases}
\tag{2}
$$

where $Q_\alpha = q_{i\alpha} + q_{j\alpha}$ and p_α and $q_{i\alpha}$ stand for the price of good α and firm i's output of good α, respectively, $\alpha = H, L, i, j = 1, 2$. Without loss of generality, we set $r = 1$, hereafter.

Moreover, suppose that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L = 0$, where $c_{i\alpha}$ is firm i's marginal and average cost of good α. This implies that a high-quality good incurs a higher cost of production than a low-quality good does. Here, without loss of generality, we assume $c_{2H} > c_{1H} = 1 > c_{iL} = 0$, which means that firm 1 is more efficient than firm 2 is. Under these assumptions, each firm's profit is defined in the following manner:

$$
\pi_i = (p_H - c_{iH})q_{iH} + p_L q_{iL} \qquad i = 1, 2. \tag{3}
$$

3 Derivation of an Equilibrium

In this section, we derive an equilibrium of Cournot duopoly game in which each firm can choose its product line and output(s) of two vertically differentiated goods with non-negativity output constraint and its expectation on rival's product line reaction. After derivation of the equilibrium, we characterize graphically firms' product line strategies through the quality superiority and the cost efficiency of high quality goods between firms in the equilibrium.

Firm $i(= 1, 2)$ chooses the output (outputs) for H or L (both) type(s) of product(s) to supply that which maximizes this profit function in Cournot fashion under non-negativitiy output constraints provided that firm $j(\neq i)$ chooses *any given* product line strategy $\mathbf{s}_j \in \mathbf{S}_j \equiv \{(0,0),(+,0),(0,+),(+,+)\}$, where $(0,0)$ implies $(q_{jH} = 0, q_{jL} = 0)$, $(+,0)$ implies $(q_{jH} > 0, q_{jL} = 0)$, and so on. Thus, for any given $\mathbf{s}_j \in \mathbf{S}_j$

$$\max_{q_{iH},q_{iL}} \pi_i = \{\mu(1-q_{iH}-q_{jH})-q_{iL}-q_{jL}-c_{iH})q_{iH}+(1-q_{iH}-q_{jH}-q_{iL}-q_{jL})q_{iL} \quad (4)$$
$$s.t. q_{iH} \geq 0, q_{iL} \geq 0, i \neq j, i,j = 1,2.$$

The necessary and complementary conditions for the above maximization problem are

$$\frac{\partial \pi_i}{\partial q_{iH}} \leq 0, \frac{\partial \pi_i}{\partial q_{iL}} \leq 0, \quad (5)$$

$$q_{iH} \cdot \frac{\partial \pi_i}{\partial q_{iH}} = q_{iL} \cdot \frac{\partial \pi_i}{\partial q_{iL}} = 0, \quad (6)$$

$$q_{iH} \geq 0, q_{iL} \geq 0, \ i = 1,2. \quad (7)$$

Each firm chooses its product line strategy for two vertically differentiated products, that is, whether it produces positive (zero) quantities of product H and L for any rival firm's product line strategy.

Note that each inequality $\partial \pi_i / \partial q_{i\alpha} \leq 0$ in (5) and the correspondent complementary slackness condition $q_{i\alpha} \cdot \partial \pi_i / \partial q_{i\alpha} = 0$ in (6) imply that if the marginal revenue of firm i of product $\alpha(= H, L)$ is below (is exactly the same as) its marginal cost of it, then firm i does not produce (does produce a positive quantity of) the product, respectively.

There are 15 cases to be solved according to each firm's product line strategies under its expectation about its rival firm's product line strategies except for the trivial case in which both firms never produce either product, H or L. After some lengthy calculations and the check of non-negativity constraints of the outputs in each equilibria, we can show that 10 out of these 15 cases have no equilibrium in the correspondent games. Owing to limitations of space of the paper, we omit these calculations and proofs of our results. We observe that the following five cases have an equilibrium in the corresponding games.

