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Population Aging, Labor Market Frictions, and PAYG Pension

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Abstract

Employing a two-period OLG model with labor market frictions and PAYG pension, this paper examines the effects of population aging on the unemployment rate and the per capita output of the economy. We show that in economies in which the population growth rate is already low and the size of PAYG pension is relatively large, a further decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy in the short run, but it increases the unemployment rate and reduces the per capita output of the economy in the long run.

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1 Introduction

Most advanced countries facing rapidly aging populations are expected to confront a decline in their labor force in the next few decades. Several countries such as Japan and Germany are already experiencing a fall in their labor force.[1] Due to a low and/or declining fertility rate in past decades, the inflow of young people into the labor force will be smaller than the outflow of older workers who retire, resulting in a natural decline of the population of working age. If activity rates remain constant, this will result in a decrease of the labor force. Several research reports provided by some think tanks and international organizations argue that such a decline in the working-age population may cause a tight labor market in which labor demand exceeds labor supply (e.g., Richard and Amico, 1997; OECD, 2003, 2013; Ganelli and Miyake, 2015). In the medium term, the increasing number of retirees will, in some occupations, lead to a replacement demand that will be hard to fill from domestic labor supplies, which may result in a labor force shortage or a reduced unemployment rate. However, because the projections of long-term labor demand and supply are highly conjectural, there has been intense debate on the plausibility of these labor shortage arguments (e.g., Freeman, 2006; Garloff and Wapler, 2016).

To the best of our knowledge, except for a few papers mentioned later, these existing studies of population aging and labor markets are based on partial equilibrium models and restrict their analyses to the direct effects of population aging within the labor market. However, population aging, triggered by a rise in life expectancy and a decline in fertility rates, induces changes in individuals' saving and investment behaviors, which may provide non-negligible indirect influences on the labor market through its general equilibrium effects. Therefore, in this paper, we propose a tractable general equilibrium growth model to analyze the effects of population aging on the labor market and the macro economy with emphasis on the role of pay-as-you-go (PAYG) pension systems. Because wage incomes are the primary sources of financing for public pension systems, interactions between labor market and pension systems are of particular relevance (e.g., Corneo and Marquardt, 2000; Kemnitz, 2003; Bräuninger, 2005; Ono, 2010). This paper develops a simple two-period overlapping generations (OLG) model with labor market frictions and PAYG pension and examines how population aging caused by a decline in the population growth rate influences the unemployment rate and

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[1]According to government latest projections, Japan's labor force is projected to shrink in the next two decades. The labor force is expected to decline from 66.3 million in 2010 to 56.8 million in 2030 under a "negative" scenario in which real growth remains near 0 percent and the labor force participation rate drops from 59.6 percent in 2010 to 54.3 percent in 2030. Even in a "positive scenario" with real average growth at 2 percent and the labor participation rate increasing to 60.1 percent by 2030, the labor force would still decrease to 62.9 million.

the per capita output of the economy.

In the model presented here, we first consider the case where a PAYG pension system is financed by a defined-benefit scheme. Under this pension payout scheme, we show that in economies in which the population growth rate is already low and the size of PAYG pension is relatively large, a further decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy in the short run, but it increases the unemployment rate and reduces the per capita output of the economy in the long run. These results are intuitively explained as follows. In our model, the unemployment rate and the capital per operating firms are negatively related. In the short run, a decline in the population growth rate mitigates the dilution of savings by the previous generation, increasing the level of capital per operating firms and thereby reducing the unemployment rate and increasing the per capita output of the economy. However, in the long run, a further decline in the population growth rate leads to a surge in the social security tax rate, decreasing the level of capital per operating firms and thereby increasing the unemployment rate and reducing the per capita output of the economy.

Furthermore, in the model presented here, we consider the two different types of pension payout schemes: a defined-contribution scheme and a tax adjusted defined-benefit scheme. Then, we examine how the introduction of these two types of pension payout schemes affects the short run as well as the long run effects of population aging on the unemployment rate and the per capita output of the economy. Under these two types of PAYG pension payout schemes, we find that a decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy not only in the short run, but also in the long run. Therefore, we can confirm that the design of pension payout scheme matters to mitigate the long run negative effects of population aging on the employment rate and the per capita output of the economy.

This paper is related to several branches of the literature.[2] First, this paper relates to the literature on pensions and unemployment in the context of two-period OLG growth models (e.g., Corneo and Marquardt, 2000; Kemnitz, 2003; Bräuninger, 2005; Ono, 2010). However, most of these studies employ collective wage-bargaining setting, and rather concern the growth and welfare implications of pension and unemployment insurance policies. This paper differs from these

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[2]This paper also shares the broad research question with the literature on the aging of labor force (i.e., the rise in average age of people in the labor force). This literature examines how the changes in the age structure of the labor force influence aggregate or age-specific unemployment rates and wage rates. For example, using data on most OECD countries from 1970 and 1994, Korenman and Neumark (2000) estimate that large youth cohorts lead to large increases in the relative unemployment rates of youth, with elasticities as high as 0.5 or 0.6. See Dixson (2003) for a literature review.

studies in that we employ the model with labor market frictions as in Bean and Pissarides (1993), which rather concerns the impacts of population aging on the unemployment rate and the per capita output of the economy.

Second, this paper relates to the series of two-period OLG studies that discuss the growth impacts of population aging, often paying attention to the adjustments required in the PAYG pension system (e.g., Zhang et al. 2001, 2003, 2005; Fanti and Gori, 2012; Artige et al, 2014; Tabata, 2015). Among these studies, those by Artige et al. (2014) and Tabata (2015) are closely related to our contributions because they emphasize the design of the PAYG pension payout scheme as a factor to determine the effects of population aging on capital accumulation. Artige et al. (2014) and Tabata (2015) show that the effect of population aging on capital accumulation is always positive under a defined-contribution scheme, whereas the effect is ambiguous under a defined-benefit scheme. Although our theoretical results are partly indebted to their contributions, in contrast to them, we consider the effects of population aging on the unemployment rate through its impact on capital accumulation, which is not examined explicitly in these studies.

Third, this paper is related to numerical simulation studies that quantify the impact of demographic changes on the macroeconomy (e.g., Börsch-Supan, 2001; Attanasio et al. 2006, 2007; Krueger and Ludwig, 2007; de la croix et al., 2013). These studies employ large-scale OLG models similar to those of Auerbach and Kotolicoff (1987). Among them, this paper is closely related to de la croix et al. (2013), which introduce labor market frictions into a standard OLG set up and examine the effects of population aging and pension reform on the unemployment rate and other macroeconomic variables. Using recent demographic projections in France, they show that population aging reduces the unemployment rate through its effect on capital accumulation, the interest rate and job openings. They also show that introducing labor market frictions changes the quantitative effects of pension reforms. These existing numerical simulation studies are quite appealing and plausible. We share the same broad question with this literature; however, because their purpose is to provide realistic quantitative projections of several macroeconomic variables, these models possess relatively complicated structures and mechanisms. To achieve this result, the values of several exogenous variables are changed simultaneously in their simulations. Consequently, it is sometimes difficult to understand which specifications or assumptions are responsible for deriving the simulation results. In this sense, these numerical models are not very tractable. Therefore, to complement these existing numerical studies, we construct a tractable two-period OLG model that enables us to examine analytically how population aging caused by a decline in the population growth rate influences the unemployment rate and the per capita output of the economy through its impact on capital accumulation.

This paper is organized as follows. Section 2 establishes the basic model.

Section 3 investigates the dynamic behavior in equilibrium and derives the relationship between the unemployment rate and the capital accumulation under a defined-benefit PAYG pension scheme. Section 4 focuses on the steady-state equilibrium and examines the long run effects of population aging on the unemployment rate and the per capita output of the economy. Section 5 focuses on the transitional phase and examines numerically the short run effects of population aging on the unemployment rate and the per capita output of the economy. Section 6 examines how the alternative design of pension payout schemes affects the main results of this paper. Section 7 compares the theoretical results obtained in Sections 4 to 6. Section 8 concludes the paper.

2 The model

We consider a two-period-lived overlapping generations model where the economy comprises many identical firms, ex ante identical individuals and a government. We denote the generation born in period t as the generation t. Time is discrete and denoted by $t = 0, 1, 2, \cdots$. In each period t, N_t individuals are born and live for two periods, youth and old age. The population grows at the constant rate n: $N_t = (1 + n)N_{t-1}$, where $n \in (-1, \infty)$. In each period t, there exist only two generations: the active working young and the retired old. Only young individuals are endowed with one unit of labor and have an opportunity to work by matching with a firm, so N_t is also the size of the total labor force potentially supplied in each period t. Unemployment in this model occurs only in youth.

The old-age dependency ratio (i.e., the ratio of old dependents to the young working population) in period t is given by $\frac{N_{t-1}}{N_t} = \frac{1}{1+n}$. Thus, population aging is triggered if there is a decrease in the population growth rate n. Further, the shrinking workforce (i.e., $N_t < N_{t-1}$) occurs when the population growth rate n is sufficiently low to satisfy $n < 0$.

