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Price and quality decision of a monopoly platform for transaction with shipping *

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Abstract

This paper theoretically examines pricing and quality decisions of a monopoly platform facilitating transactions that involve physical shipping. In our model, the platform provides two types of transaction services (a standard service and a "premium" service with high-quality delivery of a transacted item) and decides a membership fee, transaction fees, and the quality of the premium service. We conduct comparative statics with respect to shipping costs. It is shown that when shipping costs are increased, the directions of changes in the platform's decision variables are ambiguous, depending on the nature of the increased shipping costs. For example, an increase in shipping costs may *increase* the quality and *decrease* the membership fee.

Keywords: Platform monopoly; Menu-pricing; Quality decisions; Two-sided market.
JEL Classification Codes: D21, D43, L13, L15

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1 Introduction

In 2022, Amazon raised its "Prime" membership fees in European countries and the United States (*the Nikkei, July 26, 2022*).[1] The rate of increase is substantial, ranging from 20% to 43%. The article states that this rise of fees may be attributable to the increases in logistics costs. However, most part of those costs, including shipping costs, would be rather variable than fixed. Accordingly, it seems natural that those costs are passed through to fees per transaction rather than membership fees, which are annual and thus fixed regardless of the number of transactions. Indeed, the aforementioned article also reports that the membership fee is left unchanged in Japan, though logistics costs are rising there too. Our question is whether and how, if any, variable shipping costs are passed through to the membership fee in this market.

To answer this question, we develop a theoretical model of a monopoly platform that facilitates transactions accompanying physical shipping. We consider how shipping costs affect the prices and quality that the platform chooses. To address the question, the theoretical model has the following features with Amazon in mind. First, the model explicitly incorporates both a membership fee and transaction fees. To distinguishes them, buyers are divided into two statuses: existing members and nonmembers. While buyers of both statuses must pay the transaction fees, only nonmembers have to pay the membership fee. Second, the platform provides two kinds of transaction services: a "spot" service and a "premium" service with a higher-quality delivery, such as a fast delivery and delivery at the designated time. Nonmembers cannot use the premium transaction service unless they pay the membership fees. Third, the quality of the premium transaction is determined by the platform.

[1] https://www.nikkei.com/article/DGXZQOGN264TF0W2A720C2000000/ (in Japanese, accessed July 5, 2023)

We show that a change in shipping costs can affect the membership fee through the change in the quality. In addition, the direction of the change in the membership fee is ambiguous, depending on the nature of increased costs. If shipping costs regarding external careers (e.g., FedEx), which are borne by sellers when they use the spot transaction service, are increased, the membership fee is increased, while the transaction fees are decreased. In contrast, if the increased shipping costs are associated with the premium transactions and are levied on the platform, the membership fee is decreased, while a transaction fee is increased. In addition, if costs for increasing the quality of delivery are increased, both the membership and transaction fees are decreased.

This study builds on a large amount of existing works in the literature on multi-sided markets.[2] The purpose of this study is not to develop a general model of platforms, but rather to construct and analyze a theoretical model to describe a marketplace platform that facilitates transactions associated with physical shipping. Wang and Wright (2017, 2018) are closely related to this study. They study the pricing of a platform for transactions of different kinds of items, including online marketplaces (such as Amazon) and payment card networks (such as Visa). They show that, in their setting, ad valorem transaction fees achieve the same level of profit that could be obtained under third-degree price discrimination.

This study is complementary to theirs and has the following three features. First, our model is more specific to an online marketplace for transactions with physical shipping

[2]For example, Rochet and Tirole (2003,2006), Caillaud and Jullien (2003), Armstrong (2006), and Bellflamme and Peitz (2018, 2019a, 2019b, 2021, 2023). Rochet and Tirole (2006) present a rough definition of two-sided (more generally, multi-sided) markets 'as markets in which one or several platforms enable interactions between end-users and try to get two (or multiple) sides "on board" by appropriately charging each side.' They state that the theory of two-sided markets is related to the theories of network externalities and of (market or regulated) multiproduct pricing. Thus, the multiproduct pricing literature does not consider externalities in the consumption of assorted products or services. This study is in the flow of the latter multiproduct pricing literature.

and thus explicitly includes shipping costs. We then conduct comparative statics on these costs. Second, the platform in our model decides not only on fees but also on the quality of the transaction service. Third, our model describes second-degree price discrimination for buyers with different valuations for an item.

Recently, there has been a growing body of research on platform price discrimination. There are studies that examine second-degree price discrimination by platforms (e.g., Böhme, 2016; Jeon et al., 2022; Belleflamme and Peitz, 2023), as in our study.[3] This study examines how a platform's price and quality decision is affected by a change in costs per transaction, which are shipping costs in our setting. Since this study focuses on a platform for transactions with shipping (such as Amazon), such costs per transaction are not negligible. In contrast, previous studies are interested in more general settings of a platform and thus do not focus on per-transaction costs, which are negligible in many cases of platforms (e.g., payment cards and application stores).

2 The Model

This section introduces the model. Subsection 2.1 provides the overview. Subsections 2.2, 2.3, and 2.4 explains in detail the modeling of buyers, sellers, and a platform, respectively.

2.1 Overview of the model

We consider a platform that facilitates transactions between sellers and buyers through intermediation on the website. Figure 1 shows the framework of the model. All sellers provide homogenous items with the same marginal cost. Buyers are heterogeneous with

[3]For a literature review on platform price discrimination, including first- and third-degree price discrimination, see, for example, Corniére et al. (2023).

respect to their valuation of the item. Modeling regarding sellers and buyers is based on the model of Wang and Wrignt (2017), while we extend it to incorporate two types of transaction services the platform provides.

The platform provides two types of transaction services: "premium" and "spot." The premium transaction provides a high-quality delivery of an item, such as a fast delivery and delivery at the designated time. The premium transaction can be used only by the "member" buyers who have paid a membership fee.[4] The spot transaction with a standard-quality delivery can be used by any buyer with no payment to the platform.

Buyers are divided by their membership status. An "existing member" (hereafter EM) has already paid the membership fee and holds the membership status. EMs can use both the premium and spot transactions without payment to the platform. A "non-member" (hereafter NM) does not hold the membership status. While NMs can use spot transaction freely, they should become new members by paying the membership fee to use the premium service. EMs are heterogeneous in their valuation of an item, as are NMs.

The platform decides on the membership fee (A), the quality of the premium service (X), and charges levied on sellers (T^P for a premium transaction and T^S for a spot transaction).[5] The service quality of the spot transaction is standardized to one. We focus on a one-shot profit maximization of the platform and conduct comparative statics

[4] In the real economy, the platform differentiates between the buying customers (consumers) who purchase various goods on the platform. For example, if a consumer is an Amazon Prime member (annual membership fee 4900 yen), the "Amazon Prime Eligible Item" is displayed on the site. The item is then shipped to the purchaser by the next day (through one-day delivery). However, if the buyer is not an Amazon Prime member, the "Amazon Prime Eligible Item" is not displayed. The buyer can view and order only the items that can be purchased as a spot transaction. The items are not shipped through one-day delivery.

[5] According to the charge system of Amazon, we assume that transaction charges are levied on sellers. The results described in the subsequent sections does not change when charges are assumed to be levied on buyers.

for the equilibrium.

2.2 Buyer side

This subsection explains the buyer side. Because we consider a platform problem to set not only the charges per transaction (T^P and T^S) but also the membership fee (A), we assume that there are two groups of buyers with different membership statuses: EMs (existing members) and NMs (nonmembers). A buyer decides whether to buy one unit of the good. The choice set is purchasing it by using the premium transaction, purchasing it by using the spot transaction, and not buying it on the platform (including purchasing it through another channel). Note that if NMs choose the premium transaction, they should become a new member by paying the membership fee. The EMs can use the premium transaction with no payment to the platform. This study analyzes a static model, which can be interpreted as a steady state of a dynamic situation discussed in Appendix B.

