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Merger Paradox in a Network Product Market: A Horizontally Differentiated Three-Firm Model

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Abstract

Using a horizontally differentiated three-firm model, we reconsider the merger paradox and externalities, i.e., the profitability of a merger, in a network product market where network externalities and compatibilities between products exists. Investigating the effect of a merger on the profits of the insider (participant) and outsider (nonparticipant) firms, we demonstrate the conditions under which the merger paradox and externalities arise in the network product market. If the degree of the merger-related network compatibility is sufficiently large, the merger paradox never arises.

Keywords: merger paradox; network externality; compatibility; horizontal product differentiation; quantity-setting game

JEL Classification: D43; K21; L13; L14; L15

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1. Introduction

We reconsider the merger paradox and externalities, in other words, the profitability of a merger, in a network product market, where we observe network externalities and compatibilities (interoperability and interconnectivity) between products and services.¹

In particular, since the 1990s, waves of global mergers and acquisitions have been observed in various industries, including telecommunications, internet businesses, banking, airlines, and railways. These industries are commonly characterized as network product markets. The characteristics imply that mergers and acquisitions in these industries may be profitable for the participants.

Theoretically, however, horizontal mergers are not profitable for insiders (participant firms), whereas they are for outsiders (nonparticipant firms). In a seminal paper, Salant et al. (1983) show that a merger in Cournot oligopoly is unprofitable unless at least 80% of the firms in the industry participate. In Cournot oligopolistic competition, where strategic substitutability relationship between the firms arises, the contraction of the outputs of the insider firms as a result of a merger increases the

¹ Gugler and Szücs (2016) empirically analyze the externalities of mergers and demonstrate that the return on assets of rival firms (outsiders) increases significantly after a merger.

outputs of the outsider firms. As a result, although the prices increase, the insider firms lose, whereas the outsider firms gain; i.e., the merger paradox emerges.

As this 80% threshold is too high to be achievable in reality, researchers have sought several ways out of the merger paradox.² For example, introducing merger-related efficiencies occurring through scale economies, such as cost saving synergies, Perry and Poster (1985) consider incentives to merge in the context of Cournot quantity-setting oligopoly in a homogeneous product market. Assuming Bertrand price-setting competition in a horizontally differentiated product market, Deneckere and Davidson (1985) demonstrate that mergers of any size are beneficial when the assumption of strategic complements holds between the firms in the case of the price-setting game. Further, based on general demand and cost functions, Farrell and Shapiro (1990) consider horizontal mergers in the case of Cournot oligopoly in a homogeneous product market. Assuming efficiencies created by scale economies or learning in the general model, they show the condition for lowering the prices and the effect on external welfare, which is the merger and premerger difference between the total surplus net of the profits of the insiders.

In the same line of literature, Faulí-Oller (1997) develops a model includes that

² Levin (1990) shows that there is a 50% threshold under some conditions.

builds on Salant et al. (1983) and Cheung (1992) and demonstrates that the profitability of a merger depends on the degree of the concavity of demand. That is, if the market share is smaller than a certain value determined by the degree of the concavity of demand, any merger is unprofitable.³

The following literature considers other ways out of the merger paradox. Creane and Davidson (2004) and Sawler (2005) deal with a parent holding company and multidivisional firms in a merger and assume that the parent holding company determines the managerial decision-incentives of the multidivisional firms. In this case, they show that mergers are profitable for the insider (multidivisional) firms, but unprofitable for the outsider firms, given Cournot quantity-setting games. Further, Miyagiwa and Wan (2016) consider the merger paradox in the presence of R&D investments. Dong, Guo, Aian, and Wang (2016) show that the merger paradox is mitigated when capacity constraints are considered.

In summary, to resolve the merger paradox, researchers who have developed the previous theoretical models—with exception of Creane and Davidson (2004) and Sawler (2005)—assume that there are merger-related efficiencies resulting from scale economies, synergies, and externalities on the supply side. However, in this paper, to

³ Hennessy (2000) also considers that the profitability of a merger depends on the properties of the demand function.

consider the merger paradox in a network product market, we focus on the efficiency of merger-related network synergies on the demand side. That is, as shown below, we assume that the degree of compatibility between the insiders' products is larger than that of the outsider firms.

The article proceeds as follows. In the next section, we develop a horizontally differentiated three-firm model and examine the case of noncooperative Cournot competition (i.e., the premerger case) and then the merger case. In Section 3, to consider the merger paradox and externalities, we examine the effect of a merger on outputs and then on profits. We demonstrate whether the merger paradox arises in the presence of merger-related network synergies. In Section 4, we conclude the paper. In the Appendix, we examine the cases of non-network externalities and a homogeneous product market.

2. A Horizontally Differentiated Three-Firm Model with Network Externalities

2.1 Preliminary

We develop a three-firm $\{i, j, k\}$ model in a network product market, where each firm provides a single, horizontally differentiated product with a network externality.