2.1 Technology and firms

Many identical firms produce final goods with the same production technology. In addition to capital, one worker is necessary for a firm to produce the final goods. More concretely, workers and firms with vacant positions search for each other in the labor market. Firms that successfully match with a worker can operate their business. Firm i produces final goods $y_{i,t}$ at period t with a Cobb-Douglas production technology: $y_{i,t} = Ak_{i,t}^\alpha x_{i,t}^{1-\alpha}$, where $\alpha \in (0, 1)$ is a capital share of output, $k_{i,t}$ is capital per firm i, which depreciates in one period, $x_{i,t}$ is labor employed by firm i, and A is productivity of the technology. Because an operating firm hires only one worker, eventually it holds that $x_{i,t} = 1$, and the production function is

expressed as follows:

$$y_{i,t} = Ak_{i,t}^{\alpha}. \qquad (1)$$

Because the capital market is competitive, capital is paid its marginal product:

$$r_t = \alpha Ak_{i,t}^{\alpha-1} = \alpha Ak_t^{\alpha-1}, \qquad (2)$$

where r_t is the rental price of capital. The firm-specific index i is dropped because each firm employs the same amount of capital, facing the common rental price of capital. Then, the remainder of output to be allotted between firm i and its worker is given by

$$\pi_t = (1 - \alpha)Ak_t^{\alpha}. \qquad (3)$$

2.2 Individuals

Each individual derives utility from his or her own consumption in both youth and old age. Thus, the lifetime utility of an individual born in period t (i.e., generation t) is expressed as

$$U_t^j = (c_{1,t}^j)^{\beta}(c_{2,t+1}^j)^{1-\beta}, \qquad (4)$$

where $c_{1,t}^j$ is consumption in youth, $c_{2,t+1}^j$ is consumption in old age and $\beta \in (0, 1)$ is the utility weight given to the consumption in youth relative to the consumption in old age. The superscript j denotes the status of labor in youth: $j = e$ and $j = u$ if an individual is employed and unemployed, respectively. The status is assigned according to the matching process between the firm and the worker (described later) at the beginning of each period. The specification of utility function makes the analysis of the bargaining process between the firm and the worker (described later) tractable.

An individual chooses consumption and savings to maximize his or her lifetime utility under the following budget constraints:

$$c_{1,t}^j + s_t^j = \omega_t^j = \begin{cases} (1 - \eta_t - \tau_t)w_t, & \text{if } j = e \\ \bar{\gamma}_t, & \text{if } j = u, \end{cases} \qquad (5)$$

$$c_{2,t+1}^j = r_{t+1}s_t^j + \bar{b}_{t+1}, \qquad (6)$$

where ω_t^j is type-j individual's net income in youth, w_t is wage, $\bar{\gamma}_t$ is unemployment-insurance benefits, s_t^j is savings, r_{t+1} is gross interest rate,[3] η_t is the tax on labor income to cover unemployment insurance benefits, τ_t is the tax on labor income

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[3]Because the depreciation rate of capital is assumed to be 1, the gross interest rate equals to the rental price of capital.

to cover pension benefits, and $\bar{b}_{t+1}$ is pension benefits for both employed and unemployed. Unemployment benefits are assumed to be exempt from taxation. To preserve tractability, as in Bräuninger (2005) and Ono (2010), we consider the case where the unemployed receives pension benefits without any contributions. Of course, the above payment scheme is rather extreme and cannot capture the complex structures of recent social security systems in their entirety. Nevertheless, this simple framework improves the tractability of the model greatly without changing the qualitative implications of this paper.[4]

By solving the utility-maximization problem, we obtain the saving function of a type-j individual as follows:

$$s_t^j = (1 - \beta)\omega_t^j - \beta\frac{\bar{b}_{t+1}}{r_{t+1}}. \tag{7}$$

The above function states that a higher wage level or unemployment-insurance benefits implies higher savings, whereas a higher tax rate or pension benefit implies lower savings. The corresponding consumption functions are $c_{1,t}^j = \beta(\omega_t^j + \frac{\bar{b}_{t+1}}{r_{t+1}})$ and $c_{2,t+1}^j = (1 - \beta)r_{t+1}(\omega_t^j + \frac{\bar{b}_{t+1}}{r_{t+1}})$. Thus, using these functions, the indirect utility function of a type-j individual is given by:

$$U_t^j = \beta^\beta[(1 - \beta)r_{t+1}]^{1-\beta}(\omega_t^j + \frac{\bar{b}_{t+1}}{r_{t+1}}). \tag{8}$$

2.3 Unemployment insurance

Unemployment insurance provides an intragenerational transfer from the employed to the unemployed. This transfer system is balanced in each period, thereby yielding the following equality:

$$(N_t - L_t)\bar{\gamma}_t = \eta_t w_t L_t, \tag{9}$$

where L_t is the number of employed and $N_t - L_t$ is the number of unemployed. The left hand side is the total expenditure for unemployed benefits, and the right hand side is the total revenue raised from the employed. The unemployed benefits are paid to unemployed workers according to the following defined-benefit payment rule:

$$\bar{\gamma}_t = \gamma(1 - \eta_t - \tau_t)w_t, \tag{10}$$

where $\gamma \in (0, 1)$. The unemployment benefit payment to each unemployed worker in period t is proportional to but less than the after tax labor income of employed worker in period t.

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[4]Even if we consider the case where government imposes lump sum taxes for both employed and unemployed to cover the both pension and unemployment benefits, the qualitative implications of this paper do not change at all.

From (9) and (10), the unemployment insurance tax rate is given by $\eta_t = \frac{\gamma(1-l_t)}{l_t+\gamma(1-l_t)}(1-\tau_t)$, where $l_t \equiv \frac{L_t}{N_t} \in [0,1]$. The higher unemployment benefit γ, the lower employment rate l_t and the lower pension tax rate τ_t lead to the higher unemployment insurance tax rate.

2.4 Pensions

Pensions are financed by a pay-as-you-go system. The pension system provides an intergenerational transfer from the young to the old. All pension benefits in period t are financed by the total amount of contributions paid by the employed in that period. The revenue constraint is given by:

$$\bar{b}_t N_{t-1} = \tau_t w_t L_t, \tag{11}$$

where the left hand side is the total expenditure composed of the payments to all the old individuals and the right hand side is the total revenue raised from the employed. The pension benefits are paid to all old individuals according to the following defined-benefit payment rule:

$$\bar{b}_t = b w_t l_t, \tag{12}$$

where $b \in [0,1)$ and $l_t \equiv \frac{L_t}{N_t} \in [0,1]$. The pension payment to each old individual in period t is proportional to but less than the average before tax labor income of the young generation in period t.[5] We denote this pension payment rule as a defined-benefit scheme. In a defined-benefit scheme, the pension benefit for each old individual is adjusted according to changes in the young generations' wages w_t or their employment environments l_t (i.e., wage indexation). However, it is not adjusted in response to changes in demographic conditions or young generations' social security burden (i.e., non-demographically modified wage indexation). This specification of pension payment rule is crucial to derive our main theoretical results. Section 6 briefly examines the cases of other pension payout schemes in which pension benefit is fully adjusted in response to changes in demographic conditions or young generations' social security burden.

From (11) and (12), the pension tax rate is given by $\tau_t = \tau(n) \equiv \frac{b}{1+n}$. The higher pension benefit b and the lower population growth rate n lead to the higher pension tax rate. To emphasize this negative relationship between population growth rate and pension tax rate, we describe τ as $\tau(n)$. Because $\tau \in (0,1)$, the following parameter conditions must hold.

$$n \in (-(1-b), \infty).$$

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[5]The average before tax labor income of the young generation in period t is given by $\frac{w_t L_t}{N_t} = w_t l_t$.

2.5 Labor market

We introduce labor-market matching frictions in the model along the same line as Bean and Pissarides (1993) and Hashimoto, Im, and Kunieda (2016). Although the matching mechanism follows from the standard unemployment model (e.g., Diamond, 1982; Mortensen and Pissarides, 1999; Petrongolo and Pissarides; 2001), there is no time lag between a match of parties and a start of business operation in the current model.

