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Abstract

Using a Constant Elasticity of Substitution (CES)-type fertility function, we examine how child allowances, subsidized policies for child care services, and child care time affect fertility rates, working hours, and demand for child care services, and assess the tax burden necessary to achieve the target fertility rate. The following results were obtained. Child allowances can achieve the target fertility rate with the lowest tax burden for policies of three types. The target fertility rate might not be achieved if only subsidies for child care services or child care time are used. Even if the target fertility rate cannot be achieved with a single policy, the target fertility rate can be achieved with a policy mix of subsidized child care services and child care time.

Keywords: Child allowance, Constant Elasticity of Substitution (CES), Fertility, Subsidy for child care services and time

JEL: H51, H55, J14

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1. Introduction

The analyses described herein use a Constant Elasticity of Substitution (CES)-type fertility function to examine how child allowances, subsidized policies for child care services, and child care time affect fertility rates, hours worked, and demand for child care services. Moreover, when the government sets a certain fertility rate as the target, this paper presents an examination of how the tax burden designed to achieve that target fertility rate differs for child allowances, subsidized policies for child care services, and child care time.¹

We analyze not only one particular policy, but also the tax burden when considering a policy mix such as a combination of policies to subsidize childcare services and childcare time. This paper presents a discussion based on findings from a simulation analysis.

The results obtained are explained below. Child allowances can achieve the target fertility rate with the lowest tax burden among the three policies. The target fertility rate might not be achieved if only subsidies for child care services or child care time are used. Even if the target fertility rate cannot be achieved using only a single policy, it is possible to achieve the target fertility rate with a policy mix of subsidized child care services and child care time. Results show that even with only slight policy spending, the child allowance increases the fertility rate the most among the three policies, given the same tax burden.

Next, we explain the related literature. As described in this paper, we apply a model in which fertility is determined by both child care services and child care time as input factors. van Groezen, Leers and Meijdam (2003) present a model in which fertility is determined by childcare services alone. Galor and Weil (1996) present a model in which fertility is determined solely by the childcare time. Yasuoka and Miyake (2010), Hirazawa and Yakita (2009), and Apps and Rees (2004) all report studies that set up a model with both child care services and child care time as input factors.

By contrast, Yasuoka and Miyake (2010), Hirazawa and Yakita (2009) and Apps and Rees (2004) assumed a function of perfect substitution, a Cobb–Douglas type function, and a function of constant returns to scale. Although analysis with a function of constant returns to scale is a desirable setting in the sense that the assumptions are not too strong, it is necessary to specify the function in the simulation analysis. A CES-type function similar to the one presented herein is a desirable setting for simulation analysis.

Child allowances are discussed by van Groezen, Leers and Meijdam (2003). Subsidized child

¹ We can consider some types of the target fertility rate. For example, the population replacement level fertility rate in Japan is 2.07. However, it has been set at 1.8 as the target fertility, as shown in a Cabinet Office document. (Data: National Institution of population and Social Security Research “Population statistics of Japan” and Cabinet Office, Government of Japan “Points of policies for fewer children in future”)

care services are discussed in Apps and Rees (2004), both of which have been shown to raise fertility rates. In addition, when considering educational investment in children, Zhang (1997) and Yasuoka and Miyake (2014) show that child allowances raise fertility but they also lower the level of educational investment. However, the model presented herein does not incorporate educational investment in children.