Applying the frameworks of Economides (1996) and Häckner (2000), we assume the following linear inverse demand function for firm i 's product:

$$p_i = A - q_i - \gamma Q_{-i} + N(S_i^e), \quad (1)$$

where $Q_{-i} = q_j + q_k$ is the sum of the rival firms' output, A is the intrinsic market size, q_i is the output of firm i , and $\gamma \in (0,1)$ is the degree of product substitutability.

Further, $N(S_i^e)$ represents a network externality function, where S_i^e is the expected network size of firm i 's product. We assume a linear network externality function, i.e., $N(S_i^e) = nS_i^e$, where $n \in [0,1)$ represents the degree of the network externality. The expected network size of product i is given by:

$$S_i^e \equiv q_i^e + \phi_h Q_{-i}^e, \quad h = C, M, \quad (2)$$

where $Q_{-i}^e = q_j^e + q_k^e$ is the sum of the rival firms' expected outputs, $\phi_h \in [0,1]$ is the degree of product i 's compatibility (interoperability and interconnectivity) with the other firms' product $-i$, subscript C denotes the case of noncooperative Cournot competition (i.e., the premerger case), and subscript M denotes the merger case.

In employing the concept of a fulfilled expectation, we assume that consumers form expectations for network sizes before firms make their output decisions (see Katz and Shapiro, 1985; Economides, 1996). Thus, when determining their outputs, the expected network sizes are given for the firms.

For the subsequent analysis, we make the following important assumptions.

Assumptions

(i) $1 \geq \Delta > 0$ where $\Delta \equiv \phi_M - \phi_C$.

(ii) $n > \gamma$.

Assumption (i) states that the degree of compatibility between the participant insiders' products in the case of a merger case is larger than that in the premerger case (or that of a nonparticipant outsider). This represents the efficiency of merger-related network synergies on the demand side.⁴ If this effect is sufficiently large, as shown below, it is possible to increase the outputs of the insider compared with the premerger situation. Assumption (ii) involves a strong network externality. If this assumption did not apply, i.e., if $n < \gamma$, irrespective of the degree of compatibility between the products, we would have the same results as yielded by the previous research analyzing horizontal mergers in the case of Cournot oligopoly.

⁴ In this paper, we do not examine the endogenous decision regarding the degree of compatibility in the case of a merger. Toshimitsu (2017) demonstrates that there is a unique subgame perfect equilibrium where collusive firms (or insiders) decide to provide perfectly compatible products (i.e., $\phi_M = 1$) to maximize their joint profits. Further, we assume that there are nil or negligible costs involved in increasing the degree of compatibility among the products of the insiders in the case of a merger.

Further, to simplify the analysis, we assume that production costs are zero. We readily observe low and even negligible marginal running costs in telecommunications and internet businesses.

2.2 Premerger: noncooperative Cournot competition

We consider the initial situation (i.e., premerger), where three firms compete on quantities à la Cournot in the market. Based on equation (1), the profit function of firm i is given by:

$$\pi_i = \{A - q_i - \gamma Q_{-i} + N(S_i^e)\}q_i. \quad (3)$$

The first-order condition of profit maximization is:

$$\frac{\partial \pi_i}{\partial q_i} = p_i - q_i = A - 2q_i - \gamma Q_{-i} + N(S_i^e) = 0. \quad (4)$$

At a fulfilled expectation, i.e., $q_i^e = q_i$, $q_j^e = q_j$, and $q_k^e = q_k$, based on equations (2) and (4), we obtain:

$$A - (2 - n)q_i - (\gamma - n\phi_C)Q_{-i} = 0. \quad (5)$$

Assuming a symmetric equilibrium, i.e., $q_i = q_j = q_k = q_C$, we derive the following fulfilled expectation Cournot equilibrium.

$$q_C = \frac{A}{2 - n + 2(\gamma - n\phi_C)}. \quad (6)$$

As it holds that $p_C = q_C$, based on equation (4), the profit in the premerger case is

expressed as $\pi_C = (q_C)^2$.

2.3 Horizontal merger

We consider the case of a horizontal merger, where a merger takes place between two firms $\{i, j\}$ in the market. Thus, we can interpret this to indicate that the merger company is composed of two divisional firms.

Without loss of generality, we assume that there is an insider (I), providing two products $\{i, j\}$ and an outsider (O), providing one product $\{k\}$. The joint profit of the insider in the merger case can be expressed as:

$$\begin{aligned}\Pi_M &= \pi_i + \pi_j \\ &= \{A - q_i - \gamma Q_{-i} + N(S_i^e)\}q_i + \{A - q_j - \gamma Q_{-j} + N(S_j^e)\}q_j.\end{aligned}\tag{7}$$

The profit of the outsider is given by:

$$\pi_O = \{A - q_k - \gamma Q_{-k} + N(S_k^e)\}q_k.\tag{8}$$

Based on equations (7) and (8), the first-order conditions for the insider and outsider firms, respectively, are given by:

$$\frac{\partial \Pi_M}{\partial q_i} = p_i - q_i - \gamma q_j = A - 2q_i - 2\gamma q_j - \gamma q_k + N(S_i^e) = 0,\tag{9}$$

$$\frac{\partial \pi_O}{\partial q_k} = p_k - q_k = A - 2q_k - \gamma Q_{-k} + N(S_k^e) = 0,\tag{10}$$

where we can obtain the first-order conditions in a similar manner to equation (9) with

respect to product j .