2.5.1 Matching mechanism

Because workers and firms face matching frictions, unemployment occurs in equilibrium although each young individual is endowed with one unit of labor that is supplied inelastically if he or she is employed. The number of successful matches are given by $F(N_t, v_t)$, which is a function of the population of workers N_t, and the number of firms with vacancy v_t, where $0 \leq F(N_t, v_t) \leq min\{N_t, v_t\}$ for $N_t \in [0, \infty)$ and $v_t \in [0, \infty)$, $F(0, v_t) = 0$, $F(N_t, 0) = 0$, $\lim_{N_t \to \infty} F(N_t, v_t) = v_t$ and $\lim_{v_t \to \infty} F(N_t, v_t) = N_t$. The matching function $F(N_t, v_t)$ is continuously differentiable, concave, homogeneous of degree one, and increasing with respect to both N_t and v_t. The tightness of the labor market is expressed by $\theta_t \equiv \frac{v_t}{N_t} \in (0, \infty)$, which is considered the jobs-to-applicants ratio, and the probability that a firm with a vacancy matches with a worker is given by $F(N_t, v_t)/v_t = F(1/\theta_t, 1) \equiv q(\theta_t)$ where $q'(\theta_t) < 0$, $\lim_{\theta_t \to 0} q(\theta_t) = 1$ and $\lim_{\theta_t \to \infty} q(\theta_t) = 0$. Because the number of employment is equal to the number of successful matches, it follows that $L_t = l_t N_t = F(N_t, v_t)$, which is rewritten as

$$l_t = \frac{F(N_t, v_t)}{N_t} = F(1, \theta_t) = \theta_t q(\theta_t) \equiv l(\theta_t). \tag{13}$$

Equation (13) yields the employment rate l_t as a function of θ_t. Because $\frac{\partial F(1, \theta_t)}{\partial \theta_t} > 0$, we can easily confirm that the relations $l'(\theta_t) > 0$, $\lim_{\theta_t \to 0} l(\theta_t) = 0$ and $\lim_{\theta_t \to \infty} l(\theta_t) = 1$ hold. Because unemployment rate u_t is given by $u_t \equiv \frac{N_t - L_t}{N_t} = 1 - l_t$, (13) is rewritten as

$$u_t = 1 - l(\theta_t) \equiv u(\theta_t), \tag{14}$$

where $u'(\theta_t) < 0$, $\lim_{\theta_t \to 0} u(\theta_t) = 1$ and $\lim_{\theta_t \to \infty} u(\theta_t) = 0$. Therefore, as shown in the upper panel of Figure 1, (13) and (14) derive a positive (resp., negative) relationship between the employment rate (resp. the unemployment rate) and the labor-market tightness, which is the so-called Beveridge curve.

A successful match enables a firm to produce the final goods. Because $q(\theta_t)$ is the probability that a firm matches with a worker in period t, the firm's expected profits V_t are given by

$$V_t = q(\theta_t)(\pi_t - w_t) - h, \tag{15}$$

where h is the search cost in the labor market that the firm incurs when searching for a worker. Because the upper limit of $q(\theta_t)$ is one, if $\pi_t - w_t < h$, no firms operate because the expected profits are negative. In other words, only if $\pi_t - w_t \geq h$, successful matches occur between workers and firms. In the following analysis, we proceed our analysis for the case in which the relation $\pi_t - w_t \geq h$ holds. The parameter conditions that ensure $\pi_t - w_t \geq h$ are discussed later. Under these assumptions, the free-entry condition for the final goods sector leads to zero profits of each firm. Accordingly, it follows that $V_t = 0$, or equivalently

$$\pi_t - w_t = \frac{h}{q(\theta_t)}. \tag{16}$$

2.5.2 Nash bargaining

The remainder of the output after payments to capital is allotted between the firm and its worker. The shares to each are determined by maximizing the following Nash product with respect to the wage:

$$w_t = \arg \max_{w_t} (U_t^e - U_t^u)^\phi (\pi_t - w_t)^{1-\phi} = \{\beta^\beta [(1-\beta)r_{t+1}]^{(1-\beta)}\}^\phi [(1-\eta_t-\tau_t)w_t - \bar{\gamma}_t]^\phi (\pi_t - w_t)^{1-\phi}, \tag{17}$$

where $\phi \in (0,1)$ is the worker's bargaining power. From the Nash bargaining solution, it follows that

$$w_t = \phi \pi_t + \frac{1-\phi}{(1-\eta_t-\tau_t)}\bar{\gamma}_t. \tag{18}$$

Note that when the firm and its worker are bargaining, they do not have any information about the government's unemployment policy concretely, and thus, the Nash product is maximized with $\bar{\gamma}_t$ given. Because the government pays the unemployment benefits to unemployed workers in such a way that $\bar{\gamma}_t = \gamma(1-\eta_t-\tau_t)w_t$, inserting (3) and (10) in (18) yields

$$w_t = \Omega \pi_t = \Omega(1-\alpha)Ak_t^\alpha, \tag{19}$$

where $\Omega \equiv \frac{\phi}{1-(1-\phi)\gamma} \in (0,1)$ is the worker's output share of π_t. Note from $\Omega = \frac{\phi}{1-(1-\phi)\gamma}$ that the larger outside option γ and the larger Nash bargaining power ϕ lead to the greater worker's share. Substituting (3) and (19) into (16) yields

$$(1-\Omega)(1-\alpha)Ak_t^\alpha = \frac{h}{q(\theta_t)}. \tag{20}$$

It is noted from (20), that given parameter values, if k_t is sufficiently small to satisfy $(1-\Omega)(1-\alpha)Ak_t^\alpha < h$, firms cannot cover a search cost h, because the upper

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limit of $q(\theta_t)$ is 1. Therefore, in the following analysis, we focus our analysis on the case in which the following inequality holds:

$$k_t > \bar{k}, \quad \forall \, t > 0, \tag{21}$$

where $\bar{k} \equiv [\frac{h}{(1-\Omega)(1-\alpha)A}]^{\frac{1}{\alpha}}$.

3   Equilibrium

The equilibrium is characterized by the optimization conditions of individuals and firms, the outcomes of the Nash bargaining in the labor market, and the market clearing conditions for capital.

3.1   Beveridge curve and capital accumulation

Equation (20) can be rewritten as follows:

$$\theta_t = q^{-1}(\frac{h}{(1 - \Omega)(1 - \alpha)Ak_t^\alpha}) \equiv \theta(k_t). \tag{22}$$

Because of the properties of $q(\theta_t)$ function explained in Section 2-5, as shown in the lower panel of Figure 1, it is straightforward to show that the relations $\theta'(k_t) > 0$, for $k_t \in (\bar{k}, \infty)$, $\lim_{k_t \to \bar{k}} \theta(k_t) = 0$ and $\lim_{k_t \to \infty} \theta(k_t) = \infty$ hold. Capital accumulation increases the demand for labor, which positively affects the tightness of the labor market. Following Pissarides (2000), we denote equation (22) as the job-creation condition.

Substituting the job-creation condition of (22) into the Beveridge curve of (13), we obtain

$$l_t = l(\theta(k_t)) \equiv l(k_t). \tag{23}$$

Because of the properties of $l(\theta_t)$ function explained in (13), we can easily confirm that the relations $l'(k_t) > 0$, for $k_t \in (\bar{k}, \infty)$, $\lim_{k_t \to \bar{k}} l(k_t) = 0$ and $\lim_{k_t \to \infty} l(k_t) = 1$ hold. As shown in Figure 1, capital accumulation promotes employment, rendering the labor market tighter.

3.2   Dynamics

The market-clearing condition for capital is given by

$$K_{t+1} = s_t^e L_t + s_t^u(N_t - L_t). \tag{24}$$

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By substituting (5), (7) and (9) into (24), we obtain

$$K_{t+1} = (1 - \beta)(1 - \tau_t)w_t L_t - \beta \frac{\bar{b}_{t+1}}{r_{t+1}} N_t. \tag{25}$$

Using $\tau(n) = \frac{b}{1+n}$, the per capita pension benefit $\bar{b}_t$ in (12) is rewritten as $\bar{b}_t = (1 + n)\tau(n)w_t l_t$. By substituting (2), (3), (19), $\bar{b}_{t+1} = (1 + n)\tau(n)w_{t+1}l_{t+1}$ and $N_{t+1} = (1 + n)N_t$ into (25), the dynamics of aggregate capital K_t is given by

$$K_{t+1} = \frac{(1 - \beta)[1 - \tau(n)]}{1 + \beta\tau(n)\Omega^{\frac{1-\alpha}{\alpha}}} \Omega(1 - \alpha)A K_t^{\alpha} L_t^{1-\alpha}, \tag{26}$$

where $K_t \equiv k_t L_t$.

We then consider the dynamics of aggregate employment L_t. Because of the labor-market matching frictions, the aggregate employment L_t is given by $L_t = l_t N_t$. Thus, using (23) and $N_{t+1} = (1 + n)N_t$, the dynamics of the aggregate employment L_t is described by

$$L_{t+1} = \frac{l(k_{t+1})}{l(k_t)}(1 + n)L_t. \tag{27}$$

Using (26) and (27), we can derive an autonomous difference equation with respect to k_t as follows:

$$l(k_{t+1})k_{t+1} = \Gamma(n, \tau(n))l(k_t)k_t^{\alpha}, \tag{28}$$

where

$$\Gamma(n, \tau(n)) \equiv \frac{(1 - \beta)[1 - \tau(n)]\Omega(1 - \alpha)A}{(1 + n)[1 + \beta\tau(n)\Omega^{\frac{1-\alpha}{\alpha}}]} = \frac{(1 + n - b)(1 - \beta)\Omega(1 - \alpha)A}{(1 + n + \beta b \Omega^{\frac{1-\alpha}{\alpha}})(1 + n)}.$$

Because we focus on the case in which $k_t > \bar{k}$ for all $t \geq 0$ as discussed in the previous section, the domain of the dynamical system in (28) is given by $k_t \in (\bar{k}, \infty)$. Given the initial aggregate capital K_0 and population size N_0, the initial capital per operating firm k_0 is determined uniquely by (23) and $k_0 l_0 N_0 = K_0$, which means that capital per operating firm is pre-determined in period 0.

3.3 Steady states and stability

From (28), we can easily confirm that the following proposition hold.