The utility of buyers is modeled following Wang and Wright (2017). Because Wang and Wright (2017) consider only one type of transaction service a platform provides, we extend the model to incorporate the two types of transaction services into the model. The utility from not buying is standardized to zero.

The net utility that a buyer purchases an item with the spot transaction is assumed to be as follows:

$$u^S = c(1+b) - p^S, \tag{1}$$

where p^S is the price that an buyer pays to an seller for an item with the spot transaction. Following Wang and Wright (2017), the valuation of an item is assumed to be proportionate to the marginal cost (c).[6] Buyers' heterogeneity in the valuation of an

[6]As discussed in Wang and Wright (2017), the assumption that buyers' values for a good can be

item is represented by b. Its cumulative distribution function is denoted by $F(b)$ with the support $[0, \bar{b}]$. Only the buyers know their own value b, and F is known to the public.

The valuation of getting an item is higher when the premium transaction is used than when the spot transaction is used, because the former is associated with a higher-quality delivery than the latter. Specifically, the net utility for the premium transaction is assumed to be as follows:

$$u^P = c(1 + b(1 + X)) - p^P, \tag{2}$$

where p^P is the price in the case of the premium transaction and X is the quality of the premium transaction (the quality of the spot is standardized to zero). This functional form indicates that the value of a higher quality is not uniform across buyers: it is evaluated more by a buyer who feels a larger value for an item (i.e., a larger b) than by a buyer with a smaller value. Because the quality of the delivery of the premium transaction is linked to how fast a buyer can obtain the item transacted, the quality is supposed to be important for buyers who highly appreciate the item.

Note that u^S and u^P are common across EMs and NMs. The only difference between EM and NM buyers is whether a buyer has paid the membership fee or not. In turn, we consider the decision problems of EMs and NMs.

EMs can use the premium transaction without a payment of the membership fee because they are assumed to have already paid it. An EM chooses one option that gives the highest utility among three options: the premium transaction (u^P), the spot transaction (u^S), and not to buy (0).

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scaled by c is plausible, and supported by previous empirical studies, for example, Einav et al. (2015). By quasiexperimentally observing a large number of auctions for various goods across the internet, they found that the distribution of buyer valuation for a good is proportional to the transaction price.

Figure 2 shows how the choices of EMs are divided into three according to their evaluation for an item (b). The solid line represents the utility from the spot transaction (u^S). The dashed line is that from the premium transaction (u^P). While both increase in b, the slope is larger for u^P than for u^S. This is because for the premium transaction, the utility from the service quality, as well as that from an item itself, increases as b increases. Figure 2 shows that the option chosen by an EM is changed according to b. A buyer chooses not to buy if b is smaller than $\tilde{b}$, where $\tilde{b}$ is the value of b such that $u^S = 0$ and $\tilde{b} = \frac{p^S - c}{c}$. A buyer uses the spot transaction if b is in the medium range of $[\tilde{b}, \hat{b}^E]$, where $\hat{b}^E$ is the value of b such that $u^S = u^P$ and $\hat{b}^E = \frac{p^P - p^S}{cX}$. If a buyer has b larger than $\hat{b}^E$, the premium transaction is chosen.

We turn to the choice problem of NMs. While NMs can use the spot transaction freely, they should pay the membership fee to the platform if they use the premium service. If they pay the fee, they become a member and assumed to have another purchase opportunity of an item in the future (in addition to the present opportunity).[7] NMs are therefore assumed to decide whether they become a member or not based on the total utility from the present and next purchase opportunities. To simplify the model, the discount rate is assumed to be zero. We assume that NMs believe that the prices and quantity in the next opportunity are the same as in the present opportunity. This assumption means that the option chosen in the next opportunity becomes the same as in the present. NMs therefore compare the utilities obtained from paying the membership fee and purchasing an item with the premium service in the present and next opportunities ($2u^P - A$), purchasing it with the spot service in the both ($2u^S$), and not buying it at all (0).

Figure 3 depicts how the option chosen by an NM is changed according to b. The solid

[7]The dynamic situation implicitly assumed behind this setting is explained in detail in Appendix B.

line represents the total utility of the two purchase opportunities for the spot transaction ($2u^S$). The dashed line is the total utility net the payment of the membership fee ($2u^P - A$). As in Figure 2, the slope of the dashed line is larger than that of the solid line. Figure 3 shows that the chosen option depends on b. An NM purchases no item if b is less than $\tilde{b}$, where $\tilde{b}$ is the value of b such that $2u^S = 0$ and $\tilde{b} = \frac{p^S - c}{c}$. This threshold is the same as in Figure 2, which is defined such that $u^S = 0$. An NM chooses the spot transaction if b is in the range of $[\tilde{b}, \hat{b}^N]$, where $\hat{b}^N = \frac{p^P - p^S + \frac{A}{2}}{cX}$ is the value of b such that $2u^S = 2u^P - A$. The premium transaction is chosen by NMs with b larger than $\hat{b}^N$.

In this study, we concentrate our analysis on the situation where both EMs and NMs are divided into three groups: choosing the premium transaction, choosing the spot transaction, and not buying. The situation can be depicted in Figures c and d.[8] We therefore assume that parameters are in the range such as that in equilibrium, p^S, p^P, A, and X satisfy 1) $0 < \tilde{b} < \hat{b}^E$ and 2) $\hat{b}^N < \bar{b}$.[9] Note that $\hat{b}^E < \hat{b}^N$ by definition. Intuitively, NMs are unlikely to choose the premium service compared to EMs due to the membership fee.

For each status of buyers (i.e., EM and NM), there is a continuum of potential buyers with mass normalized to one. The distribution of the evaluations for an item (b) is assumed to be same for each status.[10] We assume a uniform distribution of b, the distribution function of which is

$$F(b) = \lambda b. \tag{3}$$

[8] In the real world, Amazon Prime members can purchase not only the Amazon Prime Eligible Items but also other items.

[9] We can confirm that these assumptions are satisfied in the equilibrium derived in the next.

[10] An interpretation of this assumption is that the EMs who has become a member to purchase another item (c') in the previous period (see Appendix B and Figure A1) tend to have lower b. Another interpretation is that although NMs naively believe that their valuation of an item is never changed, their valuation is newly drawn in the subsequent period.

The support of the distribution is $[0, \frac{1}{\lambda}]$, that is, $\bar{b} = \frac{1}{\lambda}$.

We can derive the demand quantities as follows. For the EMs, demand for the spot and premium transactions is

$$q^{E,S} = F(\hat{b}^E) - F(\tilde{b}) = \frac{\lambda}{cX}\left(p^P - (1+X)p^S\right) + \lambda \tag{4}$$

and

$$q^{E,P} = 1 - F(\hat{b}^E) = \frac{\lambda}{cX}\left(-p^P + p^S\right) + 1, \tag{5}$$

respectively. For the NMs, demand for the spot and premium transactions is

$$q^{N,S} = F(\hat{b}^N) - F(\tilde{b}) = \frac{\lambda}{cX}\left(p^P + \frac{A}{2} - (1+X)p^S\right) + \lambda \tag{6}$$

and

$$q^{N,P} = 1 - F(\hat{b}^N) = \frac{\lambda}{cX}\left(-p^P - \frac{A}{2} + p^S\right) + 1, \tag{7}$$

respectively.

