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Collusion and welfare in the case of a horizontally differentiated duopoly with network compatibility

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Abstract

Based on a horizontally differentiated duopoly model with network externalities, in which we focus on the role of compatibility between the products, we consider the effect of collusion on social welfare. We demonstrate that collusion improves social welfare, compared to the case of noncooperative Cournot competition, if the level of compatibility between the products under collusion is sufficiently large, given that a network externality is strong. In this case, the collusion is sustainable.

Keywords: collusion; network externality; compatibility; horizontally differentiated duopoly; welfare

JEL Classifications: D43, D62, L13, L14, L15, L41

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1. Introduction

The analysis of cartel behavior and its effect on market performance in oligopoly is a theoretical issue in industrial organization and a public (competition and anti-trust) policy concern. Recently, many researches have analyzed collusion and market concentration in the context of markets influenced by the progress of information and communication technologies, in which network externalities and compatibility between products and services exist.

In network industries (e.g., video decoders, personal computers, smartphones, application software, operation systems, and Internet services), compatibility and standardization of products and services are important for both providers and users of such products. Compatibility (interoperability and interconnectivity) is a characteristic of products and services that interact with other products and services to enhance performance for users.[1]

Focusing on network externalities and compatibility, recent literature considered whether market concentration plays a role in sustaining collusion. For example, see Lambertini, et al. (1998), Pal and Scrimitore (2016), Rasch (2017), and Song and Wang (2017), all of which are related to our paper. Currently, market concentration and monopolization is observed in various forms (e.g., cartels, collusions, mergers and acquisitions, joint ventures), including in network industries such as telecommunications and Internet services.

[1] Estimating network effects and compatibility in the Polish mobile market, Grajek (2010) finds strong network effects. Furthermore, Gandal (1995) empirically analyzes complementary network externalities in PC software markets, in which users need to exchange data files between spreadsheets and database management systems.

The abovementioned papers mainly consider the sustainability (stability) of collusion by introducing an infinitely repeated game. Pal and Scrimitore (2016) examine the case of oligopoly, with firms competing on quantities in a homogenous product market. Song and Wang (2017) analyze the case of a horizontally differentiated duopoly competing in quantity. Rasch (2017) addresses the case where firms collude on prices in a Hotelling-type market.

In this paper, we consider the effect of collusion on social welfare as well as the sustainability of collusion. The welfare effect has not been analyzed in the previous literature mentioned above. In particular, we appreciate that introducing a common standard to make products and services compatible (interconnectable and interoperable) is an important consideration with network externalities; therefore, we focus on the role of compatibility under collusion. With respect to collusive behavior in our model, we make the following assumptions: (i) the output level is determined through a cooperative decision that involves maximizing joint profits; and (ii) the level of compatibility (standardization between the products) is assumed to be higher or upgraded compared to the case of noncooperative competition.

Assumption (ii) is particularly important. Given these assumptions, we demonstrate that collusion improves social welfare, compared with the case of noncooperative Cournot competition, if the level of compatibility between the products under collusion is sufficiently large, given that a network externality is strong. In this case, the collusion is sustainable.

2. The Model

2.1 Preliminary

We develop a duopoly model in a network industry, where each firm provides a horizontally differentiated product with a network compatibility effect. Applying the frameworks of Economides (1996) and Häckner (2000), we assume a linear inverse demand function of firm *i*'s product as follows:

$$p_i = A - q_i - \gamma q_j + N(S_i^e), \tag{1}$$

where A is the intrinsic market size of product *i*, q_i is the output of firm *i*, and $\gamma \in (0,1)$ represents the level of product substitutability. Furthermore, $N(S_i^e)$ is the network externality function, where S_i^e represents the expected network size of firm *i*'s product. We assume a liner network externality function, $N(S_i^e) = nS_i^e$, where $n \in [0,1)$ represents the level of network externality. Using equation (3.15) in Shy (2001, p. 62), the expected network size of product *i* is given by:

$$S_i^e \equiv q_i^e + \phi_k q_j^e, \quad k = C, N, \tag{2}$$

where $\phi_k \in [0,1]$ denotes the level of product *i*'s compatibility (interoperability and interconnectivity) with the other firms' product *j*, and subscript *C* (*N*) denotes the case of collusion (noncooperative Cournot competition).

