INTRA-SEASON PRICING OF FASHION APPAREL GOODS

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ABSTRACT

Fashion apparel retailers customarily start out with relatively high prices for their goods and gradually reduce prices over the course of a season. This paper examines retailer behavior and proposes a theory which explains why price reductions are customary. In addition to the reservation price concept, on which past explanations are based, new concepts of reservation time and response probabilities are introduced to approximate consumer behavior toward fashion apparel goods.

INTRODUCTION

To those retailers of apparel goods that are subject to seasonal fashion cycles, setting prices over a season is a routine, but bothersome task. Fashion apparel items are in most cases stocked at the stores prior to the season, and seldom replenished during the season, because re-orders are not always fulfilled by the manufacturers. Under those conditions, retailers not only endeavor to sell out all the stocked items within the season, but try to maximize the sales revenues from those items.

It is a common practice for the retailers of this type of merchandise to reduce prices gradually throughout the season. That is, they set maximally high prices at the beginning of the season and, keeping watch over the over-time changes in sales volume, are ready to reduce them whenever sales volumes do not meet prior expectations or competitors reduce their prices. At the end of the season, they may conduct "clearance" sales at special bargain prices in order to minimize the stocks carried forward to the next season. Such retailer behavior has been explained that the demand curves for fashion apparel items generally shifted downward as the season progressed (e.g., Lazear 1986). However, few theoretical explanations have been offered so far as to why and how demand curves shifted during a fashion season.

This article examines the intra-seasonal pricing methods used for fashion apparel items from a theoretical point of view. In the following, we will make some assumptions on the potential demand function (to be derived from "reservation price distribution") for a fashion

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apparel item at the beginning of a season and show how the demand function shifts downward as potential demands become realized during the season. In addition, we will introduce a new concept of "reservation time", distributions to handle the differences among consumers regarding how late they are willing to purchase the fashion item in question. We will offer theoretical explanations for the downward shifts of demand curves, and devise an optimum pricing scheme within the season (the initial price at the beginning of the season and subsequent reduction rates) with the help of those theoretical apparatuses.

BASIC ASSUMPTIONS

In this section, we shall state the basic premises and assumptions underlying consumer and retailer behavior.

Assumptions on Retailer Behavior

Merchandise stocking schemes: Thanks to the so-called "consignment" scheme which is widely used in Japan, many apparel retailers are able to return stock remaining at the end of a season to wholesalers and/or manufacturers and thus avoid carrying forward obsolete stock to the next season. In contrast, European and U. S. retailers customarily purchase their merchandise at the beginning of the season (this is to be called the "straight buying" scheme). Let us compare the implications of those two stocking schemes.

First, under the consignment scheme, the retailers return the remaining stock to the suppliers and payments are made only for that portion of consigned goods that are sold during the season. (The scheme is sometimes called the "consumed" stocking scheme because of this.) In this situation, the marginal cost of an item is always equal to the buying cost, c, and therefore the optimal price (and the quantity sold) is determined at the point where the marginal revenue is equal to c. Since there is no stock to be disposed of, the buying quantity at the beginning of the season is not a decision variable to the retailers.

Under the straight buying scheme, it is assumed that retailers buy their seasonal requirements only once at the beginning of the season and make no replenishments. (That this assumption is not entirely unreasonable has been already discussed.) With this scheme all unsold goods at the end of the season will have to carry over to the next season. Since it would be very difficult to predict how much revenue might be generated from the stock that is carried forward, the retailers would certainly strive to maximize the revenue within the current season. In other words, under this stocking scheme the marginal cost becomes totally disassociated from the buying cost of goods and therefore the retailers will try to price an item to maximize the total revenue from it. But, if some salvage value could be expected at the end of the season, an item would be priced differently. If we let c_0, s, and q be the unit cost of an item, salvage price, and sales volume in units, respectively, the cost function for the retailer is given by

$$TC = c_0 q_0 - s(q_0 - q) = (c_0 - s)q_0 + sq.$$

In this situation, the marginal cost becomes equal to c_0 and, if a demand function were given, the optimum price is given by the point where the marginal revenue is equal to c_0. The initial quantity to be bought at the beginning of the season is not a decision variable for the retailer in this case.

The foregoing discussion shows that there is no essential difference between the pricing behavior of fashion apparel retailers between two stocking, consignment and straight buying, schemes. The difference is only in the marginal cost (c_0 for consignment and s for straight buying). In practice, the marginal cost is known a priori in the consignment scheme, but it will have to be guessed in the straight buying case. We will assume that the salvage price may be guessed fairly accurately at the beginning of the season, so will not deal with the situation in which the salvage price is closely related to the amount of the end-of-season stock (though such a situation may be a common occurrence in practice).

Intra-season patterns of prices: There are several intra-season pricing patterns that retailers of fashion apparel goods may adopt. They are divided first into two groups.