2.3 Seller side

This subsection describes the seller side. Following Wang and Wright (2017), we assume that homogeneous sellers, which have a common per-unit cost (c), engage in Bertrand competition. Although we extend the model to incorporate the two types of transaction services, sellers compete in price in each transaction market.

We also extend the model of Wang and Wright (2017) to explicitly model the costs of delivery of an item. A distinguishing feature of the platform service for a transaction between a buyer and a seller, like the services provided by Amazon, is that it necessarily entails a physical delivery. An aim of this study is to provide comparative statics regarding the costs of delivery. We assume that while sellers in the spot transactions deliver an item by using an external career (e.g., FedEx), sellers in the premium transaction leave deliveries to the platform. The platform is assumed to involve delivery for the premium transaction to guarantee the high-quality delivery service.[11]

Specifically, the costs for sellers are as follows. For a premium transaction, the seller bears the marginal costs (c) and the transaction charge paid to the platform (T^P). Because the delivery is handled by the platform for the premium transaction, the delivery costs are levied not on sellers but on the platform. Note, however, that the delivery costs may be passed through to T^P. For a spot transaction, the seller bears the delivery costs (f) in addition to the marginal costs (c) and the transaction charge (T^S).

Due to the assumption of Bertrand competition, the price that a buyer pays to a seller in the equilibrium should be

$$p^P = c + T^P \tag{8}$$

for the premium transaction and

$$p^S = c + T^S + f \tag{9}$$

for the spot transaction.

[11]Indeed, in the real economy, most transactions of the Amazon Prime Eligible Item are supplied by the sellers using the Fulfillment-by-Amazon service for sellers, in which sellers collectively leave Amazon to inventory control and delivery. In contrast, spot transactions are shipped by external carriers in most cases.

2.4 Platform

The decision variables of the platform for maximizing its profit are the charges per transaction (T^P and T^S), the membership fee (A), and the quality of the premium transaction service (X). We focus on a static decision-making of the platform.[12]

The platform bears costs per transaction. For the spot transaction, costs per transaction is denoted as k^S and is likely to be almost zero. For the premium transaction, costs are assumed to be $k^P + \gamma X^2$ and depend on the service quality (X). When the quality is zero (i.e., the same quality as the spot transaction), the costs per premium transaction is k^P. This may be larger than k^S because k^P includes the delivery costs, which the platform bears for the premium transaction. The delivery costs may increase according to the quality of the premium transaction. We assume a quadratic function to represent this effect (γX^2).

The profit maximization problem of the platform is as follows:

$$\max_{T^S, T^P, A, X} \pi = (T^S - k^S)(q^{E,S} + q^{N,S}) + (T^P - k^P - \gamma X^2)(q^{E,P} + q^{N,P}) + Aq^{N,P}. \quad (10)$$

The variables chosen by the platform is expressed by capital letters. The first term of the profit (π) represents profits from the spot transaction. The second term is those from the premium transaction. The last term stands for the revenues of the membership fee. The solution of this problem is derived in the next section.

3 Derivation of a platform monopoly equilibrium

[12] This static decision-making problem can be interpreted as a problem in a steady state of the dynamic situation that Appendix B explains in detail. A complete analysis of the dynamic problem remains for future research.

Substituting (4), (6), (5) and (7) into the r.h.s. of π, (10) yields:

$$\begin{aligned}
\max_{T^S,T^P,A,X} \pi \; &= \; \lambda \left(T^S - k^S\right) \cdot \frac{4(T^P - (1+X)(T^S+f)) + A}{2Xc} + (T^P - k^P - \gamma X^2) \\
&\times \left(2 - \frac{\lambda}{2Xc}((4(T^P - T^S - f) + A)\right) \\
&+ A\left(1 - \lambda \cdot \frac{2(T^P - T^S - f) + A}{2Xc}\right). \qquad (11)
\end{aligned}$$

The first order conditions of this maximization problem are

$$\begin{aligned}
\frac{\partial \pi}{\partial T^S} \; &\equiv \; \pi_S = \frac{\lambda}{2Xc}\left(8T^P - 4k^P + 3A - 4\gamma X^2 + (1+X)(-8T^S - 4f + 4k^S)\right) = 0 \\
&\Leftrightarrow \; 8T^P - 4k^P + 3A - 4\gamma X^2 + (1+X)(-8T^S - 4f + 4k^S) = 0, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi}{\partial T^P} \; &= \; \frac{\lambda(4\gamma X^2 - 3A - 8T^P + 8T^S + 4f + 4k^P - 4k^S) + 4cX}{2Xc} = 0 \\
&\Leftrightarrow \; \lambda(4\gamma X^2 - 3A - 8T^P + 8T^S + 4f + 4k^P - 4k^S) + 4cX = 0, \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi}{\partial A} \; &= \; \frac{\lambda(\gamma X^2 - 2A - 3T^P + 3T^S + 2f + k^P - k^S) + 2cX}{2Xc} = 0 \\
&\Leftrightarrow \; \lambda(\gamma X^2 - 2A - 3T^P + 3T^S + 2f + k^P - k^S) + 2cX = 0, \qquad (14)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \pi}{\partial X} &= \frac{1}{2cX^2}[\begin{pmatrix} 4(T^P)^2-8T^ST^P+(4\gamma X^2+3A-4f-4k^P+4k^S)(T^P-T^S) \\ +4(T^S)^2+A^2+(\gamma X^2-k^P+k^S-2f)A-4f(\gamma X^2-k^P+k^S) \end{pmatrix}\lambda \\
-8c\gamma X^3] &= 0 \\
&\Leftrightarrow \left\{ \begin{array}{c} \gamma(A+4(T^P-T^S-f))X^2+4\left(T^P-T^S\right)^2-(4(k^P-k^S+f)-3A) \\ \times\left(T^P-T^S\right)+A^2-\left(k^P-k^S+2f\right)A+4\left(k^P-k^S\right)f \end{array} \right\}\lambda \\
-8c\gamma X^3 &= 0 \qquad (15)
\end{aligned}$$

Solving (14) with respect to A, we obtain

$$A=\frac{1}{2\lambda}\left(2cX+\lambda(\gamma X^2+3(T^S-T^P)+k^P-k^S+2f)\right) \qquad (16)$$

Substituting (16) into (12), (13) and (15), rearranging them, we obtain

$$5\lambda\gamma X^2+\left(8\lambda(2T^S-(k^S-f))-6c\right)X+\lambda(5(k^P-k^S)+2f-7(T^P-T^S))=0, \quad (17)$$

$$5\lambda\gamma X^2+2cX+\lambda\left(2f+5(k^P-k^S)-7(T^P-T^S)\right)=0, \qquad (18)$$

and

$$
\begin{aligned}
& 3\lambda^2\gamma^2X^4 - 24\lambda c\gamma X^3 - (4c^2 + \lambda^2\gamma(16f - 17(T^P - T^S)) - 2\lambda^3\gamma\left(k^P - k^S + 2f\right))X^2 \\
& +\lambda^3\left(\begin{array}{c} \left(2f + k^P - k^S - 3(T^P - T^S)\right)^2 - 2\left(2f + k^P - k^S\right)\left(2f + k^P - k^S - 3(T^P - T^S)\right) \\ +16f\left(k^P - k^S\right) - 2\left(T^P - T^S\right)^2 - 2\left(T^P - T^S\right)(2f + 5(k^P - k^S)) \end{array}\right) \\
= \; & 0. \qquad (19)
\end{aligned}
$$

From (18), we have

$$
-7\lambda(T^P - T^S) = -5\lambda\gamma X^2 - 2cX - \lambda(5(k^P - k^S) + 2f). \qquad (20)
$$

Substituting (20) into (17) yields

$$
8\left(\lambda(2T^S - (k^S - f)) - c\right)X = 0. \qquad (21)
$$

Thus, we obtain

Solving (20) with respect to $T^P - T^S$, we have

$$
T^P - T^S = \frac{1}{7\lambda}\left(\lambda(5(k^P - k^S) + 2f + 5\gamma X^2) + 2cX\right). \qquad (22)
$$

From (21) and (22), we obtain

$$
T^{S*} = \frac{1}{2}\left(\frac{c}{\lambda} + k^S - f\right). \qquad (23)
$$

and

$$
T^P = \frac{1}{14\lambda}\left(10\lambda\gamma X^2 + 4cX + 7c + \lambda(10k^P - 3k^S - 3f)\right) \qquad (24)
$$

Substituting (22) into (19) and rearranging it, we obtain

$$\frac{8}{7\lambda}\left(-\lambda\gamma X^2 + cX - \lambda(k^P - k^S - f)\right)\left(-3\lambda\gamma X^2 + cX + \lambda(k^P - k^S - f)\right) = 0. \tag{25}$$

Next, we present assumptions for existence of the optimal service level for buyers of the platform.