At a fulfilled expectation, i.e., $q_i^e = q_i$, $q_j^e = q_j$, and $q_k^e = q_k$, in view of equations (2), (9), and (10), we have the following equations:

$$A - (2 - n)q_i - (2\gamma - n\phi_M)q_j - (\gamma - n\phi_C)q_k = 0, \quad (11)$$

$$A - (2 - n)q_k - (\gamma - n\phi_C)q_{-k} = 0. \quad (12)$$

Assuming a symmetric equilibrium, i.e., $q_i = q_j = q_I$ and $q_k = q_O$, equations (11) and

(12) can be rewritten as:

$$A - \{2 - n + (2\gamma - n\phi_M)\}q_I - (\gamma - n\phi_C)q_O = 0, \quad (13)$$

$$A - (2 - n)q_O - 2(\gamma - n\phi_C)q_I = 0. \quad (14)$$

Thus, we derive the following fulfilled expectation equilibrium in the merger case (M).

$$q_I = \frac{2 - n - (\gamma - n\phi_C)}{D} A, \quad (15)$$

$$q_O = \frac{2 - n - (\gamma - n\phi_C) + (\gamma - n\Delta)}{D} A, \quad (16)$$

where $D \equiv (2 - n)\{2 - n + (2\gamma - n\phi_M)\} - 2(\gamma - n\phi_C)^2 > 0$.

Based on equations (15) and (16), we derive the following relationship:

$$q_I > (<)q_O \Leftrightarrow n\Delta > (<)\gamma. \quad (17)$$

where $n\Delta$ denotes the net degree of network compatibilities. Equation (17) shows that, if the net degree of network compatibilities is larger (smaller) than the degree of product substitutability, the per unit output of the insider is larger (smaller) than that of the

outsider. That is, as in previous models without network externalities, if the network externality is zero, i.e., $n = 0$ (or the net degree of compatibilities is zero, i.e., $\Delta = 0$), a merger reduces the outputs but increases the prices of the insider compared with those of the outsider. However, if $n\Delta > \gamma$, then the merger increases the outputs and prices of the insider compared with those of the outsider. Because it holds that $p_I = (1 + \gamma)q_I$ and $p_O = q_O$, the unit profit of the insider is larger than that of the outsider: $\pi_I = (1 + \gamma)(q_I)^2 > \pi_O = (q_O)^2$.

In general, with respect to the profits, we obtain the following relationship:

$$\pi_I > (<) \pi_O \Leftrightarrow (\sqrt{1 + \gamma} - 1)\{2 - n + (\gamma - n\phi_C)\} + n\Delta - \gamma > (<) 0. \quad (18)$$

Thus, if $n\Delta > \gamma$, it holds that $\pi_I > \pi_O$. Further, even with $n\Delta \leq \gamma$, it is possible for the unit profit of the insider is possible to be larger than that of the outsider. That is, equation (18) can be rewritten as follows:

$$\pi_I > (<) \pi_O \Leftrightarrow \Gamma_{IO}(\gamma) - (1 + \phi_C)\Phi_{IO}(\phi_M, \phi_C, \gamma)n > (<) 0,$$

where $\Gamma_{IO}(\gamma) \equiv \sqrt{1 + \gamma}(2 + \gamma) - 2(1 + \gamma) > 0$ and $\Phi_{IO}(\phi_M, \phi_C, \gamma) \equiv \sqrt{1 + \gamma} - \frac{1 + \phi_M}{1 + \phi_C}$.

Thus, if $\Phi_{IO}(\phi_M, \phi_C, \gamma) \leq 0$, e.g., $\phi_M = 1 > \phi_C = 0$, it holds that $\pi_I > \pi_O$. Conversely,

if $\Phi_{IO}(\phi_M, \phi_C, \gamma) > 0$, e.g., $\phi_M \approx \phi_C$, we derive the following relationship:

$$\pi_I > (<) \pi_O \Leftrightarrow N_{IO}(\phi_M, \phi_C, \gamma) > (<) n,$$

where $N_{IO}(\phi_M, \phi_C, \gamma) \equiv \frac{\Gamma_{IO}(\gamma)}{(1 + \phi_C)\Phi_{IO}(\phi_M, \phi_C, \gamma)} > 0$.