3.1 (Case A) $q_{1H}^{*A} = q_{2H}^{*A} = 0, q_{1L}^{*A} > 0, q_{2L}^{*A} > 0$

In this case, the quality superiority of high quality product H, μ is too small as compared with the relative cost efficiency of high quality goods between firms, c_{2H}, and so both of firms never produce the product H but produce only low quality product L. A duopoly market of low quality is realized in the equilibrium. In figure 1, the area IX corresponds to this case.

$$q_{1H}^{*A} = q_{2H}^{*A} = 0 < q_{1L}^{*A} = q_{2L}^{*A} = \frac{1}{3} \text{ and } 1 < \mu \leq 2, \tag{8}$$

where the last inequality has to be satisfied from the necessary condition. From (2), (3) and (8), the corresponding equilibrium price, profit of each firm are presented respectively by

$$p_H^{*A} = \frac{1}{3}(3\mu - 2), p_L^{*A} = \frac{1}{3}$$

and

$$\pi_1^{*A} = \pi_2^{*A} = \frac{1}{3}.$$

3.2 (Case B) $q_{1L}^{*B} = q_{2H}^{*B} = 0, q_{2L}^{*B} > 0, q_{1H}^{*B} > 0$

In this case, each firm specializes to the product with relative cost efficiency. In consequent, two monopoly markets are realized in the equilibrium: the monopoly of firm 1 (2) for the product H (L). In figure 1, the area VI corresponds to this case. In area VI, the relative cost inefficiency of high quality good of firm 2, c_{2H} is relatively strong as compared with μ, the quality superiority of high quality product H.

Thus, we obtain

$$q_{1L}^{*B} = q_{2H}^{*B} = 0, q_{1H}^{*B} = \frac{1}{4\mu - 1}(2\mu - 3), q_{2L}^{*B} = \frac{1}{4\mu - 1}(\mu + 1), \tag{9}$$

$$4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}),$$

where the last inequality needs to be satisfied from the necessary condition. From (2), (3) and (8), we obtain the corresponding equilibrium price, profit of each firm:

$$P_H^{*B} = \frac{(\mu + 1)(2\mu - 1)}{4\mu - 1}, P_L^{*B} = \frac{\mu + 1}{4\mu - 1}$$

and

$$\pi_1^{*B} = \frac{\mu (2\mu - 3)^2}{(4\mu - 1)^2}, \pi_2^{*B} = \frac{(\mu + 1)^2}{(4\mu - 1)^2}.$$

We also see that $q_{1H}^{*B} - q_{2L}^{*B} = \frac{\mu - 4}{4\mu - 1} \geq 0 \Leftrightarrow q_{1H}^{*B} \geq q_{2L}^{*B}$.

$$\pi_1^{*B} - \pi_2^{*B} = \frac{1}{4\mu - 1}\left(\mu^2 - 3\mu + 1\right) > 0 \text{ for } \frac{1}{2}\sqrt{5} + \frac{3}{2} < 4 < \mu.$$

3.3 (Case C) $q_{1L}^{*C} = 0, q_{2L}^{*C} > 0, q_{1H}^{*C} > 0, q_{2H}^{*C} > 0$

In the case C, firm 2 with higher unit cost of high quality product H produces both products but firm 1 which is efficient in production of product H specializes to product H. While the market for product H is a duopoly, but the market for product L becomes monopoly! In figure 1, the areas IV and V correspond to this case. In area IV, the quality superiority of high quality product μ is high as compared with the relative cost inefficiency of high quality good of firm 2 c_{2H}. Moving down the point in area IV to area V, the relative quality superiority μ reduces and becomes small as compared with the relative cost inefficiency of good H of firm 2 c_{2H}. Hence substitution of production of firm 2 occurs from high quality good H to low quality L.

$$\begin{aligned} q_{1L}^{*C} &= 0, q_{2L}^{*C} = \frac{1}{2(\mu-1)}c_{2H}, q_{1H}^{*C} = \frac{1}{3\mu}(\mu + c_{2H} - 2), \\ q_{2H}^{*C} &= \frac{1}{6\mu(\mu-1)}(2\mu^2 - 4c_{2H}\mu + c_{2H} - 2) \end{aligned} \tag{10}$$

$$q_{1H}^{*C} > q_{2H}^{*C}, q_{2L}^{*C} > 0 \text{ and } q_{2H}^{*C} \gtreqless q_{2L}^{*C} \Leftrightarrow \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) \lesseqgtr \mu, \tag{11}$$

and

$$\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu \Leftrightarrow q_{2H}^{*C} > 0 \tag{12}$$

hold. Furthermore, we obtain

$$c_{2H} \geq 2 \text{ and } \mu > 4. \tag{13}$$

For $q_{1H}^{*C} > 0$, the inequality, $\mu > 2 - c_{2H}$ is necessary to hold. This holds, since $c_{2H} \geq 2$. The corresponding equilibrium price, profit of each firm are given by

$$P_H^{*C} = \frac{1}{3}\left(\mu + c_{2H} + 1\right), P_L^{*C} = \frac{1}{6\mu}\left(2\mu - c_{2H} + 2\right)$$

and

$$\begin{aligned}
\pi_1^{*C} &= \frac{\left(\mu + c_{2H} - 2\right)^2}{9\mu}, \\
\pi_2^{*C} &= \frac{1}{36\mu(\mu-1)}(4\mu^3 - 4(4c_{2H} - 1)\mu^2 + 4(2c_{2H} - 1)(2c_{2H} + 1)\mu - (7c_{2H} - 2)\left(c_{2H} - 2\right)).
\end{aligned}$$

If $\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu, c_{2H} \geq 2,$, then $\pi_1^{*C} > \pi_2^{*C}$.

