Theoretical Analysis of Multi-Product Firm with Within-Product Network Externality

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February 14, 2017

Preface

The first draft of this thesis was written in the period from April 2013 to December 2015 while I was enrolled as a PhD student at the Graduate School of Economics, Kwansei Gakuin University.Then, I revised my draft from April to November in 2016. I am grateful to the Graduate School of Economics, Kwansei Gakuin University an excellent research environment.

There are number of people I wish to thank. First and foremost, I would like to thank my main supervisor, Tetsuya Shinkai, for encouraging me to enroll as a PhD student, giving me guidance whenever needed, for his constructive comments, for being a unique inspiring mentor.

The part of chapter 2 has been appeared in the following publication:

Kitamura, R. and Shinkai, T. (2015), "Product line strategy within a vertically differentiated duopoly, " *Economics Letters*, Volume 137, December 2015, Pages 114—117.

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Summary

A Network Externality within Goods

Over the last decade, mobile phones have spread rapidly in many developed countries. In the market for traditional mobile phones, there is just one network externality (network effect), as has been recognized since the seminal work of Katz and Shapiro (1985).[1]In addition to these standard mobile phones, smartphones, for example, the *iPhone* from Apple, have recently increased their share and importance in our daily lives.[2] One notable property of the smartphone market that differs from the market for standard mobile phones is that it contains the following two externalities.

First, there is a network externality within carriers that has been considered in the existing literature, such as Katz and Shapiro (1985) and Chen and Chen (2011). According to this externality, a consumer who purchases a product or service from a certain carrier gains a network benefit when other consumers purchase the same or different product or service from the same carrier.

Second, we should recognize the existence of another important network

[1]In Belleflamme and Peitz (2011), network effects has been formally defined as follows: "A product is said to exhibit network effects if each user's utility is increasing in the number of other users of that product or products compatible with it."

[2]For detail of the spread of *iPhone*, see West and Mace(2010)

externality within distinct types of smartphones supplied *to different carriers* by the same producer of smartphone devices.[3]In the real world, for instance, a customer of a carrier who has Apple's *iPhone* gains a network benefit when the number of *iPhone* users increases, even when these users are customers of other carriers. This network benefit takes the form of enhancement of reputation about the *iPhone*, or an increase in complementary goods, such as application software for the *iPhone*.[4] Thus, even if consumers who use the *iPhone* do not use the same carrier, all consumers gain a network benefit from the increase in the number of *iPhone* users. To the best of our knowledge, this externality has received no attention in the previous studies that consider network externality. In this thesis, I analyze a market in which only the latter network externality works. Therefore, one of the contributions of this thesis is providing some theoretical properties of a market in the presence of network externality within goods.

A Vertical or Horizontal Differentiation

Previously, I explained within-product network externality by using smartphone market. In such smarthone industry, the products are *vertically* differentiated.[5] Another example of vertical differentiated product market is bicy-

[3]In Kitamura (2013), I define the network benefit from within-product network externality as follows: "A consumer who purchases a product from a certain firm gains a network benefit when other consumers purchase the same product from the same or different firm."

[4]In this thesis, I do not mention what kinds of network effect works; Direct and indirect network effect. For these network effect, see Chou and Shy (1990), Nocke et al (2007), Clements (2004), Church and Gandal (2012).

[5]An example of vertical differentiation between iPhone and Android smartphones in found in Geekbench (see http://browser.primatelabs.com/geekbench2/1030202 and http://browser.primatelabs.com/android-benchmarks).

cle component industry. In bicycle component industry, for instance, there were one dominant firm, Shimano Inc., and four or five smaller firms. In 1993, Shimano's sales were approximately $1.275 billion, and this accounted for 75% of global sales of bicycle components, which was about $1.7 billion. For mountain bicycle market, in particular, Shimano had become approximately 80% market share in 1990. Shimano produced all six components of bicycle, Brake Lever, Shifter, Derailleur, Freewheel, Chain and Hub,[6] and each component was produced as several quality level, respectively. When the number of users who buy a certain component increases, then a user of it which is same quality level gains a network benefit because of an increase in the number of bicycle which can be equipped with it and/or an improvement of some services and a finding how to maintain it by an increase in comment on an Internet forum or web page.

In contrast this network externality works in some other industry in which the products are *horizontally* differentiated. For instance, home electronics, PC industry and so on. In a television industry, when the number of users who buy a certain television increases, then a user of it gains a network benefit because of an increase in complementary goods of it or an improvement of some services. However, in this thesis, I characterize the equilibrium outcome by looking at a monopolistic market.[7] An example of monopoly in the presence of network externality within goods is illustrated by Japan Tobacco

[6] Simano's market share of each component is seen in Fixson and Park(2008)

[7] Although only a monopolist is analyzed in this paper, in fact, I ascertained that the outcome of duopoly model is almost the same to it of monopoly model. However, in duopoly market, the interpretations of it's outcome are complicated because there are some effects on equilibrium, competition of firms, network externalities and cannibalization. Thus, I focus on only a monopoly market in the presence of network externalities with in goods in this paper.

Inc.(JT), manufactures of the tobacco and it is a monopolist in Japanese tobacco industry. Similarly to above example, if the number of consumers who subscribe a certain tobacco produced by JT in Japan increases, then a user of it gains a benefit by a network externality since the subscribers tend to give valuable feedback and reviews or it is sold in many stores in Japan.

Constitution of this thesis

This thesis consists of four self-contained chapters that all theoretically investigate issues related to the multi-product firm. In particular, chapter 3 and 4 consider a multi-product firm market in which there exist within-product network externality.

In chapter 1 and 2, "Cannibalization within the Single Vertically Differentiated Duopoly"(co-authored with Tetsuya Shinkai) and "Product line strategy within a vertically differentiated duopoly"(co-authored with Tetsuya Shinkai), we analyze multi-product duopoly market without any network externalities in which the products are vertically differentiated in order to clear some properties of such market and to prepare the benchmark model in next chapter.

In the third chapter, "Cost Reduction can Decrease Profit and Welfare in a Monopoly", I consider multi-product monopoly model with within-product network externality in which the products are vertically differentiated.

In the fourth chapter in this thesis, "A Monopoly Model in which Two Horizontally Differentiated Goods with Network Externalities", based on Bental and Spiegel (1984) in which they consider a horizontally differentiated multi-product oligopoly model without network externality, I analyze multi-product monopoly model with within-product network externality in which the products are horizontally differentiated.

Contributions of this thesis

In this thesis, I focus on a multi-product firm market in which a firm supplies two horizontally or vertically differentiated products and on only the network externality which works in product in order to simplify the model and shed light on the effect of this network externality on the market. Then, the first contribution of this study is that I propose the new network externality which works in product and find some theoretical properties concluding cannibalization. The model can be used as a benchmark of a market in the presence of network externality within product. Second, I show that the monopolist could earn more even when the production cost increases. In detail, when the goods are not horizontally but *vertically* differentiated, then the profit can be convex function of the production cost. The reason is that I adopt, in this study, the concept of equilibrium as *Fulfilled Expectation Equilibrium* and consider the multi-product monopolist. Finally, in chapter 1 and 2, I propose a duopoly model in which firms with different costs supply two vertically differentiated products in the same market and also find that change in the quality superiority of goods and the relative cost efficiency ratios characterize graphically product line strategies of firms by the two ratios relationship.

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Chapter 1

Cannibalization within the Single Vertically Differentiated Duopoly

abstract[1]

We consider cannibalization in a duopoly model in which firms with different costs supply two vertically differentiated products in the *same market*. We find that an increase in the difference in quality between the two goods or a decrease in the marginal cost of the high-quality goods leads to cannibalization. As a result, these goods keep low-quality goods from the market. Then, as the difference in quality between the two goods increases from a sufficiently small to a sufficiently large level, we find that 1) cannibalization from the low-quality good to the high-quality good of the efficient firm expands, 2) cannibalization from the high-quality good to the low-quality good of the inefficient firm shrinks and establish that 3) an increase in the production costs of the inefficient firm improves social welfare when the difference in quality between the two goods is sufficiently small.

Keywords: Multi-product firm; Duopoly; Cannibalization; Vertical product differentiation

[1]The authors are grateful to Tommaso Valletti, Federico Etro, Hong Hwang, Noriaki Matsushima, Toshihiro Matsumura, Kenji Fujiwara, and Keizo Mizuno for their useful comments on an earlier version of this paper. The second author was supported by Grants-in-Aid for Scientific Research (Nos. 23330099 and 24530255) MEXT. Furthermore, this chapter is sum of the revised version of Kitamura and Shinkai (2013) and Kitamura and Shinkai (2015)

1.1 Introduction

In a real economy, there are oligopolistic markets in which firms produce and sell multiple products that are vertically differentiated within the same market. For example, GM sells the Chevrolet Cruze and GMC Sierra PU, and Toyota sells the Camry, Corolla Matrix, and Prius—Toyota's hybrid car—in the same segment of the car market. Hyundai also sells the Elantra and Hybrid Sonata in the same segment of the U.S. car market. As another example, Apple sells the iPad Mini and the larger iPad in the tablet market. Similarly, Samsung sells the Galaxy Note and the Galaxy Tab, in both a smaller and a larger variety.[2] Since consumers believe that the quality of the firms' technology differs, each consumer places a different value on the high-quality good of each firm. Thus, these markets are horizontally and vertically differentiated. Such markets present more cases of *cannibalization*.[3] Cannibalization *within the same market* occurs when a firm increases the output of one of its products by reducing the output of a similar competing product in the same market.

The objective of this study is to examine cannibalization within the same market from strategic point of view of the multi-product firm which supplies two goods differentiated in quality.

For the purpose of our analysis, both the quality level and the number of differentiated goods supplied by each firm are given. In addition, we

[2]See "Samsung's Brand Cannibalization," http://www.indianprice.com/mobiles/articles/15-samsungs-brand-cannibalization.html.

[3]In fact, many reports suggest that the iPad Mini is cannibalizing sales of the larger iPad. See, for example, Seward (2013), "Yes, the iPad Mini is cannibalizing sales of larger iPad."

do not consider new entries to the market in our model. In our setting, both firms produce and supply two kinds of vertically differentiated goods in a market. [4] To understand the strategic aspects of cannibalization, we consider two differences: 1) the difference in the quality of the goods; and 2) the difference in the technology of the firms. Here, we characterize the cannibalization resulting from these two differences. Thus, we consider a duopoly with asymmetric marginal costs of a high-quality good.

This study offers three contributions to existing literature. First, we find that cannibalization can be seen as a business strategy characterized by a difference in the quality of vertically differentiated goods and in cost efficiency. Second, we show that, as the difference in quality between the two goods increases from a sufficiently small to a sufficiently large level, cannibalization from the low-quality to the high-quality good of the efficient firm expands, while that from the high-quality to the low-quality good of the inefficient firm shrinks. Third, we show that counter-intuitively, an increase in the production costs of the inefficient firm improves social welfare when the difference in the quality of the two goods is sufficiently small.

We illustrate the intuitive reasoning behind the second result in relation to the current tablet PC market. When the difference in the quality of the goods is sufficiently large, or the marginal cost of the high-quality good of its rival is high, the efficient firm, for example Apple, increases its output of

[4] The readers may think that our model setting in which both firms supply two vertically differentiated products in the same market, seems to be too limited. In other paper, Kitamura and Shinkai (2014), we show that when a firm (say firm 1) chooses to expand its product line or supply only one type of good, while another firm (firm 2) sells both goods, then firm 1 has an incentive to produce both goods. Thefore, we focus on the model in which both firms supply two vertically differentiated products to the same market.

the high-quality iPad. In contrast, if its rival, the inefficient firm (for example, Samsung), can produce a high-quality tablet (owing to its research and development efforts) at a lower cost than that of Apple, or if the difference in the quality of the goods becomes small, then Apple expands production of the lower-quality iPad Mini, which cannibalizes the larger iPad. Then, Samsung's new tablet cannibalizes sales of its existing 10.1-inch tablet. However, unless the market has goods that are extremely differentiated or extremely similar in terms of quality, cannibalization does not keep one of the firms' products from the market.[5]

In typical models of horizontal or vertical product differentiation, each firm produces only one kind of good, given exogenously, which differs from that of its rival. For example, Ellison (2005), whose study is closely related to the present study, analyzes a market in which each firm sells a high-end and low-end version of the same product. Although each firm produces two differentiated goods, the two goods are sold in different markets, each with different types of consumers.[6]

In existing literature on vertical product differentiation, the quality of goods that firms produce is treated as an endogenous variable. For example, in Bonanno (1986) and Motta (1993), firms initially choose a quality level

[5]From the article in the web news,"Samsung's Brand Cannibalization," Samsung occasionally improves its products, which kills its existing product in the market. The launch of the 10.1 inch Galaxy Note (Samsung's latest tablet) will most likely cannibalize sales of the existing 10.1 inch tablet. However, Samsung does not mind, as one of the best ways to continue to exist in a competitive market is to eradicate your own goods. See http://www.indianprice.com/mobiles/articles/15-samsungs-brand-cannibalization.html for more detail.

[6]This model combines vertical differentiation (two distinct qualities) and horizontal differentiation (two firms located at distinct points in a linear city).

and then compete in Cournot or Bertrand fashion in an oligopolistic market.[7]

However, all of these studies stated above do not consider firms that sell *multiple products*, differentiated in terms of quality (vertically), in the *same* market. In dealing with cannibalization in such a market, our model needs to allow for a multi-product firm that differs in terms of its features or characteristics. Few previous studies address an oligopolistic market with such firms, although Johnson and Myatt (2003) are a notable exception.[8]

According to Johnson and Myatt (2003), firms that sell multiple quality-differentiated products frequently change their product lines when a competitor enters the market. They explain the common strategies of using "fighting brands" and "pruning" product lines. That is, unlike this study, they endogenize not only the quality level of each good, but also the number of goods that each firm supplies in the market.

In literature on product line design, Desai(2001) considers two segments duopoly markets for high-quality and low-quality goods represented by Hotelling type model. He examines whether the cannibalization problem affects a firm's price and quality decision. He characterises such effects by consumers' differences in quality valuations and in their taste preferences. Gilbert and Matutes (1993) explore vertically differentiated products' competition in the two segment market by focusing the product lines of two spatially differentiated firms. Under the exogenous quality levels assumption, they examine whether both of firms would specialize to serve one segment each and characterize this by the differentiation between two firms.

[7]For detain on Cournot model and Bertrant model, see Cornot(1838) and Bertrand(1883).

[8]For the sake of simplicity, we focus on a duopoly model.

Our study's results are also related to those of marketing studies on product segmentation and product distribution strategies. For example, Calzada and Valletti (2012) study a model of film distribution and consumption. They consider a film studio that can release two versions of one film—one for theatres and one for video– although they do not consider oligopolistic competition between film studios. In their model, a film studio decides on its versioning strategy and sequencing strategy. The versioning strategy involves the simultaneous release of the two versions, while the sequencing strategy involves the sequential release of the versions. They show that the optimal strategy for the studio is to introduce versioning if their goods are not close substitutes for each other. The "versioning strategy" in their model corresponds to the simultaneous supply of high- and low-quality goods as in our model. In the case of sequential supply in their model, the film studio supplies the high-quality film version in theatres and then launches the low-quality DVD version to the same market although we do not consider "sequential strategy" in this paper.

We establish a result which indirectly supports the above result in Calzada and Valletti (2012). Thus, when the difference in quality between the high-quality good and the low-quality good is large to some extent and so they are not close substitutes for each other, we show that both of firms had better supply both of goods in the market, that is, they should obey 'versioning strategy.'

The remainder of this paper is organized as follows. In section 2, we present our model and derive a duopoly equilibrium with two vertically differentiated products in a market. Furthermore, we use comparative statistics of the equilibrium output to explore how the quality of goods, cost asymmetry, and cannibalization are related. In section 3, we conduct a welfare analysis of the duopoly model that we present in section 2. Finally, section 4 concludes the paper and offers suggestions for possible future research.

1.2 The Model and the Derivation of an Equilibrium

Suppose there are two firms, $i = 1, 2$, and each produce two goods (good H and good L) that differ in terms of quality, where 1 and 2 imply firm 1 and firm 2 in the duopoly case, respectively. Let V_H and V_L denote the quality level of the two goods. Then, the maximum amount consumers are willing to pay for each good is assumed to be $V_H > V_L > 0$. Further, we assume $V_H = (1+\mu)V_L$, where μ represents the difference in quality between the two goods, and we normalize the quality of the low-quality good as $V_L = 1$, for simplicity. Good $\alpha(= H, L)$ is assumed to be homogeneous for any consumer.

First, we describe the consumers' behavior in our model.

Following the standard specification in the literature, for example, Katz and Shapiro (1985), we assume there is a continuum of consumers characterized by a taste parameter, θ, which is uniformly distributed between 0 and $r(> 0)$, with density 1. We further assume that a consumer of type $\theta \in [0, r]$,for $r > 0$, obtains a net surplus from one unit of good α from firm i at price $p_{i\alpha}$. Thus, the utility (net benefit) of consumer θ who buys good

α $(= H, L)$ from firm i $(= 1, 2)$ is given by

$$U_{i\alpha}(\theta) = V_\alpha \theta - p_{i\alpha} \qquad i =, 1, 2 \quad \alpha = H, L. \tag{1.1}$$

Each consumer decides to buy either nothing or one unit of good α from firm i to maximize his/her surplus.

Before deriving the inverse demand of each good, we present three further assumptions about the consumers in our model.

First, there exists a consumer, $\hat{\theta}_i \in [0, r]$, who is indifferent between the two goods of the same firm; that is,

$$U_{iH}(\hat{\theta}_i) = U_{iL}(\hat{\theta}_i) > 0, i = 1, 2. \tag{1.2}$$

Second, there always exists a consumer, $\underline{\theta}_{iL}, i = 1, 2,$, who is indifferent between purchasing good L and purchasing nothing in the duopoly.