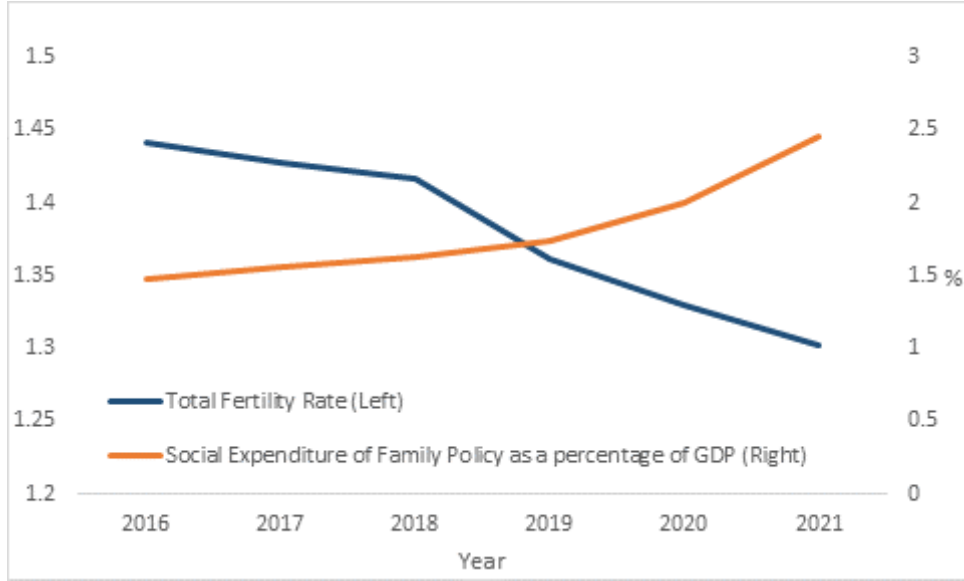


Fig. 1: Total Fertility Rate and Social Expenditure of Family Policy as a Percentage of GDP. (Data: Ministry of Health, Labour and Welfare “Vital statistics” and National Institution of population and Social Security Research “Social expenditure database 2021”)

Although the effects of each policy have been analyzed using comparative statics analysis, no report of a study describes examination of which policy minimizes the tax burden for a given target fertility rate, or describes the effects of multiple policies combined on fertility rates and the size of the tax burden at that time. Japan and other countries have set a target fertility rate and have implemented aggressive child care support policies to achieve that rate. The analyses described herein can provide an evaluation of such policies. Fig. 1 shows that fertility is declining despite increased policy spending. It would be useful to show what policies are desirable as long as the situation in Fig. 1 exists.

The remainder of this paper is organized as follows: Section 2 sets up the model. Section 3 presents examination of whether each policy can achieve the fertility target. Section 4 shows that child care services and subsidized child care time can achieve the fertility target. Section 5 examines changes in fertility rates after a differential level of policy spending. Section 6 summarizes the important findings and conclusions.

2. Model

There exist three types of agents in this model economy: households, firms, and government.

2.1 Households

The individual in the households lives in two periods: a young and an old period. At each period, there exist younger generation and older generation individuals. During the young period, the individuals care for children. Child care entails not only the child care time, but also child care services. As described herein, we assume the following fertility function²:

$$n_t = (Ae_t^\rho + Bl_t^\rho)^{\frac{1}{\rho}}, 0 < A, 0 < B, \rho < 1. \quad (1)$$

The number of children n_t , which is regarded as fertility, is determined by the input of child care services e_t and childcare time l_t . With $\rho = 0$, the fertility function is a Cobb–Douglas form function. With $\rho = 1$, the fertility function is a perfect substitution form function. With $\rho = -\infty$, the fertility function is a perfectly complementary form function.

The utility function is assumed as the following logarithm function:

$$u_t = \alpha \ln c_{1t} + \beta \ln c_{2t+1} + (1 - \alpha - \beta) \ln n_t, 0 < \alpha < 1, 0 < \beta < 1, \alpha + \beta < 1. \quad (2)$$

Therein, c_{1t} and c_{2t+1} respectively denote consumption during the young and old period.

In the young period, the individual budget constraint is

$$s_t + c_{1t} + (1 - \gamma)ze_t = (1 - l_t)w + \varepsilon l_t w + qn_t - T_t. \quad (3)$$

The individual has savings s_t for consumption during the old period. The price of childcare services is given as z . The subsidy rate for childcare services is γ . The individual has a unit of time and allocates labor time as $1 - l_t$ and childcare time as l_t . The wage rate is given as w . The government provides for the subsidy for childcare time at rate ε . In addition, the government provides a child allowance q for a child. These children are supported by policies financed by lump-sum taxation T_t .

