DISCUSSION PAPER SERIES

Discussion paper No. 260

School Choice and Private Tutoring in an Endogenous Fertility Model

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September 2023

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School Choice and Private Tutoring in an Endogenous Fertility Model[†]

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Abstract

Our paper sets a human capital accumulation model with endogenous fertility to examine how demand for school education and private tutoring and fertility are determined. Generally, high-income households choose private school education. However, if the relation between public school education and private tutoring is complementary, then low-income households are unable to choose public school education because of the necessary payments for private tutoring. Instead, they choose private school education to reduce education costs. Given a certain condition, the fertility of households that choose private school education is less than the fertility of households that choose public school education.

Keywords: Fertility, Human capital accumulation, Private tutoring, School education
JEL Classifications: I22, H52

[†] The study described in this paper was financially supported by JSPS KAKENHI No. 21K01531. We would like to thank Akira Yakita and seminar participants for the beneficial comments. Any errors are our responsibility.
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1. INTRODUCTION

The aim of this study is to set the human capital accumulation model with endogenous fertility and to examine how demand for school education, private tutoring, and fertility are determined. As in the models reported by Cardak (1992) and Glomm (1997), we specifically consider school choice between public school education and private school education. The public school education is financed by taxation. Then, a household need not pay for public school education costs. However, because public school education is distributed equally among children, the parents who want to pay for a high level of education investment choose private school education. As shown by Glomm and Ravikumar (1992), Cardak (2004), and others, the human capital growth rate of private school education is greater than that of public school education. The inequality of human capital of children decreases because of equally provided public school education investment. However, as demonstrated by Glomm and Ravikumar (2003), public school education does not always decrease the inequality of the human capital stock of children. In fact, inequality is increased in the short run.

In addition to school education, our model considers private tutoring. Bearse, Glomm and Patterson (2005) consider school education and private tutoring and derive demand for school education and private tutoring; they thereby obtain the median voting equilibrium. The model presented by Bearse, Glomm and Patterson (2005), a static model, does not consider the dynamics of equilibrium. By contrast, our manuscript sets the model based on Gamlath and Lahiri (2018) and derives the dynamics of equilibrium. This paper presents consideration of education investment decided by parents. Based on empirical results derived by Houtenville and Conway (2008), this setting is justified.

The results obtained from our study are presented as described hereinafter. First, if the relation between school education and private tutoring is substitutive, then the households which have low income choose public school education to avoid school education costs. The fertility of the households which choose private school education is constant over time. However, the fertility of households which choose public school education depends on the household income and the investment level of public school education.

Second, in the case of complementarity between public school education and private tutoring, the low-income households choose private school education to avoid payments for private tutoring. Then, middle-income households can choose public school education. Given a certain condition, the fertility of lower income and higher income households are less than the fertility of middle-income households.

The remainder of our paper is presented as described hereinafter. Section 2 sets the model with education choice. Section 3 derives the equilibrium of the model. The final section concludes our paper.

2. THE MODEL

Households in t period pay for child care, education investment for children, and consumption. There exists education of two types: one for school education and the other for private tutoring. With school education, there exist public school education and private school education. We assume inequality of the human capital stock between households. Defining $h_{i,t}$ as the human capital stock ith households in t period, then the human capital stock $h_{i,t}$ is assumed to be distributed uniformly in $\left[\underline{h}_t, \overline{h}_t\right]$

2.1 Public education

The utility function of households is assumed as

$$u_t = \alpha ln n_t h_{i,t+1} + (1-\alpha) ln c_t, 0 < \alpha < 1. \quad (1)$$

In that equation, n_t, $h_{i,t+1}$, and c_t respectively denote the number of children, the human capital stock of children of ith households, and consumption.

Human capital accumulation function is assumed as

$$h_{i,t+1} = A\left(e_{G,t}^{\beta}\left(e_{s,t} + be_{G,t}\right)^{1-\beta}\right)^{\delta} h_{i,t}^{1-\delta}, 0 < \beta, \delta < 1, 0 < A. \quad (2)$$

Therein, $e_{G,t}$ and $e_{s,t}$ respectively denote the investment level of public school education and the investment level of private tutoring. b denotes the parameter of substitution or compementary between school education and private tutoting. the This human capital accumulation form is assumed by Gamrath and Lahiri (2018).