Proposition 1 *In the dynamic system of equation (28), there exists a unique (non-trivial) steady-state k^* such that*

$$k^* = [\Gamma(n, \tau(n))]^{1/(1-\alpha)} \equiv k^*(n). \tag{29}$$

To emphasize the relationship between the steady-state capital per operating firm and the population growth rate, we describe k^* as $k^*(n)$. Because $l(k_{t+1})k_{t+1} - l(k_t)k_t = l(k_t)[\Gamma(n, \tau(n))k_t^\alpha - k_t]$, the inequality $l(k_{t+1})k_{t+1} \geq (<)l(k_t)k_t$ holds, if $k_t \leq (>)k^*$. Further, noting $1 = \Gamma(n, \tau(n))k^{*\alpha-1}$ and $\alpha \in (0, 1)$, the differentiation of (28) with respect to k_t around the steady-state k^* yields

$$\frac{dk_{t+1}}{dk_t}\bigg|_{k_{t+1}=k_t=k^*} = \frac{l'(k^*)k^* + l(k^*)\alpha}{l'(k^*)k^* + l(k^*)} < 1. \tag{30}$$

The following proposition summarizes the result.

Proposition 2 *In the dynamical system of equation (28), the steady-state k^* is stable.*

Figure 2 illustrates the possible dynamic behavior of the economy. Given the initial aggregate capital K_0 and population size N_0, the initial capital per operating firm k_0 is determined uniquely, and thus, the economy eventually converges to the unique steady-state equilibrium E with k^*. The parameter conditions that ensures $k^* > \bar{k}$ is given by

$$A > \frac{h^{1-\alpha}}{(1 - \Omega)^{1-\alpha}(1 - \alpha)}[\frac{(1 + n)(1 + \beta\tau\Omega\frac{1-\alpha}{\alpha})}{(1 - \beta)(1 - \tau)\Omega}]^\alpha. \tag{31}$$

4   Long run effects of population aging

In this section, we focus on the steady-state equilibrium and examine the long run effects of population aging on the unemployment rate and the per capita output of the economy under a defined-benefit PAYG pension scheme. To understand this issue, we need to understand the relationship between the population growth rate and the steady-state capital per operating firm.

4.1   Steady-state capital per operating firm

In this subsection, we examine how population aging caused by a decline in the population growth rate influences the steady-state capital per operating firm under a defined-benefit PAYG pension scheme. From (29), regarding the effect of the population growth rate n on the steady-state capital per operating firm k^*, we obtain the following proposition.

Proposition 3 *Under PAYG pension system with a defined-benefit scheme, the following statements hold:*

(1) Suppose $b \in (0, 1)$, then there exists an unique $\hat{n} \in (-(1-b), \infty)$ such that $k^*(\hat{n}) \geq k^*(n)$, $\forall n \in (-(1-b), \infty)$, $\frac{\partial k^*(n)}{\partial n} > 0$ $\forall n \in (-(1-b), \hat{n})$, $\frac{\partial k^*(n)}{\partial n} < 0$ $\forall n \in (\hat{n}, \infty)$, where

$$\hat{n} \equiv -(1-b) + b \sqrt{1 + \beta\Omega\frac{1-\alpha}{\alpha}}.$$

(2) Suppose $b = 0$, then $\frac{\partial k^*(n)}{\partial n} < 0$ holds.

Proof of Proposition 3 is given in Appendix. Proposition 3 indicates that when a PAYG pension system is financed by a defined-benefit scheme (i.e., $b \in (0, 1)$), there is an inverted U-shaped relationship between the population growth rate and the steady-state capital per operating firm. However, when there is no PAYG pension system (i.e., $b = 0$), the relationship between the population growth rate and the steady-state capital per operating firm is always negative.

Figure 3-1 shows numerical examples of the relationship between the population growth rate and the steady-state capital per operating firm under alternative values b of pension payout (i.e., $b = 0$, 0.2, 0.4 and 0.6). In the simulation, the matching function is specialized as $F(N_t, \upsilon_t) = (N_t\upsilon_t)/(N_t^\sigma + \upsilon_t^\sigma)^{1/\sigma}$, following Den Haan et al. (2000). Under this matching function, from (22) and (23), the employment rate l_t is given by $l_t = \{1 - [h/(1-\Omega)(1-\alpha)Ak_t^\alpha]^\sigma\}^{1/\sigma}$. The parameters used in the baseline simulations are given in footnote 6, and its explanation is provided in the Appendix.[6] Note that the objective of these numerical examples is not to calibrate our simple model to actual data but to supplement the qualitative results. The quantitative results obtained in this paper should be interpreted with caution. Consistent with Proposition 3, when $b \in (0, 1)$, there is an inverted U-shaped relationship between the population growth rate and the steady-state capital per operating firm. However, when $b = 0$, the relationship between the population growth rate and the steady-state capital per operating firm is always negative.

The mechanism of this inverted U-shaped relationship between n and k^* under a defined-benefit scheme is explained as follows. Equation (28) implies that a decline in the population growth rate n exerts two competing influences on capital accumulation. Under a defined-benefit scheme, a decline in the population growth rate n increases the old-age dependency ratio $\frac{1}{1+n}$, which positively affects the pension tax rate τ. This higher pension tax rate leads to a lower rate of saving by young individuals, which negatively affects capital accumulation. We denote this negative effect of a decline in n on capital accumulation as the "tax burden effect". This tax burden effect does not exist when there is no PAYG pension system (i.e., $b = 0$). Conversely, a decline in the population growth rate n mitigates the dilution

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[6]$\alpha = 0.5$, $\gamma = 0.6$, $\phi = 0.5$, $n = 0$ $b = 0.4$, $h = 1$ and $A = 8.907$

of savings and thus positively affects capital accumulation. We denote this positive effect of a decline in n on capital accumulation as the "anti-dilution effect". This anti-dilution effect always exists irrespective of the existence of PAYG pension system. Therefore, a decline in the population growth rate always increases the steady-state capital per operating firm when there is no pension system (i.e., $b = 0$).

These results suggest that in economies in which the population growth rate is already low and the size of PAYG pension is relatively large under a defined-benefit scheme, a further decline in the population growth rate reduces the steady-state capital per operating firm, because the negative "tax burden effect" on capital accumulation dominates the positive "anti-dilution effect". The existence of an inverted U-shaped relationship between the population growth rate and the steady-state per capita capital under a defined-benefit PAYG pension scheme has been recognized in the existing literature (e.g., Artige et al., 2014; Tabata, 2015). This paper confirms that the analogous prediction holds in the model with labor market frictions.

4.2 Steady state unemployment rate and the per capita output of the economy

In this subsection, we examine how population aging caused by a decline in the population growth rate influences the steady-state unemployment rate and the per capita output of the economy under a defined-benefit PAYG pension scheme.

From (14) and (23), the equilibrium unemployment rate u_t is given by

$$u_t = 1 - l(k_t). \tag{32}$$

Because $l'(k_t) > 0$, the unemployment rate is negatively related to the capital per operating firm k_t, because capital accumulation enhances employment rate by rendering the labor market tighter. Therefore, from (32) and Proposition 3, we can easily confirm that when a PAYG pension system is financed by a defined-benefit scheme (i.e., $b \in (0, 1)$), there is an U-shaped relationship between the population growth rate and the steady-state unemployment rate. However, when there is no PAYG pension system (i.e., $b = 0$), the relationship between the population growth rate and the steady-state unemployment rate is always positive. Numerical simulation results in Figure 3-2 confirm that these predictions hold under plausible benchmark parameter values.

Because the capital per operating firm is the dominant factor to determine the economy's unemployment rate from (32), the mechanism of the U-shaped relationship between the population growth rate and the steady-state unemployment rate is the same as that of the inverted U-shaped relationship between the

population growth rate and the steady-state capital per operating firm. Thus, when a PAYG pension system is financed by a defined-benefit scheme (i.e., $b \in (0, 1)$), there is a U-shaped relationship between the population growth rate and the steady-state unemployment rate due to the two competing influences of a decline in n on capital accumulation (i.e., the "tax burden effect" vs. the "anti-dilution effect"). However, when there is no PAYG pension system (i.e., $b = 0$), the only positive "anti-dilution effect" on capital accumulation prevails, and thus, a decline in the population growth rate always leads to the lower steady-state unemployment rate.

We then consider the long run effects of population aging on the per capita output of the economy. The per capita output $\tilde{y}_t$ in this economy is given by $\tilde{y}_t \equiv \frac{Y_t}{N_t + N_{t-1}} = \frac{Y_t}{L_t} \frac{L_t}{N_t} \frac{N_t}{N_t + N_{t-1}}$. Thus, using (1), (23) and $N_t = (1 + n)N_{t-1}$, $\tilde{y}_t$ is rewritten as follows:

$$\tilde{y}_t = \frac{Y_t}{L_t} \frac{L_t}{N_t} \frac{N_t}{N_t + N_{t-1}} = A k_t^\alpha l(k_t) \frac{1 + n}{2 + n}, \tag{33}$$

where $A k_t^\alpha$ expresses the output per employed worker, $l(k_t)$ expresses the employment rate and $\frac{1+n}{2+n}$ expresses the share of young working-age population in the total population.