The budget constraint in old period is

$$c_{2t+1} = (1 + r)s_t. \quad (4)$$

In that equation, r denotes the interest rate. We consider the utility (1) maximization problem subject to the budget constraints (3) and (4) and the fertility function (2). Then, the optimal allocations are derived as

$$c_{1t} = \alpha(w - T_t), \quad (5)$$

$$c_{2t+1} = (1 + r)\beta(w - T_t), \quad (6)$$

² This setting of the fertility function is assumed from earlier work by Yasuoka (2014, 2024).

$$e_t = \frac{(1-\alpha-\beta)(w-T_t)}{(1-\gamma)z+(1-\varepsilon)w\left(\frac{(1-\gamma)zB}{(1-\varepsilon)wA}\right)^{\frac{1}{1-\rho}} - q\left(A+B\left(\frac{(1-\gamma)zB}{(1-\varepsilon)wA}\right)^{\frac{\rho}{1-\rho}}\right)^{\frac{1}{\rho}}}, \text{ and} \quad (7)$$

$$l_t = \left(\frac{(1-\gamma)zB}{(1-\varepsilon)wA}\right)^{\frac{1}{1-\rho}} e_t. \quad (8)$$

2.2 Firms

As described in this paper, the production function is assumed as

$$y_t = k_t^\theta, 0 < \theta < 1. \quad (9)$$

Therein, y_t and k_t respectively denote the output per labor input and the capital–labor ratio.

Considering a perfectly competitive market, the wage rate and the interest rate are given as

$$w_t = (1-\theta)k_t^\theta, \quad (10)$$

$$1+r_t = \theta k_t^{\theta-1}. \quad (11)$$

We assume a small open economy with the interest rate and wage rate given by the world interest rate.

2.3 Government

The government provides child care policies of three types: a child allowance, a subsidy for child care services, and the child care time. If these policies are financed by lump-sum taxation, the government budget constraint is shown as

$$T_t = q_t n_t + \gamma_t z e_t + \varepsilon_t w l_t. \quad (12)$$

3. Simulation

This section presents an examination of the effects of childcare policies (child allowance, the subsidy for child care services and childcare time) on fertility in the different cases of fertility functions with the simulation. Before simulation, we calculate the parameters used for the simulation.

3.1 Parameters

We set the parameters as shown by Table 1. Among those parameters, β is considered by de la Croix and Doepke (2003). Also, they consider the quarterly discount factor as 0.99. Considering a period of overlapping generations model as 30 years, we calculate it as 0.99^{30} . As given by Oshio (2001), $1-\alpha-\beta$ is denoted as the preference for children. It sets the same preference for the consumption. We set $\alpha = 0.35031$. Also, θ stands for the capital–income ratio. We set $\theta = 0.3$ because the capital income ratio in OECD countries is about 30%. Because the annual interest rate is about 1% in Japan and because a period in an overlapping generations model is considered as thirty

years, we set $1 + r = 1.01^{30}$. Considering (11), we can obtain the capital–labor ratio k_t . Then (10) gives the wage level as 0.36766.

Table 1: Parameters

Parameter	Value
α	0.35031
β	0.29938
$1 - \alpha - \beta$	0.35031
$1 + r$	1.347849
w	0.36766
θ	0.3

Table 2: Childcare cost z

	z	ρ	A	B	e	l	n
Case 1	0.028419	0	0.5	0.5	2.265987	0.175155	0.629999
Case 2	0.079427	0	0.75	0.25	1.216158	0.087577	0.63
Case 3	0.003707	0	0.25	0.75	8.686015	0.262732	0.63
Case 4	0.081423	0.75	0.5	0.5	1.564797	0.003764	0.63
Case 5	0.139338	0.75	0.75	0.25	0.923711	0.000235	0.629999
Case 6	0.059361	0.5	0.5	0.5	1.868079	0.048697	0.63
Case 7	0.119136	0.5	0.75	0.25	1.043506	0.012174	0.63
Case 8	0.010467	-0.5	0.5	0.5	2.878443	0.268366	0.63
Case 9	0.05034	-0.5	0.75	0.25	1.323775	0.16906	0.63
Case 10	0.004782	-0.75	0.5	0.5	3.624694	0.303161	0.629999
Case 11	0.039112	-0.75	0.75	0.25	1.37523	0.204012	0.63