Considering the concept of a fulfilled expectation, we assume that consumers develop expectations for network sizes before the firms make their output decisions.[2]

[2] See Katz and Shapiro (1985) and Economides (1996). In the Appendix, we examine the case of consumers' *ex post* expectations, i.e., where consumers' expectations for network size are determined after the firms make their output decisions and where the

Thus, when deciding the output level, the expected network sizes are given for the firms.

For the following analysis, we make some important assumptions:

Assumption

(i) $1 \geq \phi_C > \phi_N \geq 0$.

(ii) $n > \gamma$.

Assumption (*i*) implies that the level of compatibility (interconnectivity) under collusion is larger than that in the case of noncooperative competition, i.e., the level of compatibility among the firms' products is upgraded as a result of collusion. This implies the existence of one kind of scale economy and of synergy effects on the demand side, i.e., consumption externalities. Assumption (*ii*) implies a strong network externality. Otherwise, i.e., if $n < \gamma$, irrespective of the level of compatibility between the products, we have the same results as in the related literature analyzing collusion in the case of Cournot oligopoly (e.g., Song and Wang, 2017).

Furthermore, we assume that production costs are zero, because we observe low and even negligible marginal running costs in Internet businesses.

2.2 Noncooperative Cournot competition

We consider the initial situation where the firms noncooperatively compete on quantities à la Cournot in the market. Based on equation (1), the profit function of firm *i* is given by:

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firms affect the network size.

$$\pi_i = \{A - q_i - \gamma q_j + N(S_i^e)\}q_i. \tag{3}$$

The first-order condition (FOC) of profit-maximization is:

$$\frac{\partial \pi_i}{\partial q_i} = p_i - q_i = A - 2q_i - \gamma q_j + N(S_i^e) = 0. \tag{4}$$

At the point of a fulfilled expectation, i.e., when $q_i^e = q_i$ and $q_j^e = q_j$, in view of equations (2) and (4), we obtain the following:

$$A - (2-n)q_i - (\gamma - n\phi_N)q_j = 0. \tag{5}$$

Assuming a symmetric equilibrium, i.e., $q_i = q_j = q_N$, we derive the following fulfilled expectation Cournot equilibrium:

$$q_N = \frac{A}{2 - n + (\gamma - n\phi_N)}. \tag{6}$$

Note that $n\phi_N$ denotes the level of network compatibility effect under noncooperative competition. Because it holds that $p_N = q_N$, based on equation (4), the profit in the case of noncooperative Cournot competition is expressed as $\pi_N = (q_N)^2$.

2.3 Collusion

Here, we examine the case of collusion in the market where each firm determines output to maximize the following joint profits:

$$\begin{aligned}\Pi_C &= \pi_i + \pi_j \\ &= \{A - q_i - \gamma q_j + N(S_i^e)\}q_i + \{A - q_j - \gamma q_i + N(S_j^e)\}q_j.\end{aligned} \tag{7}$$

Equation (7) implies that a multiproduct monopoly decides the output of product *i* and *j* to maximize its profit. The FOC is given by:

$$\frac{\partial \Pi_C}{\partial q_i} = p_i - q_i - \gamma q_j = A - 2q_i - 2\gamma q_j + N(S_i^e) = 0. \tag{8}$$

At the point of a fulfilled expectation, i.e., when $q_i^e = q_i$ and $q_j^e = q_j$, in view of equations (2) and (8), we obtain the following:

$$A - (2-n)q_i - (2\gamma - n\phi_C)q_j = 0. \tag{9}$$

Assuming a symmetric equilibrium, i.e., $q_i = q_j = q_C$, we derive the following collusive fulfilled expectation equilibrium:

$$q_C = \frac{A}{2 - n + (2\gamma - n\phi_C)}. \tag{10}$$

Note that $n\phi_C$ denotes the level of a network compatibility effect under collusion. In this case, both firms ensure standardization of their products, so that the level of compatibility (interconnectivity) rises, compared to the case of noncooperative competition.

Using equation (8), because the collusive price is expressed as $p_C = (1+\gamma)q_C$, the profit in the case of collusion is given by $\pi_C = (1+\gamma)(q_C)^2$.

Taking equations (6) and (10), we obtain the following relationship:

$$q_C > (<) q_N \Leftrightarrow n(\phi_C - \phi_N) > (<) \gamma. \tag{11}$$

Equation (11) indicates that if the net level of network compatibility effects is larger than the level of product substitutability, the output in the case of collusion is larger than that in the case of noncooperative competition.