Assumption 2

$k^P - k^S - f > 0.$

Assumption 3

$c^2 > 4\gamma\lambda^2\left(k^P - k^S - f\right).$

Lemma 1 *Under the assumptions 1, 2 and 3, there exists a unique optimal service level X^* for buyers on the platform. This satisfies the first order condition, (25) and second order condition of the reduced optimization problem (11),*

$$X^* = \frac{1}{6\gamma\lambda}\left(c + \sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)}\right). \tag{26}$$

For the proof of the lemma, see Appendix 1.

Substituting (26) into (22), we obtain $T^{P*} - T^{S*}$. By substituting it and (26) into (16), we obtain A^*. From the expressions $T^{P*} - T^{S*}$ and (23), we can also derive T^{P*}. Hence, we present the following proposition without proof:

Proposition 1 *Suppose that the demand function for a particular product in the market sold on the internet shopping site of the platform monopolist is given by $X_c(p_d) =$*

$1 - F(\frac{p_d}{c}) = 1 - \lambda(\frac{p_d}{c} - 1)$, *where* p_d *is the demand price,* $\lambda > 0$. *Under assumptions 1, 2 and 3, there exists an optimal annual membership fee of* A^* *to be paid by premium buyers, optimal per-transaction services fees* T^{P*} and T^{S*} *to be paid by type* P and S *sellers for providing optimal service level* X^* *to buyers on the platform, as given in Lemma 1. These optimal choices of the platform are*

$$A^* = \frac{1}{63\gamma\lambda^2}\left(4c\sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)} - 48\gamma\lambda^2\left(k^P - k^S - f\right) + 4c^2\right), \tag{27}$$

$$T^{P*} = \frac{1}{126\gamma\lambda^2}\left(11c\sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)} + 57\gamma\lambda^2\left(\frac{40}{19}k^P - k^S - f\right) + 63\gamma\lambda c + 11c^2\right), \tag{28}$$

$$X^* = \frac{1}{6\gamma\lambda}\left(c + \sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)}\right),$$

and

$$T^{S*} = \frac{1}{2}\left(\frac{c}{\lambda} + k^S - f\right).$$

In the proposition, we see that the optimal service fee per transaction intermediation for type S seller on the platform, T^{S*} depends on the service costs k^S and f. The shipping fee that a type S's seller has to pay to the transportation agents is not reflected in the service costs k^P and γ. However, X^*, A^*, T^{P*} depend on all of its services costs, k^S, f, k^P and γ. This illustrates the real pricing behavior of a platform owing to the real economy, such as Amazon, which not only supplies the intermediary services of transactions between sellers and buyers through an internet platform both but also substitutes transportation and inventory service to sellers.

Through tedious and troublesome calculation, we can see that optimal solutions, X^*, A^*, T^{P*} and T^{S*} satisfy the second order condition.

In the next section, we investigate how the unit changes in services costs for both types of sellers and buyers on the platform and the quality costs of the premium services for premium buyers affect the optimal service fees for both types of sellers and buyers and the demand price of the transaction.

4 Comparative Statics with respect to cost parameters

In this section, we explore how the optimal service fees for both types of sellers, T^{P*}, T^{S*}, X^* and A^* change when k^P, f, γ, and λ change.

We assume that the quality of the service for type P sellers is higher than that for type S sellers on a realistic basis. In a real economy, Amazon offers its P type sellers (i.e., for the FBA system users), inventory storage, packing, and shipping services, and commission sales services. However, to type S sellers it only offers the commission sale services. Accordingly, our assumption that $k^P > k^S > 0$, is plausible.

Lemma 2 $\frac{\partial T^P}{\partial X} > 0$, if $c, \gamma, \lambda > 0$ and $X \geq 0$, then $\frac{\partial A}{\partial X} \gtreqless 0 \Leftrightarrow \frac{c}{2\lambda\gamma} \gtreqless X \geq 0$, $\frac{\partial X^*}{\partial k^P} > 0$, and $\frac{\partial X^*}{\partial f} < 0$.

From Lemma 2, we obtain the following proposition on the comparative statistics results of optimal service fees T^{P*}, A^*, and X^* with respect to f, and k^P. For the proof, see appendix.

Proposition 2 *Suppose that c is fixed and both assumptions 2 and 3 hold, then the following statements also hold.*

(i) *If the cost of the services of the platform for type P sellers, k^P, increases, then the optimal quality level, X^* for type P buyers and the service fee for premium sellers, T^{P*}, also increase; however, the optimal annual membership fee for type P buyers, A^*, decreases.*

(ii) *When the shipping fee charged by the transportation agents (firms) per unit of the item for each type S seller, f, increases, the optimal quality level, X^*, for type P buyers and the service fee for premium/ spot sellers, T^{P*} / T^{S*}, also decrease; however, the optimal annual membership fee for type P buyers, A^*, increases.*

(iii) *If the quality marginal cost of the premium services paid by premium buyers, γ, increases, the optimal quality level X^* for type P buyers, optimal per-transaction services fee T^{P*} for type P sellers, and optimal annual membership fee for type P buyers, A^*, all decrease, except for the optimal per-transaction services fee T^{S*} for type S sellers. It is unchanged since it does not depend on γ.*

(iv) *If the scale parameter of the probability distribution λ decreases (or the upper bound of the distribution of the valuation of consumers $\overline{b}(1+X)$, equivalently increases), then X^*, T^{P*}, T^{S*} and A^*, all increase.*

The intuition of the proposition is as follows. In (16), (23) and (24), when k^P changes, we see that its effect on A can be divided into *two reciprocal effects*, namely, *the quality reaction effect* of T^P through X and *demand reaction effect*. Formally, we can express these changes as $\frac{\partial A^*}{\partial k^P} = \frac{\partial A}{\partial X^*} \cdot \frac{\partial X^*}{\partial k^P} + \frac{\partial A}{\partial k^P}$. The quality and demand reaction effects stand for the first and second terms in the right hand side of the formula, respectively. The former effect is shown to be positive but the latter is negative. As shown in the proof of Proposition 2, the latter effect surpasses the former so that $\frac{\partial A^*}{\partial k^P} < 0$. Therefore, as k^P, the cost of the discriminate services for premium (type P) sellers, including packing and shipping services by the platform, increases, A the service price (annual membership fee) for the new premium buyer members, also increases, but it may decrease X^{NP}, the demand for the new premium buyers of the item. To avoid shrinking in the entry of new premium members, the platform may increase X^*, the premium service level provided to premium buyers. This, in turn, increase A and T^{P*}, the service fee for the premium sellers (*the quality reaction effect*). Note that the premium service for the premium buyers *is no more than the service for the premium sellers.* Both X^{EP*} and X^{P*} decrease when X^* increases because the increase in T^{P*} implies that o $p_d^{P*} = p_c^{P*}$ (**??**). To recover the demand for item c, the platform has to reduce A^*, the price of new premium buyers (the annual membership fee). (*the demand reaction effect*) The effects for X^*, A^*, and T^{P*} *are the opposite* when the per unit shipping fee charged by the transportation agents (firms) foe each item sold by type S sellers*,* f increases. This is because the service demands for type S and P sellers on the platform are substitutes for each other.