We calculate child care cost z such that the total fertility rate is 1.26 in a recent Japanese case. For this model, the fertility rate 1.26 shows $n = 0.63$. We calculation childcare cost z in the cases of the different types of function form and the different parameter settings (Table 2). Cases 1–3, Cases 4–7, and Cases 8–11 respectively denote the cases of a Cobb–Douglas form, a substitution form, and complementary form functions.

If A is large, then the relative demand for childcare services e to childcare hours l is expected to be large. The effect will be larger in other cases, and vice versa. However, it is apparent that the effect is weaker in the complementary case.

3.2 Simulation results

In this subsection, we set the goal to increase the fertility such that $n = 1$, which is population replacement level. Then we examine the tax revenue necessary for providing the policy.

3.2.1 Child allowance

Table 3: Child allowance case

	z	ρ	A	B	e	l	n	q	qn
Case 1	0.028419	0	0.5	0.5	3.596811	0.278024	1	0.116428	0.116428
Case 2	0.079427	0	0.75	0.25	1.93041	0.139012	1	0.116427	0.116427
Case 3	0.003707	0	0.25	0.75	13.78733	0.417036	1	0.116427	0.116427
Case 4	0.081423	0.75	0.5	0.5	2.483801	0.005975	0.999998	0.116427	0.116427
Case 5	0.139338	0.75	0.75	0.25	1.466204	0.000373	0.999996	0.116427	0.116427
Case 6	0.059361	0.5	0.5	0.5	2.965202	0.077297	1	0.116427	0.116427
Case 7	0.119136	0.5	0.75	0.25	1.656357	0.019324	0.999999	0.116427	0.116427
Case 8	0.010467	-0.5	0.5	0.5	4.568946	0.425977	0.999997	0.116427	0.116427
Case 9	0.05034	-0.5	0.75	0.25	2.101226	0.268349	0.999998	0.116427	0.116427
Case 10	0.004782	-0.75	0.5	0.5	5.753467	0.481206	0.999997	0.116427	0.116427
Case 11	0.039112	-0.75	0.75	0.25	2.182903	0.323828	0.999999	0.116427	0.116427

It is readily apparent that the target fertility rate $n = 1$ is achieved in all cases (Table 3). We also find that the level of child allowance and the tax burden to achieve the target fertility rate are equal in all cases.

3.2.2 Subsidy for child care services

In the case of a subsidy for childcare services, in some cases the policy is inadequate to achieve the goal of $n = 1$ (Table 4). The reason is explainable as follows. The subsidy for child care services raises the demand for child care services. In addition, an increase in tax burden γze has a negative effect on the child care time. Therefore, because an increase in child care services reduces the marginal productivity of child care services, this subsidy policy is insufficient to achieve $n = 1$. If the subsidy for child care services increases more than the maximization level of fertility, then fertility decreases because of an increased tax burden and decreased child care time.

In Case 5, we can obtain $n = 1$. The tax burden is nearly the same as in the case of a child allowance. This case is nearly the same as that of perfect substitution between child care services and child care time. The subsidy for child care services is equivalent to the child allowance. In this case, there is nearly zero child care time.