3.4 (Case D) $q_{1L}^{*D} > 0, q_{2L}^{*D} > 0, q_{1H}^{*D} > 0, q_{2H}^{*D} = 0$

In this case, contrary to the case C, firm 1 with efficient in production of product H supplies both product but the inefficient firm 2 specializes to product L. Consequently, the market for low quality product L becomes a duopoly but the one for high quality product H is a monopoly. The areas VII and VIII correspond to this case. the relative superiority of high quality good μ is relatively small as compared with the relative cost inefficiency of high quality good of firm 2, c_{2H},especially in case VIII.

$$q_{1L}^{*D} = \frac{1}{6(\mu-1)}(4-\mu), q_{2L}^{*D} = \frac{1}{3}, q_{1H}^{*D} = \frac{1}{(\mu-1)}(\mu-2), q_{2H}^{*D} = 0. \tag{14}$$

Because q_{1L}^{*D} and q_{1H}^{*D} are positive values, we have

$$2 < \mu < 4.$$

In addition, we have

$$q_{1L}^{*D} \gtreqless q_{1H}^{*D} \Leftrightarrow \mu \lesseqgtr \frac{5}{2} \text{ and } \mu \leq 2c_{2H},$$

where the last inequality has to be held for the necessary condition. From (2), (3) and (8), the corresponding equilibrium price, profit of each firm are

$$P_H^{*D} = \frac{3\mu + 2}{6}, P_L^{*D} = \frac{1}{3}$$

and

$$\pi_1^{*D} = \frac{1}{36(\mu - 1)}\left(9\mu^2 - 32\mu + 32\right), \pi_2^{*D} = \frac{1}{9}.$$

$$\pi_1^{*D} - \pi_2^{*D} = \frac{1}{36\mu - 36}\left(9\mu^2 - 32\mu + 32\right) - \frac{1}{9} = \frac{1}{4(\mu - 1)}(\mu - 2)^2 > 0, \text{for } \mu \leq 2c_{2H}, 2 < \mu < 4.$$

3.5 (Case E) $q_{1L}^{*E} > 0, q_{2L}^{*E} > 0, q_{1H}^{*E} > 0, q_{2H}^{*E} > 0$

In case E, both firms produce both products and both markets become duopoly. In this case, c_{2H} is so small in comparison with μ. In Figure 1, this case corresponds to the areas I', I, II and III. Moving down from area III to I', the inefficient firm 2 of high quality product H reduces the quantity output of product H.

$$q_{1L}^{*E} = \frac{1}{3(\mu-1)}(2-c_{2H}), q_{2L}^{*E} = \frac{1}{3(\mu-1)}(2c_{2H}-1), \tag{15}$$
$$q_{1H}^{*E} = \frac{1}{3(\mu-1)}(\mu+c_{2H}-3), q_{2H}^{*E} = \frac{1}{3(\mu-1)}(\mu-2c_{2H}).$$

For $q_{1L}^{*E} > 0$ and $q_{1L}^{*E} > 0$,

$$1 < c_{2H} < 2$$

is necessary to be held. We observe that $q_{1H}^{*E} > q_{2H}^{*E}$ under this condition. For $q_{1H}^{*E} > 0$ and $q_{2H}^{*E} > 0$, we observe that

$$\mu > 3 - c_{2H} \text{ and } \mu > 2c_{2H}$$

are necessary to be held, respectively. In addition, we obtain

$$q_{1H}^{*E} \gtreqless q_{1L}^{*E} \Leftrightarrow \mu \gtreqless 5-2c_{2H}, \; q_{2H}^{*E} \gtreqless q_{1L}^{*E} \text{ and } q_{2L}^{*E} \gtreqless q_{1H}^{*E} \Leftrightarrow \mu \gtreqless c_{2H}+2$$