The household budget constraint is shown as the equation below.

$$(1-\tau)(1-\phi n_t)h_{i,t} = c_t + e_{s,t} n_t \quad (3)$$

In that equation, ϕ time is needed for child care ($0 < \phi < 1$). Then, the labor time of household is given as $1 - \phi n_t$. This setting is normal in the endogenous fertility model (Galor and Weil (1996), and others). Also, τ denotes the income tax rate to finance for the public school education ($0 < \tau < 1$). Therefore, the household disposable income is given as $(1-\tau)(1-\phi n_t)h_{i,t}$. Finally, $h_{i,t}$ represents the household human capital stock, which is regarded as the labor income.

We consider the maximization of utility function (1) subject to constraints (2) and (3). Then, we can obtain the following household optimal allocations in the case of $(1-\beta)\delta(1-\tau)\phi h_{i,t} > be_{G,t}$.

$$e_{s,t} = \frac{(1-\beta)\delta(1-\tau)\phi h_{i,t} - be_{G,t}}{1-(1-\beta)\delta} \quad (4)$$

$$n_t = \frac{\alpha(1-(1-\beta)\delta)(1-\tau)h_{i,t}}{(1-\tau)\phi h_{i,t} - be_{G,t}} \quad (5)$$

$$c_t = (1-\alpha)(1-\tau)h_{i,t} \quad (6)$$

Because of $n_t \geq 0$, $(1-\tau)\phi h_{i,t} - be_{G,t} > 0$ must hold. With $b > 0$, an increase in h_t reduces fertility n_t. However, with $b < 0$, an increase in h_t raises fertility n_t. If $(1-\beta)\delta(1-\tau)\phi h_{i,t} < be_{G,t}$, then the household optimal allocations are shown as the following.

$$e_{s,t} = 0 \tag{6}$$

$$n_t = \frac{\alpha}{\phi} \tag{7}$$

Therefore, by defining $\acute{h}_t$ to hold $(1-\beta)\delta(1-\tau)\phi h_t = be_{G,t}$, households of $h_{i,t} < \acute{h}_t$ choose no private tutoring and choose fertility (7). Otherwise, households of $h_{i,t} > \acute{h}_t$ choose positive demand for private tutoring and fertility (5). As shown by the following figure, there exist two types of households that choose public education.

[Insert Fig.1 around here.]

2.2 Private education

This subsection presents consideration of the case of private school education. In the case of public school education, the school education cost is financed by taxation. By contrast, in the case of private school education, the school education cost should be paid by the households.

The human capital accumulation function is assumed to be the following.[1]

$$h_{i,t+1} = Ae_{P.t}^{\delta} h_{i,t}^{1-\delta} \tag{8}$$

Therein, $e_{P,t}$ denotes the investment level of private school education.

The household budget constraint is shown as the following.

$$(1-\tau)(1-\phi n_t)h_{i,t} = c_t + e_{P,t}n_t \tag{9}$$

We consider the maximization of the utility (1) subject to constraints (8) and (9). Then, we can obtain the following household optimal allocations.

$$e_{P,t} = \frac{\delta(1-\tau)\phi h_{i,t}}{1-\delta} \tag{10}$$

$$n_t = \frac{\alpha(1-\delta)}{\phi} \tag{11}$$

$$c_t = (1-\alpha)(1-\tau)h_{i,t} \tag{12}$$

Fertility n_t is independent of human capital and is constant over time.

[1] As shown by Bearse et al. (2005), human capital accumulation is produced by school education and private tutoring in public school education. However, in private school, human capital accumulation is produced only by private education (including school education and private tutoring).

3 SCHOOL CHOICE

In this section, we consider the school choice of a household. Households choose school education to consider utility maximization. We consider two cases: $b \geq 0$ and $b < 0$.