To derive a duopoly equilibrium, we need one other key assumption.

Finally, in the duopoly, for an arbitrary type-θ_α consumer,

$$U_{1\alpha}(\theta_\alpha) = U_{2\alpha}(\theta_\alpha), \alpha = H, L. \tag{1.3}$$

This last assumption implies that the net surplus of consumer θ_α must be the same whether buying a good produced by firm 1 or a good produced by firm 2, as long as the two firms produce the same quality of good α and have positive sales.

From these assumptions, we can derive and illustrate the demand for good H and good L using a line segment, as shown in Figure 1.1, where

$Q_\alpha = q_{i\alpha} + q_{j\alpha}, \alpha = H, L, i, j = 1, 2.$[9]

Here, $\widehat{\theta}^*$, the threshold between the demand for product H and for L, is given by

$$\widehat{\theta}^* = \frac{1}{\mu}(p_H^* - p_L^*). \tag{1.4}$$

Then, the inverse demand functions can be obtained in the following manner:

$$\begin{cases} p_H = (1+\mu)(r - Q_H) - Q_L \\ p_L = r - Q_H - Q_L. \end{cases} \tag{1.5}$$

Moreover, suppose that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L = 0$, where $c_{i\alpha}$ is firm i's marginal and average cost of good α. This implies that a high-quality good incurs a higher cost of production than a low-quality good.[10] Under these assumptions, each firm's profit is defined in the following manner:

$$\pi_i = (p_{iH} - c_{iH})q_{iH} + p_{iL}q_{iL} \qquad i = 1, 2, \tag{1.6}$$

where $p_{i\alpha}$ is the price of good α sold by firm i, and $q_{i\alpha}$ is the firm's output of good α. Each firm chooses the quantity to supply that maximizes this profit function in Cournot fashion.

To maximize profit function (1.6), each firm determines the quantity of

[9] The demand function is similar to that derived in Bonanno (1986), but it is different from that in Bonnano in that both firms supply two vertically differentiated products in the same market. For the derivation of the demand, see Kitamura and Shinkai (2013) in detail.

[10] For details on the symmetric costs version of our analysis, see Kitamura and Shinkai (201b).

goods to produce, q_{iH} and q_{iL}, in the following manner:

$$\max_{q_{iH}, q_{iL}} \pi_i.$$

Here, we set $c_{2H} > c_{1H} > c_{iL} = 0$, which means that firm 1 is more efficient than firm 2. The first-order conditions for profit maximization are as follows:

$$\begin{aligned}
-(1+\mu)q_{1H} + (1+\mu)(r - Q_H) - Q_L - c_{1H} - q_{1L} &= 0 \\
-(1+\mu)q_{2H} + (1+\mu)(r - Q_H) - Q_L - c_{2H} - q_{2L} &= 0 \\
-q_{1H} + r - Q_H - Q_L - q_{1L} &= 0 \\
-q_{2H} + r - Q_H - Q_L - q_{2L} &= 0.
\end{aligned}$$

Solving this system, we obtain the following Nash equilibrium quantities:

$$\begin{cases} q_{1H}^* = \frac{r}{3} - \frac{2c_{1H} - c_{2H}}{3\mu}, & q_{1L}^* = \frac{2c_{1H} - c_{2H}}{3\mu} \\ q_{2H}^* = \frac{r}{3} - \frac{2c_{2H} - c_{1H}}{3\mu}, & q_{2L}^* = \frac{2c_{2H} - c_{1H}}{3\mu}. \end{cases} \tag{1.7}$$

For q_{iH}^* and q_{iL}^* to be positive, we assume that

$$\mu > \frac{2c_{2H} - c_{1H}}{r} \quad \textit{and} \quad c_{1H} > \frac{1}{2}c_{2H}. \tag{1.8}$$

Hence, the total equilibrium output, Q^*, becomes constant:

$$Q^* = Q_1^* + Q_2^* = Q_H^* + Q_L^* = \frac{2}{3}r, \tag{1.9}$$

where $Q_i^* = Q_{i\alpha}^* + Q_{i\beta}^*, i = 1, 2, \alpha, \beta = H, L.$

From (1.5) and (1.7), we obtain the following equilibrium prices of the goods:

$$p_H^* = \frac{(1+\mu)r + c_{1H} + c_{2H}}{3}, \; p_L^* = \frac{r}{3}. \tag{1.10}$$

We also have the equilibrium profit of firm i:

$$\pi_i^* = \frac{\mu(1+\mu)r^2 - 2\mu(2c_{iH} - c_{jH})r + (2c_{iH} - c_{jH})^2}{9\mu}, \quad i = 1, 2 \;, \quad i \neq j \tag{1.11}$$

Then, the equilibrium outputs of (1.7) lead to the following condition for cannibalization: We have

$$\begin{aligned} q_{1H}^* - q_{2H}^* &= \frac{1}{3\mu}(2c_{2H} - c_{1H} - (2c_{1H} - c_{2H})) \\ &= q_{2L}^* - q_{1L}^* \\ &= \frac{1}{\mu}(c_{2H} - c_{1H}) > 0. \end{aligned} \tag{1.12}$$

We also confirm the difference in the profits of the two firms, as follows:

$$\begin{aligned} \pi_2 - \pi_1 &= \frac{1}{3\mu}(c_{1H} - c_{2H})(2\mu r - c_{1H} - c_{2H}) < 0, \\ \text{since } \mu &> \frac{2c_{2H} - c_{1H}}{r} > \frac{c_{1H} + c_{2H}}{2\,r} \text{ and } c_{1H} < c_{2H}. \end{aligned} \tag{1.13}$$

Hence, we can easily establish the following proposition.

Proposition 1.1 *Although the efficient firm (firm 1) produces more of* *high-quality good H than the inefficient firm (firm 2), the inefficient firm sells more of the low-quality good L than the efficient firm. Furthermore, if the difference in unit costs between the two firms is sufficiently small (i.e., if $2c_{1H} = c_{2H}$), then the efficient firm does not produce the low-quality good. The efficient firm 1 earns more than the inefficient firm 2 does.*

The proposition implies that the efficient firm 1 earns more than the inefficient firm 2 because of cost efficiency of firm 1 over firm 2 on the high-quality good H under the positive outputs assumption (1.8) in the equilibrium.

Next, we examine under which conditions the cannibalization from one product to another occurs in the equilibrium. Note that we say "a product cannibalizes a similar product" when a firm increases the output of the product by reducing that of the similar product supplied in the same market.

From (1.7), we have

$$\begin{aligned} q^*_{2H} - q^*_{2L} &= \frac{1}{3}(r - \frac{2(2c_{2H} - c_{1H})}{\mu}) \gtreqless 0 \\ &\Leftrightarrow \mu \gtreqless \frac{2(2c_{2H} - c_{1H})}{r} \Leftrightarrow q^*_{2H} \gtreqless q^*_{2L} \end{aligned} \tag{1.14}$$

and

$$\begin{aligned} q^*_{2H} - q^*_{1L} &= \frac{r}{3} - \frac{2c_{2H} - c_{1H}}{3\mu} - \frac{2c_{1H} - c_{2H}}{3\mu} \\ &= q^*_{1H} - q^*_{2L} = \frac{1}{3\mu}(\mu r - (c_{2H} + c_{1H})) \\ &\gtreqless 0 \Longleftrightarrow \mu \gtreqless \frac{c_{2H} + c_{1H}}{r}. \end{aligned} \tag{1.15}$$

From (1.8), we also see that

$$\frac{c_{1H}+c_{2H}}{r} > \frac{2c_{2H}-c_{1H}}{r}.$$

Then, from the above inequality, (1.15), (1.14), and proposition 2.1, we immediately obtain

$$\begin{aligned} q_{2H}^* &\leq q_{1L}^* < q_{1H}^* \leq q_{2L}^* \text{ for } \frac{2c_{2H}-c_{1H}}{r} < \mu \leq \frac{c_{1H}+c_{2H}}{r}, \\ q_{1L}^* &< q_{2H}^* < q_{2L}^* < q_{1H}^* \text{ for } \frac{c_{1H}+c_{2H}}{r} < \mu < \frac{2(2c_{2H}-c_{1H})}{r}, \\ q_{1L}^* &< q_{2L}^* \leq q_{2H}^* < q_{1H}^* \text{ for } \frac{2(2c_{2H}-c_{1H})}{r} \leq \mu. \end{aligned} \tag{1.16}$$

Thus, we present the following proposition, without proof.

Proposition 1.2 *In the duopoly equilibrium derived above, if the difference in the quality of the two goods, μ, is sufficiently small (i.e., $\mu \in (\frac{2c_{2H}-c_{1H}}{r}, \frac{c_{1H}+c_{2H}}{r}]$), then $q_{2H}^* \leq q_{1L}^* < q_{1H}^* \leq q_{2L}^*$. As μ approaches $\frac{2c_{2H}-c_{1H}}{r}$ from above, product L of firm 2 cannibalizes product H and q_{2H}^* approaches 0. When μ* grows, *product H of both firms always cannibalizes product L. As μ grows and approaches $\frac{c_{1H}+c_{2H}}{r}$, and q_{2H}^* approaches q_{1L}^* . If μ is included in the median value range (i.e., $\mu \in (\frac{c_{1H}+c_{2H}}{r}, \frac{2(2c_{2H}-c_{1H})}{r})$), then $q_{1L}^* < q_{2H}^* < q_{2L}^* < q_{1H}^*$. As μ grows and approaches $\frac{2(2c_{2H}-c_{1H})}{r}$, q_{2H}^* approaches q_{2L}^* . However, if μ is sufficiently high (i.e., $\mu \in (\frac{2(2c_{2H}-c_{1H})}{r}, \infty)$), then $q_{1L}^* < q_{2L}^* \leq q_{2H}^* < q_{1H}^*$. As μ approaches ∞ , q_{1L}^* and q_{2L}^* vanish.*

The intuition behind Proposition 1.2 is straightforward. When the difference in the quality of the two goods is sufficiently small, the inefficient firm produces far more of low-quality good L, with no production cost, than it does of high-quality good H, which has a higher positive cost. In contrast, the efficient firm produces moderately more of its low-quality good L than it does of good H, since its production cost for good H is lower than that of its rival. However, its marginal revenue from good H is not high, because the difference in the quality of the two goods is very small.

Thus, interestingly, as μ approaches $(2c_{2H} - c_{1H})/r$ from (1.7), q^*_{2H} approaches 0. Thus, the inefficient firm 2 stops producing the high-quality good H, almost specializing in the low-quality good. Then, in equilibrium, the market approaches a three-goods market. This market is filled with large quantities of the low-quality good L supplied by both of firms, but relatively little of the high-quality good H supplied by firm 1.

This result is consistent with the result in Calzada and Valletti (2012) that the optimal strategy for the film studio is to introduce versioning if their goods are not close substitutes for each other. Thus, when the difference in quality between the high-quality good H and the low-quality good L is large to some extent, we can consider that they are not close substitutes for each other. Then, the result in the above proposition asserts that both of firms had better supply both of goods in the market, that is, to obey 'versioning strategy,' in Calzada and Valletti (2012). On the other hand, if the difference in quality of two goods reduces to nearly zero and they become close substitutes each other, the best strategy of the inefficient firm 2 is to vanish the output of its high-quality goods H and to specialize in the low-quality good L!

When the difference in the quality of the two goods becomes high, the efficient firm produces far more of the high-quality good than it does of the low-quality good, because it is profitable to do so. However, the inefficient firm also reduces the output of its low-quality good and increases that of its high-quality good, because the profitability of good H becomes large, even though the inefficient firm's production cost is higher than that of its rival.

In this case, as μ approaches $(c_{1H} + c_{2H})/r$ from (1.7), q^*_{2H} approaches q^*_{1L}. As μ increases further over $(c_{1H} + c_{2H})/r$, the cannibalization from the low-quality good to the high-quality good of efficient firm 1 increases, since the benefit to the efficient firm 1 of supplying the high-quality good over the low-quality good increases. However, the same benefit to the inefficient firm 2 decreases, until the former surpasses the latter. Then, as μ approaches $2(2c_{2H} - c_{1H})/r$, q^*_{2H} approaches q^*_{2L}. Lastly, as μ increases further over $2(2c_{2H} - c_{1H})/r$ to infinity, q^*_{1L} and q^*_{2L} vanish and both firms only produce their high-quality goods H.

Next, we analyze the comparative statics of the equilibrium outputs and profits of the firms for differences in the quality and in the marginal costs of good H.

Proposition 1.3 *In the duopoly equilibrium derived above, when the difference in the quality of the two goods,* μ, *or the marginal cost of high-quality good* H *of competitor* c_{jH} *increases* (*decreases*), *then cannibalization occurs in the outputs of firm* i *such that the supply of high-quality* (*low-quality*) *good* H (L) *increases at the expense of one of low-quality* (*high-quality*) *good* L (H). *However, if the marginal cost of its own high-quality good* H, c_{iH}, *increases (decreases), then cannibalization occurs in the outputs of firm i such that the supply of low-quality (high-quality) good L (H) increases at the expense of one of high-quality (low-quality) good H (L).*

From (1.11), we have

$$\frac{\partial \pi_i^*}{\partial \mu} = \frac{(\mu r + 2c_{iH} - c_{jH})(\mu r - (2c_{iH} - c_{jH}))}{9\mu^2} > 0, i = 1, 2. \tag{1.17}$$

Furthermore, we also check the effects of production costs on profit. From (1.11), we have

$$\frac{\partial \pi_i^*}{\partial c_{iH}} = -\frac{4}{9}(r - \frac{2c_{iH} - c_{jH}}{\mu}) < 0, \ \frac{\partial \pi_i^*}{\partial c_{jH}} = \frac{2}{9}(r - \frac{2c_{iH} - c_{jH}}{\mu}) > 0 \ .$$

Thus, we obtain the following proposition.

Proposition 1.4 *When the difference in the quality of the two goods increases, the equilibrium profits of both firms increase. Furthermore, a decrease in the marginal cost of a firm's own good H or an increase in the marginal cost of the competitor's good H increases the profit of the firm.*

This proposition is plausible. When the difference in the quality between two goods is sufficiently small, the inefficient firm produces more of the low-quality good than it does of the high-quality good, from equation (1.16), to avoid suffering from the positive marginal cost of producing the high-quality good. Then, an increase in the difference in the quality of the two goods, μ, or a decrease in the unit cost of a firm's own good H or an increase in the unit cost of its competitor's good H induces this firm to produce more of the high-quality good. Thus, it reduces the quantity of the low-quality good L because of cannibalization. However, from equations (1.7) and (1.16), the proportion of the cannibalization from the low-quality good to the high-quality good in both firms is different. That of the efficient firm 1 is lower than that of the inefficient firm 2 because of the cost efficiency of firm 1 for the high-quality good.[11] Similarly, if the difference in quality is sufficiently small, a decrease in a firm's own unit cost of good H or an increase in the unit cost of the rival firm has a similar effect on both firms' proportions of cannibalization from the low-quality good to the high-quality good.

However, if the difference in quality between the goods μ becomes sufficiently large, the efficient firm 1 produces more of the high-quality good and reduces the quantity of the low-quality good because of its cost efficiency in the case of the high-quality good. Then, the inefficient firm 2 reduces the quantity of the low-quality good and increases the output of the high-quality good to limit the reduction in its profit owing to the cannibalization from the low-quality good to the high-quality good. In the case of a decrease in a firm's own unit cost of good H or an increase in the unit cost of the rival firm when the difference in quality between the goods, μ, is large, the effect is similar to the effect on both firms' proportions of cannibalization from the low-quality good to the high-quality good. The changes in μ, c_{iH},and c_{iH}

[11]From (1.7), the proportions of the cannibalization for firm 1 and firm 2 from the low-quality good to high-quality good owing to an increase in the difference in quality are expressed by
$\Delta Canniba^1_{qL\to H(\mu)} \equiv \partial q^*_{1H}/\partial\mu - \partial q^*_{1L}/\partial\mu = \left((2c_{1H} - c_{2H}) - (2c_{2H} - c_{1H})\right)/(3\mu^2)$
$= 2(2c_{1H} - c_{2H})/(3\mu^2)$,
and
$\Delta Canniba^2_{qL\to H(\mu)} \equiv \partial q^*_{21H}/\partial\mu - \partial q^*_{2L}/\partial\mu = \left((2c_{2H} - c_{1H}) - (2c_{2H} - c_{1H})\right)/(3\mu^2)$
$= 2(2c_{2H} - c_{1H})/(3\mu^2)$, respectively. Hence,
$\Delta Canniba^1_{qL\to H(\mu)} - \Delta Canniba^2_{qL\to H(\mu)} = 2(c_{2H} - c_{1H})/\mu^2 > 0$.
Furthermore, from (1.16), we see that
$q^*_{1H} - q^*_{1L} < q^*_{2L} - q^*_{2H}$ if $\frac{2c_{2H}-c_{1H}}{r} < \mu < \frac{c_{1H}+c_{2H}}{r}$.

mean that the increase in the profit of firm 1 surpasses that of firm 2.[12]

1.3 Welfare Analysis with Asymmetric Cost

In this section, we describe the comparative statics of the social welfare in the equilibrium.