Furthermore, we show that

$$q_{2H}^{*E} \gtreqless q_{2L}^{*E} \Leftrightarrow \mu \gtreqless 4c_{2H}-1.$$

Further, we obtain the corresponding equilibrium price, profit of each firm (2), (3) and (8):

$$p_H^{*E} = \frac{1}{3}\left(\mu + c_{2H} + 1\right), p_L^{*E} = \frac{1}{3},$$

$$\pi_1^{*E} = \frac{1}{9(\mu-1)}\left(\mu^2 + (2c_{2H}-5)\,\mu + (c_{2H}-2)(c_{2H}-4)\right),$$
$$\pi_2^{*E} = \frac{1}{9(\mu-1)}\left(\mu^2 - (4c_{2H}-1)\,\mu + 4c_{2H}^2 - 1\right).$$

For $1 < c_{2H} < 2, \mu \geq 2c_{2H} > \frac{1}{2}(c_{2H}+3), \pi_1^{*E} - \pi_2^{*E} = \frac{1}{3(\mu-1)}(c_{2H}-1)(2\mu - c_{2H} - 3) > 0$.

Putting the above 5 cases together, we obtain the following proposition. Furthermore, we can classify the product line strategy of the duopoly game under the rival's nonnegative output belief in $c_{2H} - \mu$ plane in Figure 1.

[Insert Figure 1 here]

Proposition 1 *In the duopoly equilibrium of the game under rival's nonnegative quantities expectation presented above, the next inequalities hold among the outputs of high-quality good and low quality good of each firm*:

$$0 < q_{2H}^{*E} < q_{1H}^{*E} \leq q_{1L}^{*E} < q_{2L}^{*E}$$
$$\text{for } (c_{2H}, \mu) \in \{(c_{2H}, \mu) \in R^{2++} \mid \mu > 2c_{2H}, \mu \leq 5 - 2c_{2H} \text{ and } 1 < c_{2H} < \frac{5}{4}\} \; (I'),$$

$$
\begin{aligned}
0 &< q_{2H}^{*E} < q_{1L}^{*E} < q_{1H}^{*E} < q_{2L}^{*E} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \mu > 2c_{2H}, \mu > 5 - 2c_{2H}, \mu < c_{2H} + 2 \text{ and } 1 < c_{2H} < 2\} \ (I),
\end{aligned}
$$

$$
\begin{aligned}
0 &< q_{1L}^{*E} \leq q_{2H}^{*E} < q_{2L}^{*E} < q_{1H}^{*E} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \mu \leq c_{2H} + 2, \mu < 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \ (II),
\end{aligned}
$$

$$
\begin{aligned}
0 &< q_{1L}^{*E} < q_{2L}^{*E} \leq q_{2H}^{*E} < q_{1H}^{*E} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \mu \geq 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \ (III),
\end{aligned}
$$

$$
\begin{aligned}
q_{1L}^{*C} &= 0 < q_{2H}^{*C} < q_{2L}^{*C} < q_{1H}^{*C} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \mu > \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) > 4, c_{2H} \geq 2\}, \ (IV) \\
q_{1L}^{*C} &= 0 < q_{2L}^{*C} \leq q_{2H}^{*C} < q_{1H}^{*C} \text{ for } (c_{2H}, \mu) \in \\
\{(c_{2H}, \mu) &\in R^{2++} \mid \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) > \mu \geq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) > 4 \\
, c_{2H} &\geq 2\}, \ (V)
\end{aligned}
$$

$$
\begin{aligned}
q_{1H}^{*B} &\geq q_{2L}^{*B} > q_{1L}^{*B} = q_{2H}^{*B} = 0 \\
\text{for } (c_{2H}, \mu) &\in \{(c_{2H}, \mu) \in R^{2++} \mid 4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}), \mu \geq \frac{5}{2} \ (VI).
\end{aligned}
$$

$$
\begin{aligned}
q_{2L}^{*D} &= \frac{1}{3} > q_{1H}^{*D} > q_{1L}^{*D} > q_{2H}^{*D} = 0 \text{ when } \frac{5}{2} < \mu < 4,\ \mu \leq 2c_{2H} \ (VII) \\
q_{2L}^{*D} &= \frac{1}{3} > q_{1L}^{*D} \geq q_{1H}^{*D} > q_{2H}^{*D} = 0 \text{ when } 1 < \mu \leq \frac{5}{2}, \mu \leq 2c_{2H}, (VIII).
\end{aligned}
$$

$$
q_{1H}^{*A} = q_{2H}^{*A} = 0 < q_{1L}^{*A} = q_{2L}^{*A} = \frac{1}{3} \text{ when } 1 < \mu \leq 2 \ (IX).
$$

where Roman numbers imply the area in $c_{2H} - \mu$ plane in Figure 1, respectively.