Statement (i) of the proposition is an example of the result of Doi and Shinkai (2023), because the result shows that a cost increase may decrease price (i.e., a negative pass-through) and increase quality when quality and prices are endogenously chosen in a monopoly.

Statement (i) also explains why only Amazon Japan has not raised the annual membership fee for premium buyers even though its delivery (transportation) service cost for premium buyers has rapidly increased. The reason is the short supply of delivery services over demand of them, which occured during te COVID-19 pandemic that took place in the recent years.

In Japan, The default option in doorstep delivery services of small parcels of transport companies has been "attended delivery," in which the delivery is completed by a signature when the recipient receives a parcel. Therefore, if the recipient is absent, the transport company has to conduct redelivery. Redeliveries increase the delivery service costs of the transport companies. As the delivery service cost per item for premium buyers of Amazon Japan has tremendously increased because of the COVID-19 pandemic, Amazon has decreased its level of premium services X^* provided to premium buyers. It has also reduced the k^P by changing its default delivery preference services for premium buyers to "unattended delivery," where the item is delivered to the specified location regardless of whether the premium buyer is at home or absent. Therefore, delivery can be completed without requiring the recipient's signature, as a result, there is no need to condut rederivery. Specifically, except for fresh or perishable foods or frozen foods, Amazon Japan has replaced its outsourcing company of premium delivery services from Yamato Transport or Japan Post to AZ-COM Maruwa Inc.. It manages the Third-party logistics (3PL) services system" as a platform system tying small personal transport companies and linking them to Amazon Japan.

To make this system function well, Amazon needs to change its default delivery preference services for premium buyers from "attended delivery" to "unattended delivery," so that it can remove the cost-up factor, redelivery and requiring the signature of recipient for small transport companies. By doing so, Amazon Japan can reduce k^P and small transport companies can stably provide plenty of daily opportunities to the workers delivering services to Amazon's premium buyers[13]. In statement (i) in proposition 2, if

[13]By the TBS TV program, "Gacchirri Monday," broadcasted on February 19th, 2023, AZ-COM Maruwa Inc. has been intermediating on small parcel delivery services of premium buyers between Amazon Japan and small transport companies using 3PL services systems through construction of a platform systems in recent years. Masaru Wasami, President and CEO, in an interview, said that AZ-COM Maruwa Inc. has increased its group sales, which are estimated to reach 171.5 billion yen in March 2023! https://note.com/gacchiri/n/nf49db8c80369

k^P decreases, then X^* and T^{P*} in consequent p_d^{P*} also decrease. Hence premium sellers' demand for of the item increases. This provides sufficient room for A^* to increase in the future, subsequently boosting the platform's profit.

The intuitions of the statements (ii), (iii) and (iv) are all straightforward, which is why we do not mention them.

5 Conclusion

In this study, we analyze price discrimination of a platform monopolist that serves both sellers and buyers with their intermediary services by menu-pricing and quality control. For this purpose, we extended Wang and Wright's (2017) model to include the quality level X (≥ 0) chosen by the platform and one-day delivery services without delivery fees offered to premium buyers, for goods indexed by c. We assume that the distribution of buyers' valuation of the transaction is presented by the generalized Pareto distribution. The platform serves both sides of the market, premium sellers (exhibitors) and premium buyers, through its internet shopping site. It offers intermediation services for transactions among both sides, inventory storage and packing and shipping services of level X to premium sellers based on a pay-as-you-go system and premium services of level X to premium buyers including one-day delivery services without a delivery fee.

Under certain assumptions, we derive the optimal annual membership fee of premium buyers, optimal per-transaction services fees for type P and S sellers and the optimal service level for buyers on the platform. We also investigate how the unit changes in services costs for both types of sellers and buyers on the platform and the quality costs paid by the premium buyers affect the optimal service fees for both types of sellers and buyers. We also examine the demand price of the transaction, on the platform. Consequently, we find a counterintuitive result that *an increase in the cost of the services* of platform for premium type sellers *increases the optimal quality level for premium type buyers* and the service fees for premium sellers: *however,it also decreases of the optimal service fee* (*annual membership fee*) *for the premium buyers.*

Given the limitation of space, we do not derive the first best (socially optimal) equilibrium and conduct the comparison using our monopoly equilibrium in this study. Our model does not consider network externalities either within-group or cross-group which are important in research on the platform economy. These extension of our study is left for future research.

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Appendix A

Proof of Lemma 1

From the quartic equation of X, (25) we have

$$-\lambda\gamma X^2 + cX - \lambda(k^P - k^S - f) = 0 \tag{29}$$

or

$$-3\lambda\gamma X^2 + cX + \lambda(k^P - k^S - f) = 0. \tag{30}$$

For the quadratic equation (29) of X,, from assumption 2, we have the determinant

$$D_1^X = c^2 - 4\gamma\lambda^2(k^P - k^S - f) > 0, \tag{31}$$

so that it has two real solutions

$$X_1 = \frac{c + \sqrt{c^2 - 4\gamma\lambda^2(k^P - k^S - f)}}{2\gamma\lambda} > 0$$

and

$$X_2 = \frac{c - \sqrt{c^2 - 4\gamma\lambda^2(k^P - k^S - f)}}{2\gamma\lambda} > 0.$$

For the quadratic equation (30) of X,, from assumption 1, we have the determinant

$$D_2^X = c^2 + 12\gamma\lambda^2(k^P - k^S - f) > c^2 - 4\gamma\lambda^2(k^P - k^S - f) > 0,$$

so that it also has two real solutions.

Thus, we have

$$X_3 = \frac{c + \sqrt{c^2 + 12\gamma\lambda^2(k^P - k^S - f)}}{6\gamma\lambda} > 0$$

and

$$X_4 = \frac{c - \sqrt{c^2 + 12\gamma\lambda^2(k^P - k^S - f)}}{6\gamma\lambda} < 0.$$

We can easily show that

$$X_1 > X_3 > X_2 > 0 > X_4.$$

The quartic equation of X, (25) can be rewritten by

$$3\gamma^2\lambda^2X^4 - 4c\gamma\lambda X^3 + 2\left(3\gamma\lambda^2(k^P - k^S - f) + c^2 - \gamma\lambda^2(k^P - k^S - f)\right)X^2 - 2\lambda^2(k^P - k^S - f)^2 = 0.$$

Let the l.h.s of the above by $g(X)$, i.e. ,

$$
\begin{aligned}
g(X) &= 3\gamma^2\lambda^2X^4 - 32c\gamma\lambda X^3 + 2\left(3\gamma\lambda^2(k^P - k^S - f) + c^2 - \gamma\lambda^2(k^P - k^S - f)\right)X^2 \\
-2\lambda^2(k^P - k^S - f)^2 &= 0. \qquad (32)
\end{aligned}
$$

$g''(X) = 4c^2 - 8\lambda^2\gamma\left(f - k^P + k^S\right) + 36X^2\lambda^2\gamma^2 - 192cX\lambda\gamma$