The social surplus in equilibrium, derived in the preceding section, is given by

$$\begin{aligned} W^* &= \int_{\frac{r}{3}}^{\hat{\theta}^*} \theta d\theta + \int_{\hat{\theta}^*}^{r} (1+\mu)\theta d\theta - c_{1H}q_{1H}^* - c_{2H}q_{2H}^* \\ &= -\frac{\mu}{2}(\hat{\theta}^*)^2 - \frac{r^2}{18} + \frac{(1+\mu)r^2}{2} - c_{1H}q_{1H}^* - c_{2H}q_{2H}^*. \end{aligned} \tag{1.18}$$

First, we explore the effect of a change in unit cost on social welfare. From (1.4) and (1.7)

$$\frac{\partial W^*}{\partial c_{iH}} = \frac{11c_{iH} - 7c_{jH} - 4\mu r}{9\mu} \qquad i = 1, 2.$$

Thus,

$$\begin{cases} \frac{\partial W^*}{\partial c_{1H}} < 0 \\ \frac{\partial W^*}{\partial c_{2H}} > 0 \qquad if\ \frac{2c_{2H}-c_{1H}}{r} \leq \mu < \frac{11c_{2H}-7c_{1H}}{4r} \\ \frac{\partial W^*}{\partial c_{2H}} \leq 0 \qquad if\ \frac{11c_{2H}-7c_{1H}}{4r} \leq \mu. \end{cases} \tag{1.19}$$

Finally, we show that a change in the difference in quality between the

[12]For an increase in μ, we see that $\frac{\partial \pi_1^*}{\partial \mu} - \frac{\partial \pi_2^*}{\partial \mu} = (c_{1H} + c_{2H})(c_{2H} - c_{1H})/(3\mu^2) > 0$, since $c_{2H} > c_{1H} > 0$, from (1.17). The argument is similar for a decrease in c_{iH} and an increase in c_{jH}.

two goods always has a positive effect on social welfare, as follows:

$$\frac{\partial W^*}{\partial \mu} = \frac{8\mu^2 r^2 - 11c_{1H}^2 - 11c_{2H}^2 + 14c_{1H}c_{2H}}{18\mu^2} \tag{1.20}$$

The sign of $\partial W^*/\partial\mu$ is determined by the sign of the numerator of (1.20), where we define the numerator by $W_\mu^n(r)$, and $W_\mu^n(r)$ is a quadratic in r. Evaluating $W_\mu^n(r)$ at $r = (2c_{2H} - c_{1H})/\mu$, we have

$$\begin{aligned} W_\mu^n(\frac{2c_{2H} - c_{1H}}{\mu}) &= 3(7c_{2H}^2 - c_{1H}^2 - 6c_{1H}c_{2H}) \\ &= 3(c_{2H} - c_{1H})(7c_{2H} + c_{1H}) > 0, (\because c_{2H} > c_{1H}) \end{aligned} \tag{1.21}$$

and we see that the slope of $W_\mu^n(r)$ with respect to r is

$$\left.\frac{\partial W_\mu^n(r)}{\partial r}\right|_{r=\frac{2c_{2H}-c_{1H}}{\mu}} = 16(2c_{2H} - c_{1H}) > 0.$$

Then, we obtain

$$\frac{\partial W^*}{\mu} > 0. \tag{1.22}$$

Thus, we show that an increase in the difference between the two goods improves social welfare. From (1.19) and (1.22), we have following proposition.

Proposition 1.5 *The social surplus in equilibrium increases with*

1. *a decrease in the marginal cost of the efficient firm for the high-quality good.*

2. *a decrease (increase) in the unit cost of the inefficient firm when pro-*

ducing the high-quality good if the difference in quality is sufficiently large (small).

Moreover, an increase in the difference between the two goods always increases the social surplus in equilibrium.

The second part of this proposition is both interesting and counter-intuitive, because we may think that an increase in the production cost would lead to a decrease in social welfare. However, a case exists in which social welfare improves if there is an increase in the marginal cost of the high-quality good. The reason is that when the difference in quality is small, the increase in the marginal cost of the inefficient firm leads to a reduction in the total cost; $(\partial Total\ cost)/\partial c_{2H} < 0$. This has a positive effect on social welfare. On the other hand, the effect on total consumer utility is always negative; $(\partial Total\ utility)/\partial c_{2H} < 0$. Thus, when the positive effect of the former dominates the negative effect of the latter, the social surplus in equilibrium increases because the unit cost to the inefficient firm of producing good H is high and the difference in quality is sufficiently small. In Lahiri and Ono (1988), they show that a marginal cost reduction of a firm with a sufficiently low share can decrease welfare by production substitution. This proposition reappears their finding by multi-product firm and cannibalization.

1.4 Concluding Remarks

In this study, we considered and proposed a duopoly model of cannibalization in which two firms each produce and sell two distinct products that are differentiated vertically in the same market. Then, we showed that in the market equilibrium, the efficient firm produces more of the high-quality good and the inefficient firm produces more of the low-quality good. When the difference in the quality of the two types of goods is small (large), cannibalization for firm 2 (firm 1) is stronger than that for firm 1 (firm 2).

Furthermore, we presented several comparative statics and established that an increase in the difference in the quality of the two types of goods (a reduction in the marginal cost of producing its own high-quality good) leads to cannibalization such that the high-quality good drives the low-quality good out of the market. Similarly, a decrease in the difference in the quality of the two goods (an increase in the marginal cost of the high-quality good of the competitor) causes cannibalization such that the low-quality good drives the high-quality good out of the market. However, unless the market has goods that are extremely differentiated or extremely similar in terms of quality, cannibalization does not keep one product of a firm from the market, and firms supply both goods. Furthermore, we characterize graphically product line strategies of firms by the two ratios relationship and established that the change in the quality superiority and the relative cost efficiency ratios causes cannibalization, so that it crucially affects the decision making of firm's product line.

We also presented an intuitive explanation for these comparative statics. In relating to the results in marketing studies on product segmentation and product distribution strategies, we also establish a result which is consistent with the result in Calzada and Valletti (2012) that the optimal strategy for the film studio is to introduce versioning if their goods are not close substitutes for each other. Thus, when the difference in quality between the high-quality good and the low-quality good is large to some extent and so they are not close substitutes for each other, we show that both of firms had better supply both of goods in the market, that is, they should obey 'versioning strategy.'

Then, we conducted a welfare analysis and showed that an increase in the difference between the two goods and a decrease in the production costs of the high-quality good for the efficient firm always increase social welfare. However, an increase in the marginal cost of producing the high-quality good for the inefficient firm does not always harm social welfare. In particular, if the difference in quality is sufficiently small, rather counter-intuitively, an *increase in the unit cost* of the high-quality good for the inefficient firm *improves social welfare.*

Extensions to this study in future research are possible. For example, it would be useful to analyze a case in which each firm can choose its quality level as well as the number of goods it produces. In addition, in this study, we do not consider a market with network externality, which would be worth studying if we consider a market such as the tablet PC industry described in section 2. Indeed, we are analyzing such a market in another study.

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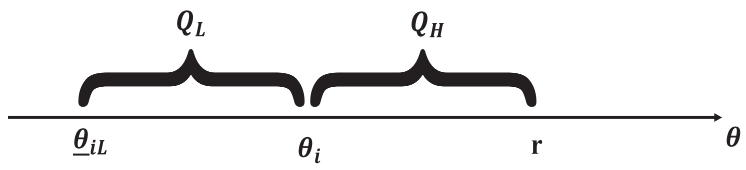
Figure 1.1



Chapter 2

Product Line Strategy in a Vertically Differentiated Duopoly

abstract[1]

In real oligopolistic market, we often firms supply several own products differentiated in quality in a same market. To explore why oligopolistic firms do so, we consider a duopoly model in which firms with different costs supply two vertically differentiated products in the *same market*. We characterize graphically product line strategies of firms by the change in the quality superiority and the relative cost efficiency ratios.

Keywords: Multi-product firm; Duopoly; Cannibalization; Vertical product differentiation

[1]The authors are grateful to Tommaso Valletti, Federico Etro, Hong Hwang, Noriaki Matsushima, Toshihiro Matsumura, Kenji Fujiwara, and Keizo Mizuno for their useful comments on an earlier version of this paper. The second author was supported by Grants-in-Aid for Scientific Research (Nos. 23330099 and 24530255) MEXT. Furthermore, this chapter is sum of the revised version of Kitamura and Shinkai (2015a) and a part of Kitamura and Shinkai (2015b).

2.1 Introduction

As a mentioned in previous chapter, there are oligopolistic markets in which firms produce and sell multiple products that are vertically differentiated within the same market. Such markets present more cases of cannibalization. Cannibalization within the same market occurs when a firm increases the output of one of its products by reducing the output of a similar competing product in the same market. The objective of this study is to examine cannibalization within the same market from strategic point of view of the multi-product firm which supplies two goods differentiated in quality. We do not consider new entries to the market and choice of quality level as considered in Johnson and Myatt (2003). We consider a duopoly in which each firm produces and supplies two kinds of vertically differentiated high-quality and low-quality goods in a market. Then, we explore the condition under which both or either of firms specialize(s) in one of the high or low-quality goods. To understand how cannibalization affects product line strategies of firms, we consider two ratio indicators: (1) the predominance quality ratio of high-quality good to that of lowquality; and (2) the relative marginal cost efficiency of high-quality good between the two firms. We find that cannibalization can be seen as a product line control strategy characterized by the quality superiority of high-quality good to low-quality and the relative cost efficiency of an efficient firm. By limiting at most two vertically differentiated goods that each firm can supply to the same market, we succeed in characterizing product line strategies of firms through cannibalization graphically in the plane of these two ratio indicators.

2.2 Product Line Strategy

[2]The objective of this section is to examine more correctly substitution of products within the same market from strategic point of view of the multi-product firm which supplies two goods differentiated in quality. For this purpose, we consider a duopoly game with two vertically differentiated products under nonnegative outputs constraints, provided that any rival's product line strategies are given.

At first, we set $r = 1$, $c_{2H} > c_{1H} = 1$ and $V_H = \mu^{'} V_L = \mu^{'} > V_L = 1$. In this section, each firm simultaneously chooses the output (outputs) of for H or L (both) type(s) of product(s) to supply that maximizes this profit function in Cournot fashion under nonnegativitiy outputs constraints provided that its rival also chooses nonnegativity output(s). Thus firm i has a belief on its rival's any product line strategies $\mathbf{s}_j \in \mathbf{S}_j \equiv \{(0,0),(+,0),(0,+),(+,+)\}$, where $(0,0)$ implies $(q_{jH} = 0, q_{jL} = 0)$, $(+,0)$ implies $(q_{jH} > 0, q_{jL} = 0)$ and so on. For any given $\mathbf{s}_j \in \mathbf{S}_j$

$$\begin{aligned} \max_{q_{iH}, q_{iL}} \pi_i &= \{\mu^{'}(1 - Q_H) - Q_L - c_{iH})q_{iH} + (1 - Q_H - Q_L)q_{iL} \quad (2.1) \\ s.t.\ q_{iH} &\geq 0, q_{iL} \geq 0,\ i \neq j, i, j = 1, 2. \end{aligned}$$

Kuhn-Tucker conditions are

$$\frac{\partial \pi_i}{\partial q_{iH}} \leq 0, \frac{\partial \pi_i}{\partial q_{iL}} \leq 0, \quad (2.2)$$

[2]This section is a revised version of Kitamura and Shinkai (2015b).

$$q_{iH} \cdot \frac{\partial \pi_i}{\partial q_{iH}} = q_{iL} \cdot \frac{\partial \pi_i}{\partial q_{iL}} = 0, \tag{2.3}$$

$$q_{iH} \geq 0, q_{iL} \geq 0. \tag{2.4}$$

Each firm chooses its product line strategy of two vertically differentiated products, that is, whether it produces positive (zero) quantities of product H and L under its belief on its rival firm's product line strategies.

There are sixteen cases to be solved according to each firm's product line strategies under its beliefs on its rival firm's product line strategies except for the trivial case in that both firms never produces both products H and L. After some tiresome calculations, we can show that ten cases out of these sixteen cases have no equilibrium in the correspondent games. Hence, we have the following.[3]

Proposition 2.1 *In the duopoly equilibrium of the game under rival's nonnegative quantities belief presented above, the following five cases have an equilibrium in the correspondent games.*

(Case A) $q_{1H}^{*A} = q_{2H}^{*A} = 0, q_{1L}^{*A} > 0, q_{2L}^{*A} > 0,$ *iff* $\mu^{'} \leq 2.$

(Case B) $q_{1L}^{*B} = q_{2H}^{*B} = 0, q_{2L}^{*B} > 0, q_{1H}^{*B} > 0$ *iff*

$$4 \leq \mu^{'} \leq \frac{1}{2}(2c_{2H} + \sqrt{2(2c_{2H}^2 - c_{2H} + 2)}).$$

[3] See Appendix for these calculations.

(Case C) $q_{1L}^{*C} = 0, q_{2L}^{*C} > 0, q_{1H}^{*C} > 0, q_{2H}^{*C} > 0$ *iff*

$$\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu^{'},\ \mu^{'} > 2 - c_{2H}\ and\ c_{2H} \geq 2.$$

(Case D) $q_{1L}^{*D} > 0, q_{2L}^{*D} > 0, q_{1H}^{*D} > 0, q_{2H}^{*D} = 0$ *iff*

$$2 < \mu^{'} < 4\ and\ \mu^{'} \leq 2c_{2H}.$$

(Case E) $q_{1L}^{*E} > 0, q_{2L}^{*E} > 0, q_{1H}^{*E} > 0, q_{2H}^{*E} > 0$ *iff*

$$1 < c_{2H} < 2,\ \mu^{'} > 3 - c_{2H}\ \text{ and } \mu^{'} > 2c_{2H}.$$

The details of Proposition 2.1 is as follows.

(Case A) $q_{1H}^{*A} = q_{2H}^{*A} = 0, q_{1L}^{*A} > 0, q_{2L}^{*A} > 0$

$$q_{1H}^{*A} = q_{2H}^{*A} = 0 < q_{1L}^{*A} = q_{2L}^{*A} = \frac{1}{3} \text{ and } \mu^{'} \leq 2, \tag{2.5}$$

where the last inequality needs for the Kuhn-Tucker condition to be satisfied.

(Case B) $q_{1L}^{*B} = q_{2H}^{*B} = 0, q_{2L}^{*B} > 0, q_{1H}^{*B} > 0$

We have

$$q_{1L}^{*B} = q_{2H}^{*B} = 0, q_{1H}^{*B} = \frac{1}{4\mu^{'} - 1}(2\mu^{'} - 3), q_{2L}^{*B} = \frac{1}{4\mu^{'} - 1}(\mu^{'} + 1) \tag{2.6}$$

and

$$4 \leq \mu^{'} \leq \frac{1}{2}(2c_{2H} + \sqrt{2(2c_{2H}^2 - c_{2H} + 2)}),$$

where the last inequality needs for the Kuhn-Tucker condition to be satisfied.

(Case C) $q_{1L}^{*C} = 0, q_{2L}^{*C} > 0, q_{1H}^{*C} > 0, q_{2H}^{*C} > 0$

$$\begin{aligned} q_{1L}^{*C} &= 0, q_{2L}^{*C} = \frac{1}{2(\mu^{'} - 1)} c_{2H}, q_{1H}^{*C} = \frac{1}{3\mu^{'}}(\mu^{'} + c_{2H} - 2), \\ q_{2H}^{*C} &= \frac{1}{6\mu^{'}(\mu^{'} - 1)}(2\mu^{'}(\mu^{'} - 1) - (4\mu^{'} - 1)c_{2H} + 2(\mu^{'} - 1)) \end{aligned} \tag{2.7}$$

$$q_{1H}^{*C} > q_{2H}^{*C}, q_{2L}^{*C} > 0 \text{ and } q_{2H}^{*C} \gtreqless q_{2L}^{*C} \Leftrightarrow \frac{1}{4}(7c_{2H} + \sqrt{49c_{2H}^2 - 8c_{2H} + 16}) \lesseqgtr \mu^{'},$$

and

$$\frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu^{'} \Leftrightarrow q_{2H}^{*C} > 0$$

hold. Furthermore, from the Kuhn-Tucker condition, we have

$$c_{2H} \geq 2. \tag{2.8}$$

For $q_{1H}^{*C} > 0$, the inequality, $\mu^{'} > 2 - c_{2H}$ is necessary to hold. This is hold since $c_{2H} \geq 2$.

(Case D) $q_{1L}^{*D} > 0, q_{2L}^{*D} > 0, q_{1H}^{*D} > 0, q_{2H}^{*D} = 0$

$$q_{1L}^{*D} = \frac{1}{6(\mu^{'} - 1)}(4 - \mu^{'}), q_{2L}^{*D} = \frac{1}{3}, q_{1H}^{*D} = \frac{1}{(\mu^{'} - 1)}(\mu^{'} - 2), q_{2H}^{*D} = 0. \quad (2.9)$$

For q_{1L}^{*D} and q_{1H}^{*D} are positive values, we have

$$2 < \mu^{'} < 4.$$

We also have

$$q_{1L}^{*D} \gtreqless q_{1H}^{*D} \Leftrightarrow \mu^{'} \lesseqgtr \frac{5}{2} \text{ and } \mu^{'} \leq 2c_{2H},$$

where the last inequality has to hold for the Kuhn–Tucker condition to be satisfied.