3. Merger Paradox and Externalities

3.1 The effect on output

With respect to the per unit output of the insider and of the outsider in the merger case, compared with the premerger case, we derive the following Lemma 1, based on equations (6), (15), and (16):

Lemma 1

$$(i) \quad q_I > (<)q_C \Leftrightarrow n\Delta > (<)\gamma.$$

$$(ii) \quad q_O > (<)q_C \Leftrightarrow (\gamma - n\phi_C)(\gamma - n\Delta) > (<)0.$$

Suppose that the degree of the network externality is zero, i.e., $n = 0$. It holds that $q_I < q_C$ and $q_O > q_C$. Without network externalities, as is well known in the case of Cournot oligopoly in a homogeneous product market, the per unit output of the insider decreases, whereas that of the outsider increases, compared with the premerger situation.

Now, we consider the effect of the merger on outputs in the presence of network externalities. With respect to equation (5), which determines the fulfilled expectation equilibrium in the noncooperative Cournot competition, i.e., the premerger situation, we assume that the outputs of firms i and j are symmetrical and produced by the insider, whereas the output of firm k is that of the outsider, $q_i = q_j = q_{I_c}$ and $q_k = q_{O_c}$. In this case, we derive the following reaction functions:

$$q_{I_c} = \frac{A}{2-n+(\gamma-n\phi_C)} - \frac{\gamma-n\phi_C}{2-n+(\gamma-n\phi_C)} q_{O_c}, \quad (19.1)$$

$$q_{O_c} = \frac{A}{2-n} - \frac{2(\gamma-n\phi_C)}{2-n} q_{I_c}. \quad (19.2)$$

Similarly, based on equations (13) and (14), the following reaction functions can be derived:

$$q_I = \frac{A}{2-n+(2\gamma-n\phi_M)} - \frac{\gamma-n\phi_C}{2-n+(2\gamma-n\phi_M)} q_O, \quad (20.1)$$

$$q_O = \frac{A}{2-n} - \frac{2(\gamma-n\phi_C)}{2-n} q_I. \quad (20.2)$$

The reaction functions in equations (19.2) and (20.2) are the same. The position of firm k in the premerger case is the same as that of the outsider in the merger case. Thus, if $n\phi_C > (<)\gamma$, then the reaction curve is upward (downward) sloping.

For the first terms on the right-hand sides of equations (19.1) and (20.1), we obtain the following relationship:

$$\frac{A}{2-n+(\gamma-n\phi_C)} > (<) \frac{A}{2-n+(2\gamma-n\phi_M)} \Leftrightarrow \gamma > (<) n\Delta.$$

Further, the reaction curve of the insider is upward (downward) sloping if $n\phi_C > (<)\gamma$.

Thus, we have the following four cases:

Case 1: $n\phi_C > \gamma$, where the slope of the reaction functions is upward. This implies the existence of strategic complements. See Figure 1.

Insert Figure 1 here.

(a) If $n\Delta > \gamma$, the reaction curve of the insider shifts up compared with the premerger case. As a result, the outputs of the insider and outsider increase compared with the premerger case.

(b) If $n\Delta < \gamma$, the reaction curve of the insider shifts down compared with the premerger case. As a result, the outputs of the insider and outsider decrease compared with the premerger case.

Case 2: $n\phi_C < \gamma$, where the slope of the reaction curves is downward. This implies the existence of strategic substitutes. See Figure 2.

Insert Figure 2 here.

(a) If $n\Delta > \gamma$, the reaction curve of the insider shifts up compared with the premerger case. As a result, the output of the insider increases, whereas that of the outsider decreases, compared with the premerger case.

(b) If $n\Delta < \gamma$, the reaction curve of the insider shifts down compared with the

premerger case. As a result, the output of the insider decreases, whereas that of the outsider increases, compared with the premerger case. As discussed above, this case is identical to the case without a network externality. That is, the merger paradox and externalities arise.

As will be examined below, in terms of the effect on the per unit profit of the insider, irrespective of the slope of the reaction functions (i.e., $n\phi_C > (<)\gamma$), if $n\Delta > \gamma$, then not only the output of the insider increases, but also the per unit profit increases. This is because an increase in the degree of network compatibilities resulting from the merger positively affects the output and profit of the insider. Accordingly, in the following analysis, we define $n\Delta > (<)\gamma$ as the merger-related network efficient (inefficient) synergies.

3.2 Loss or gain from the merger

To examine the per unit profit of the insider and that of the outsider in the merger case compared with the premerger case, we define the following: $\Delta\Pi_M \equiv 2(\pi_I - \pi_C)$,

$$\Delta\pi_O \equiv \pi_O - \pi_C, \text{ where } \pi_I = (1 + \gamma)(q_I)^2, \pi_O = (q_O)^2, \text{ and } \pi_C = (q_C)^2.$$

Taking equations (6), (15), and (16), we have the following Lemma 2:⁵

⁵ See the Appendix, where we consider the cases of non-network externalities, i.e., $n = 0$, and a homogeneous product, i.e., $\gamma = 1$.