Note that the equilibrium output of each firm presented in Proposition 1 at each equilibrium is that of the duopoly game under its expectation about its rival's nonnegative output(s).

The result presented in Proposition 1 makes firms conjecture *correctly* the state of nature regarding the quality superiority and the relative cost efficiency ratios *ex post*, by observing realized equilibrium output strategies in the equilibrium.

Note that we assume $c_{2H} > c_{1H} = 1$ and $V_H = \mu V_L = \mu > V_L = 1$. Thus, the horizontal and vertical axes variables in Figure 1 imply the relative cost ratio c_{2H} and quality value ratio μ. At any point (c_{2H}, μ) in the areas I', I, II, and III in Figure 1, relative cost ratio c_{2H} is between 1 and 2, so the difference of unit cost of both firms is small. The equilibrium in the case E above corresponds to these areas. In the areas I', I, II, the relative superiority of high quality good μ is also not so large, both firms are likely to supply high- and low-quality goods. However, as the quality value ratio μ increases and becomes sufficiently high and the relative cost ratio c_{2H} sufficiently low in the area III , the inefficient firm 2 produces far more of the low-quality good with no production cost than it does of the high-quality good, which has a higher positive cost. In contrast, the efficient firm 1 produces moderately more of the high-quality good H than it does of the low-quality good L, since its production cost for good H is lower than that of its rival firm. However, its marginal revenue from good H is not high, because its quality superiority μ is not very large. As the point (c_{2H}, μ) moves from Area *I'* to Areas *I*, *II* and *III* in Figure 1, substitution of production proceeds from the low-quality good to the high-quality good in both firms. Such substitution of production is stronger for the efficient firm than for the inefficient one.

This result is consistent with Calzada and Valletti (2012), where the optimal strategy for a film studio is to introduce versioning if their goods are not close substitutes for each other. Thus, when the predominance in quality value of the high-quality good H is large compared to good L to some extent, we can conclude that they are not close substitutes for each other. Then, the result in the above proposition confirms that it would be better for both firms to supply both goods in the market, that is, to obey the "versioning strategy" in Calzada and Valletti (2012).

As at any point (c_{2H}, μ) in the areas IV and V, the relative superiority μ is large as compared with the relative cost ratio c_{2H}, the margin of the efficient firm 1 for high quality good H is not so large, consequently substitution of production of firm 1 from good H to good L occurs, so the efficient firm 1 specializes good L with relatively large margin because of zero unit cost of good L. Moving from the area IV to the area V, the inefficient firm 2 starts to lose incentive to supply high quality good, firm 2 reduces the output of high quality good and increases the output of low quality good instead. The equilibrium in the case C corresponds to these areas. In the area VI, the relative superiority μ is moderate level, but is smaller than that, and the relative cost ratio c_{2H} is larger than the level of it in the areas, IV and V. Hence, firm 2 with the inefficient in production technology for high quality good stops producing good H and specializes in low quality good L. Two monopoly markets of both goods appear in this case. The equilibrium in the case B corresponds to this area. As the relative superiority μ reduces from the point (c_{2H}, μ) from the area VII and VIII, the efficient firm in production for high quality good reduces the output of good H and augment the output of low quality good. Thus substitution of production from high quality to low quality good advances steadily. At last, the equilibrium in the case A, thus in the area IX, firm 1 ceases to produce high quality good H and specializes in low quality good. In consequent, the market in the equilibrium becomes a duopoly of low quality good!

Next we provide a proposition on the equilibrium profits of firms for five nontrivial equilibria.

Proposition 2 *In the duopoly equilibrium of the game under rival's nonnegative quantities expectation presented above, the next inequalities hold among the profits of each firm*:

If $1 < \mu \leq 2$, $c_{2H} \geq 1$, then $\pi_1^{*A} = \pi_2^{*A}$.
If $4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{2(2c_{2H}^2 - c_{2H} + 2)})$, then $\pi_1^{*B} > \pi_2^{*B}$.
If $\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu, c_{2H} \geq 2$, then $\pi_1^{*C} > \pi_2^{*C}$.

If $\mu \leq 2c_{2H}$, $2 < \mu < 4$, then $\pi_1^{*D} > \pi_2^{*D}$.

Suppose that $1 < c_{2H} < 2$. If $1 < c_{2H} < 2$ and $\mu \geq 2c_{2H}$, then $\pi_1^{*E} > \pi_2^{*E}$.