(Case E) $q_{1L}^{*E} > 0, q_{2L}^{*E} > 0, q_{1H}^{*E} > 0, q_{2H}^{*E} > 0$

$$\begin{aligned} q_{1L}^{*E} &= \frac{1}{3(\mu^{'} - 1)}(2 - c_{2H}), q_{2L}^{*E} = \frac{1}{3(\mu^{'} - 1)}(2c_{2H} - 1), \quad (2.10) \\ q_{1H}^{*E} &= \frac{1}{3(\mu^{'} - 1)}(\mu^{'} + c_{2H} - 3), q_{2H}^{*E} = \frac{1}{3(\mu^{'} - 1)}(\mu^{'} - 2c_{2H}). \end{aligned}$$

For $q_{1L}^{*E} > 0$ and $q_{1L}^{*E} > 0$,

$$1 < c_{2H} < 2$$

is necessary to hold. We see that $q_{1H}^{*E} > q_{2H}^{*E}$ under this condition. For $q_{1H}^{*E} > 0$ and $q_{2H}^{*E} > 0$, we see that

$$\mu^{'} > 3 - c_{2H} \text{ and } \mu^{'} > 2c_{2H}$$

are necessary to hold, respectively. We also have

$$q_{1H}^{*E} \gtreqless q_{1L}^{*E} \Leftrightarrow \mu^{'} \gtreqless 5 - 2c_{2H},\ q_{2H}^{*E} \gtreqless q_{1L}^{*E} \text{ and } q_{2L}^{*E} \gtreqless q_{1H}^{*E} \Leftrightarrow \mu^{'} \gtreqless c_{2H} + 2$$

Furthermore we also show that

$$q_{2H}^{*E} \gtreqless q_{2L}^{*E} \Leftrightarrow \mu^{'} \gtreqless 4c_{2H} - 1.$$

Summarizing above results, we have the following proposition:

Proposition 2.2 *In the duopoly equilibrium of the game under rival's nonnegative quantities belief presented above, the next inequalities hold among the outputs of high-quality good and low quality good of each firm*:

$$\begin{aligned} 0 &< q_{2H}^{*E} < q_{1H}^{*E} \leq q_{1L}^{*E} < q_{2L}^{*E} \\ \text{for } (c_{2H}, \mu^{'}) &\in \{(c_{2H}, \mu^{'}) \in R^{2++} \mid \mu^{'} > 2c_{2H}, \mu^{'} \leq 5 - 2c_{2H} \text{ and } 1 < c_{2H} < \frac{5}{4}\} \ (I'), \end{aligned}$$

$$\begin{aligned} 0 &< q_{2H}^{*E} < q_{1L}^{*E} < q_{1H}^{*E} < q_{2L}^{*E} \text{ for } (c_{2H}, \mu^{'}) \in \\ \{(c_{2H}, \mu^{'}) &\in R^{2++} \mid \mu^{'} > 2c_{2H}, \mu^{'} > 5 - 2c_{2H}, \mu^{'} < c_{2H} + 2 \text{ and } 1 < c_{2H} < 2\} \ (I), \end{aligned}$$

$$
\begin{aligned}
0 &< q^{*E}_{1L} \leq q^{*E}_{2H} < q^{*E}_{2L} < q^{*E}_{1H} \text{ for } (c_{2H}, \mu^{'}) \in \\
\{(c_{2H}, \mu^{'}) &\in R^{2++} \mid \mu^{'} \leq c_{2H} + 2, \mu^{'} < 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \; (II),
\end{aligned}
$$

$$
\begin{aligned}
0 &< q^{*E}_{1L} < q^{*E}_{2L} \leq q^{*E}_{2H} < q^{*E}_{1H} \text{ for } (c_{2H}, \mu^{'}) \in \\
\{(c_{2H}, \mu^{'}) &\in R^{2++} \mid \mu^{'} \geq 4c_{2H} - 1, \text{ and } 1 < c_{2H} < 2\} \; (III),
\end{aligned}
$$

$$
\begin{aligned}
q^{*C}_{1L} &= 0 < q^{*C}_{2L} \leq q^{*C}_{2H} < q^{*C}_{1H} \text{ for } (c_{2H}, \mu^{'}) \in \\
&\{(c_{2H}, \mu^{'}) \in R^{2++} \mid \\
&\frac{1}{4}(7c_{2H} + \sqrt{49c^2_{2H} - 8c_{2H} + 16}) > \mu^{'} \geq \frac{1}{2}(2c_{2H} + \sqrt{4c^2_{2H} - 2c_{2H} + 4}) > 4 \\
&, c_{2H} \geq 2\} \; (VI),
\end{aligned}
$$

$$
\begin{aligned}
q^{*C}_{1L} &= 0 < q^{*C}_{2H} < q^{*C}_{2L} < q^{*C}_{1H} \text{ for } (c_{2H}, \mu^{'}) \in \\
&\{(c_{2H}, \mu^{'}) \in R^{2++} \mid \mu^{'} > \frac{1}{4}(7c_{2H} + \sqrt{49c^2_{2H} - 8c_{2H} + 16}) > 4, c_{2H} \geq 2\} \; (V),
\end{aligned}
$$

$$
\begin{aligned}
q^{*B}_{1H} &\geq q^{*B}_{2L} > q^{*B}_{1L} = q^{*B}_{2H} = 0 \text{ for } (c_{2H}, \mu^{'}) \in \\
&\{(c_{2H}, \mu^{'}) \in R^{2++} \mid 4 \leq \mu^{'} \leq (2c_{2H} + \sqrt{4c^2_{2H} - 2c_{2H} + 4})/2\} \; (IV),
\end{aligned}
$$

$$q_{2L}^{*D} = \frac{1}{3} > q_{1L}^{*D} \geq q_{1H}^{*D} > q_{2H}^{*D} = 0 \text{ when } 1 < \mu^{'} \leq \frac{5}{2}, \mu^{'} \leq 2c_{2H} \quad (VIII),$$
$$q_{2L}^{*D} = \frac{1}{3} > q_{1H}^{*D} > q_{1L}^{*D} > q_{2H}^{*D} = 0 \text{ when } \frac{5}{2} < \mu^{'} < 4, \ \mu^{'} \leq 2c_{2H} \ (VII),$$

$$q_{1H}^{*A} = q_{2H}^{*A} = 0 < q_{1L}^{*A} = q_{2L}^{*A} = \frac{1}{3} \ when 1 < \mu^{'} \leq 2 \ (IX),$$

where Roman numbers imply the area in $c_{2H} - \mu^{'}$ *plane in Figure 2.1, respectively.*

We present classification of product line strategy of the duopoly game under rival's nonnegative output belief in $c_{2H} - \mu^{'}$ plane in Figure 2.1.

Hence, the horizontal and the vertical axes variable in Figure 2.1 implies the relative cost ratio c_{2H} and the quality value ratio $\mu^{'}$. In any point $(c_{2H}, \mu^{'})$ belonging to Areas I, II and III in Figure 2.1, both firms supply high and low-quality goods. Thus, as the quality value ratio $\mu^{'}$ is sufficiently high and the relative cost ratio c_{2H} is also small in these areas, the inefficient firm produces far more of low-quality good, with no production cost, than it does of high-quality, which has a higher positive cost. In contrast, the efficient firm produces moderately more of its high-quality good H than it does of good L, since its production cost for good H is lower than that of its rival. However, its marginal revenue from good H is not high, because the quality superiority $\mu^{'}$ is not so large. As the point $(c_{2H}, \mu^{'})$ moves from area I to areas II and III, the cannibalization from low-quality to high-quality of both firms proceeds. Such cannibalization of the efficient firm is stronger than that of the inefficient firms.

This result is consistent with the result in Calzada and Valletti (2012) that the optimal strategy for the film studio is to introduce versioning if their goods are not close substitutes for each other. Thus, when the predominance in quality value of the high-quality good H is large to some extent, we can consider that they are not close substitutes for each other. Then, the result in the above proposition asserts that both of firms had better supply both of goods in the market, that is, to obey 'versioning strategy,' in Calzada and Valletti (2012).

In contrast, when relative cost efficiency c_{2H} is large (Areas from IV to IX) the efficient firm never supplies its low-quality good, thus in equilibrium, the market becomes a three-goods market at first. In this market is filled with large quantities of the low-quality good L supplied by both of firms, but relatively little of the high-quality good H supplied by firm 1. As the quality superiority μ' reduces further, the inefficient firm 2 stops producing the high-quality good H specializing in the low-quality good. Then,the efficient firm 1 specializes in high-quality good supply and the inefficient firm 2 does in low-quality good supply, respectively.

2.3 Concluding Remarks

In this study, we considered a duopoly model of cannibalization in which two firms each produce and sell two distinct products that are differentiated vertically in the same market.

Then, we established that the change in the quality superiority and the relative cost efficiency ratios causes cannibalization, so that it crucially affects the decision making of firm's product line. Furthermore, we consider a duopoly game with two vertically differentiated products under nonnegative outputs constraints and the belief on its rival's product line strategies. Further, we derive an equilibrium for the game and characterize graphically firms' product line strategies through the quality superiority and the relative cost efficiency ratios.

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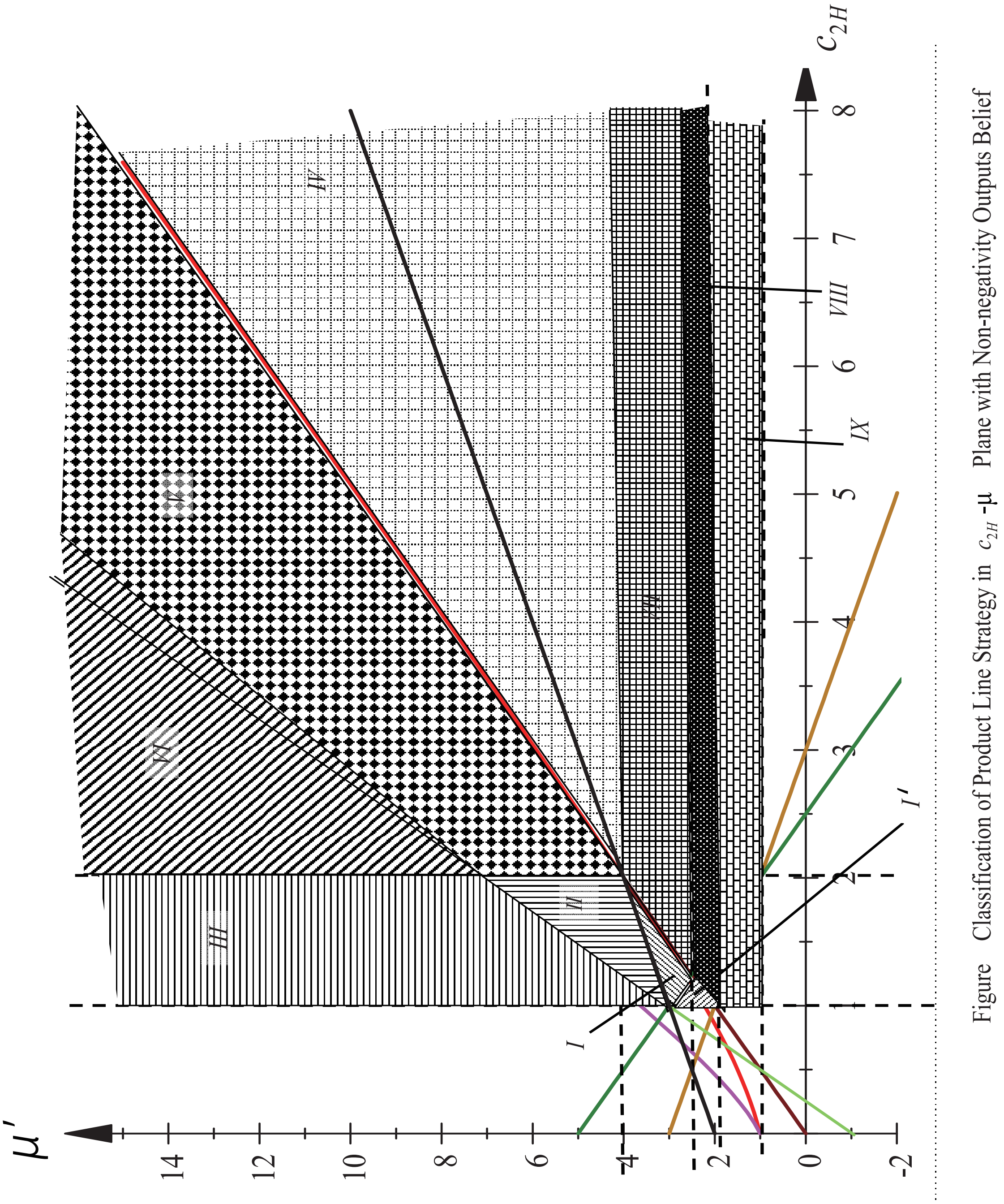
Figure 2.1 Classification of Product Line Strategy in $c_{2H} - \mu'$ Plane with Non-negativity Outputs Belief ($r = 1$)



Appendix

In this model, there are following sixteen types according to each firm's product line strategies.

(1) $q_{1H} = q_{2H} = q_{1L} = q_{2L} = 0$

(2) $q_{1H} > 0, q_{2H} = q_{1L} = q_{2L} = 0$

(3) $q_{1H} > 0, q_{2H} > 0, q_{1L} = q_{2L} = 0$

(4) $q_{1H} > 0, q_{2H} > 0, q_{1L} > 0, q_{2L} = 0$

(5) $q_{1H} > 0, q_{2H} > 0, q_{1L} > 0, q_{2L} > 0$

(6) $q_{2H} > 0, q_{1H} = q_{1L} = q_{2L} = 0$

(7) $q_{2H} > 0, q_{1L} > 0, q_{1H} = q_{2L} = 0$

(8) $q_{2H} > 0, q_{1L} > 0, q_{2L} > 0, q_{1H} = 0$

(9) $q_{1L} > 0, q_{1H} = q_{2H} = q_{2L} = 0$

(10) $q_{1L} > 0, q_{2L} > 0, q_{1H} = q_{2H} = 0$

(11) $q_{2L} > 0, q_{1H} = q_{2H} = q_{1L} = 0$

(12) $q_{2L} > 0, q_{1H} > 0, q_{2H} = q_{1L} = 0$

(13) $q_{1H} > 0, q_{1L} > 0, q_{2H} = q_{2L} = 0$

(14) $q_{2H} > 0, q_{2L} > 0, q_{1H} = q_{1L} = 0$

(15) $q_{1H} > 0, q_{1L} > 0, q_{2L} > 0, q_{2H} = 0$

(16) $q_{1H} > 0, q_{2H} > 0, q_{2L} > 0, q_{1L} = 0$

However, from Kuhn-Tucker conditions(2.2), (2.3) and (2.4), we have the five cases of equilibrium. Here, note that these Kuhn-Tucker conditions are a necessary and sufficient condition for existence of five cases of equilibrium since objective functions are concave and constraint conditions are linear in this model. These calculations are as follows.

The inequalities (2.2) are rewritten for all types as

$$\mu' - 2\mu' q_{1H} - \mu' q_{2H} - 2q_{1L} - q_{2L} - 1 \leq 0 \tag{2.11}$$

$$1 - 2q_{1H} - q_{2H} - 2q_{1L} - q_{2L} \leq 0 \tag{2.12}$$

$$\mu' - 2\mu' q_{2H} - \mu' q_{1H} - q_{1L} - 2q_{2L} - c_{2H} \leq 0 \tag{2.13}$$

$$1 - 2q_{2H} - q_{1H} - q_{1L} - 2q_{2L} \leq 0 \tag{2.14}$$

- The type (1):$q_{1H} = q_{2H} = q_{1L} = q_{2L} = 0$.

  Then, since (2.12) implies $1 \leq 0$, type (1) is in contradiction with Kuhn -Tucker condition.

- The type (2): $q_{1H} > 0, q_{2H} = q_{1L} = q_{2L} = 0$.

  From (2.3), we have
  $$q_{1H} = \frac{\mu' - 1}{2\mu'}.$$
  Then, since (2.12) implies $1 \leq 0$, type (2) is in contradiction with Kuhn -Tucker condition.

- The type (3): $q_{1H} > 0, q_{2H} > 0, q_{1L} = q_{2L} = 0$.

  From (2.3), we have
  $$q_{1H} = \frac{\mu' + c_{2H} - 2}{3\mu'}, \quad q_{2H} = \frac{\mu' - 2c_{2H} + 1}{3\mu'}.$$

Then, since (2.12) implies $3 \leq 0$, type (3) is in contradiction with Kuhn -Tucker condition.

- The type (4): $q_{1H} > 0, q_{2H} > 0, q_{1L} > 0, q_{2L} = 0$.

  From (2.3), we have

$$\begin{aligned} q_{1H} &= \frac{2(\mu^{'})^2 - 6\mu^{'} + 2(\mu^{'} - 1)c_{2H} + 1}{6\mu^{'}(\mu^{'} - 1)},\ q_{1L} = \frac{1}{2(\mu^{'} - 1)}, \\ q_{2H} &= \frac{1 + \mu^{'} - 2c_{2H}}{3\mu^{'}}. \end{aligned}$$

  Then, although (2.14) implies $\mu^{'} + 1 + 2(\mu^{'} - 1)c_{2H} \leq 0$, it is not satisfied since $\mu^{'} > 1$. Thus, type (4) is in contradiction with Kuhn -Tucker condition.

- The type (5): $q_{1H} > 0, q_{2H} > 0, q_{1L} > 0, q_{2L} > 0$.

  From (2.3), we have

$$\begin{aligned} q_{1H} &= \frac{\mu^{'} + c_{2H} - 3}{3(\mu^{'} - 1)},\ q_{1L} = \frac{2 - c_{2H}}{3(\mu^{'} - 1)}, \\ q_{2H} &= \frac{\mu^{'} - 2c_{2H}}{3(\mu^{'} - 1)},\quad q_{2L} = \frac{2c_{2H} - 1}{3(\mu^{'} - 1)}. \end{aligned}$$

  Then, each equilibrium output is positive when $2c_{2H} < \mu^{'}$, $3 - c_{2H} < \mu^{'}$ and $c_{2H} < 2$. Thus, the equilibrium of type (15) exists iff $(\mu^{'}, c_{2H})$ satisfy these three inequalities. This corresponds to the equilibrium in the Case E.