3.1 Case of $b \geq 0$

We consider indirect utility functions of two types. We define V_t^{pub} and V_t^{pri} respectively as the indirect utility function by which households choose public school education with private tutoring and the indirect utility function by which households choose private school education.

In the case of $b \geq 0$, which is the substitution case between public school and private tutoring, households of two types choose public schools: households that demand private tutoring and the households that do not demand private tutoring.

Actually, V_t^{pub} is shown by the following:

$$\begin{aligned} V_t^{pub} = {} & \alpha ln \frac{\alpha(1-(1-\beta)\delta)(1-\tau)h_{i,t}}{(1-\tau)\phi h_{i,t} - be_{G,t}} + \alpha lnA + \alpha\beta\delta lne_G \\ & + \alpha(1-\beta)\delta ln \frac{(1-\beta)\delta(1-\tau)\phi h_{i,t} - (1-\beta)\delta be_{G,t}}{1-(1-\beta)\delta} \\ & + \alpha(1-\delta)lnh_{i,t} + (1-\alpha)ln(1-\alpha)(1-\tau)h_{i,t}. \end{aligned} \tag{13}$$

Also, V_t^{pri} is shown by the following

$$\begin{aligned} V_t^{pri} = {} & \alpha ln \frac{\alpha(1-\delta)}{\phi} + \alpha lnA + \alpha\delta ln \frac{\delta(1-\tau)\phi h_{i,t}}{1-\delta} + \alpha(1-\delta)lnh_{i,t} \\ & + (1-\alpha)ln(1-\alpha)(1-\tau)h_{i,t}. \end{aligned} \tag{14}$$

Although V_t^{pri} increases with an increase in $h_{i,t}$, V_t^{pub} is ambiguous. By calculating $\frac{dV_t^{pub}}{dh_{i,t}}$, one can obtain

$$\frac{dV_t^{pub}}{dh_{i,t}} = \frac{1+\alpha(1-\delta)}{h_{i,t}} - \frac{\alpha(1-\tau)\phi}{(1-\tau)\phi h_{i,t} - be_{G,t}} + \frac{\alpha(1-\beta)\delta(1-\tau)\phi}{(1-\tau)\phi h_{i,t} - be_{G,t}}. \tag{15}$$

By calculating $\frac{dV_t^{pri}}{dh_{i,t}}$, we can obtain the equation presented below:

$$\frac{dV_t^{pri}}{dh_{i,t}} = \frac{1}{h_{i,t}}. \tag{16}$$

Therefore, if $\frac{dV_t^{pri}}{dh_{i,t}} - \frac{dV_t^{pub}}{dh_{i,t}} > 0$ for any $h_{i,t}$, the households of low human capital stock $h_{i,t}$ choose public school education and the households of high human capital stock $h_{i,t}$ choose private school education, as depicted in Fig. 2.

[Insert Fig.2 around here.]

We assume the parameter condition such that $\frac{dV_t^{pri}}{dh_{i,t}} - \frac{dV_t^{pub}}{dh_{i,t}} > 0$ holds.[2] We define $\hat{h}_t$ such that human capital stock h_t of V_t^{pub} and V_t^{pri}. Then, on the one hand, households for which the human capital stock is $[\underline{h}_t, \hat{h}_t]$ choose public school education. On the other hand, households for which the human capital is $[\hat{h}_t, \overline{h}_t]$ choose private school education. Moreover, there exist households of two types that choose public school education. On the one hand, households of $[\underline{h}_t, \acute{h}_t]$ do not pay for private tutoring. On the other hand, households of $[\acute{h}_t, \hat{h}_t]$ pay for private tutoring. Fig.3 shows the two types of households: one for the household thac chooses public education and the other for the household that chooses private education.

[Insert Fig.3 around here.]

The government budget constraint is shown as

$$\tau \int_{\underline{h}}^{\overline{h}} f(h_{i,t}) h_{i,t} dh_t = e_{G,t} F(\hat{h}_t). \tag{17}$$

In this case, we consider households of three types.