From (33), the lower population growth rate leads to the smaller share of young working-age population in the total population (i.e., $\frac{1+n}{2+n}$), which negatively affects the per capita output of the economy $\tilde{y}_t$. We denote this direct negative effect of a decline in n on the per capita output $\tilde{y}_t$ as the "old-age dependency ratio effect". On the one hand, because the both output per employed worker $A k_t^\alpha$ and employment rate $l(k_t)$ are positively related to the capital per operating firm k_t, a decline in the population growth rate influences the per capita output $\tilde{y}_t$ indirectly through its effect on the capital per operating firm (i.e., k_t). We denote these indirect effects of a decline in n on the per capita output $\tilde{y}_t$ through k_t as the "capital accumulation effect". From Proposition 3, we can easily confirm that when a PAYG pension system is financed by a defined-benefit scheme (i.e., $b \in (0, 1)$), there is an inverted U-shaped relationship between the population growth rate and the steady-state output per employed worker $A(k^*)^\alpha$ or the steady-state employment rate $l(k^*)$. However, when there is no PAYG pension system (i.e., $b = 0$), the relationship between the population growth rate and the steady-state output per employed worker $A(k^*)^\alpha$ or the steady-state employment rate $l(k^*)$ is always negative. From (33), these results suggest that when the indirect "capital accumulation effects" are large enough to dominate the direct negative "old-age dependency ratio effect", we may find an inverted U-shaped relationship between the population growth rate and the steady-state per capita output $\tilde{y}^*$ in the case where a PAYG pension system is financed by a defined-benefit scheme (i.e., $b \in (0, 1)$). However, in the case where there is no PAYG pension system (i.e., $b = 0$), the relationship

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between the population growth rate and the steady-state per capita output might be negative.

Unfortunately, it is difficult to organize these direct and indirect effects of a decline in n on the steady-state per capita output $\tilde{y}^*$ analytically. Therefore, we provide only numerical examples. Figure 3-3 shows numerical examples of the relationship between the population growth rate and the steady-state per capita output under alternative values b of pension payout. The figure shows that when $b \in (0,1)$, there is an inverted U-shaped relationship between the population growth rate and the steady-state per capita output. However, when $b = 0$, the relationship between the population growth rate and the steady-state per capita output is always negative. The numerical simulation results under benchmark parameter values suggest that the indirect "capital accumulation effects" play a significant role to explain the relationship between the population growth rate and the steady state per capita output. As inferred from (33), the mechanism of the inverted U-shaped relationship between the population growth rate and the steady-state per capita output is the same as that of the inverted U-shaped relationship between the population growth rate and the steady-state capital per operating firm.

These results obtained in this section suggest that in economies in which the population growth rate is already low and the size of PAYG pension under a defined-benefit scheme is relatively large, a further decline in the population growth rate increases the steady-state unemployment rate and reduces the steady-state per capita output of the economy because the negative "tax burden effect" on capital accumulation dominates the positive "anti-dilution effect".

5 Short run effects of population aging

In this section, we focus on the transitional phase and examine numerically the short-term effects of population aging on the unemployment rate, the per capita output of the economy and the welfare level of the current and future generations, when a PAYG pension system is financed by a defined-benefit scheme.

From (8) and (13), the welfare measure of agents in generation t is given by the weighted average of the lifetime utility level of the employed U_t^e and the unemployed U_t^u in generation t:

$$U_t = l_t U_t^e + (1 - l_t)U_t^u.$$

We consider the following experiment. Initially, we assume that the economy is in the steady-state equilibrium where the population growth rate is already low and is given by $n_t = 0$ (i.e., $n_t = 0$ for all period $t < 15$). Then, the population growth rate in period 15 and subsequent periods are decreased from 0 to -0.3 (i.e., $n_t = -0.3$ for all period $t \geq 15$). These experiments imply that total fertility

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rate of generation 15 and subsequent generations are decreased from 2 per couple (i.e., $n_t = 0$) to 1.4 per couple (i.e., $n_t = -0.3$). Other parameter values are held constant at their baseline values in footnote 6. Because the population growth rate is time-variant, the dynamical system of (28) is rewritten as

$$l(k_{t+1})k_{t+1} = \Gamma(n_t, \tau(n_{t-1}), \tau(n_t))l(k_t)k_t^{\alpha},$$

where

$$\Gamma(n_t, \tau(n_{t-1}), \tau(n_t)) \equiv \frac{(1 - \beta)[1 - \tau(n_{t-1})]\Omega(1 - \alpha)A}{(1 + n_t)[1 + \beta\tau(n_t)\Omega\frac{1-\alpha}{\alpha}]},$$

$\tau(n_{t-1}) \equiv \frac{b}{1+n_{t-1}}$ and $\tau(n_t) \equiv \frac{b}{1+n_t}$. Moreover, to avoid lexicographic unintuitive explanations, we focus our analysis on the case where the pension payout is relatively large and is given by $b = 0.4$. In this case, as shown in the $b = 0.4$ lines in Figures 3-1 to 3-3, a decline in the population growth rate from 0 to -0.3 negatively affects both the steady-state capital per operating firm and the per capita output of the economy, and positively affects the steady-state unemployment rate.
[7]

Figures 4-1 to 4-3 show the dynamic transition path of the capital per operating firm, the unemployment rate and the per capita output, respectively. The decline in population growth rate in period 15 mitigates the dilution of aggregate savings by generation 15 and thus positively affects the accumulation of capital per operating firm from period 15 to period 16. In addition, because the old-age dependency ratio in period 15 remains unchanged, the social security tax burden of generation 15 also remains unchanged. These factors induce a substantial rise in the capital per operating firm in period 16. Note that the negative "tax burden effect" on capital accumulation does not work in period 15, and thus, the only positive "dilution effect" prevails in period 15. From (32) and (33), because the capital per operating firm is negatively related to the unemployment rate and is positively related to the per capita output, as shown in Figures 4-2 and 4-3, the unemployment rate in period 16 decreases and the per capita output in period 16 increases, respectively.[8] However, because the decline in population growth rate from period 15 increases the social security tax burden of generation 16 and subsequent generations, the capital per operating firm in period 17 and subsequent periods decreases gradually. Along with these gradual declines in the capital per operating firm from period 17, the unemployment rate increases, the per capita output decreases, and the economy gradually converges to the new steady-state equilibrium. Therefore, consistent with the result of $b = 0.4$ line in Figure 3-2,

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[7]The case where pension out is relatively small (i.e., $b = 0.2$) is analyzed in the Appendix.

[8]Although the direct negative "old-age dependency ratio effect" on the per capita output starts to work from period 16, the indirect "capital accumulation effects" dominate the "old-age dependency ratio effect" in period 16, which induces a rise in the per capita output in period 16.

the unemployment rate in the new steady-state equilibrium becomes higher than that in the original steady-state equilibrium.

Figure 4-4 shows the welfare level of agents belonging to generations 12 to 22. From the figure, the decline in population growth rate from period 15 negatively affects the welfare level of generation 15 and all subsequent generations. The net welfare losses of future generations tend to be larger than those of the current generation. Because the rise in capital per operating firm in period 16 has a negative effect on the rate of return of saving that agents of generation 15 receive in their old age, it lowers the welfare level of generation 15. Although the rise in capital per operating firm in period 16 has a positive welfare effect on the agents in generation 16 through the hike in their wage income, the decline in population growth rate from period 15 increases the social security tax burden of generation 16, which negatively affects the welfare level of agents in generation 16. Because the latter negative welfare effect dominates the former positive welfare effect, the welfare level of generation 16 also becomes lower than that in the original steady-state equilibrium. Moreover, because the level of capital per operating firm decreases in period 17 and in all subsequent periods, it lowers the welfare level of all subsequent generations, and the economy gradually converges to the new steady-state equilibrium. The welfare level in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium.

These numerical simulation results suggest that even in economies in which the population growth rate is already low and the size of PAYG pension is relatively large under a defined-benefit scheme, a further decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy in the short run because the only positive "anti-dilution effect" on capital accumulation prevails in the short run (i.e., period 16). However, because the negative "tax burden effect" on capital accumulation dominates the positive "anti-dilution effect" in the long run, a further decline in the population growth rate increases the unemployment rate and reduces the per capita output of the economy in the long run. Moreover, our baseline simulation result shows that in economies in which the population growth rate is already low and the size of PAYG pension is relatively large under a defined-benefit scheme, a further decline in the population growth rate lowers the welfare level of both current and future generations. The decline in interest rate in period 16 due to the positive "anti-dilution effect" on capital accumulation lowers the welfare level of generation 15 (i.e., initial old), while the rise in social security tax burden lowers the welfare level of generation 16 and all subsequent generations. Because the "tax burden effect" on capital accumulation negatively affects the entire capital accumulation process, the net welfare losses of future generations tends to be larger than those of the current generations.

6 Alternative design of pension payout schemes

In this section, we examine how the alternative design of pension payout schemes affects the main results of this paper. In the previous sections, the negative "tax burden effect" on capital accumulation plays a key role to derive our main theoretical result: the U-shaped relationship between the population growth rate and the steady-state unemployment rate. However, the significance of this negative "tax burden effect" on capital accumulation heavily depends upon the design of PAYG pension payout scheme. Suppose that the pension benefit for each old individual is fully adjusted in response to changes in demographic conditions or young generation's social security burden; the rise in pension tax rate due to a decline in n becomes smaller, which weakens the negative "tax burden effect" on capital accumulation. To confirm this issue, this section examines the two different types of PAYG pension payout scheme: a defined-contribution scheme and a tax adjusted defined-benefit scheme.