$g''(X_3) = f''(\frac{c+\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}}{6\gamma\lambda}) = 4c^2 - 32c\left(c + \sqrt{c^2 + 12\lambda^2\gamma\left(k^P - k^S - f\right)}\right)$
$+\left(c + \sqrt{c^2 + 12\lambda^2\gamma\left(k^P - k^S - f\right)}\right)^2 + 8\lambda^2\gamma\left(k^P - k^S - f\right) = 20\lambda^2\gamma k^P - 26c^2 - 20\lambda^2\gamma f -$
$30c\sqrt{c^2 - 12\lambda^2\gamma f + 12\lambda^2\gamma k^P - 12\lambda^2\gamma k^S} - 20\lambda^2\gamma k^S$
$= -26c^2 - 30c\sqrt{c^2 + 12\lambda^2\gamma\left(k^P - k^S - f\right)} + 20\lambda^2\gamma(k^P - k^S - f)$
$= -2(13c^2 + 15c\sqrt{c^2 + 12\lambda^2\gamma\left(k^P - k^S - f\right)} - 10\lambda^2\gamma(k^P - k^S - f))$
$< -2(13 \cdot 4\lambda^2\gamma(k^P - k^S - f) + 15c\sqrt{c^2 + 12\lambda^2\gamma\left(k^P - k^S - f\right)} - 10\lambda^2\gamma(k^P - k^S - f))$
$= -6(14\lambda^2\gamma(k^P - k^S - f) + 15c\sqrt{c^2 + 12\lambda^2\gamma\left(k^P - k^S - f\right)}) < 0$, where the inequality holds from (31).

We can easily show that $g''(X_1) > 0$ and $g''(X_2) > 0$. Hence, only $X^* = X_3 > 0$ satisfies the second order condition out of four real solutions X_1, X_2, X_3 X_4 of the quartic equation (32). ■

The parameters have the following constrains: $\lambda > 0, 0 \leq \sigma < 2, \sigma \neq 1, c, k_L > 0$.

Proof of Lemma 2

From (23) and (22), we have

$T^P = \frac{5}{7}\gamma X^2 + \frac{2}{7}\frac{c}{\lambda}X + \left(\frac{5}{7}k^P - \frac{3}{14}f - \frac{3}{14}k^S + \frac{1}{2}\frac{c}{\lambda}\right)$

$\frac{\partial T^P}{\partial X} = \frac{2}{7\lambda}\left(c + 5X\lambda\gamma\right) > 0$, since $c, \gamma, \lambda > 0$ and $X \geq 0$. From (16) and (22), we obtain $A = \frac{4}{7\lambda}(cX - \lambda(\gamma X^2 + \left(k^P - k^S - f\right)))$. Hence we see that $\frac{\partial A}{\partial X} = \frac{4}{7\lambda}\left(c - 2X\lambda\gamma\right) \gtreqless 0 \Leftrightarrow \frac{c}{2\lambda\gamma} \gtreqless X \geq 0$. From (26) and assumption 2, we have

$\frac{\partial X^*}{\partial k^P} = \frac{\lambda}{\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}} > 0$, and $\frac{\partial X^*}{\partial f} = -\frac{\lambda}{\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}} < 0$. ■

Proof of Proposition 2

(i) From (26), we can show that

$\frac{c}{2\lambda\gamma} - X^* = \frac{1}{6\lambda\gamma}\left(2c - \sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)}\right) > 0$, since

$(2c)^2 - (\sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)})^2$

$= \left(2c - \sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)}\right)\left(2c + \sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)}\right) > 0$

and $2c + \sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)} > 0$. Therefore, from Lemma 2, $\frac{\partial A}{\partial X^*} > 0$.

$\frac{\partial A^*}{\partial k^P} = \frac{\partial A}{\partial X^*}\cdot\frac{\partial X^*}{\partial k^P} + \frac{\partial A}{\partial k^P} = \frac{4}{7}\frac{c-2X^*\lambda\gamma}{\lambda}\cdot\frac{\lambda}{\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}} + (-\frac{4}{7}) = -\frac{1}{21}\frac{8(2\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}-c)}{\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}}$

.

(ii) From (26), (28), and (23), we obtain

$\frac{\partial X^*}{\partial f} = -\frac{\lambda}{\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}} < 0$, $\frac{\partial T^{P*}}{\partial f} = -\frac{1}{42}\frac{22c+19\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}}{\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}} < 0$, and

$\frac{\partial T^{S*}}{\partial f} = \frac{\partial}{\partial f}(\frac{1}{2}\left(\frac{c}{\lambda} + k^S - f\right)) = -\frac{1}{2} < 0$. From (16), (26)

and assumption 2, we have $\frac{\partial A^*}{\partial f} = (\frac{\partial A}{\partial X} + \frac{\partial A}{\partial(T^S-T^P)}\cdot(\frac{\partial(T^S-T^P)}{\partial X}))\cdot\frac{\partial X^*}{\partial f} + \frac{\partial A}{\partial f}$

$= (\frac{1}{\lambda}\left(c + X^*\lambda\gamma\right) + \frac{3}{2}\cdot(-\frac{1}{7\lambda}\left(2c + 10X^*\lambda\gamma\right)))\cdot\left(-\frac{\lambda}{\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}}\right) + \frac{4}{7}$

$= -\frac{8}{21}\frac{c-2\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}}{\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}} = \frac{8}{21}\frac{-c+2\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}}{\sqrt{c^2+12\lambda^2\gamma(k^P-k^S-f)}} > 0.$

(iii) From (26) (28), (23), (27) and assumption 2, we obtain

$\frac{\partial X^*}{\partial \gamma} = -\frac{1}{6\lambda\gamma^2\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}}\left(c\sqrt{c^2 + 12\gamma\lambda^2\left(k^P - k^S - f\right)} + c^2 + 6\gamma\lambda^2\left(k^P - k^S - f\right)\right) < 0,$

$\frac{\partial T^{P*}}{\partial \gamma} = -\frac{1}{42}\frac{22c+19\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}}{\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}} < 0$, $\frac{\partial T^{S*}}{\partial \gamma} = 0$, $\frac{\partial A^*}{\partial \gamma} = -\frac{4}{63}\frac{c}{\lambda^2\gamma^2\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}}$ $\left(c\sqrt{c^2+12\gamma\lambda^2\left(k^P-k^S-f\right)}+c^2+6\lambda^2\gamma\left(k^P-k^S-f\right)\right) < 0.$

(iv) From (26) (28), (23), (27) and assumption 2, we have

$\frac{\partial X^*}{\partial \lambda} = -\frac{c(\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}+c)}{6\lambda\gamma^2\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}} < 0$, $\frac{\partial T^{P*}}{\partial \lambda} = -\frac{1}{63}\frac{11c((\frac{63\gamma\lambda}{22}+c)(\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}+c^2+6\gamma\lambda^2\left(k^P-k^S-f\right)))}{\gamma\lambda^3\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}}$ < 0,

$\frac{\partial T^{S*}}{\partial \lambda} = -\frac{c}{2\lambda^2} < 0$, $\frac{\partial A^*}{\partial \lambda} =$ $-\frac{8}{63}\frac{c}{\lambda^2\gamma^2\sqrt{c^2+12\gamma\lambda^2(k^P-k^S-f)}}\left(c\sqrt{c^2+12\gamma\lambda^2\left(k^P-k^S-f\right)}+c^2+6\lambda^2\gamma\left(k^P-k^S-f\right)\right)$ $< 0.$ ■

Appendix B

This appendix explains a dynamic situation that the static model of this study implicitly assumes. The dynamic situation is depicted in Figure A1.[14] In each time period, NMs enter the market and decide whether to buy one unit of the good. A buyer decides whether to buy one unit of the good. The choice set is purchasing it by using the premium transaction, purchasing it by using the spot transaction, and not buying it on the platform (including purchasing it through another channel). If NMs want to choose the premium transaction, they should become a new member by paying the membership fee. Membership is effective during two periods, that is, the period when an NM pays the membership fee and the subsequent period. This means that the NMs who choose the premium service by paying the fee in period $t-1$ become EMs in period t. The EMs can use the premium transaction with no payment to the platform, because they have already paid the membership fee. In addition, the platform provide the transaction

[14]On this idea of time flow of purchase decision of buyers, we owe to the comment of Hiroaki Ino.

service for not only the good focused on here (c) but also another good (c'). The EMs in the market of good c in period t include buyers who has become a member for purchasing the good c' by using the premium transaction in period $t-1$.