Type I: Households that choose public school education without private tutoring
Type II: Households that choose public school education with private tutoring
Type III: Households that choose private school education

The human capital accumulation of Type I, Type II and Type III household are given by the follows,

$$h_{i,t+1} = A e_{G,t}^{\delta} h_{i,t}^{1-\delta}, \quad \text{(Type I)} \tag{18}$$

$$h_{i,t+1} = A\left(e_{G,t}^{\beta}\left(\frac{(1-\beta)\delta(1-\tau)\phi h_{i,t} - (1-\beta)\delta e_{G,t}}{1-(1-\beta)\delta}\right)^{1-\beta}\right)^{\delta} h_{i,t}^{1-\delta}, \text{(Type II)} \tag{19}$$

$$\frac{h_{i,t+1}}{h_{i,t}} = A\left(\frac{\delta(1-\tau)\phi}{1-\delta}\right)^{\delta}, \text{(Type III)}. \tag{20}$$

$e_{G,t}$ is given by (17) and the fertility of each type household are given by (7) (Type I), (5) (TypeII) and (11) (Type III), respectively.

We assume that $\underline{h}_t$ is enough small and $\overline{h}_t$ is enough large and then the following proposition

[2] In the related literature, the household with low human capital stock chooses the public school education. On the other hand, the household with high human capital stock chooses the private school education. This assumption is consistent with the setting of the related literatures.

can be established.[3]

Proposition 1

We assume the case of $b \geq 0$. Then, households of $[\underline{h}_t, \acute{h}_t]$ choose public school education without private tutoring (Type I). Households of $[\acute{h}_t, \hat{h}_t]$ choose public school education with private tutoring (type II). Households of $[\hat{h}_t, \overline{h}_t]$ choose private school education (Type III), as shown by Fig.4.

[Insert Fig.4 around here.]

We examine the fertility of each type of household. The fertility of Type I is greater than the fertility of Type III. However, it remains ambiguous whether fertility of Type II is less than the fertility of Type III or Type I. In the case of $b > 0$, an increase in human capital $h_{i,t}$ reduces fertility. Therefore, if the fertility at the human capital $\hat{h}_t$ is greater than the fertility of Type III (11), then the fertility of the households which choose public school education is always greater than that of households which choose private school education. The condition can be shown as

$$\frac{e_{G,t}}{\hat{h}_t} > -\frac{(1-\tau)\phi\beta\delta}{(1-\delta)b}. \tag{21}$$

Because the inequality (21) is always held, then the fertility in the case of public school education is always greater than the fertility in the case of private school education for human capital stock $[\acute{h}_t, \hat{h}_t]$. Then, the following proposition can be established.

Proposition 2

With $b > 0$, the fertility in the case of public school education is greater than the fertility in the case of private school education.

Now, we consider proposition 1. In proposition 1, $\acute{h}_t < \hat{h}_t$ is held.[4] As shown by the footnote 4, if the lebel of e_G is large, we can establish the following proposition.

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[3] Otherwise, there exists the equilibrium without Type I or Type II or Type III.

[4] With $\acute{h}_t$, the inditect utility V_t^{pub} is given by $V_t^{pub} = \alpha ln\frac{\alpha}{\phi} + \alpha lnA + \alpha\beta\delta lne_G + \alpha(1-\delta)ln\acute{h}_t + (1-\alpha)ln(1-\alpha)(1-\tau)\acute{h}_t$. The indirect utility V_t^{pri} is given by $V_t^{pri} = \alpha ln\frac{\alpha(1-\delta)}{\phi} + \alpha lnA + \alpha\delta ln\frac{\delta(1-\tau)\phi}{1-\delta}\acute{h}_t + \alpha(1-\delta)ln\acute{h}_t + (1-\alpha)ln(1-\alpha)(1-\tau)\acute{h}_t$. Therefore, depending on the lebel of e_G, we obtain the case $\acute{h}_t > \hat{h}_t$.