1. Maintain a constant price throughout a season.
2. Set a maximally high price at the beginning of the season, and reduce the price gradually over the season.

We will not pursue the first (i.e., constant price) pattern further because it can be shown that it will not maximize the over-all revenue within a season under our assumptions in the next section. We shall only deal with variations in the second (i.e., declining price) pattern.

Assumptions on Consumer Behavior

Distribution of Reservation Price: We assume that each consumer has in his/her mind a level of price for a fashion apparel item at the beginning of a season "over which he/she will never purchase it". We call this price level the "reservation price" for this item, and assume further for simplicity that the reservation price does not change throughout the season. Reservation prices of a group of customers of a retailer have a statistical distribution, whose density function, $f(r)$, might look bell-shaped, as shown in Figure 1. The solid curve gives a normal density and the broken curve a log-normal density. The reasons for choosing bell-shaped density curves are:

1. Those customers whose reservation prices are either 0 or near 0 cannot be considered as potential customers of this item. Only those customers whose reservation prices are substantially higher than 0 may be thought of as potential customers.
2. Customers of a particular retailer often have a fairly good expectation of the intra-season price range for a particular item sold in that store.

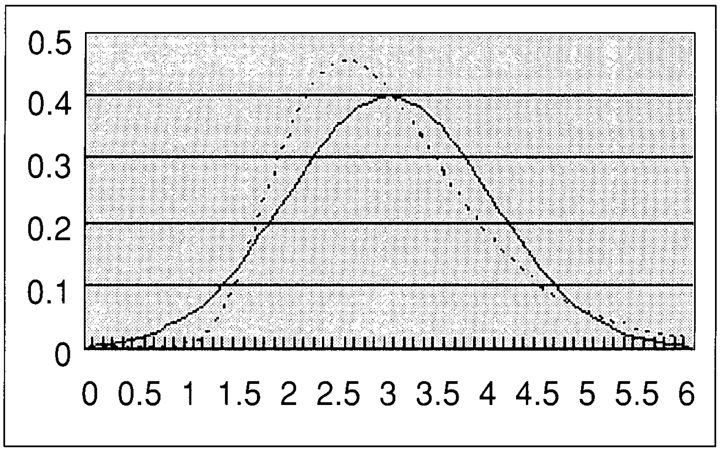


Figure 1 Normal and Lognormal Distributions

3. Because consumers are known to have the so-called "price-quality association," they lose interest in an item if its price is significantly below their expectations, thinking that its quality is low.

For these reasons, we will assume that reservation price distributions are either normal or log-normal.

We further assume that each customer buys a fashion apparel item only once in a season, which is reasonable if one considers the nature of the merchandise. Hence those customers who have made a purchase exit from the market as well as from the reservation price distribution for this item. By gradual reductions in the price of this item from the beginning to the end of a season, the reservation price distribution loses its constituents from the right-hand (high price) positions, and thus shifts to the left.

Demand functions: With the assumption that each customer buys an item only once in a season, the demand function for this item at the beginning of the season is obtained by integrating the reservation price distribution from the high-price side, that is,

$$(1) \qquad q(p) = n \int_p^{\infty} f(r)\, dr$$

where p is the price of the item and $q(p)$ is the quantity sold at p. In the above equation, n is the number of potential customers (to be estimated) who may purchase the item in question. Since the magnitude of n is expected to vary greatly depending on the store size, n will have to be estimated for each store even in a single chain.

In order to describe the shifting of reservation price distributions as customers who purchased the item exit from the market, we divide the season into T periods, and assume that the price is held constant within a period. The demand function at the beginning of the season is given equation (1) and the unit sales volume in period 1 is given by

$$q_1(p_1) = n\int_{p_1}^{\infty} f(r)\, dr$$

where p_1 and q_1 is the price and quantity sold in the first period. Those customers who are included in $q_1(p_1)$ all exit from the market and will have to be excluded from the demand function for period 2. Letting p_2 and q_2 be the price and quantity sold in the period 2, we have

$$q_2(p_2) = n\int_{p_2}^{p_1} f(r)\, dr.$$

Similarly, the quantity sold in period t, $q_t(p_t)$, is expressed as

$$q_t(p_t) = n\int_{p_t}^{p_{t-1}} f(r)\, dr.$$

Reservation Time: There are problems in deriving demand functions only from reservation time distributions. If customers knew that prices would be reduced as the season progressed, it is difficult to explain why they should purchase apparel items in the earlier part of the season at higher prices. But we know that some consumers of fashion apparel items lose interest in them in the latter part of the season. Of course there are others who postpone their purchases until the last part of the season. We will try to capture those differences in purchase timing by a new concept of "reservation time." Reservation time, t, is defined as "the time elapsed from the beginning of the season to the last time point after which a consumer will not purchase an apparel item." Let $f(t/r)$ be the density function of reservation time t conditional on reservation price r. We choose this conditional density formulation because we suspect the existence of a high correlation between t and r. Those consumers who wish to purchase a fashion apparel item in the earlier part of the season (that is, those with small values of t) do not hesitate to pay higher prices, while those consumers who wait till the last part of the season (that is, those with large values of t) will not purchase unless the price is significantly reduced. In the following $f(t/r)$ will be assumed to be a normal density function with mean a – br and standard deviation st. With the introduction of reservation time the demand function is redefined as follows. The initial (period 1) demand function is given by