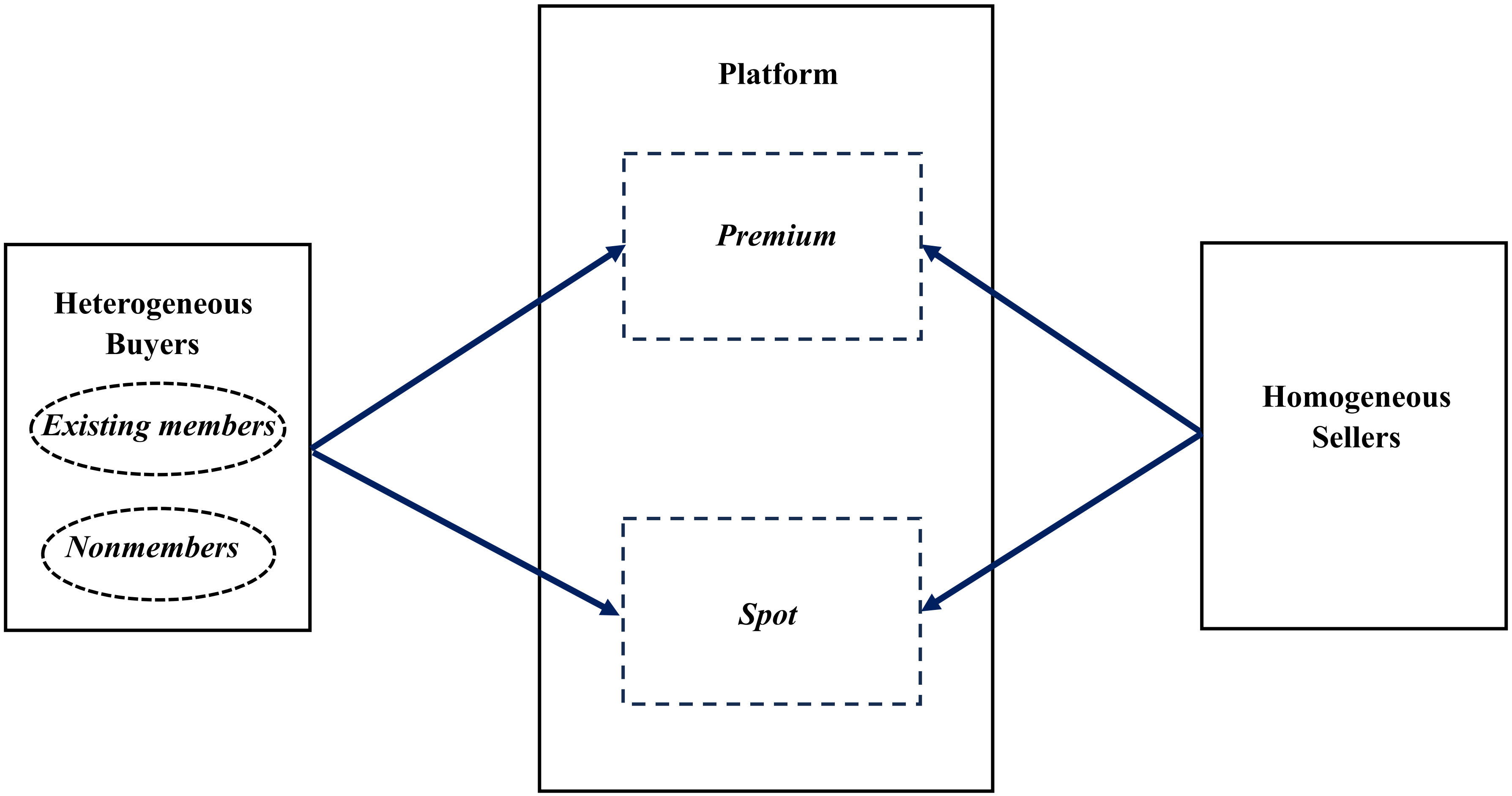

Figure 1

Framework of the model

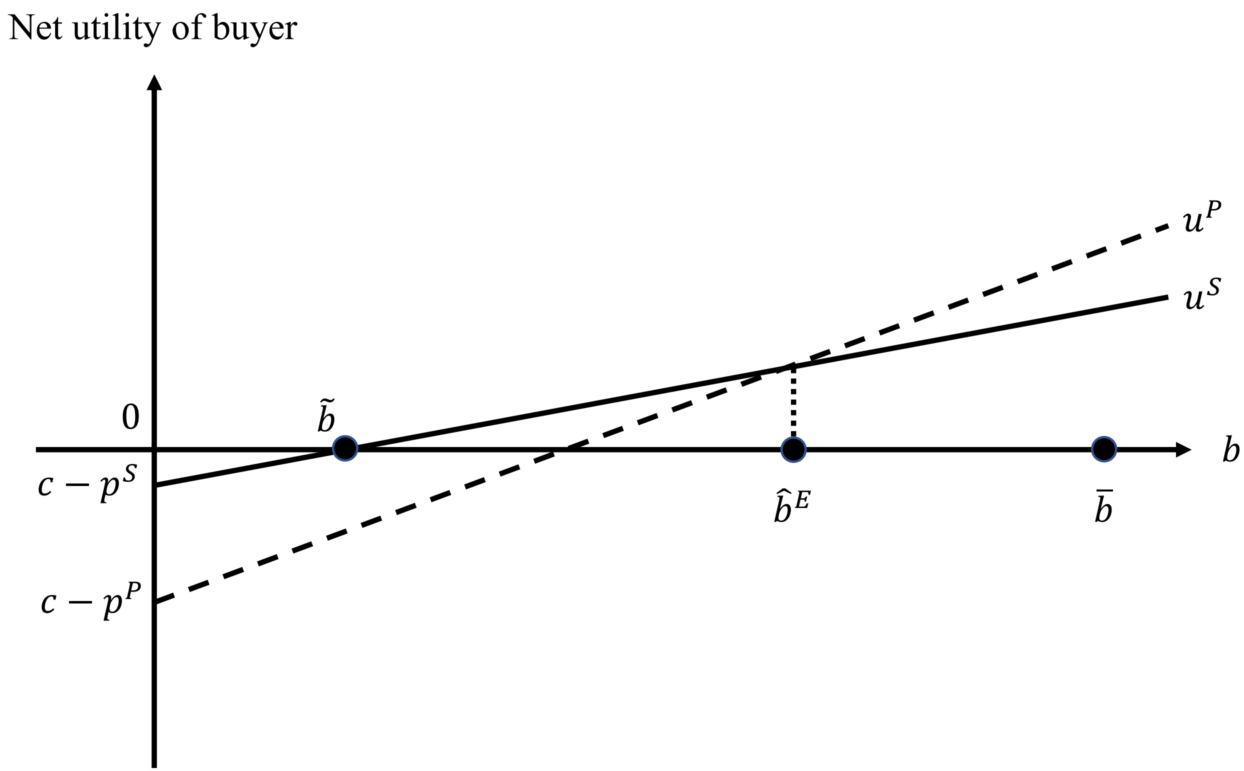

Figure 2
Choice of existing members