Proposition 3

We assume the case of $b \geq 0$ and large level of e_G. Then, there exist two types of households. households of $[\underline{h}_t, \hat{h}_t]$ choose public school education without private tutoring (Type I). Households of $[\hat{h}_t, \overline{h}_t]$ choose private school education (Type III), as shown by Fig.5.

[Insert Fig.5 around here.]

3.2 Case of $b < 0$

We notify the case of $b < 0$ because $e_{s,t} + be_{G,t}$ must not be negative. Especially, in the case of public school education, there exists the difficulty of the negative sign of $e_{s,t} + be_{G,t}$. To avoid the negative sign of $e_{s,t} + be_{G,t}$, households for which the human capital stock h_t is low if $\underline{h}_t$ is too low, they can not choose public school education because of the negative sign of $e_{s,t} + be_{G,t}$. Then, they choose private school education and pay for a low level of private school education.

Then, the households of $[\underline{h}_t, \acute{h}_t]$ choose private school education. Households for which $[\acute{h}_t, \hat{h}_t]$ choose public school education with private tutoring. Households for which $[\hat{h}_t, \overline{h}_t]$ choose private school: there exist households of two types.

Type II: Households that choose public school education with private tutoring
Type III: Households that choose private school education

Then, the following proposition can be established.

Proposition 4

There exist households of two types in the case of $b < 0$. The households of $[\underline{h}_t, \acute{h}_t]$ choose the private school education (Type III). The households of $[\acute{h}_t, \hat{h}_t]$ choose public school education with private tutoring (Type II). The households of $[\hat{h}_t, \overline{h}_t]$ choose private school education (Type III) as shown by Fig.6.

[Insert Fig.6 around here.]

The fertility of households that choose public school education with private tutoring increases with an increase in the human capital stock $h_{i,t}$. Compared (5) with (11), if the following inequality (21) holds, then the fertility of the households which choose the public school education is greater than the fertility of the households which choose private tutoring. In the case of $b < 0$, if $\hat{h}_t <$

$-\frac{(1-\delta)be_{G,t}}{(1-\tau)\phi\beta\delta}$, then the fertility in the case of private school education is greater than the fertility in the case of public school education. Then, the following proposition can be established.

Proposition 5

With $b < 0$ and $\hat{h}_t < -\frac{(1-\delta)be_{G,t}}{(1-\tau)\phi\beta\delta}$, the fertility in the case of public school education is less than the fertility in the case of private school education.

This proposition shows that the fertility of households which choose public school education is less than the fertility of households which choose private school education. The reason can be explained as follows. Because of the complementarity between public school education and private tutoring, the households need to pay for private tutoring more than in the case of substitution. If b is smaller, the range of human capital stock to hold $\hat{h}_t < -\frac{(1-\delta)be_{G,t}}{(1-\tau)\phi\beta\delta}$ increases.

As shown by footnote 4, we can obtain the case of $\acute{h}_t > \hat{h}_t$. Then, the following proposition can be established.

Proposition 6

There is no type II household. Then, every household choose the private school education.

4. CONCLUSIONS

Our study sets a human capital accumulation model with endogenous fertility and subsequently examines how demand for school education investment, private tutoring, and fertility are determined. The results obtained from this study are the following. In the households which choose private school education, fertility is constant over time and is independent of the human capital stock. However, in households which choose public school education, fertility depends on human capital accumulation and the investment in public school education. Depending on the parameters that show the substitutability and complementarity, the fertility increases or decreases with the change of the human capital stock and the level of investment in public school education.

In addition, if the relation between school education and the private tutoring is complementary, then the households for which the human capital stock is too low can not choose public school education. Furthermore, these households pay for a small amount of private school education. Given certain conditions, middle-income households choose to use public school education. Then, low-income households and high-income households choose private school education.