$$q_1(p_1) = n\int_{p_1}^{\infty} (1 - F(t/r)) f(r)\, dr.$$

The demand function in period t is given by

$$(2) \qquad q_t(p_t) = n\int_{p_t}^{p_{t-1}} (1 - F(t/r)) f(r)\, dr.$$

$F(t/r)$ in the above equations are the cumulative distribution function of t given r.

SEQUENTIAL REDUCTION IN PRICES

In this section we will try to find a pattern of price reductions over a season which maximizes the following profit function on the basis of the preceding assumptions.

$$(3) \qquad \underset{\{p_1, p_2, \ldots, p_T\}}{Max} \ \pi = \sum_{t=1}^{T} (p_t - c)\, q_t\, (p_t) \qquad \text{subject to } p_1 > p_2 > \ldots > p_T > 0$$

We will call the above equation "the objective function." In this equation c is the average buying price, but, in the case of straight buying, s (=salvage value) may have to be used instead.

One of the useful features of this type of problem formulation is that, because n in equation (2) is a simple multiplier, one may safely assume that n=1 without loss of generality. The following proposition summarizes this point.

Proposition 1: Under equations (2) and (3), the optimal price path for the sequential price reduction method does not depend on the purchase volume at the beginning of a season.

We will examine the optimum pricing patterns for the following four cases.

1. The reservation price distribution is normal.
   a. The prices for different periods are dealt with as independent decision variables.
   b. The prices for successive periods are values of a single function whose parameters are determined to maximize the objective function.
2. The reservation price distribution is log-normal.
   c. The prices for different periods are dealt with as independent decision variables.
   d. The prices for successive periods are values of a single function whose parameters are determined to maximize the objective function.

By comparing normal and log-normal distributions, we will see if the optimal pricing pattern over a season is dependent on the assumption on reservation price distribution. As is shown below, finding the optimal pricing pattern is greatly facilitated by treating prices in successive periods as values from a single function. If the maximized values of the objective function in cases b and d are found to be close enough to the maximized values in cases a and c above, then we will be able to use this "optimal function" approach in actual applications with more confidence. We will use the following simple functional form for this approach. We call this functional form the "three-parameter model".

$$p_t = \alpha - \beta t^{\gamma} \qquad (\alpha, \beta, \text{ and } \gamma \text{ are parameters to be optimized.})$$

Computation of Optimum Pricing Patterns

Computations are made by assuming that the average reservation price and standard deviation are 30,000 yen and 10,000 yen, respectively, at the beginning of a season. The conditional average of reservation time is assumed to be

$$\mu(t/r) = 60{,}000 - r$$

and its standard deviation, 5,000 yen. Also assume that the buying cost (or salvage value), c, of the item in question be 10,000 yen.

Table 1 and Figures 2 and 3 give comparisons among the four afore-mentioned cases (a, b, c and d) in terms of optimal pricing patterns over six periods of a season.

Table 1 Comparison of Optimum Price Paths (Normal vs. Lognormal) **Estimated Parameters**

	1	2	3	4	5	6	α	β	γ
Case a	3.77	3.07	2.47	1.94	1.52	1.25	—	—	—
Case b	3.77	3.04	2.48	1.98	1.51	1.06	3.7660	0.7285	0.8163
Case c	3.81	3.02	2.42	1.92	1.52	1.29	—	—	—
Case d	3.74	2.95	2.43	1.98	1.57	1.19	3.7418	0.7938	0.7260

Figure 2 Comparison of Optimal Paths (Normal distribution)