- The type (6): $q_{2H} > 0, q_{1H} = q_{1L} = q_{2L} = 0$.

From (2.3), we have

$$q_{2H} = \frac{\mu^{'} - c_{2H}}{2\mu^{'}}.$$

Then, since (2.14) implies $c_{2H} \leq 0$, it is in contradiction with $c_{2H} \geq 1$.

- The type (7): $q_{2H} > 0, q_{1L} > 0, q_{1H} = q_{2L} = 0$.

  From (2.3), we have

$$q_{1L} = \frac{\mu^{'} + c_{2H}}{4\mu^{'} - 1}, \; q_{2H} = \frac{2\mu^{'} - 2c_{2H} - 1}{4\mu^{'} - 1}.$$

  Then, (2.11) and (2.14) require following two inequalities;

$$1 < \mu^{'} \leq \frac{3 - c_{2H} + \sqrt{c_{2H}^2 - 2c_{2H} + 7}}{2}$$
$$1 + 3c_{2H} \leq \mu^{'}.$$

  However, it is not satisfied because $(3c_{2H} + \sqrt{c_{2H}^2 - 2c_{2H} + 7})/2 < 1 + 3c_{2H}$. Thus, type (7) is in contradiction with Kuhn -Tucker condition.

- The type (8): $q_{2H} > 0, q_{1L} > 0, q_{2L} > 0, q_{1H} = 0$.

  From (2.3), we have

$$\begin{aligned} q_{1L} &= \frac{1}{3}, \\ q_{2H} &= \frac{\mu^{'} - 1 - c_{2H}}{2(\mu^{'} - 1)}, \quad q_{2L} = \frac{1 - \mu^{'} + 3c_{2H}}{6(\mu^{'} - 1)}. \end{aligned}$$

  Then, although (2.11) implies $(\mu^{'})^2 - 4\mu^{'} + 3 + c_{2H}(\mu^{'} - 1) \leq 0$, it is not satisfied for any $\mu^{'}$. Thus, type (8) is in contradiction with Kuhn -Tucker condition.

- The type (9): $q_{1L} > 0, q_{1H} = q_{2H} = q_{2L} = 0$.

  From (2.3), we have$q_{1L} = 1/2$. Then, since (2.14) implies $1/2 \leq 0$, type (9) is in contradiction with Kuhn -Tucker conditions.

- The type (10): $q_{1L} > 0, q_{2L} > 0, q_{1H} = q_{2H} = 0$.

  From (2.3), we have

$$q_{1L} = q_{2L} = \frac{1}{3}.$$

  Then, (2.11) and (2.13) require following two inequalities;

$$\mu^{'} \leq 2$$
$$\mu^{'} \leq 1 + c_{2H}.$$

  Therefore, the equilibrium of type (10) exists iff $(\mu^{'}, c_{2H})$ satisfy these two inequalities. This corresponds to the equilibrium in the Case A.

- The type (11): $q_{2L} > 0, q_{1H} = q_{2H} = q_{1L} = 0$.

  From (2.3), we have$q_{2L} = 1/2$. Then, since (2.12) implies $1/2 \leq 0$, type (11) is in contradiction with Kuhn -Tucker condition.

- The type (12): $q_{2L} > 0, q_{1H} > 0, q_{2H} = q_{1L} = 0$.

  From (2.3), we have

$$q_{1H} = \frac{2\mu^{'} - 3}{4\mu^{'} - 1}, \ q_{2L} = \frac{\mu^{'} + 1}{4\mu^{'} - 1}.$$

Then, (2.12) and (2.13) require following two inequalities;

$$4 \le \mu'$$
$$\mu' \le \frac{2c_{2H} + \sqrt{2(2c_{2H}^2 - c_{2H} + 2)}}{2}.$$

Therefore, the equilibrium of type (12) exists iff (μ', c_{2H}) satisfy these two inequalities. This corresponds to the equilibrium in the Case B.

- The type (13): $q_{1H} > 0, q_{1L} > 0, q_{2H} = q_{2L} = 0$.

  From (2.3), we have

$$q_{1H} = \frac{\mu' - 2}{2(\mu' - 1)}, \quad q_{2L} = \frac{1}{2(\mu' - 1)}.$$

  Then, since (2.14) implies $\mu' \le 1$, it is in contradiction with $\mu' > 1$.

- The type (14): $q_{2H} > 0, q_{2L} > 0, q_{1H} = q_{1L} = 0$.

  From (2.3), we have

$$q_{2H} = \frac{\mu' - c_{2H} - 1}{2(\mu' - 1)}, \quad q_{2L} = \frac{c_{2H}}{2(\mu' - 1)}.$$

  Then, since (2.12) implies $\mu' \le 1$, it is in contradiction with $\mu' > 1$.

- The type (15): $q_{1H} > 0, q_{1L} > 0, q_{2L} > 0, q_{2H} = 0$.

$$q_{1H} = \frac{\mu' - 2}{2(\mu' - 1)}, \quad q_{1L} = \frac{4 - \mu'}{6(\mu' - 1)},$$
$$q_{2L} = \frac{1}{3}.$$

Then, each equilibrium output is positive when $2 < \mu^{'} < 4$. Moreover, (2.13) requires $\mu^{'} \leq 2c_{2H}$. Thus, the equilibrium of type (15) exists iff $(\mu^{'}, c_{2H})$ satisfy these two inequalities. This corresponds to the equilibrium in the Case D.

- The type (16): $q_{1H} > 0, q_{2H} > 0, q_{2L} > 0, q_{1L} = 0$.

$$
\begin{aligned}
q_{1H} &= \frac{\mu^{'} - 2 + c_{2H}}{3\mu^{'}}, \\
q_{2H} &= \frac{2\mu^{'}(\mu^{'} - 1) - (4\mu^{'} - 1)c_{2H} + 2(\mu^{'} - 1)}{6\mu^{'}(\mu^{'} - 1)}, \ q_{2L} = \frac{c_{2H}}{2(\mu^{'} - 1)}.
\end{aligned}
$$

Then, each equilibrium output is positive when $(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4})/2 < \mu^{'}$.Furthermore, (2.12) requires $2 \leq c_{2H}$. Thus, the equilibrium of type (16) exists iff $(\mu^{'}, c_{2H})$ satisfy these two inequalities. This corresponds to the equilibrium in the Case C.

Chapter 3

A Monopoly model with Two Vertically Differentiated Goods under Within-Product Network Externalities

abstract[1]

Developing a monopoly model with two vertically differentiated products and a within-product network externality, this study examines the effect of falling cost of high-quality goods. The result shows that both firm profit and welfare become U-shaped in the cost, that is, cost reduction can decrease profits. Further, I discuss how cannibalization between products plays a key role in this counter-intuitive result.

Keywords: Multi-product firm, Monopoly, Cannibalization, Network externality

[1] I thank Noriyuki Doi, Kenji Fujiwara, Hiroaki Ino, Noriaki Matsushima, Akira Miyaoka, Tetsuya Shinkai, and Tommaso Valletti as well as the other participants at the workshop at Kwansei Gakuin University for their useful comments. Any remaining errors are my own. Further, this chapter is revised version of Kitamura (2015) presented at EARIE 2015.

3.1 Introduction

The majority of smartphone carriers sell both high-and low-quality smartphones.[2] Network externalities in this industry exist across products supplied by one firm and within products, that is, all consumers of a good gain, as the number of users purchasing the same smartphone increases. Although prior literature has explored former network externality, no study has analyzed a market with a within-product network externality.[3] This study focuses on a within-product network externality and examines its positive and normative consequences by considering a market with a multi-product firm.

Incorporating a within-product network externality into a multi-product monopoly model, this study examines firm and consumer behavior, and the resulting market configurations.[4] First, I find that cannibalization happens under certain conditions; namely, an increase in consumers of one good occurs at the expense of consumers of other goods sold by same firm (Copulsky, 1976).[5] Second, I demonstrate a counterintuitive result; a decrease in the marginal cost of a high-quality good can *reduce* firm profit. More precisely, profit becomes U-shaped in the marginal cost of the high-quality good. Third,

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[2]An example of vertical differentiation between the iPhone and Android smartphones is found in Geekbench (see http://browser.primatelabs.com/geekbench2/1030202 and http://browser.primatelabs.com/android-benchmarks).

[3]I define this externality as follows: "A consumer who purchases a product from a certain firm gains a network benefit when other consumers purchase the same product from the same or different firm."(Kitamura, 2013)

[4]I use a monopoly model to isolate the implication of a within-product network externality and a multi-product firm, and to stress that the result holds, even in the absence of strategic interactions among oligopolistic firms. The oligopoly case is left to future research.

[5]The relevance of cannibalization has been established empirically. For instance, Ghose et al. (2006) and Smith and Telang (2008) find that 16% of used books, 24% of used CDs, and 86% of used DVDs directly cannibalize new product sales on Amazon.com.

the relationship between welfare and marginal cost also becomes U-shaped.[6]

A U-shaped profit with respect to marginal cost implies cost reduction, for instance, through innovation or an R&D subsidy, can decrease firm profit. Under the U-shaped profit curve, monopoly profit decreases if the production cost of the high-quality good is high and the degree of cost reduction is small. In other words, a sufficiently significant cost reduction is required to increase profit. When the fulfilled expectation, explored below, is reasonable, a small R&D subsidy can be detrimental rather than beneficial.

Two assumptions play a key role behind these remarkable results. The first important assumption is that of a multi-product firm. In this background market structure, cost reduction leads to cannibalization and the transition of network within firm affects profit and welfare. The second key assumption is a fulfilled expectations equilibrium, where (i) consumers' expected network sizes are equal to actual (*rational expectation*), and (ii) " consumers' expectations of the network sizes are given to all firms" (Katz and Shapiro, 1985, pp. 427–428).[7] This second definition implies that the firms' announcement of its planned level of output has no effect on consumer expectations. In this case, the firm cannot commit itself and is unable to transfer the network sizes optimally in response to the change in marginal cost. This property of fulfilled expectation equilibrium is the key rationale behind the counter intuitive relationship between monopoly profit and falling

[6] While Lahiri and Ono(1988)find that under Cournot *oligopoly*, marginal cost reduction in a firm with a sufficiently low share decreases welfare, in this study, under *monopoly*, I show the a similar result is caused by two key assumptions: fulfilled expectations equilibrium and multi-product firm.

[7] Newbery and Stiglitz (1981, pp. 134–135) defend the rational expectation hypothesis, claiming that if consumers' past expectations are not rational, they are still modifying their expectations.

cost. The study clarifies how assumption (ii) works by comparing the fulfilled expectation equilibrium where the firm takes the consumers expectation into consideration, that is, when it commits its own network size/output level.[8]

This equilibrium concept, proposed by Katz and Shapiro (1985), has been used in the literature on network industries (e.g., Barrett and Yang, 2001; Hahn, 2003). Katz and Shapiro (1985) find no problem regarding firm commitment because their main result holds irrespective of the firm behavior for consumers' expectation. Most prior studies have not focused on the difference caused by the firm's commitment. However, my analysis results in a good model, where the result crucially depends on firms' commitment. This implies that equilibrium concepts should be chosen carefully and a reconsideration of formalizing the effects of one's action on expected network sizes of others.

A large body of literature exists on network externalities and multi-product firms. Katz and Shapiro (1985) are the first to formulate a duopoly model with a network externality across both firms' products.[9] Baake and Boom (2001) and Chen and Chen (2011) consider an oligopoly and a duopoly model of vertical product differentiation with a network externality, in which firms decide their degree of product compatibility. However, each firm only supplies only one and not multiple products. In this study, the degree of compatibility is exogenous but a single firm produces two types of products. In contrast, Haruvy and Prasad (1998) analyze a market in which a monopolist sells high- and low-end versions of the same product and derive the

[8]Indeed, our U-shaped relation can be obtained if the firm cannot take the consumers' expectation into consideration. See Remark 2 in Section 3.

[9]For more extensive surveys, see Katz and Shapiro (1994) and Shy (2001).

conditions under which producing both goods is optimal with a network externality. On the other hand, Desai(2001) considers a two segments duopoly market for high-quality and low-quality goods represented by a Hotelling type model without network externality. He examines whether the cannibalization problem affects a firm's price and quality decision. However, in both their models, the two goods are sold in different markets, each with different types of consumers. Instead, I assume that both goods are supplied to the *same* market.

This chapter is organized as follows. Section 2 presents the model and Section 3 derives the main results. Section 4 contains the comparative statics. Section 5 concludes, and the Appendix provides proofs of the results.

3.2 The Model

This section presents the model. While I basically follow Katz and Shapiro (1985), who consider an oligopolistic network industry, I modify their model in two ways. First, I assume a monopoly to eliminate the strategic effect between the firms. Second, this single firm produces two vertically differentiated goods which may involve a network externality. In what follows, I describe the market equilibrium after characterizing the behavior of the firm and consumers.

I begin by considering the firm's behavior. Suppose a monopolistic firm producing two goods (H and L) that differ in their quality, and let V_H and V_L ($V_H > V_L$) denote the quality of each good. For simplicity, I assume that $V_H = (1 + \mu)V_L$, where $\mu > 0$ measures the degree of quality difference, and that the quality of good L is normalized to one (i.e., $V_L = 1$). The marginal cost of producing each good is given by c_H and c_L, respectively, which satisfy $c_H > c_L = 0$. Then, the firm's profit is defined by

$$(p_H - c_H)q_H + p_L q_L, \tag{3.1}$$

where q_α and p_α, for $\alpha = H, L$, are the output and price of good α, respectively. The monopolist chooses outputs to maximize (3.1).

To derive the inverse demand functions, I now describe the behavior of consumers. Following Katz and Shapiro (1985), consider a continuum of consumers characterized by a taste parameter θ that is uniformly distributed in $[-R, r], R, r > 0$ with density one.[10] By purchasing one unit of good α, consumer $\theta \in [-R, r]$ obtains a net surplus[11]

$$U_\alpha(\theta) = V_\alpha \theta + \nu V_\alpha g_\alpha^e - p_\alpha, \qquad \alpha = H, L, \tag{3.2}$$

where the first term in the right-hand side is the intrinsic utility of consuming the good and the second term represents a network externality. Parameter $\nu > 0$ measures the degree of the network externality and g_α^e is the expectation over the network benefit, which takes the form

$$g_\alpha^e \equiv g_\alpha(q_\alpha^e) = q_\alpha^e, \quad \alpha = H, L, \tag{3.3}$$

where, q_α^e is the expectation of output level of good α. Therefore, Eq. (3.3)

[10] I assume that R is large enough to avoid a corner solution.
[11] Baake and Boom (2001) adopt a similar expression for the consumer surplus.

represents the within-product externality.

Based on these preparations, I now derive the inverse demand functions. When consumer $\hat{\theta}$ is indifferent between purchasing good H and good L, it must hold that

$$\begin{aligned} U_H\left(\hat{\theta}\right) &= U_L\left(\hat{\theta}\right) > 0 \\ \iff (1+\mu)\hat{\theta} + \nu(1+\mu)g_H^e - p_H &= \hat{\theta} + \nu g_L^e - p_L. \end{aligned}$$

Thus, the index of this consumer is obtained as

$$\hat{\theta} = \frac{1}{\mu}\{p_H - p_L - \nu((1+\mu)g_H^e - g_L^e)\}. \tag{3.4}$$

Furthermore, there should be a consumer $\underline{\theta}_L$ who is indifferent between purchasing good L and nothing. The index of such a consumer satisfies

$$U_L\left(\underline{\theta}_L\right) = 0,$$

and, hence, is obtained as

$$\underline{\theta}_L = p_L - \nu g_L^e. \tag{3.5}$$

Then, from (3.2), (3.4), and (3.5), and given that $U_L(\cdot)$ is increasing in θ, I have

$$U_H(\hat{\theta}) = U_L(\hat{\theta}) > U_L(\underline{\theta}_L) = 0,$$

which is equivalent to

$$\hat{\theta} > \underline{\theta}_L. \tag{3.6}$$

The following lemma follows from this result.[12]

Lemma 3.1. *Any consumer* $\theta \in (-R, \underline{\theta}_L)$ *buys nothing, while consumer* $\theta \in (\underline{\theta}_L, \widehat{\theta})$ $(\theta \in [\widehat{\theta}, r])$ buys *good* L *(good* H*).*

From Lemma 3.1, the market-clearing conditions of goods H and L are

$$r - \hat{\theta} = q_H, \quad r - \underline{\theta}_L = q_H + q_L.$$

Substituting (3.4) and (3.5) into these equations and solving for p_H and p_L yields the inverse demand functions:

$$p_H = (1+\mu)(r + \nu g_H^e - x q_H) - q_L, \quad p_L = r + \nu g_L^e - q_H - q_L.$$

Thus, the profit in (3.1) can be rewritten as

$$\{(1+\mu)(r + \nu g_H^e - q_H) - q_L - c_H\} q_H + \{r + \nu g_L^e - q_H - q_L\} q_L. \tag{3.7}$$

Having described the behavior of the firm and consumers, I now derive the market equilibrium. For this purpose, I employ Katz and Shapiro's (1985) concept of the fulfilled expectations equilibrium, which requires that consumers' expected quantities equal the actual outputs. In addition, the firm

[12]See the Appendix for the proof.

chooses the outputs *after* taking consumers' expectations about the network size as given. From (3.7), the first-order conditions for profit maximization are

$$\begin{aligned} -(1+\mu)q_H + (1+\mu)(r + \nu g_H^e - q_H) - q_L - q_L - c_H = 0, \\ -q_H - q_L + r + \nu g_L^e - q_H - q_L = 0. \end{aligned} \tag{3.8}$$

In addition, to guarantee positive outputs in equilibrium, I make two additional assumptions:

$$0 < \nu < \frac{2(1+\mu-\sqrt{1+\mu})}{1+\mu}, \tag{3.9}$$

and

$$\underline{c}_H < c_H < \bar{c}_H, \tag{3.10}$$

where $\underline{c}_H = \nu(1+\mu)r/2$ and $\bar{c}_H = (2\mu - \nu - \nu\mu)r/(2-\nu)$.