Lemma 2

$$(i) \quad \begin{aligned} \Delta\Pi_M > (<)0 &\Leftrightarrow \pi_I > (<)\pi_C \\ &\Leftrightarrow (\sqrt{1+\gamma}-1)\{2-n+2(\gamma-n\phi_C)\}\{2-n-(\gamma-n\phi_C)\} + (2-n)(n\Delta-\gamma) > (<)0. \end{aligned}$$

$$(ii) \quad \Delta\pi_O > (<)0 \Leftrightarrow (\gamma-n\phi_C)(\gamma-n\Delta) > (<)0.$$

First, we consider the case of strategic complements, i.e., $n\phi_C > \gamma$.

If $n\Delta > \gamma$, i.e., $n\phi_M > n\phi_C + \gamma > n\phi_C > \gamma$, the merger increases the profits of the insider and of the outsider compared with the premerger case. As in Case 1.a, the outputs and prices of the insider and outsider increase because of the merger-related network efficient synergies under the strategic complementary relationship. Thus, both the insider and outsider gain from the merger. In other words, not only does the merger paradox not arise, but also a positive merger externality does.

If $n\Delta < \gamma$, i.e., $n\phi_C + \gamma > n\phi_M > n\phi_C > \gamma$, then merger-related network inefficient synergies arise under a strategic complementary relationship, i.e., Case 1.b. In this case, the merger decreases the profit of the outsider compared with the premerger situation. This case involves a negative merger externality. Further, the effect on the per unit profit of the insider is ambiguous. That is, in view of Lemma 1, the merger decreases the output, but it can increase the price, i.e., $p_I = (1+\gamma)q_I$, compared with those in the

premerger case. That is, if $\gamma q_I > q_C - q_I > 0$, it holds that $p_I > p_C$. In this case, it is possible to increase the per unit profit of the insider compared with that in the premerger case. For example, see Case 1 in the numerical examples in Table 1.

Second, we consider the case of strategic substitutes, i.e. $n\phi_C < \gamma$.

If $n\Delta > \gamma$, i.e., $n\phi_M > n\phi_C + \gamma > \gamma > n\phi_C$, the merger increases the unit profit of the insider, whereas it decreases that of the outsider, compared with the premerger case. As in Case 2.a, the reaction curve of the insider shifts up compared with the premerger case. As a result, although the outputs and prices of the insider increase, those of the outsider decrease, compared with the premerger case. Thus, the insider gains from merger, whereas the outsider loses. In other words, the merger paradox does not arise, but a negative merger externality does.

If $n\Delta < \gamma$, i.e., $n\phi_C + \gamma > n\phi_M$ and $\gamma > n\phi_C$, then merger-related network inefficient synergies arise under a strategic substitutionary relationship, i.e., Case 2.b. In this case, the reaction curve of the insider shifts down compared with the premerger case. As a result, because the output and price of the outsider increase, its profit always increases. In other words, this case involves a positive merger externality. However, as mentioned above, the effect on the unit profit of the insider is ambiguous. That is, although the outputs decrease, it is possible that the prices are possible to increase.

Cases 2 and 4 in the numerical examples in Table 1 provide examples of the unit profit of an insider increasing. On the other hand, the merger paradox arises in Case 3. This is because the net degree of compatibilities in Case 4 is larger than in Case 3, i.e., $\Delta_4 = 0.6 > \Delta_3 = 0.02$. In other words, the absolute value of the merger-related network inefficient synergies in Case 4 is smaller than that in Case 3.

Insert Table 1 here.

We summarize the analysis above as the following Proposition.

Proposition

(1) *In the presence of merger-related network efficient synergies, i.e., $n\Delta > \gamma$, the merger paradox does not necessarily arise. Further, if the degree of the outsider's network compatibility is larger (smaller) than that of product substitutability, i.e., $n\phi_C > (<)\gamma$, then the outsider gains (losses) from the merger; i.e., a positive (negative) merger externality arises.*

(2) *In the presence of merger-related network inefficient synergies, i.e., $n\Delta < \gamma$, it is possible for the merger paradox to arise. Further, if the degree of the outsider's network compatibility is larger (smaller) than that of product substitutability, i.e., $n\phi_C > (<)\gamma$, then the outsider loses (gains) from the merger, i.e., a negative (positive) merger*

externality arises.

In particular, Proposition (2) implies that the presence of a merger-related network compatibility, even though it is inefficient, mitigates the loss from the merger. Further, when merger-related network efficient synergies exist and the degree of the outsider's network compatibility is sufficiently large, the total outputs are larger in the merger case than in the premerger case. Accordingly, this result implies that the merger increases consumer surplus and thus social surplus, compared with the premerger case. Therefore, we note, although we do not discuss, antitrust issues as the focus of this paper, that mergers in such a case should be allowed by antitrust authorities.

4. Concluding Remarks

Focusing on the role of merger-related network compatibility, which induces synergies on the demand side, we have reconsidered the profitability of mergers in the case of a network externality. In particular, we have found that the merger paradox never arises in the presence of merger-related network efficient synergies, where the net degree of

network compatibilities is larger than the degree of product substitutability. Further, even in the case of merger-related network inefficient synergies, the merger paradox does not necessarily arise.