Suppose that there is either no network externality or the same level of compatibility under both collusion and noncooperative competition, i.e., $n = 0$ or $\phi_C = \phi_N$. In this case, as is well known, collusion reduces output but increases prices compared to those in the case of noncooperative competition. Furthermore, even with a positive network externality, these well-known results hold in the model of Song and Wong (2017), in which they assume that product substitutability is equal to compatibility, i.e., that $\gamma = \phi_C = \phi_N$, as expressed in the notation of our model.

However, given a strong network externality, i.e., $n > \gamma$, based on assumption (*ii*), if the level of compatibility under collusion is sufficiently larger, collusive outputs do not necessarily decrease compared to those under noncooperative competition. For example, we can image the case of perfect compatible products under collusion (i.e., $\phi_C = 1$) and incompatible products under noncooperative competition ($\phi_N = 0$). Then, it holds that $q_C > q_N$.

With respect to the profits, we can derive the following relationships:

$$\pi_C > (<) \pi_N \Leftrightarrow \sqrt{1+\gamma}(q_C) > (<) q_N$$

$$\Leftrightarrow \left(\sqrt{1+\gamma} - 1\right)\{2 - n + (\gamma - n\phi_N)\} + n(\phi_C - \phi_N) - \gamma > (<) 0.$$

In view of (11), if $n(\phi_C - \phi_N) \geq \gamma$, then it holds that $\pi_C > \pi_N$. The above relationship can be also expressed as:

$$\pi_C > (<) \pi_N \Leftrightarrow \Gamma(\gamma) + \left[(1+\phi_C) - \sqrt{1+\gamma}(1+\phi_N)\right]n > (<) 0, \tag{12}$$

where $\Gamma(\gamma) \equiv \left[\sqrt{1+\gamma}(2+\gamma) - 2(1+\gamma)\right] > 0$ and $1 > n > \gamma > 0$. Regarding equation (12), even with $n(\phi_C - \phi_N) < \gamma$, if $\dfrac{1+\phi_C}{1+\phi_N} \geq \sqrt{1+\gamma}$, then it holds that $\pi_C > \pi_N$. Furthermore, if $\dfrac{1+\phi_C}{1+\phi_N} < \sqrt{1+\gamma}$, equation (12) can be rewritten as:

$$\pi_C > (<) \pi_N \Leftrightarrow \mathrm{N}(\gamma, \phi_C, \phi_N) > (<) n, \tag{13}$$

where $\mathrm{N}(\gamma,\phi_C,\phi_N)\equiv\frac{\Gamma(\gamma)}{\sqrt{1+\gamma}(1+\phi_N)-(1+\phi_C)}>0$. Therefore, if $\mathrm{N}(\gamma,\phi_C,\phi_N)>n$, the firms have an incentive to collude.[3]

We summarize the results analyzed above regarding the quantities and profits under collusion and noncooperative Cournot competition as Lemma1.

Lemma 1

(i) If $n(\phi_C-\phi_N)\geq\gamma$, *it holds that* $q_C\geq q_N$ *and* $\pi_C>\pi_N$.

(ii-a) If $n(\phi_C-\phi_N)<\gamma$ *and* $\frac{1+\phi_C}{1+\phi_N}\geq\sqrt{1+\gamma}$, *it holds that* $q_C<q_N$ *and* $\pi_C>\pi_N$.

(ii-b) If $n(\phi_C-\phi_N)<\gamma$ *and* $\frac{1+\phi_C}{1+\phi_N}<\sqrt{1+\gamma}$, *it holds that* $q_C<q_N$ *and* $\mathrm{N}(\gamma,\phi_C,\phi_N)>(<)n\Leftrightarrow\pi_C>(<)\pi_N$.

Lemma 1 (ii-b) is similar to Song and Wong (2017), where they assume that product substitutability is equal to compatibility, i.e., $\gamma=\phi_C=\phi_N$. However, Lemma 1 (i) implies that market concentration resulting from collusion does not necessarily negatively affect market performance. We will demonstrate this point in Section 3.

2.4 The sustainability of collusion

Here, we examine whether the firms have an incentive to deviate from collusion. Without generality, we assume that firm i deviates from collusion, given that firm j decides the collusive output level, i.e., $q_i=q_D$ and $q_j=q_C$. In this case, the profit

[3] See equation (10) of Song and Wang (2017).

function can be represented by:

$$\pi_D = \{A - q_D - \gamma q_C + N(S_D^e)\}q_D, \tag{14}$$

where $S_D^e \equiv q_D^e + \phi_D q_M^e$.