6.1 Defined-contribution scheme

Under a defined-contribution scheme, the pension tax rate τ_t is given by $\tau_t = \tau$ for all $t > 0$. In this case, the per capita pension benefit $\bar{b}_t$ is determined to meet the government's budget constraint of (11) and satisfies the following relationship:

$$\bar{b}_t = \tau(1 + n)w_t l_t. \tag{34}$$

Compared with (12) (i.e., pension payment rule under a defined-benefit scheme), the pension benefit for each old individual is fully adjusted in response to changes in demographic conditions. The lower population growth rate leads to the lower pension benefit. Under this type of pension payment rule, the dynamical system of equation (28) is rewritten as

$$l(k_{t+1})k_{t+1} = \Gamma(n, \tau)l(k_t)k_t^\alpha. \tag{35}$$

where

$$\Gamma(n, \tau) \equiv \frac{(1 - \beta)(1 - \tau)\Omega(1 - \alpha)A}{(1 + n)[1 + \beta\tau\Omega\frac{1-\alpha}{\alpha}]}.$$

Note that τ is constant irrespective of the value of n. From (35), we can easily confirm that the negative "tax burden effect" on capital accumulation disappears and the only positive "anti-dilution effect" on capital accumulation prevails. Therefore, under a defined-contribution scheme, a decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy not only in the short run but also in the long run.

6.2 Tax adjusted defined-benefit scheme

Under a tax adjusted defined-benefit scheme, the pension benefits are paid to all old individuals according to the following defined-benefit rule:

$$\bar{b}_t = b(1 - \tau_t)w_t l_t. \tag{36}$$

In this case, the pension payment to each old individual in period t is proportional to but less than the average after tax labor income of the young generation in period t.[9] Compared with (12) (i.e., pension payment rule under a defined-benefit scheme), the pension benefit for each old individual is fully adjusted in response to changes in the young generations' social security tax burden. The higher pension tax rate leads to the lower per capita pension benefit. Under this type of pension payment rule, from (11) and (36), the pension tax rate is given by $\tau_t = \tilde{\tau}(n) = \frac{b}{1+n+b}$. The lower population growth rate leads to the higher pension tax rate. However, compared with the case of $\tau_t = \tau(n) = \frac{b}{1+n}$, the marginal increase in pension tax rate due to a decline in n becomes smaller. Under this type of pension payment rule, the dynamical system of equation (28) is rewritten as

$$l(k_{t+1})k_{t+1} = \Gamma(n, \tilde{\tau}(n))l(k_t)k_t^{\alpha}. \tag{37}$$

where

$$\Gamma(n, \tilde{\tau}(n)) \equiv \frac{(1 - \beta)[1 - \tilde{\tau}(n)]\Omega(1 - \alpha)A}{(1 + n)[1 + \beta\tilde{\tau}(n)\Omega\frac{1-\alpha}{\alpha}]} = \frac{(1 - \beta)\Omega(1 - \alpha)A}{1 + n + b + \beta b\Omega\frac{1-\alpha}{\alpha}}.$$

From (37), we can easily confirm that the positive "anti-dilution effect" on capital accumulation dominates the negative "tax burden effect" in the long run. Therefore, under a tax adjusted defined-benefit scheme, a decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy not only in the short run but also in the long run.

7 Discussion

Before concluding this paper, we briefly discuss the theoretical results obtained in Sections 4 to 6. On the one hand, Sections 4 and 5 examine the one extreme case in which pension benefit is not entirely adjusted in response to changes in demographic conditions or young generations' social security burden (i.e., a defined-benefit scheme). In this case, we show that in economies in which the population growth rate is already low and the size of PAYG pension is relatively large, a

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[9]The average after tax labor income of the young generation in period t is given by $\frac{(1-\eta_t-\tau_t)w_t L_t + \bar{\gamma}_t(N_t - L_t)}{N_t} = \frac{(1-\tau_t)w_t L_t}{N_t} = (1 - \tau_t)w_t l_t.$

further decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy in the short run, but it increases the unemployment rate and reduces the per capita output of the economy in the long run. On the other hand, Section 6 examines the opposite extreme case in which the pension benefit is fully adjusted in response to changes in demographic conditions (i.e., a defined-contribution scheme) or young generations' social security burden (i.e., a tax adjusted defined-benefit scheme). In this case, we show that a decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy not only in the short run but also in the long run.

In general, because the actual PAYG pension payment rule has complex structures with many policy purposes, it is difficult to judge precisely whether PAYG pension benefit is fully adjusted in response to changes in demographic conditions or young generations' social security burden. Moreover, intergenerational conflicts of interests between young and old generations influence the political decision regarding the PAYG pension payment rule. For example, the 2004 pension reform in Japan abandoned the traditional practice of continuously increasing contributions to maintain the ratio of pensions to average wages of working generations at a constant value (i.e., 59% benefit level). Instead, Japan capped future contributions at 18.3% and introduced a demographically modified indexation program to ensure that the size of pension benefits was consistent with the new contribution cap, which is a so-called "macroeconomic formula". However, this pension cut mechanism was designed to activate only when price and wages were rising steadily. Consequently, the ratio of pensions to average wages of workers eventually increased from 59% in 2007 to 62.7% in 2016. Oguro (2014) notes that the strong opposition of older generations motivates politicians to introduce such a restrictive precondition for pension cut reform, which waters down the 2004 pension reform plan.

To the best of our knowledge, the actual pension payment rule may lie between the two extreme cases analyzed in this paper. The current framework omits many important pension payout design details to obtain intuitive and manageable results. We also employed rather restrictive preference and production specifications. The application of our simple framework to assess the likely impact of policy reform is obviously limited. Therefore, the development of a more elaborate numerical version of the growth model that fully accounts for many important pension payout design details are promising directions for future research.

8   Concluding remarks

Employing a two period overlapping generations model with labor market frictions and pay-as-you-go (PAYG) pension, this paper examined how population

aging caused by a decline in the population growth rate influences the unemployment rate and the per capita output of the economy under a defined-benefit PAYG pension scheme. We showed that in economies in which the population growth rate is already low and the size of PAYG pension is relatively large, a further decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy in the short run, but it increases the unemployment rate and reduces the per capita output of the economy in the long run. We also noted that the design of the pension payout scheme matters to mitigate the long run negative effects of population aging on the employment rate and the per capita output of the economy.

Appendix

The proof of Proposition 3-1

Noting the fact that $sign[\frac{\partial k^*(n)}{\partial n}] = sign[\frac{\partial \Gamma(n,\tau(n))}{\partial n}]$ from (29), by differentiating $\Gamma(n, \tau(n))$ with respect to n, we find:

$$\frac{\partial \Gamma(n, \tau(n))}{\partial n} = \frac{(1 - \beta)\Omega(1 - \alpha)A}{(1 + n + \beta b\Omega\frac{1-\alpha}{\alpha})^2(1 + n)^2}G(1 + n),$$

where

$$G(1 + n) = -(1 + n)^2 + 2b(1 + n) + \beta b^2\Omega\frac{1 - \alpha}{\alpha}.$$

Note that $G(1 + n)$ is quadratic function with respect to $1 + n$ and satisfies the following properties.

$$sign[\frac{\partial k^*(n)}{\partial n}] = sign[G(1 + n)],$$

$$\lim_{1+n \to b} G(1 + n) > 0,$$

$$\lim_{1+n \to \infty} G(1 + n) = -\infty,$$

$$b = \arg\max_{1+n} G(1 + n).$$

Therefore, given $b \in (0, 1)$, there exists a unique $\hat{n} \in (-(1 - b), \infty)$ such that $G(1 + \hat{n}) = 0, G(1 + n) > 0 \ \forall n \in (-(1 - b), \hat{n}), G(1 + n) < 0 \ \forall n \in (\hat{n}, \infty)$, where

$$\hat{n} \equiv -(1 - b) + b\sqrt{1 + \beta\Omega\frac{1 - \alpha}{\alpha}}.$$

Recalling the fact that $sign[\frac{\partial k^*(n)}{\partial n}] = sign[G(1+n)]$, there exists a unique $\hat{n} \in (-(1 - b), \infty)$ such that $k^*(\hat{n}) \geq k^*(n), \ \forall n \in (-(1 - b), \infty), \frac{\partial k^*(n)}{\partial n} > 0 \ \forall n \in (-(1 - b), \hat{n})$, $\frac{\partial k^*(n)}{\partial n} < 0 \ \forall n \in (\hat{n}, \infty)$.

The proof of Proposition 3-2

Suppose $b = 0$, $\Gamma(n, \tau(n))$ in (28) is rewritten as

$$\Gamma(n, \tau(n)) = \frac{(1 - \beta)\Omega(1 - \alpha)A}{1 + n}.$$

Recalling the fact that $sign[\frac{\partial k^*(n)}{\partial n}] = sign[\frac{\partial \Gamma(n,\tau(n))}{\partial n}]$ from (29), we can easily confirm that the relation $\frac{\partial k^*(n)}{\partial n} < 0$ holds.