We appreciate that our model is based on specific assumptions, e.g., a three-firm model, with linear functions and limited parameters. In future research, it would be useful to relax these assumptions to analyze more general cases. Our results demonstrated in the model are related to mergers and competition policies in network industries, including, but not limited to, telecommunications and internet businesses. We consider that, if the merger-related network efficient synergies can be evaluated, a proposed merger should be allowed by antitrust authorities because the consumer surplus, as well as the producer surplus, increases as a result of the merger compared with the premerger case. Thus, in future, we intend to examine merger policy in the presence of network compatibility.

Appendix

1. The case of non-network externalities

Using Lemma 2, we obtain as follows:

$$(i) \quad \pi_I > (<) \pi_C \Leftrightarrow \gamma^\# > (<) \gamma, \quad \text{where } \gamma^\# \approx 0.555 > \frac{1}{2}.^6$$

$$(ii) \quad \pi_O > \pi_C.$$

The above results are well established in the previous literature. For (i), the degree of product differentiation is sufficiently large (small), i.e., $\gamma \Rightarrow 0$ (1), so that a merger increases (decreases) the unit profit of the insider compared with the premerger case. As is well known in the case of a homogeneous product market, i.e., where $\gamma = 1$, a merger decreases the unit profit of the insider; i.e., the merger paradox arises. On the other hand, for (ii), a merger increases the profit of the outsider compared with the premerger case; i.e., a positive merger externality arises.

⁶ $\gamma^\# = \{\gamma \in (0,1) \mid \Gamma_{IC}(\gamma) = 0\}$, where $\Gamma_{IC}(\gamma) \equiv (\sqrt{1+\gamma} - 1)(1+\gamma)(2-\gamma) - \gamma$. the continuous function $\Gamma_{IC}(\gamma)$ has the following properties:

$$(i) \quad \lim_{\gamma \rightarrow 0} \Gamma_{IC}(\gamma) = 0, \quad \lim_{\gamma \rightarrow 1} \Gamma_{IC}(\gamma) = (2\sqrt{2} - 3) < 0,$$

$$(ii) \quad \Gamma_{IC}(\gamma) > 0 \quad \text{for } 0 < \gamma < 0.555, \quad \Gamma_{IC}(\gamma) = 0 \quad \text{for } \gamma \approx 0.555, \quad \text{and } \Gamma_{IC}(\gamma) < 0 \quad \text{for } 0.555 < \gamma < 1.$$

$$(iii) \quad \frac{d\Gamma_{IC}(\gamma)}{d\gamma} > 0 \quad \text{for } 0 < \gamma < 0.366, \quad \frac{d\Gamma_{IC}(\gamma)}{d\gamma} = 0 \quad \text{for } \gamma \approx 0.366, \quad \text{and } \frac{d\Gamma_{IC}(\gamma)}{d\gamma} < 0 \quad \text{for } 0.366 < \gamma < 1.$$

2. The case of a homogeneous product market with a network externality

Based on Lemma 2, when $\gamma = 1$, we obtain the following relationships:

$$(i) \quad \Delta\Pi_M > (<)0 \Leftrightarrow (\sqrt{2} - 1)\{4 - (1 + 2\phi_C)n\}\{1 - (1 - \phi_C)n\} + (2 - n)(n\Delta - 1) > (<)0.$$

$$(ii) \quad \Delta\pi_O > (<)0 \Leftrightarrow (n\phi_C - 1)(n\Delta - 1) > (<)0.$$

With respect to the level of compatibility in the merger and premerger cases, we assume that $\phi_M = 1$, and $\phi_C = 0$, and, thus, $\Delta = 1$. In the premerger case, the firms provide incompatible products, whereas, in the case of a merger, the insider provides the perfectly compatible products and the outsider provides the incompatible one. The above relations can be rewritten as:

$$(i) \quad \Delta\Pi_M > (<)0 \Leftrightarrow (1 - n)\left\{(2 - \sqrt{2})n - 2(3 - 2\sqrt{2})\right\} > (<)0 \Leftrightarrow n > (<)\frac{2(3 - 2\sqrt{2})}{2 - \sqrt{2}} \approx 0.59.$$

$$(ii) \quad \Delta\pi_O > 0 \Leftrightarrow (1 - n) > 0,$$

where $1 > n > 0$.

Therefore, we have the following results: if the degree of the network externality is sufficiently large, i.e., $n > 0.59$, then the merger is profitable for the insider and outsider firms.