The equilibrium outcomes are obtained from $g_\alpha^e = q_\alpha^e = q_\alpha$ and (3.8):

$$\begin{cases} -(1+\mu)q_H + (1+\mu)(r + \nu g_H^e - q_H) - 2q_L - c_H = 0 \\ -2q_H - 2q_L + r + \nu g_L^e = 0 \\ g_H^e = q_H \\ g_L^e = q_L. \end{cases}$$

Then, the equilibrium outputs and prices are

$$q_H^* = \frac{(2-\nu)\{(1+\mu)r - c_H\} - 2r}{Z}, q_L^* = \frac{-(1+\mu)\nu r + 2c_H}{Z}, \tag{3.11}$$

and

$$p_H^* = \frac{r(1+\mu)(2\mu-2\nu-\mu\nu)+\{(1+\mu)\nu^2-3(1+\mu)\nu+2\mu\}c_H}{Z}, p_L^* = \frac{2r(\mu-\nu-\mu\nu)+\nu c_H}{Z}, \quad (3.12)$$

where $Z = (1+\mu)(2-\nu)^2 - 4 > 0$ by (3.9). These outcomes lead to the equilibrium profit:

$$\pi^* = \frac{1}{Z^2}\Big[\{\mu(2-\nu)^2+\nu^2\}c_H^2 - 2r\{\mu^2(2-\nu)^2+2\mu^2+\mu\nu(3\nu-4)\}c_H$$
$$+r^2(1+\mu)(\mu^2(\nu-2)^2+4\nu^2+\mu\nu(-8+5\nu)\Big]. \quad (3.13)$$

This completes the description of the model.

3.3 U-Shaped Profit

Based on the results in the previous section, this section demonstrates that the firm profit is U-shaped in the marginal cost of the high-quality good. The proof of the results are left in Appendix.

3.3.1 Output

First, I consider the effects of an increase in the marginal cost of producing the high-quality good on each quantity, as described in the following proposition.

Proposition 3.1. *An increase (decrease) in c_H leads to cannibalization, such that it reduces (raises) the output of the high-quality good and raises (reduces) the output of the low-quality good.*

This proposition is a natural result, since the firm would like to produce a relatively efficient product.[13]

3.3.2 Profit

Next, I address the effect on the firm profit, which can be stated in

Proposition 3.2. *Suppose a within-product network externality exists. Then, the firm profit is U-shaped in* c_H*.*

This is illustrated in Figure 3.1. This result implies that a small cost reduction can decrease the monopoly profit. When c_H is high enough, the firm does not moderate cost reduction. In other words, the firm does not accept an innovation or subsidy unless it is able to drastically reduce c_H. This proposition suggests that if c_H is sufficiently high, a decrease in it *reduces* the firm's profit.

As emphasized in the Introduction, the assumption that consumers form their expectations *before* the output decision is crucial to the above result.[14] To see why, let us drop this assumption. That is, I compare this case with the case in which the firm can control both its output and the expected network size; it maximizes the profit with taking $g^e_\alpha = x_\alpha$ into consideration. Then, I have the following lemma.

[13]The same property is confirmed in Kitamura and Shinkai (2013), who consider a duopoly market without a network externality.

[14]This assumption implies that a monopolist's announcement of its planned level of output has no effect on consumer expectations.

Lemma 3.2. *In the monopoly model with the fulfilled expectations equilibrium derived above, if c_H increases, then marginal changes in the equilibrium quantities of good H and L are less than when the firm can control the expected network size.*

I have assumed that the firm takes the expected network size as given (i.e., it cannot control the expected network size). However, the expected network size must coincide with the actual network size in equilibrium. In other words, the monopolist choose outputs to maximize the profit without recognizing that the expected network size is equal to the actual network size. This lack of information leads the firm to either under-produce or over-produce compared with the case in which the firm can control the expected network size. To check this result, let us compute the first-order conditions when the firm can control the expected network size:

$$(1+\mu)(\nu\frac{\partial g_H}{\partial q_H}-1)q_H+(p_H-c_H)-q_L=0,\ (\nu\frac{\partial g_L}{\partial q_L}-1)q_L+p_L-q_H=0.$$

By contrast, if the firm cannot control the expected network size, the corresponding conditions are

$$-(1+\mu)q_H+(p_H-c_H)-q_L=0,\ \ -q_H+p_L-q_L=0.$$

When the monopolist can control the expected network size, an increase in output affects the network externality as represented by $\partial g_\alpha/\partial q_\alpha=1$. This difference in the first-order conditions results in Lemma 3.2. In fact, when the firm can control the expected network size, the equilibrium outputs are as follows:

$$q_H^{*C} = \frac{(1-\nu)\{(1+\mu)r - c_H\} - r}{2(1+\mu)(1-\nu)^2 - 2}, \; q_L^{*C} = \frac{-(1+\mu)\nu r + c_H}{2(1+\mu)(1-\nu)^2 - 2},$$

where superscript $*C$ indicates the case in which the firm can control the expected network size. Then, I can show that

$$\left|\frac{\partial q_H^{*C}}{\partial c_H}\right| > \left|\frac{\partial q_H^{*}}{\partial c_H}\right|, \quad \left|\frac{\partial q_L^{*C}}{\partial c_H}\right| > \left|\frac{\partial q_L^{*}}{\partial c_H}\right|.$$

The intuition behind Proposition 3.2 is explained from Proposition 3.1 and Lemma 3.2. According to these, a decrease in c_H increases the output of good H and decreases that of good L. However, these changes are not as drastic as in the case when the firm can control the expected network size. Thus, the firm cannot aggressively transfer the network of good L to that of good H in spite of the decrease in c_H, and the positive effect on the profit from good H is not able to dominate the negative effect of good L. This finding is impossible, however, if the firm can control the expected network size.

Indeed, we can observe this fact more plausibly as follows. I consider the effect of an increase in c_H on the profit from producing each individual good: $\pi^* = \pi_H^* + \pi_L^* \equiv (p_H^* - c_H)q_H^* + p_L^* q_L^*$. Using this decomposition of profits, I have the following lemma.[15]

[15] Note that the lemma requires the existence of positive equilibrium outputs: (3.9) and (3.10).

Lemma 3.3. *π_H^* is monotonically decreasing in c_H, and π_L^* is monotonically increasing in c_H.*

Figure 3.2 illustrates this lemma. Given this lemma and Figure 3.2, when c_H decreases by a sufficiently large amount, the negative effect on π_L^* (i.e., $\frac{\partial \pi_L^*}{\partial c_H}$) dominates the positive effect on π_H^* (i.e., $\frac{\partial \pi_H^*}{\partial c_H}$). Accordingly, if c_H is initially high, a decrease in c_H reduces the overall profit. The opposite holds when c_H is low enough.

Remark 1. One natural question regarding to Proposition 3.2 is whether the profit continues to be U-shaped in c_H even if the two goods have some compatibility. To answer it, I modify the form of network externality (3.3) as follows:

$$g_\alpha^e \equiv g_\alpha(q_H^e, q_L^e, \phi) = q_\alpha^e + \phi q_\beta^e \quad \alpha,\ \beta = H, L, \alpha \neq \beta,\ 0 < \phi \leq 1,$$

where ϕ is a parameter that measures the degree of compatibility between the two goods. The following proposition gives an affirmative answer to the above question.

Proposition 3.3. *Suppose that a within-product network externality and partial compatibility ($\phi < 1$) exist between the two differentiated goods. Then, the firm's profit is U-shaped in c_H.*

This proposition implies that the firm's profit can decrease when c_H decreases except for the case of $\phi = 1$ as long as a within-product network externality exists.

If $\phi = 1$, then $g^e_\alpha = q^e_H + q^e_L$ $(\alpha = H, L)$. Because the two goods are fully compatible, this case corresponds to the case analyzed by Katz and Shapiro (1985), that is there is the within-*firm* network externality. Then, we find that the firm's profit is a monotonically decreasing function of c_H. However, the case of fully compatible goods is a special situation,[16] because I consider the within-product network externality, and fully compatible products do not have individual networks. This result implies that the within-product network externality offers different equilibrium outcomes and properties to the within-firm network externality established in Katz and Shapiro (1985).

Remark 2. Thus far, I have assumed that a monopolist's announcement of its planned level of output has no effect on consumer expectations. Then, another natural question is whether the profit continues to be U-shaped in c_H even when its announcement of output level partially affects consumer expectations. In order to address it, I modify the form of network externaity (3.3) as follows:

$$g^e_\alpha \equiv g_\alpha(q^e_\alpha, q_\alpha, \epsilon) = \epsilon q_\alpha + (1 - \epsilon) q^e_\alpha \quad \alpha = H, L, \ 0 \leq \epsilon \leq 1.$$

In this formulation, the monopolist's announcement of its output level has ϵq_α influence on consumer expectations. For instance, if $\epsilon = 0$ then it has no effect on consumer expectations, on the other hand, if $\epsilon = 1$ then the firm

[16]See the Appendix for a special case, that is, $\frac{\partial \pi^*}{\partial c_H}|_{c_H = \bar{c}_{H\phi}} = 0$ only if $\phi = 1$.

perfectly control the consumer expectations. With this generalization, I can obtain:

Proposition 3.4. *Suppose that a within-network externality exists between the two differentiated goods and the monopolist's announcement of its planned level of output partially affects ($\epsilon < 1$) consumer expectations. Then, the firm's profit is U-shaped in c_H.*

Thus, the firm's profit is U-shaped in so far as its announcement of outputs imperfectly (that is when $0 \leq \epsilon < 1$) effects on consumer expectations.

When $\epsilon = 1$, $g^e_\alpha = q_\alpha(\alpha = H, L)$. As mentioned in Lemma 3.2, this implies that the monopolist can perfectly control the expected network size. Then, it chooses the output levels to maximize the profit with understanding that the consumer expectations are equal to the actual network size. Thus in the same way as reasons of Proposition 3.2, the firm's profit is monotonically decreasing in c_H only when $\epsilon = 1$.

3.4 Further Discussion

In this section, I address two issues that are important but have not been discussed in the last section.

3.4.1 Welfare

First, I examine the welfare effect of a change in c_H. Noting that welfare is equal to the sum of the consumer surplus and the firm's profit, it is defined by

$$\begin{aligned} W^* &\equiv \int_{\underline{\theta}_L}^{\hat{\theta}^*} (\theta + \nu g_L^*) d\theta + \int_{\hat{\theta}^*}^{r} (1+\mu)(\theta + \nu g_H^*) d\theta - c_H q_H^* \\ &= \frac{(1+\mu)r^2}{2} + \nu(1+\mu)(r - \hat{\theta}^*) g_H^* + \nu(\hat{\theta}^* - \underline{\theta}_L^*) g_L^* - \frac{(\underline{\theta}_L^*)^2}{2} - \frac{\mu(\hat{\theta}^*)^2}{2} - c_H q_H^* \\ &= \frac{(1+\mu)r^2}{2} + \nu(1+\mu) x_H^* g_H^* + \nu x_L^* g_L^* - \frac{(r - x_H^* - x_L^*)^2}{2} - \frac{\mu(r - x_H^*)^2}{2} - c_H q_H^*, \end{aligned}$$

where superscript * indicates the equilibrium outcome. Lengthy manipulations allow me to have a notable relationship $W^* = 3\pi^*/2$. Hence, the following result is immediately obtained.

Proposition 3.5. *Suppose that a within-product network externality exists. Then, social welfare is U-shaped in* c_H.

This proposition is natural since the consumer surplus is larger when c_H takes an extremely large or small value and only one side of the network is larger than it is when c_H takes an intermediate value and each network size is small.[17] Recalling Remark 1 and discussion after Proposition 3.3, I immediately find that welfare with fully compatible products ($\phi = 1$) is a monotonically decreasing function of c_H because, in that case, the network size of each product is always the sum of the network sizes of both products.

[17] The consumer surplus is also U-shaped in c_H.

Proposition 3.2 and 3.5 imply that a drastic cost reduction is needed to increase the profit and welfare when the production cost of the high-quality good is high. Then, as a mentioned in Section 1, these suggest that if the production subsidy is insufficient, subsization can reduce both the firm's profit and welfare.

3.4.2 Effect of μ on Outputs

Throughout this paper, I have focused on the effect of c_H. Finally, I consider the effect of an increase in the quality of the high-quality good μ on each quantity, as stated in the following proposition.

Proposition 3.6. *An increase in μ leads to an increase (decrease) in $x_H^*(x_L^*)$.*

This proposition is also interesting because cannibalization occurs as a result of not only c_H but also μ.[18] That is, an increase in μ has a contrasting effect in the sense that it raises (reduces) $q_H(q_L)$. The intuition for this proposition is as follows. A larger difference in the quality of the two goods implies that the high-quality good is superior to the low-quality good, which has a positive effect on the utility of the consumer. Thus, when the quality difference of the two goods becomes large , the monopolist has an incentive to increase q_H. In such a case, cannibalization occurs as it raises q_H while q_L decreases. Conversely, when the difference in the quality of the two goods decreases, the consumer does not value the high-quality good over the

[18] In Proposition 4.1, the change in the parameter of supply side c_H causes cannibalization, while in Proposition 4.6, that of demand side μ leads to cannibalization.

low-quality good. Thus, the monopolist will expand q_L since it is costly to produce q_H. In this case, cannibalization occurs such that the firm produces more of good L and less of good H. For example, the iPad Mini cannibalized sales of the larger iPad.[19]

3.4.3 Symmetric Cournot Oligopoly

Finally, I modify previous model from monopolistic market to the oligopolistic one. Suppose, there are n firms ($i = 1, 2, ...n$), each firm producing two goods (H and L) that differ in their quality but having same production technology ($c_H > c_L = 0$), and let V_H and V_L ($V_H > V_L$) denote the quality of each good. Then, the firm's profit π_i is defined by

$$\pi_i \equiv (p_{iH} - c_H)x_{iH} + p_{iL}x_{iL}, \tag{3.14}$$

where $x_{i\alpha}$ and $p_{i\alpha}$, for $\alpha = H, L$, are the output and price of firm i's good α, respectively. The oligopolistic firm i chooses outputs to maximize (3.14). By purchasing one unit of good α, consumer $\theta \in [-R, r]$ obtains a net surplus

$$U_{i\alpha}(\theta) = V_\alpha\theta + \nu V_\alpha g^e_{i\alpha} - p_{i\alpha}. \qquad \alpha = H, L, \tag{3.15}$$

The expectation over the network benefit takes the form

$$g^e_{i\alpha} \equiv g_{i\alpha}(x^e_{i\alpha}) = X^e_\alpha, \quad \alpha = H, L. \tag{3.16}$$

[19]See the internet articles by Keizer (2012) and Seward (2013).

Where, $X^e_\alpha = nx^e_{i\alpha}$ and $x^e_{i\alpha}$ is the expectation of output level of good α. Therefore, Eq. (3.16) represents the within-product externality.

Based on these preparations, I have the inverse demand functions;

$$p_{iH} = (1+\mu)(r + \nu g^e_{iH} - X_H) - X_L, \quad p_{iL} = r + \nu g^e_{iL} - X_H - X_L.$$

Then, the first-order conditions for profit maximization are

$$\begin{aligned} -(1+\mu)x_{iH} + (1+\mu)(r + \nu g^e_{iH} - X_H) - X_L - x_{iL} - c_H = 0, \\ -x_{iH} - x_{iL} + r + \nu g^e_{iL} - X_H - X_L = 0. \end{aligned} \tag{3.17}$$

In addition, to guarantee the positive outputs and the stability condition of Cournot-Nash equilibria , I make two additional assumptions:

$$0 < \nu < \frac{(1+n)(1+\mu-\sqrt{1+\mu})}{(1+\mu)n}, \tag{3.18}$$

and

$$\underline{c}_H < c_H < \bar{c}_H, \tag{3.19}$$

where $\underline{c}_H = (1+\mu)n\nu r/(n+1)$ and $\bar{c}_H = (n\mu + \mu - n\nu - n\nu\mu)r/(1+n-n\nu)$.

Here, I have assumed symmetric firm and focus on symmetric equilibrium outcome, so that $x^*_{iH} = x^*_H$ for all firm in equilibrium. The equilibrium outputs are obtained from $g^e_{i\alpha} = X^e_\alpha = X_\alpha$ and (3.17):

$$\begin{cases} -(1+\mu)x_H + (1+\mu)(r + \nu g^e_{iH} - X_H) - X_L - x_L - c_H = 0 \\ -x_H - x_L + r + \nu g^e_{iL} - X_H - X_L = 0 \\ g^e_{iH} = X_H \\ g^e_{iL} = X_L. \end{cases}$$

Then, the equilibrium outputs are

$$x^*_H = \frac{(n+1-n\nu)\{(1+\mu)r - c_H\} - (n+1)r}{Z}, x^*_L = \frac{-(1+\mu)n\nu r + (n+1)c_H}{Z}. \tag{3.20}$$

Thus, the prices and the profit in equilibrium $(p^*_H(x^*_H, x^*_L), p^*_L(x^*_H, x^*_L), \pi^*(x^*_H, x^*_L))$ are also obtained from (3.20). Based on the results in these, this demonstrates that the firm profit is U-shaped in the marginal cost of the high-quality good. The proof of the results are left in Appendix. I address the effect on the firm profit, which can be stated in

Proposition 3.7. *Suppose a within-product network externality exists and there are n firms. Then, the firm profit can be U-shaped in c_H.*

This result implies that a small cost reduction can decrease the each profit. As mentioned in Appendix, the U-shaped profit in c_H requires that $1-(1-\nu)n$ is positive. That is, this holds if the number of firm n is sufficiently small and the value of network externality ν is too large. Further, note that $1-(1-\nu)n$ is necessarily positive when $n = 1$, that is, monopoly case in Proposition 3.3.