The parameters for the simulation

Following Den Haan et al.(2000), we set the value of α, the capital share of output, to 0.36, and the value of ϕ, the workers' Nash bargaining power, to 0.5. To investigate the effect of the decline in population growth rate, we set the value of n, population growth rate, to 0 in the base-case simulation, as is observed in Japan, and changed it from -0.6 to 0.6 in increments of 0.1. In addition, to investigate the effect of the pension policy, we set the value of b at 0.4 in the base case simulation and changed it from 0 to 0.6 in increments of 0.2. According to the OECD (2015), the net pension replacement rate in Japan was 40% for both men and women.

   Additionally, the 45%-80% of the average wage for the last six months is paid to the unemployed people in Japan for the unemployed benefit. Accordingly, we set the value of γ to 0.6. The utility weight given to the consumption in youth β is set to 0.5 so that the worker's output share Ω achieves the equilibrium values of 0.714, which is analogous to the results obtained in Hashimoto Im and Kunieda (2016). We also set the search cost h as a relatively low value, $h = 1$, such that the economy is feasible and production occurs in the analysis. Regarding the remaining parameter values of A and σ, the productivity of technology and the parameters for search cost function, we set $A = 1$ and $\sigma = 4$ such that the steady-state unemployment rates are approximately 4 % under the benchmark parameter values.

Short run effects of population aging in the case where $b = 0.2$

In this Appendix, we briefly consider the robustness of our numerical simulation results obtained in Section 5. For comparison, we focus on the case where the pension payout is relatively small and is given by $b = 0.2$. In this case, as shown in the $b = 0.2$ lines in Figures 3-1 to 3-3, a decline in the population growth rate from 0 to -0.3 positively affects both the steady-state capital per operating firm and the per capita output of the economy, and negatively affects the steady-state unemployment rate.

Figures 5-1 to 5-3 show the dynamic transition path of the capital per operating firm, the unemployment rate and the per capita output, respectively. The decline in the population growth rate in period 15 mitigates the dilution of aggregate savings by generation 15 and thus positively affects the accumulation of capital per operating firm from period 15 to period 16. In addition, because the old-age dependency ratio in period 15 remains unchanged, the social security tax burden of generation 15 also remains unchanged. These factors induce a substantial rise in the capital per operating firm in period 16. Note that the negative "tax burden effect" on capital accumulation does not work in period 15, and thus, the only positive "anti-dilution effect" prevails in period 15. From (32) and (33), because the capital per operating firm is negatively related to the unemployment rate and is positively related to the per capita output, as shown in Figures 5-2 and 5-3, the unemployment rate in period 16 decreases, and the per capita output in period 16 increases, respectively.[10]

Similar to the analyses of Section 5, the "tax burden effect" on capital accumulation starts to work from period 16, and thus, the social security tax burden of generation 16 and subsequent generations increases. These factors potentially negatively affect the capital per operating firm in period 17 and subsequent periods. However, because the positive "anti-dilution effect" on capital accumulation dominates the negative "tax burden effect", as shown in Figure 5-1, the capital per operating firm in period 17 and subsequent periods increases steadily. Along with these gradual increases in the capital per operating firm from period 17, the unemployment rate decreases, the per capita output increases, and the economy gradually converges to the new steady-state equilibrium. Therefore, consistent with the result of the $b = 0.2$ line in Figure 3-2, the unemployment rate in the new steady-state equilibrium becomes lower than that in the original steady-state equilibrium.

Figure 5-4 shows the welfare level of agents belonging to generations 12 to 22. From the figure, the decline in the population growth rate from period 15 negatively affects the welfare level of generation 15 and all subsequent generations. The net welfare losses of the current generations tend to be larger than those of the future generations. Because the rise in capital per operating firm in period 16 has a negative effect on the rate of return of saving that agents of generation 15 receive in their old age, it lowers the welfare level of generation 15. Although the rise in capital per operating firm in period 16 has a positive welfare effect on the agents in generation 16 through the hike in their wage income, the decline in population growth rate from period 15 increases the social security tax burden of

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[10]Although the direct negative "old-age dependency ratio effect" on the per capita output starts to work from period 16, the indirect "capital accumulation effects" dominate the "old-age dependency ratio effect" in period 16 and in subsequent periods, which induces a rise in the per capita output in period 16 and in subsequent periods.

generation 16, which negatively affects the welfare level of agents in generation 16. Because the latter negative welfare effect dominates the former positive welfare effect, the welfare level of generation 16 also becomes lower than that in the original steady-state equilibrium. However, because the level of capital per operating firm increases in period 17 and in all subsequent periods, the welfare level of all subsequent generations increases, and the economy gradually converges to the new steady-state equilibrium. The welfare level in the new steady-state equilibrium is lower than that in the original steady-state equilibrium.

These numerical simulation results suggest that in economies in which the size of PAYG pension is relatively small under a defined-benefit scheme, a decline in the population growth rate reduces the unemployment rate and increases the per capita output of the economy not only in the short run, but also in the long run, because the positive "anti-dilution effect" on capital accumulation always dominates the negative "tax burden effect". Moreover, our base line simulation shows that in economies in which the size of PAYG pension is relatively small under a defined-benefit scheme, a decline in the population growth rate lowers the welfare level of both current and future generations. The decline in the interest rate in period 16 due to the positive "anti-dilution effect" on capital accumulation lowers the welfare level of generation 15 (i.e., initial old), while the rise in social security tax burden lowers the welfare level of generation 16 and all subsequent generations. However, because the positive "anti-dilution effect" on capital accumulation dominates the negative "tax burden effect", the net welfare losses of future generations tends to be smaller than those of the current generations.