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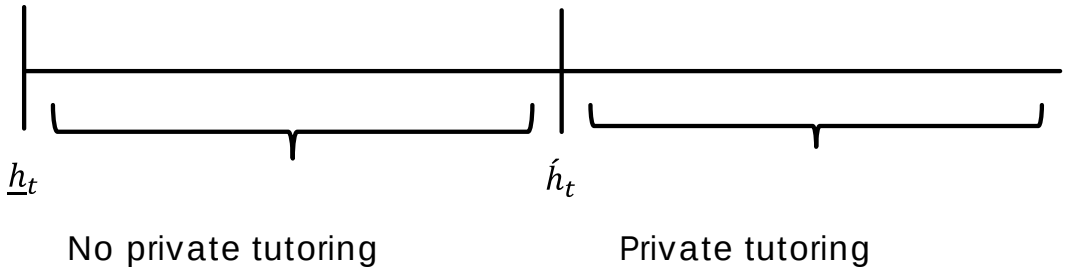

Fig.1: Two types of public education

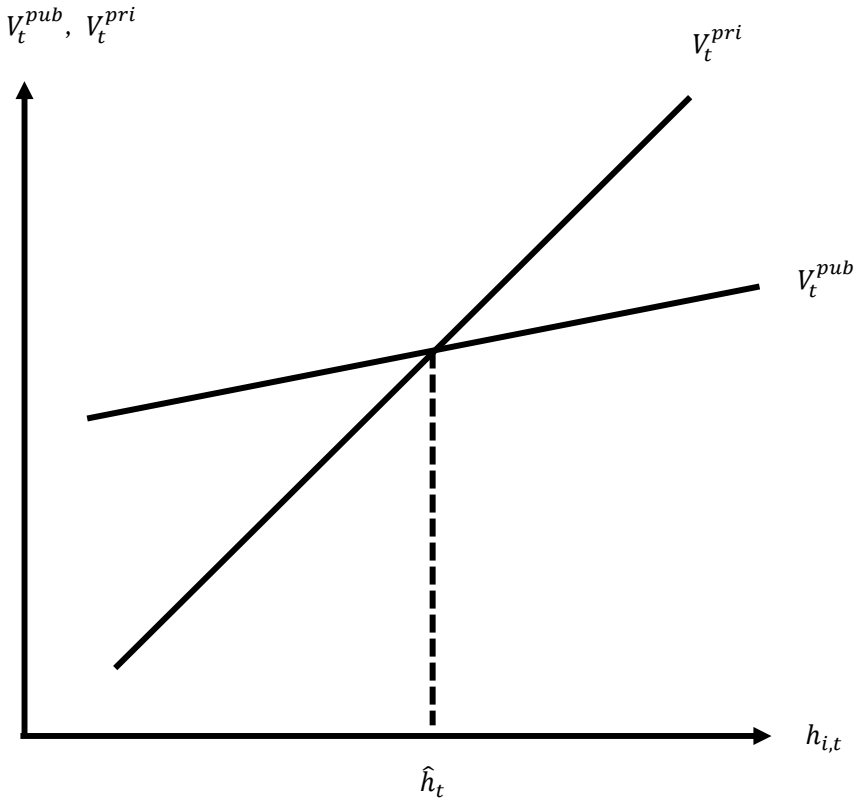

Fig. 2: School choice.