We assume that $\phi_C > \phi_D \geq \phi_N$. This assumption implies that the level of compatibility decreases if firm *i* deviates from collusion, but the level of compatibility is at least equal to that in the case of noncooperative Cournot competition.

The FOC of profit maximization is:

$$\frac{\partial \pi_D}{\partial q_D} = p_i - q_i = A - 2q_D - \gamma q_C + N(S_i^e) = 0. \tag{15}$$

When expectations are fulfilled, i.e., when $q_D^e = q_D$ and $q_C^e = q_C$, we obtain the following:

$$A - (2-n)q_D - (\gamma - n\phi_D)q_C = 0. \tag{16}$$

Substituting equation (10) into equation (16), we derive the following output in the case of deviation.

$$q_D = \frac{\{2 - n + \gamma - n(\phi_C - \phi_D)\}A}{(2-n)\{2 - n + (2\gamma - n\phi_C)\}} = \frac{\{2 - n + \gamma - n(\phi_C - \phi_D)\}}{2-n} q_C. \tag{17}$$

Given equations (10) and (17), we obtain the following:

$$q_C > (<) q_D \Leftrightarrow n(\phi_C - \phi_D) > (<) \gamma. \tag{18}$$

Because it holds that $p_D = q_D$, the profit in the case of deviation is expressed as $\pi_D = (q_D)^2$. Thus, with respect to the profits under collusion and deviation, we can derive the following relationship.

$$\pi_C > (<)\pi_D \Leftrightarrow \sqrt{1+\gamma}(q_C) > (<)q_D$$

$$\Leftrightarrow \left(\sqrt{1+\gamma}-1\right)(2-n) + n(\phi_C - \phi_D) - \gamma > (<)0.$$

The above relationship can be rewritten as:

$$\pi_C > (<)\pi_D \Leftrightarrow -\Gamma_D(\gamma) + \left[(\phi_C - \phi_D) - \left(\sqrt{1+\gamma}-1\right)\right]n > (<)0, \tag{19}$$

where $\Gamma_D(\gamma) \equiv \left[(2+\gamma) - 2\sqrt{1+\gamma}\right] > 0.$

If $\phi_C - \phi_D \leq \sqrt{1+\gamma}-1$, it holds that $\pi_C < \pi_D$. Thus, the firm has an incentive to deviate from collusion because the difference in the level of compatibility is small between the cases of collusion and deviation. Conversely, if $\phi_C - \phi_D > \sqrt{1+\gamma}-1$, equation (19) can be rewritten as:

$$\pi_C > (<)\pi_D \Leftrightarrow n > (<)\mathrm{N}_D(\gamma, \phi_C, \phi_D), \tag{20}$$

where $\mathrm{N}_D(\gamma, \phi_C, \phi_D) \equiv \dfrac{\Gamma_D(\gamma)}{(\phi_C - \phi_D) - \left(\sqrt{1+\gamma}-1\right)} > 0.$ If $\mathrm{N}_D(\gamma, \phi_C, \phi_N) > (<)n$, the firm will (not) deviate from the collusion.

Taking equations (18) and (19), regarding the incentive to deviate from the collusion, we present the following Lemma 2.

Lemma 2

(i) If $n(\phi_C - \phi_D) \geq \gamma$, *it holds that* $\pi_C > \pi_D$. *Thus, the firms do not have an incentive to deviate.*

(ii-a) If $n(\phi_C - \phi_D) < \gamma$ *and* $\phi_C - \phi_D \leq \sqrt{1+\gamma}-1$, *it holds that* $\pi_C < \pi_D$. *Thus, the firms have an incentive to deviate.*

(ii-b) If $n(\phi_C - \phi_D) < \gamma$ *and* $\phi_C - \phi_D > \sqrt{1+\gamma}-1$, *it holds that*

$n > (<)\mathrm{N}_D(\gamma, \phi_C, \phi_D) \Leftrightarrow \pi_C > (<)\pi_D$. *Thus, the firms have an incentive to deviate if* $n < \mathrm{N}_D(\gamma, \phi_C, \phi_D)$.

Lemma 2 (i) and (ii-b) imply that, for firms to sustain collusion, it is necessary that the level of compatibility under collusion and the degree of the network externality are sufficiently large.