Table 4: Subsidy for child care services cases

	z	ρ	A	B	e	l	n	γ	γze
Case 1	0.028419	0	0.5	0.5	6.468637	0.106173	0.828731	0.787658	0.144798
Case 2	0.079427	0	0.75	0.25	4.036295	0.015207	1.000001	0.94768	0.303819
Case 3	0.003707	0	0.25	0.75	24.76618	0.216048	0.706931	0.711598	0.06533
Case 4	0.081423	0.75	0.5	0.5	2.516992	0.000202	1	0.572453	0.11732
Case 5	0.139338	0.75	0.75	0.25	1.467418	1.28E-05	1	0.569686	0.116482
Case 6	0.059361	0.5	0.5	0.5	3.568515	0.012309	0.999999	0.636235	0.134774
Case 7	0.119136	0.5	0.75	0.25	1.725765	0.003475	1	0.584556	0.120185
Case 8	0.010467	-0.5	0.5	0.5	7.086416	0.249267	0.707001	0.76826	0.056982
Case 9	0.05034	-0.5	0.75	0.25	3.322106	0.136839	0.845848	0.81683	0.136601
Case 10	0.004782	-0.75	0.5	0.5	8.381977	0.296412	0.672782	0.778304	0.031199
Case 11	0.039112	-0.75	0.75	0.25	3.279992	0.185964	0.792523	0.814227	0.104455

3.2.3 Subsidy for childcare time

Table 5: Subsidy for childcare time cases

	z	ρ	A	B	e	l	n	ε	εwl
Case 1	0.028419	0	0.5	0.5	1.373533	0.50002	0.82873	0.787668	0.144803
Case 2	0.079427	0	0.75	0.25	0.999793	0.249905	0.70693	0.711904	0.06541
Case 3	0.003707	0	0.25	0.75	4.794539	0.59304	1	0.755457	0.164718
Case 4	0.081423	0.75	0.5	0.5	1.519275	0.017543	0.631407	0.324409	0.002092
Case 5	0.139338	0.75	0.75	0.25	0.922069	0.001081	0.630087	0.31735	0.000126
Case 6	0.059361	0.5	0.5	0.5	1.418232	0.190643	0.662208	0.559631	0.039226
Case 7	0.119136	0.5	0.75	0.25	0.985731	0.045686	0.636909	0.498279	0.00837
Case 8	0.010467	-0.5	0.5	0.5	2.363246	0.549101	1	0.74582	0.150568
Case 9	0.05034	-0.5	0.75	0.25	1.094118	0.423511	0.824687	0.810486	0.126199
Case 10	0.004782	-0.75	0.5	0.5	1.078618	0.93089	1	0.983167	0.33649
Case 11	0.039112	-0.75	0.75	0.25	1.171223	0.49806	0.893812	0.84165	0.15412

In the case of a subsidy for childcare time, the policy can not achieve the goal of $n = 1$ in some cases (Table 5). The reason is explainable as follows. The subsidy for child care time raises demand for child care time. In addition, an increase in the tax burden εwl has a negative effect on child care services. Therefore, because an increase in child care time reduces the marginal productivity of child

care time, this subsidy policy is unable to achieve $n = 1$. If the subsidy for child care time increases more than the maximization level of the fertility, then fertility decreases because of an increase in tax burdens and a decrease in child care services.

Then, the following proposition can be established.

Proposition 1

A child allowance can achieve fertility of $n = 1$. However, in the case of the subsidy for childcare services and childcare time, some cases can not have fertility of $n = 1$. The tax burden to have $n = 1$ is smallest in the case of a child allowance.

A child allowance is the subsidy for fertility: child care services and child care time. Because of subsidies to both inputs, the marginal productivity of both inputs is not reduced. Because the increase in demand for child care services increases the marginal productivity of child care time, the opposite is also true. Therefore, a child allowance can achieve fertility of $n = 1$ while imposing the smallest tax burden.

In the next section, we consider mixed policies of the subsidy for child care services and childcare time to examine whether fertility of $n = 1$ can be achieved, or not.

4. Mixed policies

Table 6: Subsidy for both child care services and child care time

	e	l	n	γ	γze	ε	εwl	T
Case 1	3.597003	0.278009	1	0.569527	0.058219	0.56948	0.058208	0.116427
Case 2	1.93041	0.139012	1	0.569502	0.08732	0.569502	0.029107	0.116427
Case 3	13.78711	0.417038	1	0.569496	0.029106	0.569505	0.087321	0.116427
Case 4	2.483827	0.00597	1	0.569503	0.115177	0.569414	0.00125	0.116427
Case 5	1.466212	0.000372	1	0.569503	0.116349	0.569185	7.79E-05	0.116427
Case 6	2.965279	0.077285	1	0.569507	0.100246	0.569468	0.016181	0.116427
Case 7	1.65636	0.019324	1	0.569502	0.112381	0.569497	0.004046	0.116427
Case 8	4.568944	0.425974	0.99999	0.569502	0.027234	0.569497	0.089191	0.116425
Case 9	2.101226	0.268347	0.999994	0.569502	0.060239	0.569497	0.056187	0.116426
Case 10	5.753457	0.481202	0.999989	0.569502	0.01567	0.569497	0.100755	0.116425
Case 11	2.182899	0.323826	0.999993	0.569502	0.048623	0.569497	0.067803	0.116426