[Insert Figure 2 here]

Note that the cost-efficient firm on high quality product earns more than the cost-inefficient firm at the equilibria in the cases except for cases A.

In the case A, taking the results of Proposition 1 and 2 into consideration together, in this area, we see that the relative superiority of high quality good μ is too small as compared with both of unit costs for high quality good H, so both firms specialize to good L and the market for good L becomes Cournot duopoly. Hence, two firms' equilibrium profits are exactly same.

4 Conjecture on the Relative Superiority and the Relative Cost Ratios

In the preceding section, we derived the equilibria of duopoly games in which each firm can choose its product line and output(s) of two vertically differentiated goods with non-negativity constraints and its expectation on its rival's product lie reaction. When we drive the equilibria, we assume that the relative superiority ratio μ and the relative cost ratio c_{2H} are common knowledge with both firms. However, in a real economy or a market, firms may not know μ and c_{2H} precisely. In this section, accordingly, we discuss about *ex post* firms' conjecture on the relative superiority and the relative cost ratios μ and c_{2H} from the realized outputs outcome in the equilibrium.

Suppose that both firms are sufficiently rational, so they can formulate our model and can solve it. Then, by observing the realized equilibrium their outputs levels in the market and referring proposition 1 and Figure 1, they can conjecture which equilibrium has been realized out of in the cases A, B, C, D and to which area of the point (c_{2H}, μ) that they are facing belongs out of nine areas $I, II, \cdots, IX$.

For example, suppose that both firs observe that they supply both goods H and L in the market. Then, from proposition 1, they can find that the equilibrium they face with is the one in the case E. Further, they also find that the realized outputs of the goods satisfy the inequality $0 < q_{iL}^{*E} \leq q_{jH}^{*E} < q_{jL}^{*E} < q_{iH}^{*E}$. Then, referring proposition 1 and Figure 1, they faces with the state point of (c_{2H}, μ) exactly belongs to the area II, and the relative cost ratio c_{2H} belongs to the interval $(1, 2)$ and the relative superiority μ belongs to the interval $[5/2, 7]$. Thus, they can learn that the difference unit cost of good H between own firm and rival is relatively small and their consumers' evaluation on the superiority ratio μ of good H to L is between two point five times and seven times.

Next, say, if they find that only a firm supplies good H but does not good L and its rival firm supplies only good L by observation the realized equilibrium outputs. Then, from proposition 1, they can find that the equilibrium they face with is the one in the case B! The can also learn from Figure 1 that the state point of (c_{2H}, μ) exactly belongs to the area IX. So both firms learn from this fact that the cost ratio c_{2H} is large as compared with the relative superiority of good H, μ!

Thus, they can conjecture on the state of the relative superiority and the relative cost ratio pair (c_{2H}, μ) they face with, from the realized equilibrium outputs outcome!

5 Conclusion

In this study, we consider duopoly game with two vertically differentiated products under nonnegative output constraints and its expectation about its rival's product line strategies. We derive an equilibrium for the game and characterize graphically firms' product line strategies and the realized profits of both firms in each equilibria through quality superiority and relative cost efficiency ratios.

We also show that the firm with cost efficiency for high quality good earns more than the firm with inefficient cost except for the special case in which the relative superiority of high quality good μ is too small as compared with both of unit costs for high quality good H, so both firms specialize to good L and the market for good L becomes Cournot duopoly. In this case, off course, both firms' profit is exactly same. We also illustrate that firms can correctly conjecture the *ex ante* relationship between the quality superiority of both goods and the relative cost efficiency ratios of firms on high quality good *ex post* in equilibrium by observing the realized equilibrium outputs and reffering the results on the equilibrium outputs in proposition 1 and Figure 1.