Furthermore, in view of Lemma 2 (ii-a) and (ii-b), given that $n(\phi_C - \phi_D) < \gamma$, if either $\phi_C - \phi_D \le (\sqrt{1+\gamma} - 1)$ or $\phi_C - \phi_D > (\sqrt{1+\gamma} - 1)$ and $n < \mathrm{N}_D(\gamma, \phi_C, \phi_D)$, the collusion is not sustainable. However, as Pal and Scrimitore (2016), Song and Wang (2017), and Rasch (2017) show, assuming an infinitely repeated Cournot game with a trigger strategy punishment, we can also demonstrate that there exists a certain value of discount factor composed of network compatibility effects and product substitutability that makes the collusion sustainable.

3. The Effect of Collusion on Social Welfare

We consider the effect of collusion on social welfare, in particular, consumer surplus. Taking equation (1), consumer surplus is given by $CS_k \equiv (1+2\gamma)(q_k)^2$, where $k = C, N$. Thus, in view of equations (6) and (10), we derive the following relationship directly:

$$CS_C > (<)CS_N \Leftrightarrow q_C > (<)q_N \Leftrightarrow n(\phi_C - \phi_N) > (<)\gamma. \quad (21)$$

Therefore, based on Lemmas1 (i) and 2 (i), and equation (21), we obtain the following key result:

Proposition

If $n(\phi_C - \phi_N) > \gamma$, *then the firms have an incentive to collude. In this case, the firms do not have an incentive to deviate from the collusion. The collusion increases consumer surplus, and thus social welfare, compared to the case of noncooperative Cournot competition.*

The Proposition declares that collusion in a network industry does not necessarily reduce the resulting welfare level if the level of the network compatibility effect under collusion is sufficiently large, given a strong network externality. However, the prices rise compared to those in the case of noncooperative Cournot competition. In this sense, collusion may not be procompetitive even though the consumer surplus increases because of the strong network externality.

In contrast, if $n(\phi_C - \phi_N) < \gamma$, then the effect of collusion on consumer surplus is always negative. This case is similar to the result in the previous literature considering collusion with network externalities (e.g., Song and Wang, 2017).

4. Concluding Remarks

Assuming that the level of compatibility is upgraded under collusion, in other words, there us greater standardization between products and services compared to that in the case of noncooperative competition, we considered collusive behavior and its effect on consumer surplus and social welfare.[4] In particular, we demonstrated that collusion improves social welfare if the level of compatibility in the case of collusion is sufficiently large, given that a network externality is strong. That is, in this case, the collusive output levels are larger than those of noncooperative Cournot competition. We may observe such collusive behavior by firms in the telecommunications and Internet services market.

Our result has some limitations because our duopoly model is based on specific assumptions and linear functions. In future research, we intend to discuss more general cases, relaxing the assumptions and extending the model to oligopolistic competition. In the case of oligopoly, e.g., we must consider the presence of a firm that is an outsider to the collusion between other firms. Furthermore, we should consider monopolization (market concentration) of the markets that occurs through mergers and acquisitions and its effect on market performance and welfare.

Appendix: The case of consumers' *ex post* expectations

Here, we consider the case of consumers' *ex post* expectations. This implies that the firms can commit to their output levels, and on this basis, consumers then form expectations for the network size, i.e., $q_i^e = q_i$ and $q_j^e = q_j$. Thus, it holds that $S_i^e = S_i = q_i + \phi_k q_j$, where S_i is the actual network size of firm *i*'s product. In particular, we consider subgame perfect Nash equilibria, in which consumers observe

[4] Formally, our model is related to joint ventures and strategic alliances (e.g., airlines).

output levels (capacities) before making their actual consumption decision. As consumers have to make their choice given the choices of all other consumers in the Nash equilibrium, each consumer's beliefs about the behavior of other consumers are confirmed. Thus, equation (1) can be changed as follows:

$$\tilde{p}_i = A - (1-n)q_i - (\gamma - n\phi_k)q_j, \quad k = C, N, \tag{A.1}$$

where we assume that $1 > n + \gamma$. This assumption implies that the own-price effect exceeds the cross-price effects at the point of the fulfilled expectation, i.e., $\left|\frac{\partial p_i}{\partial q_i}\right| > \left|\frac{\partial p_i}{\partial q_j}\right|$.

In the following, we confirm the robustness of our Proposition.