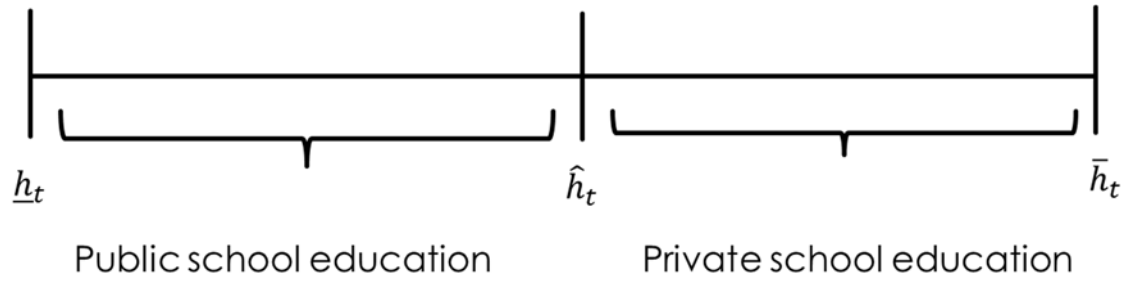

Fig.3 School choice between public school education and private school education

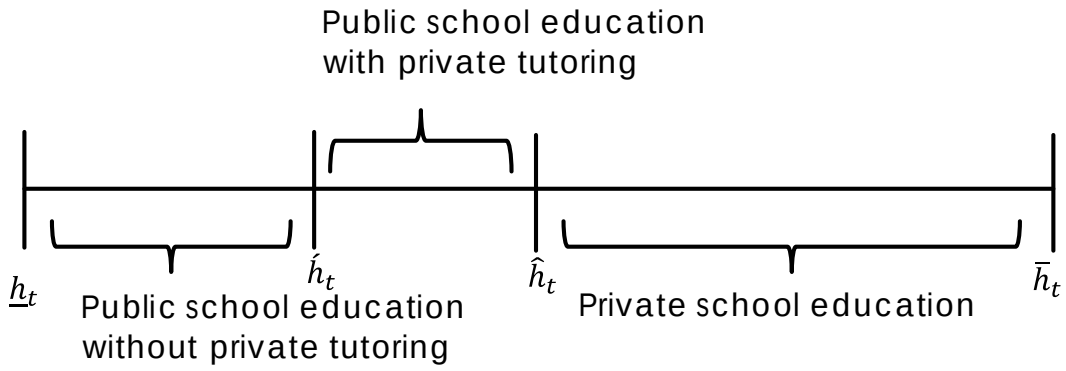

Fig.4: Three types of households

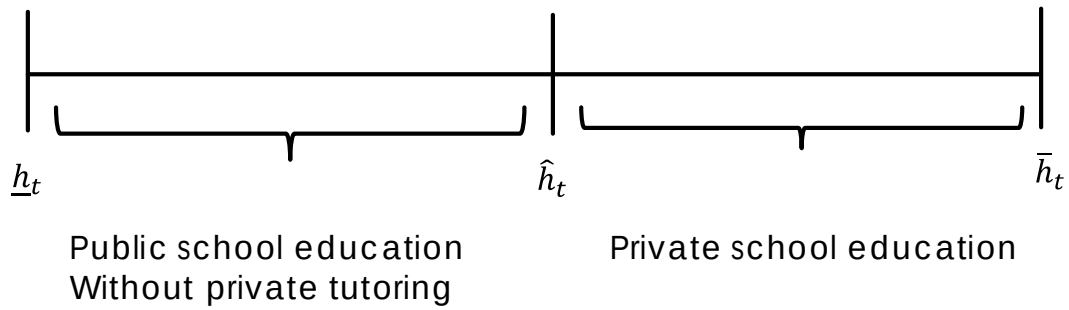

Fig.5: Two types of households

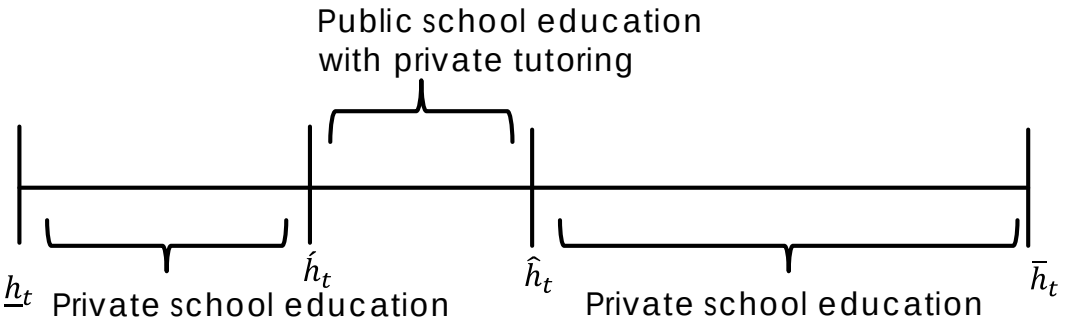

Fig.6: Three types of household