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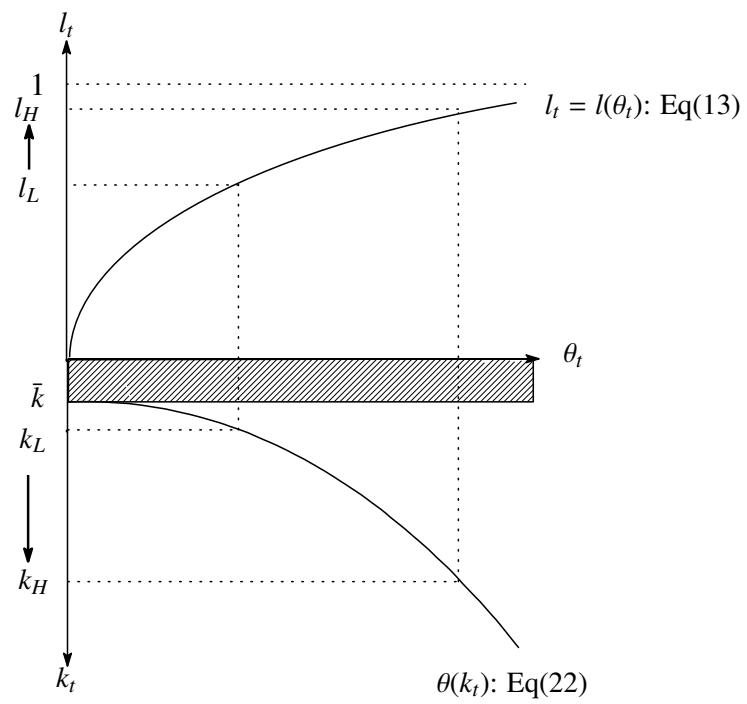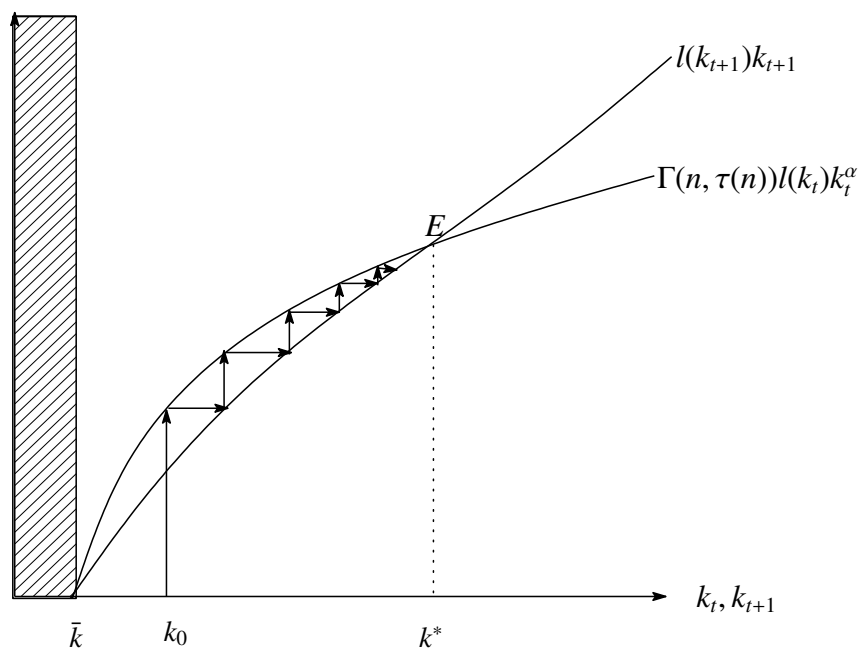
Figure 1: Beveridge curve and job-creation condition
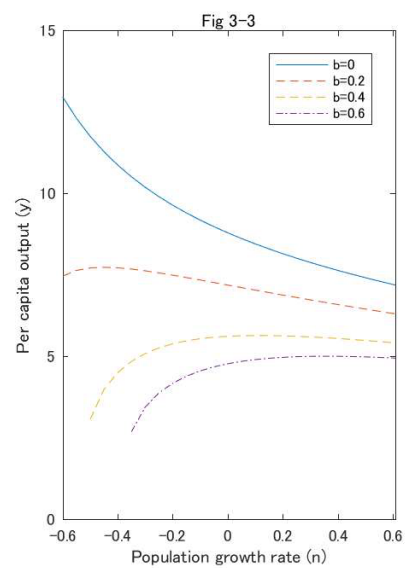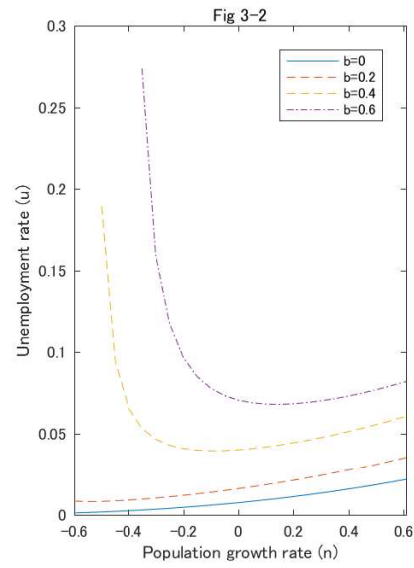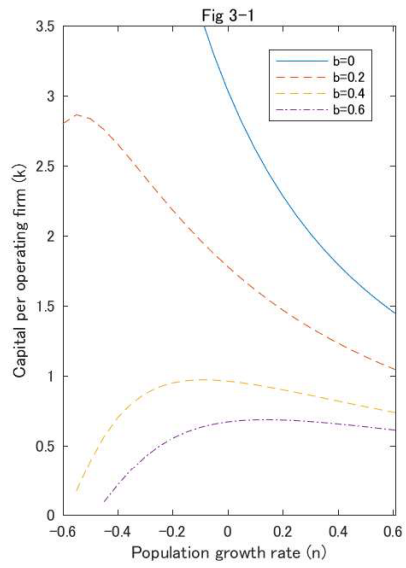
Figure 2: The possible dynamic behavior of the economy
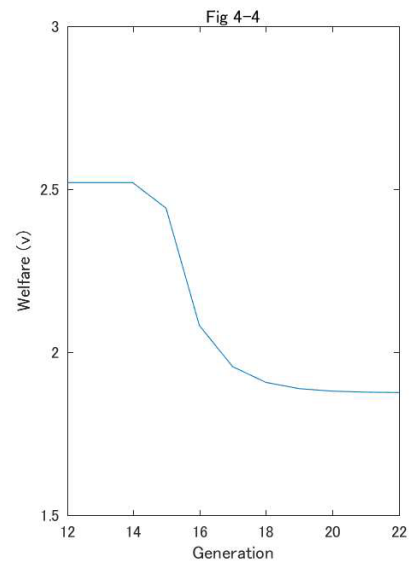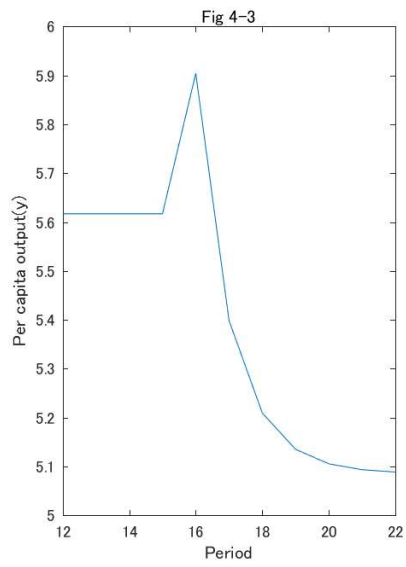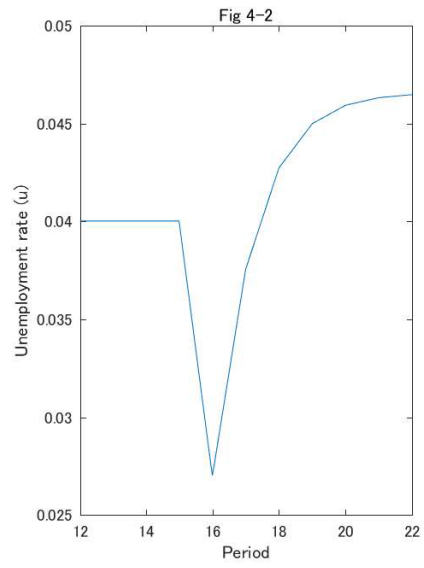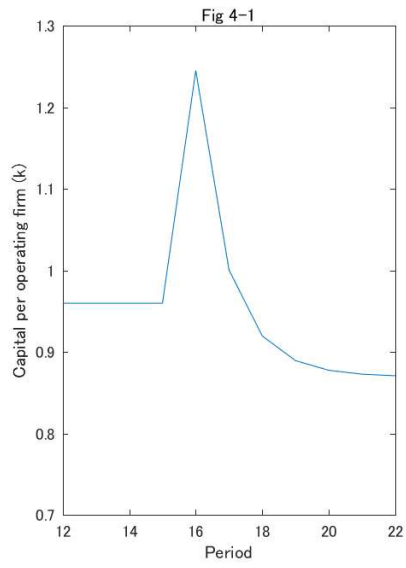
Figure 3: The long run effect of population aging
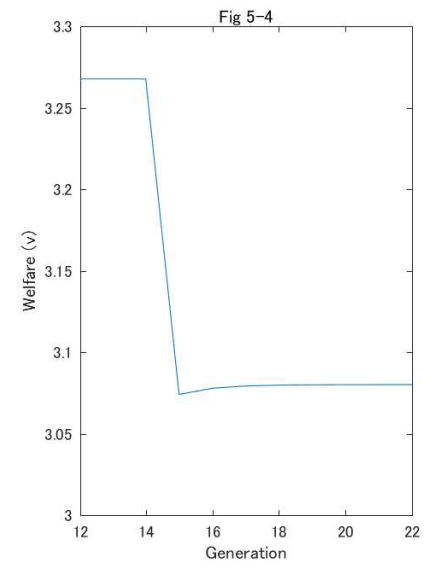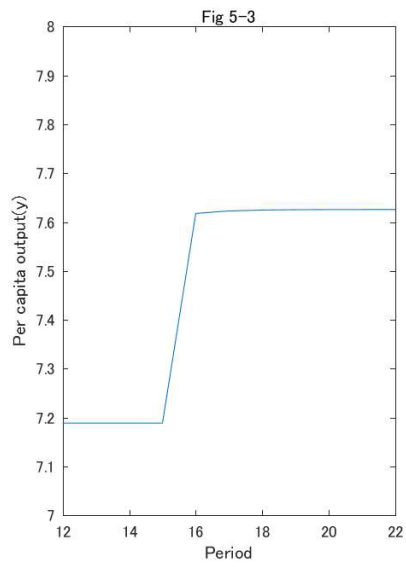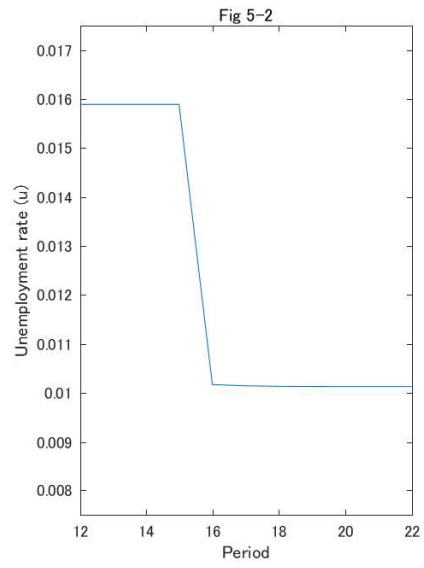
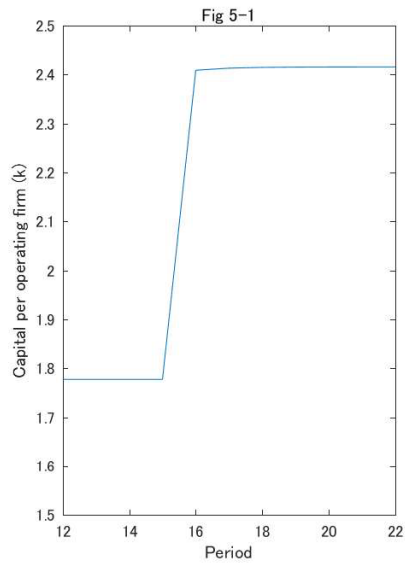Figure 4: The short run effect of population aging ($b = 0.4$)

Figure 5: The short run effect of population aging ($b = 0.2$)