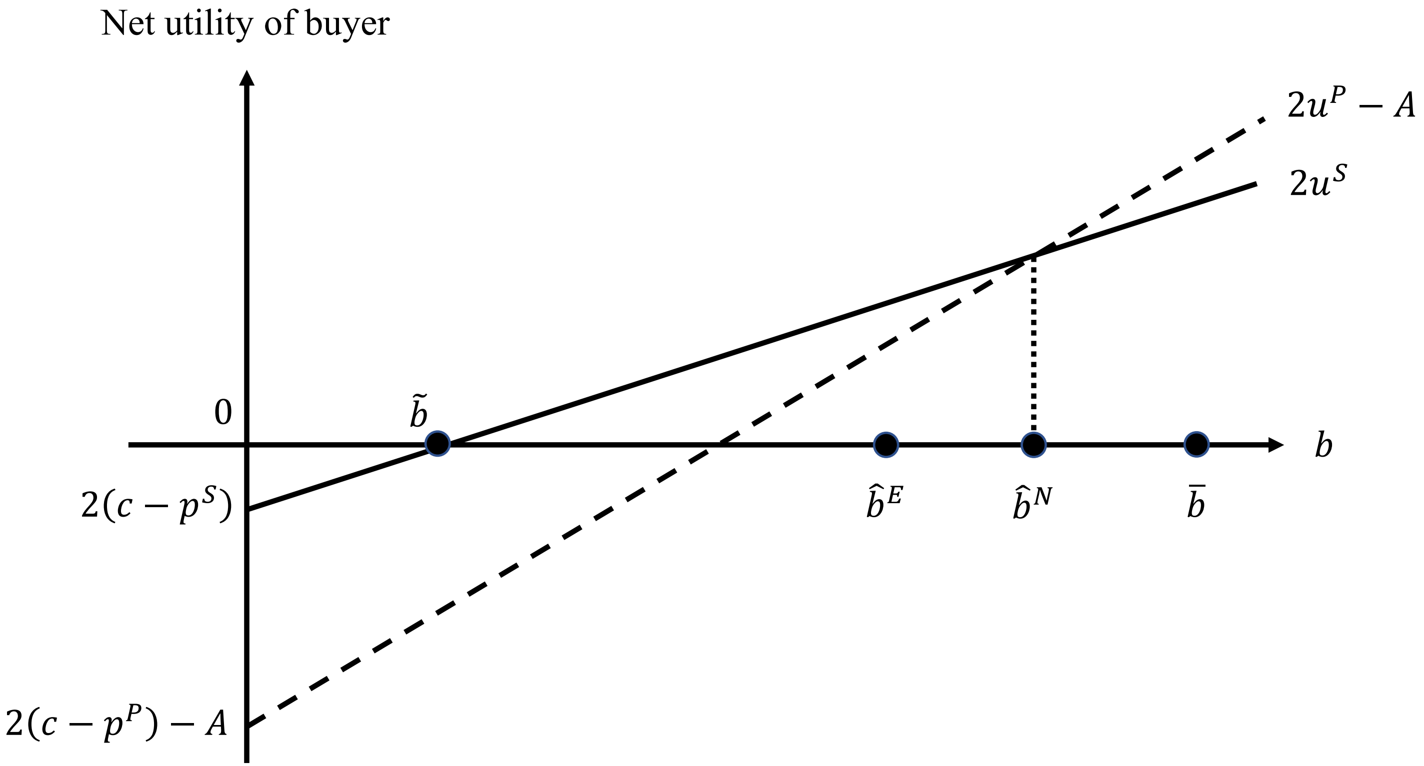

Figure 3
Choice of nonmembers

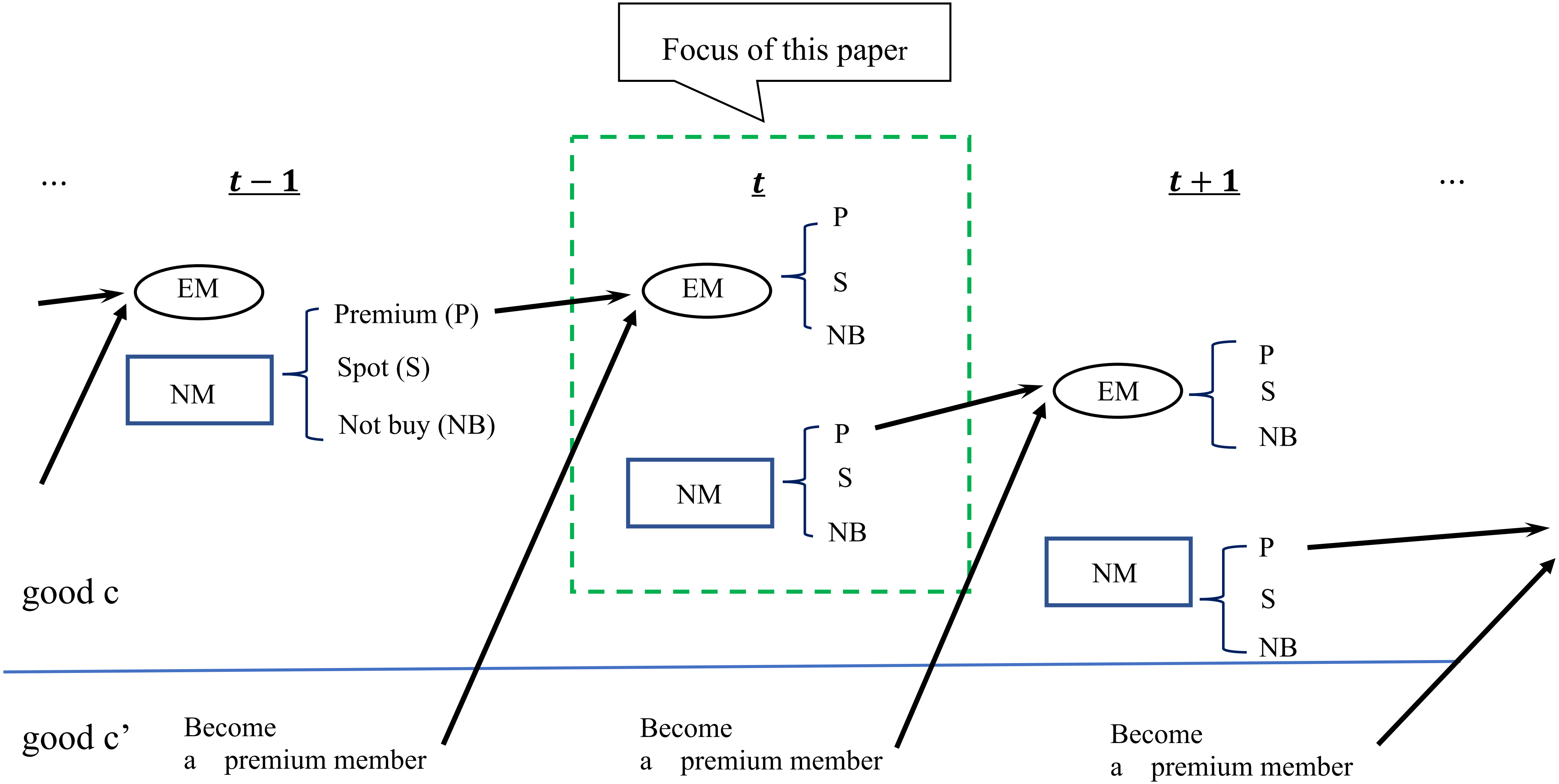

Figure A1

Dynamics behind the model