This proposition suggests that if c_H is sufficiently high, a decrease in it *reduces* the firm's profit. Although the background process of this result is same way in Proposition 3.3, it is not easy to be U-shaped profit in c_H since the existence of competition weakens an irrelevant transition of network sizes by each firm.

Next, I modify the form of network externality (3.16) as follows:

$$g^e_{i\alpha} \equiv g_{i\alpha}(x^e_{i\alpha}\phi) = x^e_{i\alpha} + \phi x^e_{-i\alpha} \quad \alpha = H, L, \alpha \neq \beta,\ 0 < \phi \leq 1,$$

where ϕ is a parameter that measures the degree of compatibility among the same goods. The following proposition is intuitive results.

Proposition 3.8. *Suppose that a within-product network externality and partial compatibility ($\phi < 1$) exist among the same goods produced by different firm. Then, the higher compatibility leads the firm's profit to be U-shaped in c_H.*

Similarly to Proposition 4.7, the sufficiently condition for U-shaped profit in c_H is that $1-n+\nu+\nu(1-n)\phi > 0$. That is, the large degree of compatibility among the same goods ϕ leads to U-shaped profit in c_H

3.5 Concluding Remarks

Highlighting a within-product network externality, this chapter has theoretically analyzed multi-product monopoly behavior and the resulting market configurations. In particular, I focused on a monopoly model where a single firm sells two differentiated products (low- and high-quality goods) in a market with a within-goods network externality.

The notable result is that the firm profit is U-shaped in the production cost of the high-quality good. This result implies that the firm profit may decrease in spite of a cost reduction. Then, I have shown that two assumptions, the fulfilled expectations equilibrium and multi-product monopoly, yield the counterintuitive result. Moreover, I addressed the two cases in which (i) the two goods are partially and fully compatible and (ii) a firm's announcement of its output partially and perfectly affects consumer expectations, and established that when a within-product network externality exists, the firm profit is U-shaped except for two polar cases in which the two goods are completely compatible and in which a firm's announcement perfectly influences on consumer expectations. This analysis also shows that it is easy to be U-shaped profit in production cost when both the there exist a few firms in the market and the value of a within-product network externality is too high. In addition, I analyzed the effect of a change in the production cost of the high-quality good on welfare, finding that welfare is also U-shaped in the cost.

Furthermore, I highlighted that changes in the production cost and in the quality of the high-quality good affect the quantities. Moreover, by using the example of cannibalization, I found that an increase (decrease) in the production cost of the high-quality good and a decrease (increase) in its quality bring about cannibalization, such that the firm raises (reduces) the output of the high-quality good while it reduces (raises) the output of the low-quality good.

In this chapter, I exclusively focused on a monopoly model without choosing product compatibility, but future studies should aim to analyze a model when the firm can choose a compatible product with a fixed cost of making its products compatible.

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Figure 3.1 $(r = 1, \nu = 1/2, \mu = 1)$

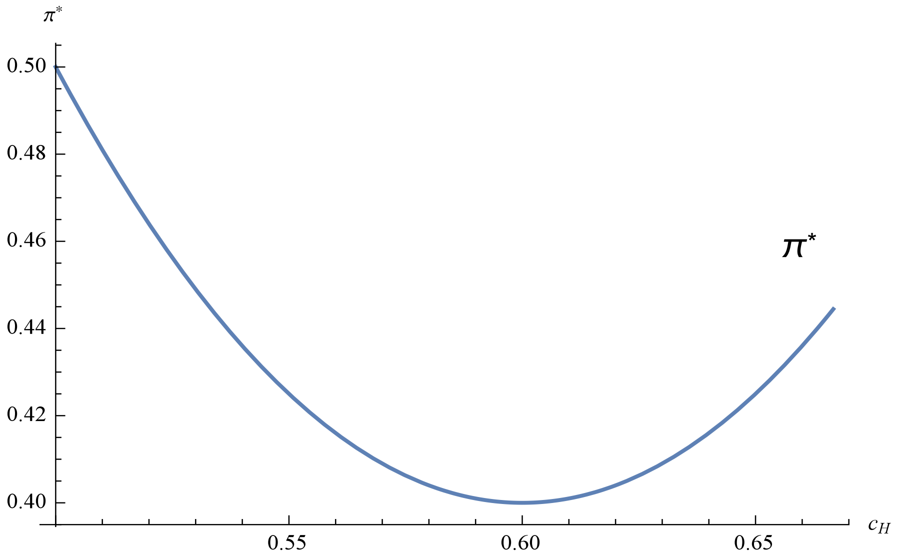
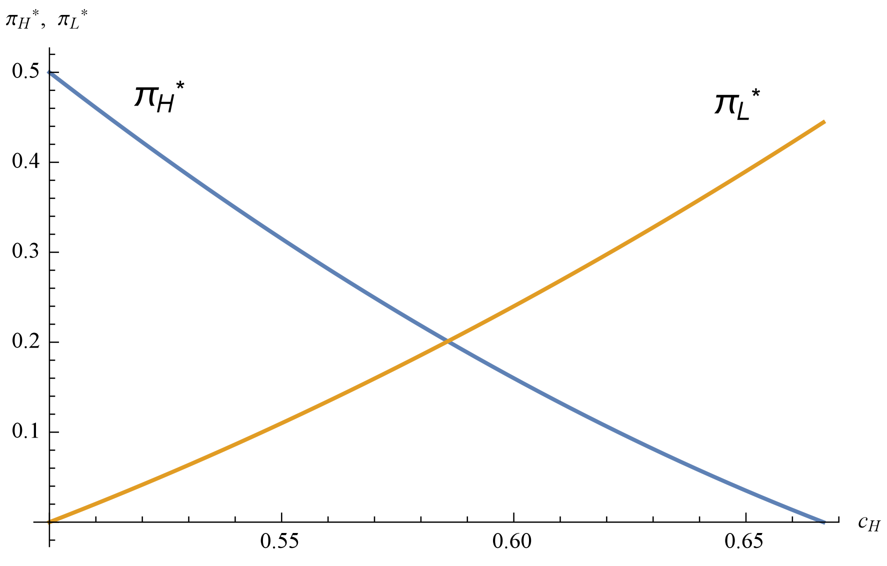


Figure 3.2 $(r = 1, \nu = 1/2, \mu = 1)$



Appendix

Proof of Lemma 3.1

According to Eqs. (4.2) and (4.4), for arbitrary $\theta > \hat{\theta}_i$, from (4.2) and (3.6), we have

$$\begin{aligned} U_L(\widehat{\theta}) - U_L(\underline{\theta}_L) &= \hat{\theta} + \nu g_L^e - p_L - (\underline{\theta}_L + \nu g_L^e - p_L) \\ &= \hat{\theta} - \underline{\theta}_L > 0, \end{aligned}$$

for arbitrary type $\theta \in (\ \underline{\theta}_L, \widehat{\theta})$. Then,

$$\begin{aligned} U_H(\theta) - U_L(\theta) \quad &= (1+\mu)\theta + \nu(1+\mu)g_H^e - p_H - \theta - \nu g_L^e + p_L \\ &= \mu\theta - \{p_H - p_L - (\nu(1+\mu)g_H^e - \nu g_L^e)\} \\ &> \mu\hat{\theta} - \{p_H - p_L - (\nu(1+\mu)g_H^e - \nu g_L^e)\} \\ &= 0. \end{aligned}$$

From (4.2) and (3.6), we have

$$\begin{aligned} U_L(\widehat{\theta}) - U_L(\underline{\theta}_L) &= \hat{\theta} + \nu g_L^e - p_L - (\underline{\theta}_L + \nu g_L^e - p_L) \\ &= \hat{\theta} - \underline{\theta}_L > 0, \end{aligned}$$

for arbitrary type $\theta \in (\ \underline{\theta}_L, \widehat{\theta})$.

Proof of Proposition 3.1

From equilibrium outcome (3.11), we have $\partial q_H^*/\partial c_H < 0$ and $\partial q_L^*/\partial c_H > 0$.

Proof of Proposition 3.2

$$\begin{cases} \frac{\partial^2 \pi^*}{\partial c_H^2} = \frac{2\{\mu(2-\nu)^2+\nu^2\}}{Z^2} > 0 \\ \\ \frac{\partial \pi^*}{\partial c_H}|_{c_H=\underline{c}_H} = \frac{-r\{\mu(2-\nu)-\nu\}}{(2-\nu)Z} < 0, \quad \frac{\partial \pi^*}{\partial c_H}|_{c_H=\bar{c}_H} = \frac{2r\nu}{(2-\nu)Z} > 0. \end{cases}$$

Proof of Lemma 3.3

The individual profits from producing goods H and L are given by

$$\pi_H^* = \frac{\{c_H(2-\nu)+r\{\mu(-2+\nu)+\nu\}\}\{c_H\{\mu(2-\nu)-\nu\}+r(1+\mu)\{\mu(-2+\nu)+2\nu\}\}}{Z^2}$$

$$\pi_L^* = \frac{\{-2c_H+r(1+\mu)\nu\}\{-c_H\nu+2r\{\mu(-1+\nu)+\nu\}\}}{Z^2},$$

respectively, so that

$$\begin{cases} \frac{\partial \pi_H^*}{\partial c_H}|_{c_H=\bar{c}_H} = \frac{-r}{Z} < 0, \quad \frac{\partial^2 \pi_H^*}{\partial c_H^2} = \frac{2(2-\nu)(2\mu-\nu-\nu\mu)}{Z^2} > 0 \\ \frac{\partial \pi_L^*}{\partial c_H}|_{c_H=\underline{c}_H} = rZ > 0, \quad \frac{\partial^2 \pi_L^*}{\partial c_H^2} = \frac{4\nu}{Z} > 0. \end{cases}$$

Proof of Proposition 3.3

The equilibrium outcomes for $0 < \phi \leq 1$ are obtained as follows.

$$
\begin{cases}
q_H^* = \frac{(2-\nu)\{r(1+\mu)-c_H\}-r\{2-\phi(1+\mu)\nu\}}{Z_\phi}, \ q_L^* = \frac{(1+\mu)(2-\nu)-\{r(1+\mu)-c_H\}(2-\phi\nu)}{Z_\phi} \\
\\
p_H^* = \frac{r(1+\mu)(2(\phi-1)\nu+\mu\{2-(1-\phi)\nu\}+c_H\{(1-\phi)\nu(-3+\nu+\phi\nu)-\nu\{-2+(3-2\phi)\nu-(1-\phi^2)\nu^2\}\}}{Z_\phi} \\
\\
p_L^* = \frac{2r\{(\phi-1)\nu+\mu\{1+\phi-1)\nu\}\}+(1-\phi)\nu c_H\}}{Z_\phi} \\
\\
\pi^* = \frac{1}{Z_\phi^2}\Big[\{\mu(2-\nu)^2+(1-\phi)^2\nu^2\}c_H^2+2r\{-2(1-\phi)^2\nu^2+\mu^2(-2+\nu)\{2-(1-\phi)\nu\}+\mu\nu(1-\phi)\{4+(2\phi-3)\nu\}\}c_H \\
\qquad +r^2(1+\mu)\{4(1-\phi)^2\nu^2+\mu^2\{2-(1-\phi)\nu\}^2-\mu\nu(1-\phi)\{8-5(1-\phi)\nu\}\}\Big],
\end{cases}
$$

where $Z_\phi = \nu(1-\phi)(\phi\nu+\nu-4)+\mu\{4-2(2-\phi)\nu+(1-\phi^2)\nu^2\} > 0$. Furthermore, $\underline{c}_{H\phi} < c_H < \bar{c}_{H\phi}$ where $\underline{c}_{H\phi} = (1+\mu)(1-\phi)r\nu/(2-\phi\nu)$ and $\bar{c}_{H\phi} = \{r\{(1+\mu)(2-\nu)-\{2-\phi(1+\mu)\nu\}\}/(2-\nu)$.

Then,

$$
\begin{cases}
\frac{\partial\pi^*}{\partial c_H}|_{c_H=\underline{c}_{H\phi}} = \frac{-r\{\mu(2-\nu)-(1-\phi)\nu\}}{(2-\phi\nu)Z_\phi} < 0 \\
\frac{\partial\pi^*}{\partial c_H}|_{c_H=\bar{c}_{H\phi}} = \frac{2(1-\phi)r\nu}{(2-\nu)Z_\phi} \geq 0.
\end{cases}
$$

Thus, the firm profit is U-shaped in c_H except for the case of $\phi = 1$.

Proof of Proposition 3.4

The equilibrium outcomes for $0 \leq \epsilon \leq 1$ are given as follows:

$$\begin{cases} q_H^* = \frac{(2-\nu-\nu\epsilon)\{(1+\mu)r-c_H\}-2r}{Z_\epsilon},\ q_L^* = \frac{2c_H-\nu(1+\mu)(1+\epsilon)r}{Z_\epsilon} \\ p_H^* = \frac{c_H\{-3\nu+\nu\epsilon(-1+\nu)+\nu^2+\mu(-1+\nu)(-2+\nu+\epsilon\nu)\}+r(1+\mu)\{(1+\epsilon)\nu(-2+\epsilon\nu)+\mu\{2-(1+3\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\}}{Z_\epsilon} \\ p_L^* = \frac{-c_H(-1+\epsilon)\nu+r\{(1+\epsilon)(-2+\epsilon\nu)\nu+\mu\{2-2(1+\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\}}{Z_\epsilon} \\ \pi^* = \frac{1}{Z_\epsilon^2}\Big[\{\mu(-1+\epsilon\nu)(-2+\nu+\epsilon\nu)^2+\nu\{\nu+\epsilon^2(5-2\nu)\nu-\epsilon^3\nu^2-\epsilon(8-6\nu+\nu^2)\}\}c_H^2 \\ \quad +2r\{(1+\epsilon)^2\nu^2(-2+\epsilon\nu)+\mu^2(-1+\epsilon\nu)(-2+\nu+\epsilon\nu)^2+\mu\nu\{4-3\nu+2\epsilon^3\nu^2+\epsilon^2\nu(-7+4\nu)+2\epsilon(4-5\nu+\nu^2)\}\}c_H \\ \quad +r^2(1+\mu)\{(-1-\epsilon)\nu\{(1+\epsilon)(-2+\epsilon\nu)\nu+\mu\{2-2(1+\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\} \\ \quad -\{(1+\epsilon)\nu+\mu(-2+\nu+\epsilon\nu)\}\{(1+\epsilon)(-2+\epsilon\nu)\nu+\mu\{2-(1+3\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\}\}\Big], \end{cases}$$

where $Z_\epsilon = (1+\mu)(2-\nu-\nu\epsilon)^2 - 4 > 0$ if and only if $0 < \nu < 2(1+\mu-\sqrt{1+\mu})/(1+\mu)(1+\epsilon)$. Furthermore, $\underline{c}_{H\epsilon} < c_H < \overline{c}_{H\epsilon}$ where $\underline{c}_{H\epsilon} = \nu(1+\mu)(1+\epsilon)r/2$ and $\overline{c}_{H\epsilon} = r\{2\mu - \nu(1+\mu)(1+\epsilon)\}/(2-\nu-\nu\epsilon)$.

Then,

$$\begin{cases} \frac{\partial\pi^*}{\partial c_H}|_{c_H=\underline{c}_{H\epsilon}} = \frac{-r\{\nu\{-1+\epsilon(\nu-3)+\nu\epsilon^2\}+\mu\{2-(1+3\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}}{Z_\epsilon} < 0 \\ \frac{\partial\pi^*}{\partial c_H}|_{c_H=\overline{c}_{H\phi}} = \frac{2(1-\epsilon)r\nu}{(2-\nu-\nu\epsilon)Z_\epsilon} \geq 0. \end{cases}$$

Thus, the firm profit is U-shaped in c_H except for the case of $\epsilon = 1$.

Proof of Proposition 3.5

Straightforward manipulations give

$$\frac{\partial q_H^*}{\partial \mu} = \frac{(2-\nu)\{(2-\nu)^2 c_H - 2\nu r\}}{Z^2} > 0, \ \frac{\partial q_L^*}{\partial \mu} = \frac{-2\{(2-\nu)^2 c_H - 2\nu r\}}{Z^2} < 0.$$

Proof of Proposition 3.7

$$\begin{cases} \frac{\partial^2 \pi^*}{\partial c_H^2} = \frac{2\mu(1+n-n\nu)^2+2n\nu}{Z^2} > 0 \\ \\ \frac{\partial \pi^*}{\partial c_H}|_{c_H=\bar{c}_H} = \frac{(1+n)\{1-(1-\nu)n\}}{(2-\nu)Z}. \end{cases}$$

Thus, if both n is sufficiently small and ν is sufficiently large, then the profit becomes U-shaped in c_H.

Proof of Proposition 3.8

The equilibrium outcomes for $0 < \phi \leq 1$ are obtained as follows.

$$\begin{cases} x_H^* = \frac{\{1+n-(1+\phi(n-1))\nu\}\{1+\mu)r-c_H\}-(1+n)r}{Z_\phi} \\ x_L^* = \frac{-r(1+\mu)\{1+\phi(n-1)\nu+(1+n)c_H}{Z_\phi} \end{cases}$$

where $Z_\phi = (1+\mu)\{1+n-(1+\phi(n-1))\nu\}^2 - (1+n)^2 > 0$. Furthermore, $\underline{c}_{H\phi} < c_H < \bar{c}_{H\phi}$ where $\underline{c}_{H\phi} = r(1+\mu)\{1+\phi(n-1)\nu/(1+n)$ and $\bar{c}_{H\phi} = \{r\{(1+\mu)\{1+n-(1+\phi(n-1))\nu\} - (1+n)\}/\{1+n-(1+\phi(n-1))\nu\}$.