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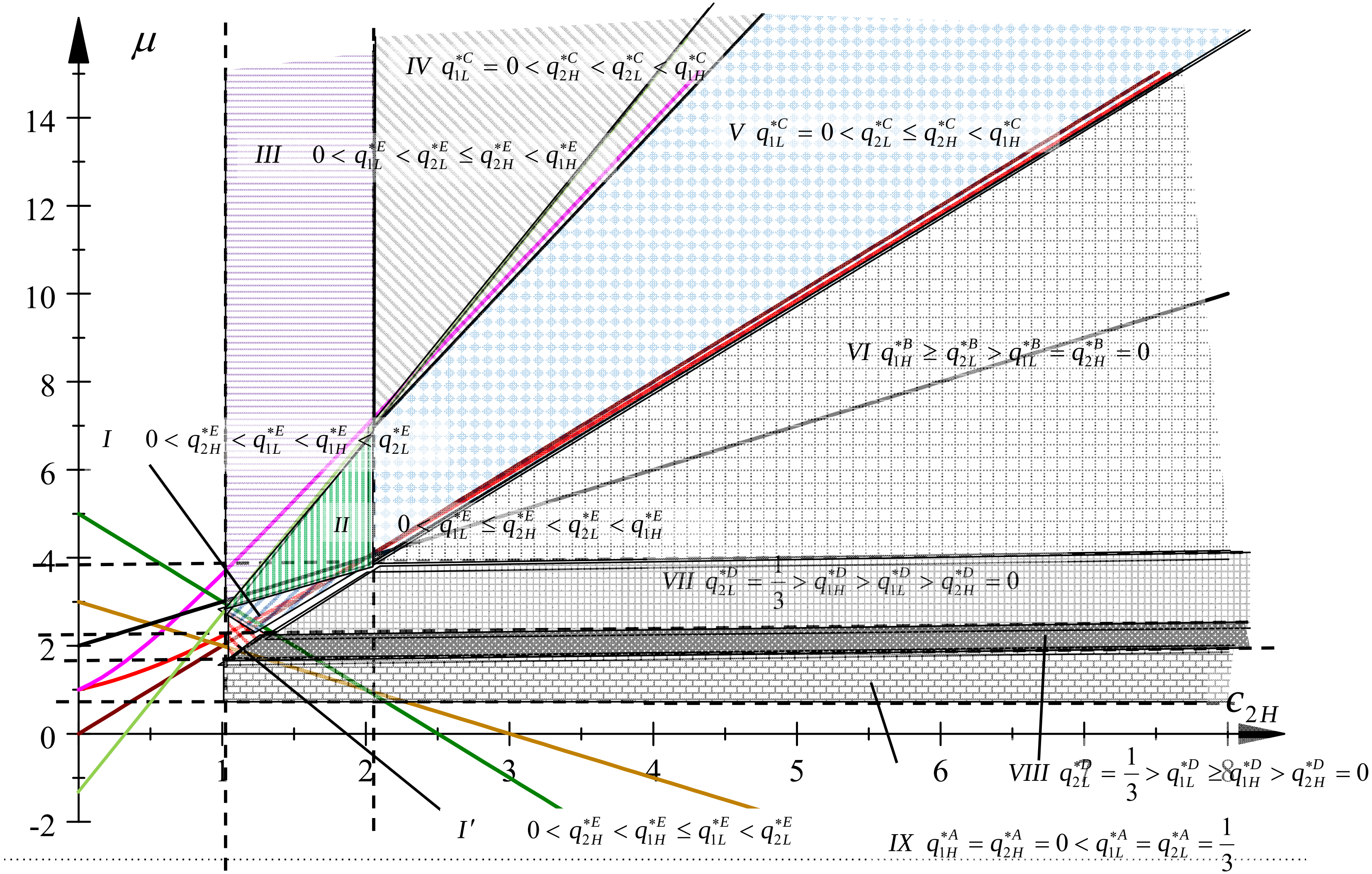

μ
14
12
10
8
6
4
2
0
-2
1
2
3
4
5
6
8
c_2H
IV $q_{1L}^{*C} = 0 < q_{2H}^{*C} < q_{2L}^{*C} < q_{1H}^{*C}$
III $0 < q_{1L}^{*E} < q_{2L}^{*E} \le q_{2H}^{*E} < q_{1H}^{*E}$
V $q_{1L}^{*C} = 0 < q_{2L}^{*C} \le q_{2H}^{*C} < q_{1H}^{*C}$
VI $q_{1H}^{*B} \ge q_{2L}^{*B} > q_{1L}^{*B} = q_{2H}^{*B} = 0$
I $0 < q_{2H}^{*E} < q_{1L}^{*E} < q_{1H}^{*E} < q_{2L}^{*E}$
II $0 < q_{1L}^{*E} \le q_{2H}^{*E} < q_{2L}^{*E} < q_{1H}^{*E}$
VII $q_{2L}^{*D} = \frac{1}{3} > q_{1H}^{*D} > q_{1L}^{*D} > q_{2H}^{*D} = 0$
VIII $q_{2L}^{*D} = \frac{1}{3} > q_{1L}^{*D} \ge q_{1H}^{*D} > q_{2H}^{*D} = 0$
I′ $0 < q_{2H}^{*E} < q_{1H}^{*E} \le q_{1L}^{*E} < q_{2L}^{*E}$
IX $q_{1H}^{*A} = q_{2H}^{*A} = 0 < q_{1L}^{*A} = q_{2L}^{*A} = \frac{1}{3}$