In this section, government provides subsidy policies of two types simultaneously: a subsidy for child care services and a subsidy for child care time. Consequently, we obtain the result of the case

of child allowance (Table 6). The tax burden is the same as that in the case of a child allowance. Then, the following proposition can be established.

Proposition 2

Even if the target fertility rate $n = 1$ cannot be achieved by subsidizing childcare services or child care time alone, the target fertility rate can be achieved by simultaneously subsidizing both child care services and child care time.

This proposition is intuitive. Subsidizing only one factor increases the input of that factor, but it decreases the marginal productivity of that factor, making it more difficult to achieve the target fertility rate. Therefore, subsidizing both factors increases the input of both factors, which prevents a decline in marginal productivity and makes it easier to achieve the target fertility rate. Of course, depending on the level of the elasticity of substitution, an increase in the input of child care services increases the marginal productivity of the child care time. An increase in the input of child care time increases the marginal productivity of child care services. Subsidies for both inputs to increase both factors' inputs impose the smallest tax burden to achieve the target fertility rate.

The tax burden then is the same as for that of the child allowance. The child allowance is a subsidy for both inputs. The tax burden of the child allowance to achieve the target fertility rate is equal to the total tax burden of the subsidies for child care services and child care time.

5. Differential effects

Table 7: Subsidy for both childcare services and childcare time

	Child allowance	Subsidy for child care services	Subsidy for child care time
Case 1	0.641662	0.641410972	0.641411
Case 2	0.641663	0.641578492	0.640936
Case 3	0.641663	0.640935968	0.641578
Case 4	0.641663	0.641652591	0.631153
Case 5	0.641662	0.641661892	0.626821
Case 6	0.641663	0.641583008	0.639307
Case 7	0.641663	0.641645185	0.635544
Case 8	0.641662	0.641112149	0.64161
Case 9	0.641663	0.641503812	0.64148
Case 10	0.641662	0.640733301	0.64164
Case 11	0.641663	0.641457712	0.641558

This section presents examination of how much the subsidy policy by a tax burden of 1% of wages as a differential effect raises the fertility rate. The fertility rate before the policy is set as 0.63. Table 7 shows the cases of child allowances, subsidies for childcare services, and subsidies for childcare time. The case of the child allowance has the largest fertility in three policies. The child allowance is a subsidy for both childcare services and childcare time. Raising both input factors efficiently raises the fertility rate.

6. Conclusion

These analyses have compared the tax burdens of achieving the target fertility rate through child allowances, subsidies for childcare services, and subsidies for childcare time when the target fertility rate is set as 1 (2 in the real economy) as the population replacement level.

The results obtained from this study are the following. Child allowances can achieve the target fertility rate with the smallest tax burden in three policies. Subsidies for childcare services and childcare time impose a larger tax burden than child allowances. Subsidies for child care services and child care time might be inadequate to achieve the target fertility rate, depending on the parameters set. The reason for this inadequacy is that the fertility rate cannot be increased effectively because of diminishing marginal productivity. Furthermore, the demand for childcare services and time for childcare is reduced because of tax burdens. Even if the target fertility rate cannot be achieved by a single policy, it is possible to achieve it by combining multiple policies. Thereby, one can avoid diminishing marginal productivity.

The results of these analyses were not obtained through comparative statics analysis, but by simulation. The simulations conducted for these analyses were done assuming various parameters to address various cases. The findings presented herein are expected to be useful for making realistic policy recommendations.

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