Then,

$$\begin{cases} \frac{\partial^2 \pi^*}{\partial c_H^2} = \frac{2\mu\{1+n-(1+\phi(n-1))\nu\}^2+2\{1+\phi(n-1)\nu}{Z_\phi^2} > 0 \\ \frac{\partial \pi^*}{\partial c_H}|_{c_H=\bar{c}_{H\phi}} = \frac{(1+n)\{1-n+(1+\phi(n-1))\nu\}}{Z_\phi}. \end{cases}$$

Thus, the firm profit is U-shaped in c_H when ϕ is too large and n is sufficiently small.

Chapter 4

A Monopoly Model with Two Horizontally Differentiated Goods under Network Externalities

abstract[1]

This chapter develops a linear model in which a monopolist supplies two horizontally differentiated goods involving a within-product network externality. Within this model, I analyze multi-product monopoly behavior. Then, I examine how a change in both the production cost and the location cost gives effect on equilibrium location, outputs, prices and profit. Furthermore, I find how a change in a degree of compatibility between two goods affects them.

Keywords: Multi-product firm, Monopoly, Cannibalization, Network externality

[1]I thank Kenji Fujiwara, Hiroaki Ino, Noriaki Matsushima and Tetsuya Shinkai as well as the other participants at the workshop at Kwansei Gakuin University for their useful comments. Any remaining errors are my own.

4.1 Introduction

Within-product network externality works in many industries in which the products are *horizontally* differentiated.[2] Mentioned in previous, for instance, a refrigerator, a television, PC industry and so on. In these industry, when the number of users who buy a certain products increases, then a user of it gains a network benefit because of an increase in complementary goods (some software, compatible goods) of it or an improvement of some services.

In order to theoretically consider these industry, I incorporate within-product network externality into the multi-product monopoly model based on Bental and Spiegel (1984) that analyze the oligopolistic multi-product market with horizontally differentiated products. Then, I find that the cost reduction of both the production cost and the location cost increases the monopoly firm's location, outputs, prices and profit. Furthermore, an increase in the value of network size also gives same effects on them. Finally, I consider the effect of degree of the compatibility between two products on equilibrium location, outputs prices and profit. Then, I show that an increase in degree of compatibility gives positive effect on them.

A lot of the existing literatures on the problem "horizontal product variety" consider consumers, who do not agree on their ranking of varieties. Lancaster (1979) and Salop (1979) among others, also incorporate this notion.[3] I expand the multi-product monopoly model by Bental and Spiegel (1984), where they assume that, 'a firm's technology is geared towards the

[2]For definition of within-product network externality, see Kitamura(2014)
[3]Mussa and Rosen (1978) consider the problem "vertical product variety".

production of a particular brand which they call the "main product". The firm may also produce varieties of the main product, so that the design of these varieties is associated with(fix) cost.'

The remainder of this paper is organized as follows. In section 1.2, I present a model and derive a monopoly equilibrium with two horizontally differentiated products in a market and with within-products network externality. In section 1.3, I use comparative statistics of the equilibrium location, outputs, prices and profit. Finally, section 1.4 concludes the paper and offers suggestions for possible future research.

4.2 Model

This section presents the model and derives the equilibria. Following the formulation of Bental and Spiegel (1984), I modify their model in two ways. First, I assume a monopoly which sells two horizontally differentiated products, 1 and 2. Second, these products potentially involve a network externality. I begin by considering the firm's behavior. Suppose a monopolistic firm producing two goods (1 and 2) that differ in their locations, and let l_1 and l_2 denote the location of each good. For simplicity, I assume that $l_1 = 0 < l_2$. The marginal cost of producing each good is given by c_1 and c_2, respectively, which satisfy $c_1 = c_2 = c$. Moreover, the firm suffers the cost from locating good 2 as $fl_2^2/2,\ f > 0$, which is increasing as the variety gets technology further from the 'main product' good 1. Then, the firm's profit is defined by

$$(p_1 - c)q_1 + (p_2 - c)q_2 - f\frac{l_2^2}{2}, \tag{4.1}$$

where q_α and p_α, for $\alpha = 1, 2$, are the output and price of good α, respectively. The monopolist chooses location and outputs to maximize (4.1).

To derive the inverse demand functions, I now describe the behavior of consumers. Following Bental and Spiegel (1984), consider a continuum of consumers characterized by a taste parameter θ that is uniformly distributed in $[0, r], r > 0$ with density one.[4] By purchasing one unit of good α, consumer $\theta \in [0, r]$ obtains a net surplus

$$U_\alpha(\theta) = R + \nu g_\alpha^e - |\theta - l_\alpha| - p_\alpha, \qquad R > 0, \quad \alpha = 1, 2, \tag{4.2}$$

where the first term in the right-hand side is the intrinsic utility of consuming the good and the second term represents a network externality. Parameter $\nu > 0$ measures the degree of the network externality and g_α^e is the expectation over the network benefit, which takes the form

$$g_\alpha^e \equiv g_\alpha(q_\alpha^e) = q_\alpha^e, \quad \alpha = 1, 2. \tag{4.3}$$

Where, q_α^e is the expectation of output level of good α. Therefore, Eq. (4.3) represents the within-product externality.

Based on these preparations, I now derive the inverse demand functions. When consumer $\hat{\theta}$ is indifferent between purchasing good 1 and good 2, it must hold that

$$\begin{aligned} U_1\left(\hat{\theta}\right) &= U_2\left(\hat{\theta}\right) > 0 \\ \iff R + \nu g_1^e - \hat{\theta} - p_1 &= R + \nu g_2^e - (l_2 - \hat{\theta}) - p_2. \end{aligned}$$

[4] I assume that r is large enough to avoid a corner solution.

Thus, the index of this consumer is obtained as

$$\hat{\theta} = \frac{1}{2}\{l_2 + p_2 - p_1 + \nu(g_1^e - g_2^e)\}. \tag{4.4}$$

Furthermore, there should be a consumer $\overline{\theta}$ who is indifferent between purchasing good 2 and nothing. The index of such a consumer satisfies

$$U_2\left(\overline{\theta}\right) = 0,$$

and, hence, is obtained as

$$\overline{\theta} \quad = R + \nu g_2^e + l_2 - p_2. \tag{4.5}$$

Thus, the market-clearing conditions of goods H and L are

$$\hat{\theta} = q_1, \quad \overline{\theta} - \hat{\theta} = q_2.$$

This is illustrated in Figure 4.1.

Substituting (4.4) and (4.5) into these equations and solving for p_1 and p_2 yields the inverse demand functions:

$$p_1 = R + 2l_2 + \nu g_1^e - 3q_1 - q_2, \quad p_2 = R + l_2 + \nu g_2^e - q_1 - q_2.$$

Thus, the profit in (4.1) can be rewritten as

$$\{R + 2l_2 + \nu g_1^e - 3q_1 - q_2 - c\}q_1 + \{R + l_2 + \nu g_2^e - q_1 - q_2 - c\}q_2 - f\frac{l_2^2}{2}. \tag{4.6}$$

4.3 Analysis

Having described the behavior of the firm and consumers, I now derive the market equilibrium. For this purpose, I employ Katz and Shapiro's (1985) concept of the fulfilled expectations equilibrium, which requires that consumers' expected quantities equal the actual outputs. In addition, the firm chooses the outputs and the location *after* taking consumers' expectations about the network size as given. From (4.6), the first-order conditions for profit maximization on q_1, q_2, l_2 are

$$\begin{aligned} R + 2l_2 + \nu g_1^e - 6q_1 - 2q_2 - c = 0, \\ R + l_2 + \nu g_2^e - 2q_1 - 2q_2 - c = 0, \\ 2q_1 + q_2 - f l_2 = 0. \end{aligned} \tag{4.7}$$

In addition, to guarantee positive outputs in equilibrium, I make an additional assumption:

$$0 < \nu < 1, \tag{4.8}$$

$$0 < c < R, \tag{4.9}$$

$$\frac{5 - 3\nu}{2(2-\nu)} < f < \frac{1}{\nu}. \tag{4.10}$$

The equilibrium outcomes are obtained from $g^e_\alpha = q^e_\alpha = q_\alpha$ and (4.7):

$$\begin{cases} R + 2l_2 + \nu g^e_1 - 6q_1 - 2q_2 - c = 0 \\ R + l_2 + \nu g^e_2 - 2q_1 - 2q_2 - c = 0 \\ 2q_1 + q_2 - fl_2 = 0 \\ g^e_1 = q_1 \\ g^e_2 = q_2. \end{cases} \tag{4.11}$$

Then, the equilibrium location, outputs and prices are

$$l^*_2 = \frac{(4-3\nu)(R-c)}{Z}$$

$$q^*_1 = \frac{(1-f\nu)(R-c)}{Z}, \quad q^*_2 = \frac{(4f - f\nu - 2)(R-c)}{Z},$$

$$p^*_1 = \frac{\{1+4f(1-\nu)\}R + c\{5\nu - 7 + f(2-\nu)^2\}}{Z}, \quad p^*_2 = \frac{\{-1+2(2-\nu)\}R + c\{-5(1-\nu) + f(\nu^2 - 6\nu + 4)\}}{Z},$$

where $Z = f\nu^2 - (8f-5)\nu + 8f - 6 > 0$ by (4.10). These outcomes lead to the equilibrium profit:

$$\pi^* = \frac{\{4f^2(3\nu^2 - 8\nu + 8) - f(9\nu^2 - 24\nu + 32) + 6\}(R-c)^2}{2Z^2}$$

This completes the description of the model.[5] Then,

$$q^*_1 - q^*_2 = \frac{(R-c)(3-4f)}{Z} < 0$$

[5] In this setting, it is easy to confirm that the monopoly profit from suppling two products is higher than that from suppling only one product (good 1 or good 2).

and

$$p_1^* - p_2^* = \frac{2\{f(2(1-\nu)r + \nu c) - (1-\nu)R - c\}}{Z} > 0$$

, where note that $f > 5 - 3\nu 2(2-\nu) > \{(1-\nu)R - c\}/\{2(1-\nu)r + \nu c\}$ by $\nu > 0$ and (4.10).

Proposition 4.1 *The monopolist produces good* 2 *more than good* 1 *and sets the high-price on good* 1 *and low-price on good* 2*.*

This proposition is natural since it is easy for firm to take the demand of good 2 than the demand of good 1. Because the demand of good 1 depends on $\hat{\theta}$, it is always faced with cannibalization of the two goods. However the demand of good 2 is sum of the consumers $\theta \in [\hat{\theta}, l_2]$ and $\theta \in [l_2, \overline{\theta}]$. The latter demand is not affected by cannibalization of two goods contrary to the former demand, so that the firm can easily capture the demand of good 2 than the demand of good 1.

Here, I consider the effect of a change in location cost of producing the good 2 and on location, each quantity, each price and profit.

The results of comparative statics illustrated in following table.[6]

Table 1

	l_2^*	x_1^*	x_2^*	p_1^*	p_2^*	π^*
f	$-$	$-$	$-$	$-$	$-$	$-$

Proposition 4.2 *A decrease in* f *leads to an increase in* l_2^**,* q_1^**,* q_2^**,* p_1^**,* p_2^* *and* π^**.*

[6] See Appendix for these calculations.

This implies that the cost reduction of good 2's location gives positive effects on the monopoly firm's location, outputs, prices and profit. In Chapter 3, I show that a decrease in the marginal cost of high-quality good leads to cannibalization such that an increase in the number of users of high-quality good occurs at the expense of those of low-quality good, so that firm profit can decreases.[7]

Remark. Finally, I consider the effect of degree of compatibility between two goods on equilibrium location, outputs, prices and profit. To analyze it, I modify the form of network externality (4.3) as follows:

$$g^e_\alpha \equiv g_\alpha(q^e_1, q^e_2, \phi) = q^e_\alpha + \phi q^e_\beta, \quad \alpha = 1, 2. \tag{4.12}$$

where ϕ is a parameter that measures the degree of compatibility between two goods. For simplicity, I assume that $c = 0$ and $f = 1$. The following table gives the results of comparative statics:

Table 2

	l_2^*	x_1^*	x_2^*	p_1^*	p_2^*	π^*
ϕ	+	+	+	+	+	+

Proposition 4.3 *Suppose that there exists partial compatibility between two goods. An increase in the degree of compatibility leads to an increase in* l_2^*, q_1^*, q_2^{**}, p_1^*, p_2^* *and* π^*.

This proposition implies that when compatibility cost is zero, then the move to completely compatibility raises the firm profit.

[7]In this paper, I use the concept of cannibalization defined by Copulsky(1976).

4.4 Concluding Remarks

Highlighting a within-product network externality, this thesis has theoretically analyzed multi-product monopoly behavior and the resulting market configurations. In this chapter, I focused on a monopoly model where a single firm sells two *horizontally* differentiated products in a market with a within-goods network externality.

The notable result is that the cost reduction of both the production cost and the location cost increases the monopoly firm's location, outputs, prices and profit. Furthermore, an increase in the value of network size also gives same effects on them. Finally, I address the effect of degree of the compatibility between two goods on equilibrium location, outputs prices and profit. Then, I find that an increase in degree of compatibility gives positive effect on them. Therefore, The firm would like to make two products more compatible if the compatibility cost is small enough.

In this chapter, I exclusively focused on a monopoly model so that, in future works, it should be analyzed oligopolistic markets at same framework.

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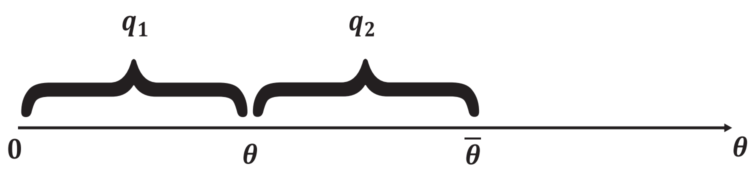
Figure 4.1



Appendix

Comparative statics without compatibility

The results of each calculation are as follows

$$\frac{\partial l_2^*}{\partial f} = \frac{-(R-c)(4-3\nu)(\nu^2-8\nu+8)}{Z^2} < 0,$$

$$\frac{\partial q_1^*}{\partial f} = \frac{-2(R-c)(3\nu^2-7\nu+4)}{Z^2} < 0,$$

$$\frac{\partial q_2^*}{\partial f} = \frac{-(R-c)(3\nu^2-10\nu+8)}{Z^2} < 0,$$

$$\frac{\partial p_1^*}{\partial f} = \frac{-(R-c)(21\nu^2-52\nu+32)}{Z^2} < 0,$$

$$\frac{\partial p_2^*}{\partial f} = \frac{-(R-c)(4-3\nu)^2}{Z^2} < 0,$$

$$\begin{cases} \frac{\partial \pi^*}{\partial f} = \frac{(4-3\nu)(R-c)^2 N}{2Z^3} \\ \frac{\partial N}{\partial f} = -(32-40\nu+12\nu^2+3\nu^3) > 0 \\ N|_{f=\frac{1}{\nu}} = -64 + \frac{32}{\nu} + 46\nu - 12\nu^2 > 0 \;\; if \;\; 0 < \nu < 1. \end{cases}$$

where $N = 24 - 34\nu + 15\nu^2 - f(32 - 40\nu + 12\nu^2 + 3\nu^3)$.

Equilibrium outcomes with partial compatibility

The equilibrium location, quantities, prices and profit are obtained as

$$l_2^* = \frac{R(4-3\nu+3\nu\phi)}{Z_\phi},$$

$$q_1^* = \frac{R(1-\nu+\nu\phi)}{Z_\phi},$$

$$q_2^* = \frac{R(2-\nu+\nu\phi)}{Z_\phi},$$

$$p_1^* = \frac{R(5-4\nu+4\nu\phi)}{Z_\phi},$$

$$p_2^* = \frac{R(3-2\nu+2\nu\phi)}{Z_\phi},$$

$$\pi^* = \frac{R^2\{3(1-\phi)^2\nu^2 - 8(1-\phi)\nu + 6\}}{2Z_\phi^2},$$

where $Z_\phi = (1-\phi^2)\nu^2 - 3\nu + 2$.

Comparative statics with compatibility

$(c = 0, f = 1, 0 \leq \phi \leq 1)$:

$$\frac{\partial l_2^*}{\partial f} = \frac{R\nu\{3(1-\phi)^2 - (9-8\phi)\nu + 6\}}{Z_\phi^2} > 0,$$

$$\frac{\partial q_1^*}{\partial f} = \frac{R\nu\{(1-\phi)^2\nu^2 - (3-2\phi)\nu + 2\}}{Z_\phi^2} > 0,$$

$$\frac{\partial q_2^*}{\partial f} = \frac{R\nu\{(1-\phi)^2\nu^2 - (3-4\phi)\nu + 2\}}{Z_\phi^2} > 0,$$

$$\frac{\partial p_1^*}{\partial f} = \frac{2R\nu\{2(1-\phi)^2\nu^2 - (6-5\phi)\nu + 4\}}{Z_\phi^2} > 0,$$

$$\frac{\partial p_2^*}{\partial f} = \frac{2R\nu\{(1-\phi)^2\nu^2 - (9-8\phi)\nu + 6\}}{Z_\phi^2} > 0,$$

$$\frac{\partial \pi^*}{\partial f} = \frac{R^2\nu\{-3(1-\phi)^3\nu^3 + (13-25\phi+12\phi^2)\nu^2 - 18(1-\phi)\nu + 8\}}{Z_\phi^3} > 0.$$