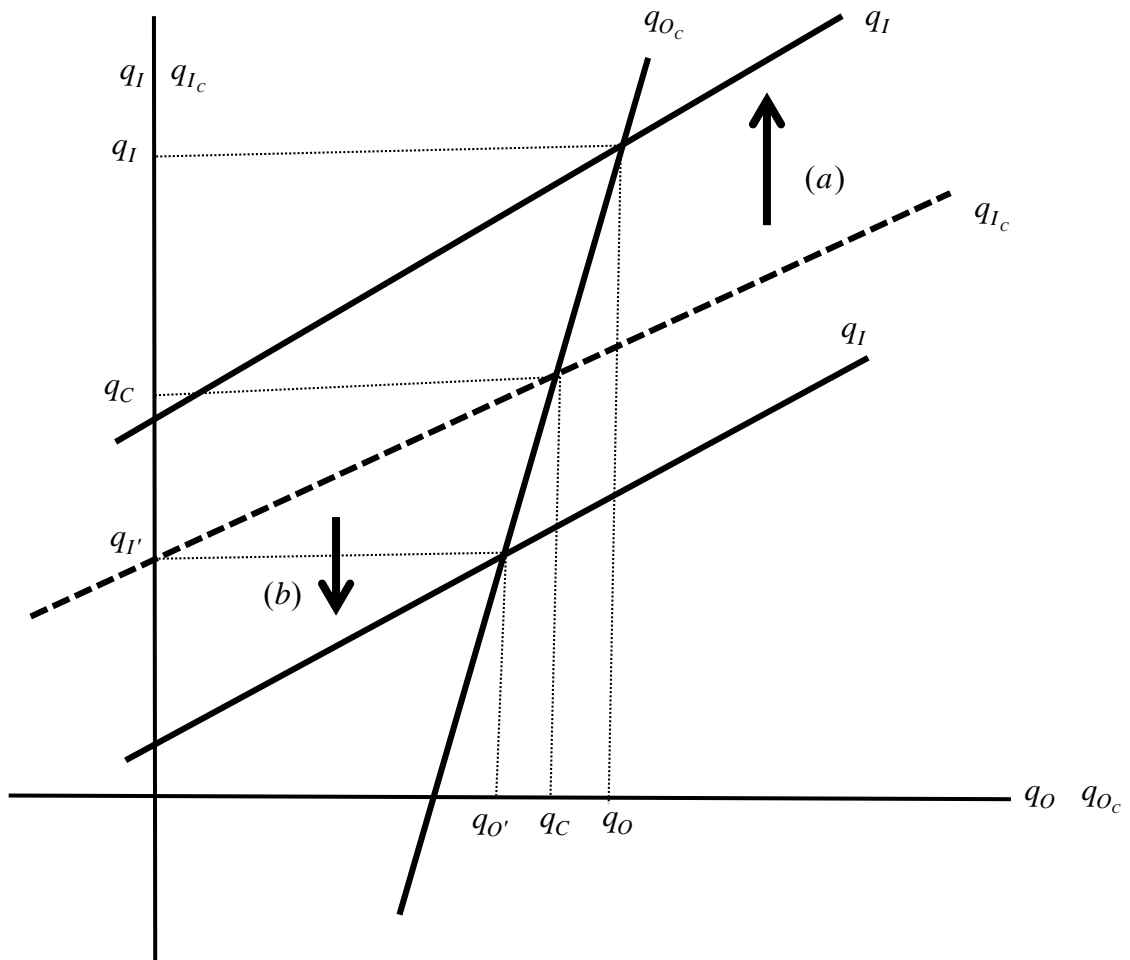
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Figure 1

Case 1 $n\phi_C > \gamma$ (Strategic Complements)