(i) Cournot competition

Taking equation (A.1), the profit function is $\tilde{\pi}_i = \{A - (1-n)q_i - (\gamma - n\phi_N)q_j\}q_i$. The FOC is given by:

$$\frac{\partial \tilde{\pi}_i}{\partial q_i} = \tilde{p}_i - (1-n)q_i = A - 2(1-n)q_i - (\gamma - n\phi_N)q_j = 0. \tag{A.2}$$

At the symmetric equilibrium, i.e., $q_i = q_j = \tilde{q}_N$, we have:

$$\tilde{q}_N = \frac{A}{2(1-n) + (\gamma - n\phi_N)}. \tag{A.3}$$

In this case, because $\tilde{q}_N = (1-n)\tilde{q}_N$, the profit is represented as $\tilde{\pi}_N = (1-n)(q_N)^2$.

(ii) Collusion

The joint profit under collusion is given by:

$$\tilde{\Pi}_C = \tilde{\pi}_i + \tilde{\pi}_j$$

$$= \{A - (1-n)q_i - (\gamma - n\phi_C)q_j\}q_i + \{A - (1-n)q_j - (\gamma - n\phi_C)q_i\}q_j.$$

Thus, the FOC is:

$$\frac{\partial \tilde{\Pi}_C}{\partial q_i} = \tilde{p}_i - (1-n)q_i - (\gamma - n\phi_C)q_j$$

$$= A - 2(1-n)q_i - 2(\gamma - n\phi_M)q_j = 0. \tag{A.4}$$

At the symmetric equilibrium, i.e., $q_i = q_j = \tilde{q}_C$, we have:

$$\tilde{q}_C = \frac{A}{2(1-n) + 2(\gamma - n\phi_C)}. \tag{A.5}$$

Furthermore, the collusive price is $\tilde{p}_C = \{(1-n) + (\gamma - n\phi_C)\}\tilde{q}_C$. Thus, the profit per product is $\tilde{\pi}_C = \{(1-n) + (\gamma - n\phi_C)\}(\tilde{q}_C)^2$.

Using equations (A.3) and (A.5), we can derive the following relationships:

$$\tilde{q}_C > (<) \tilde{q}_N \Leftrightarrow n(2\phi_C - \phi_N) > (<) \gamma. \tag{A.6}$$

Thus, (A.6) implies that if $n(\phi_C - \phi_N) > \gamma$, as in the condition in Proposition, then collusion increases the consumer surplus compared to that in the case of noncooperative Cournot competition.

Regarding the comparison of the profits, that is, the condition of incentive to collude, we have:

$$\tilde{\pi}_C > (<) \tilde{\pi}_N \Leftrightarrow \{1 - n + (\gamma - n\phi_C)\}(\tilde{q}_C)^2 > (<)(1-n)(\tilde{q}_N)^2.$$

Substituting equations (A.3) and (A.5) into the relationship, we can derive the following:

$$\tilde{\pi}_C > \tilde{\pi}_N \Leftrightarrow 4(1-n)n(\phi_N - \phi_C) + (\gamma - n\phi_N)^2 > 0. \tag{A.7}$$

Thus, in the case of consumers' *ex post* expectations, the firms always have an incentive to collude.

Furthermore, given that the other firm's output is at the collusive level, the output of the deviating firm is given by $\tilde{q}_D = \frac{\{2(1-n)+\gamma-n(2\phi_C-\phi_D)\}}{2(1-n)}\tilde{q}_C$. Thus, with respect to the profits under collusion and deviation, we can derive the following relationship:

$$\tilde{\pi}_C > (<) \tilde{\pi}_D \Leftrightarrow 4(1-n)n(\phi_C-\phi_D) - \{(\gamma - n\phi_C) - n(\phi_C-\phi_D)\}^2 > (<) 0.$$

In this case, if $n(\phi_C-\phi_N) > \gamma$ and $\phi_N = \phi_D$, we have $(1-n)n(\phi_C-\phi_N) > \gamma^2$, given the assumption, i.e., that $1-n > \gamma$. It holds that $\tilde{\pi}_C > \tilde{\pi}_D$ because $4(1-n)n(\phi_C-\phi_D) > \gamma^2 > \{(\gamma-n\phi_C)-n(\phi_C-\phi_D)\}^2$. Therefore, the firms do not have an incentive to deviate from collusion.

Based on the analysis above, in the case of consumers' *ex post* expectations, we can demonstrate that the Proposition is robust.

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