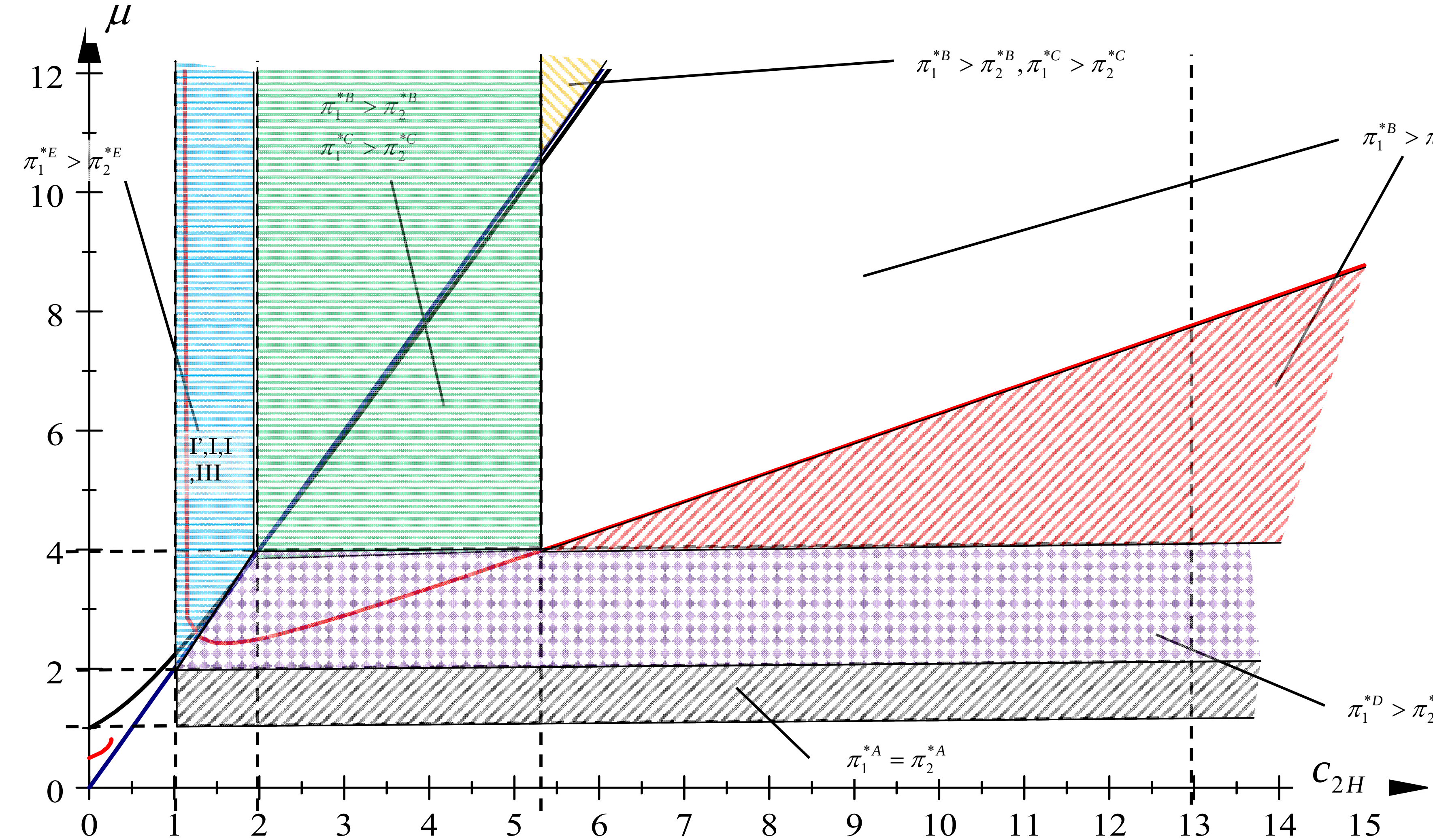

Figure 2   Profits Comparison in the Equilibia

$\mu = \frac{1}{2}\left(2c_{2H} + \sqrt{2(2c_{2H}^2 - c_{2H} + 2)}\right)$ Black

$\mu = \frac{1}{8(c_{2H}-1)}\left(2(c_{2H}+3)(c_{2H}-1) + \sqrt{4(c_{2H}-1)(3c_{2H}+3c_{2H}^2+c_{2H}^3-1)}\right)$ Red Light

$\mu = 2c_{2H}$ Blue