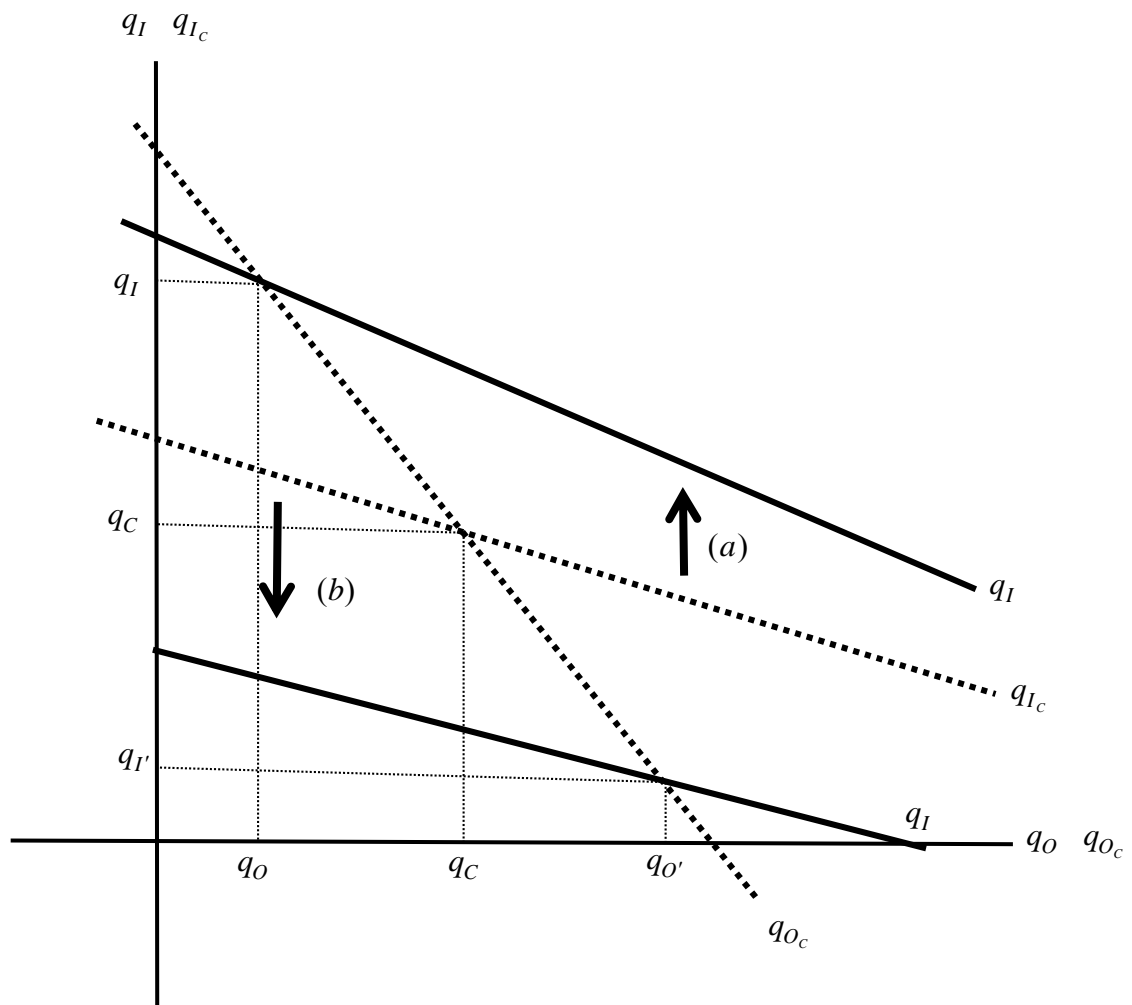
Note

(a) $n\Delta > \gamma$: $q_I > q_C$ and $q_O > q_C$

(b) $n\Delta < \gamma$: $q_I' < q_C$ and $q_{O'} < q_C$

Figure 2

Case 2 $n\phi_C < \gamma$ (Strategic Substitutes)



Note

(a) $n\Delta > \gamma$: $q_I > q_C$ and $q_O < q_C$

(b) $n\Delta < \gamma$: $q_I' < q_C$ and $q_O' > q_C$

Table 1: Numerical examples where $\gamma > n\Delta$

<p>Case 1: $\gamma = 0.1$ and $\Delta = 0.02$ then $0.1 > 0.02n$ and $0.9 > n > 0.1$</p> $H_M \approx 0.18 - 0.08n + 0.03n^2 > 0 \Rightarrow \Delta\Pi_M > 0$ $H_O = (0.1 - 0.2n)(0.1 - 0.02n) \Rightarrow \Delta\pi_O < (>)0 \Leftrightarrow n > (<) \frac{1}{2}$
<p>Case 2: $\gamma = 0.1$ and $\Delta = 0.6$ then $0.1 > 0.6n$ and $0.9 > n > 0.1$</p> $H_M \approx 0.004 + 1.26n - 0.55n^2 > 0 \Rightarrow \Delta\Pi_M > 0$ $H_O = (0.1 - 0.2n)(0.1 - 0.6n) \Rightarrow \Delta\pi_O > 0 \Leftrightarrow 0.1 < n < \frac{1}{6}$
<p>Case 3: $\gamma = 0.4$ and $\Delta = 0.02$ then $0.4 > 0.02n$ and $0.6 > n > 0.4$</p> $H_M \approx (2 - n)(0.01 - 0.19n) < 0 \Rightarrow \Delta\Pi_M < 0$ $H_O = (0.4 - 0.2n)(0.4 - 0.02n) \Rightarrow \Delta\pi_O > 0$
<p>Case 4: $\gamma = 0.4$ and $\Delta = 0.6$ then $0.4 > 0.6n$ and $0.6 > n > 0.4$</p> $H_M \approx (2 - n)(0.01 + 0.39n) > 0 \Rightarrow \Delta\Pi_M > 0$ $H_O = (0.4 - 0.2n)(0.4 - 0.6n) \Rightarrow \Delta\pi_O > 0$

Note

We assume that $\phi_C = 0.2$ and either $\phi_M = 0.22$ or $\phi_M = 0.8$.

It holds that $1 - \gamma > n > \gamma$, from Assumptions (ii) and (iii).

$$H_M(n; \gamma, \phi_C, \Delta) \equiv (\sqrt{1 + \gamma} - 1)\{2 - n + 2(\gamma - n\phi_C)\}\{2 - n - (\gamma - n\phi_C)\} + (2 - n)(n\Delta - \gamma).$$

$$H_O(n; \gamma, \phi_C, \Delta) \equiv (\gamma - n\phi_C)(\gamma - n\Delta).$$