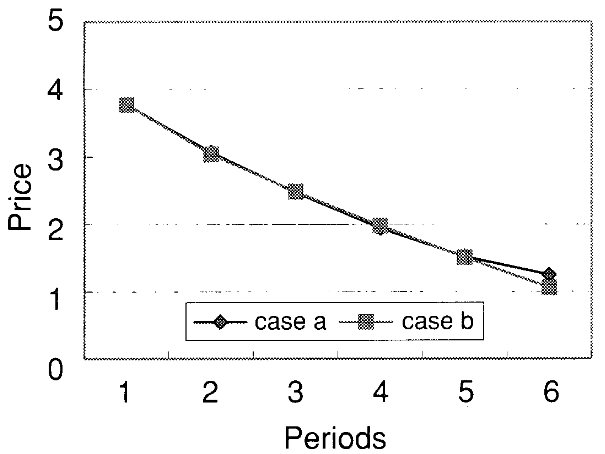
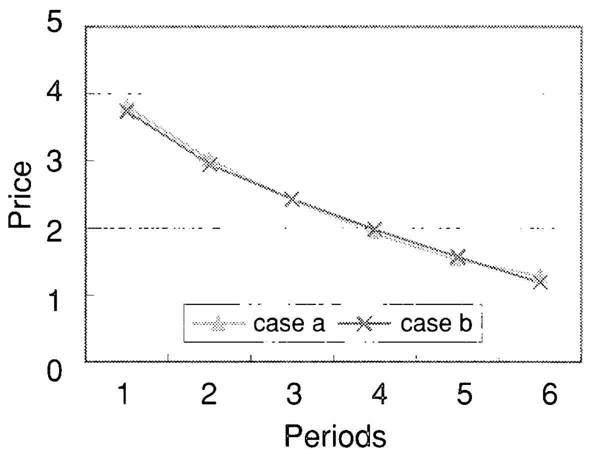


Figure 3 Comparison of Optimal Paths (Lognormal distribution)



That the results for cases a and c and are virtually indistinguishable suggests that, if the average and standard deviation are equal, the choice between normal and log-normal distributions is immaterial, especially in practice. Similar conclusions are reached from the comparison of optimal paths (b and d) that are generated by the three-parameter model. Curves for those two cases are virtually indistinguishable. Though estimated parameters are slightly different between normal and log-normal distributions, the differences among optimal prices in a period are so small that the distributional assumptions could not critically affect the shape of optimal price paths.

3. Introducing Response Probabilities

There is a problem in the formulation of optimum price decision in the previous section. The demand quantity, q_t, computed from equation (2) may not be totally realized within period t. Even if the price is reduced to the level that is attractive to consumers, they may not be aware of the price reduction, or have no time to go to shop and therefore may not purchase the product. In other words, equation (2) may only give the potential demand quantity for the period.

In order to take this fact into account, we will introduce response probability, ϕ_t, on the right hand-side of equation (2). ϕ_t is the probability that the potential demand quantity for period t will be realized within that period. We expect that ϕ_t will vary from one period to the next in response to the level of promotional activities. Also, if much mouth-to-mouth communication were to be generated by satisfied customers, then ϕ_t might increase with the passage of time.

Let us assume that

$$(4) \qquad q_t(p_t) = n\phi_t \int_{p_t}^{p_{t-1}} (1 - F(t-1/r)) f(r)\, dr$$

and put (4) in equation (3). The unique characteristic of this new formulation is that the optimal price path is only affected by the relative sizes of ϕ_t ($t = 1, 2, \dots, T$), because the objective function (3) is not affected by multiplying any constant to ϕ_t. This fact leads to the following proposition.

Proposition 2: Under assumptions (3) and (4), the optimal price path over a season depends only on the relative size of response probabilities.

This proposition gives us great flexibility in the pattern of promotional activity allocation over a season. To see some of the implications, we computed optimal price paths for the following cases.

Case 2: (Upward pattern) Response probabilities ϕ_t increase monotonically over a season.

Case 3: (Downward Pattern) Response probabilities ϕ_t decrease monotonically over a season.

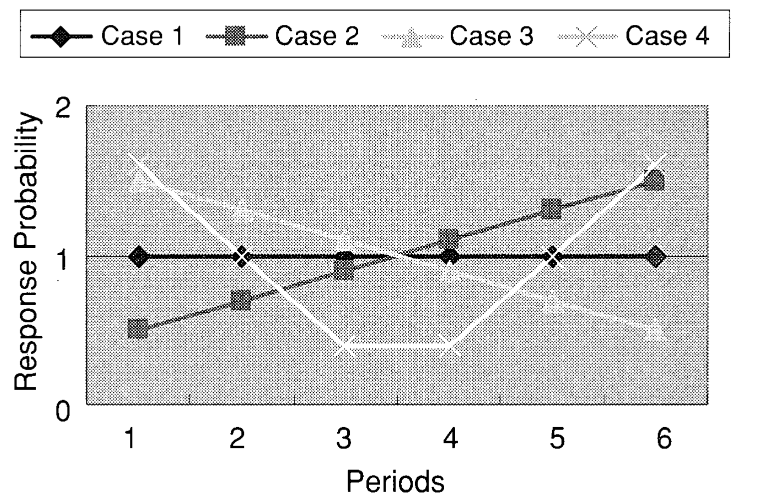
Case 4: (V-letter Pattern) Response probabilities ϕ_t are high at the beginning and the end of a season.

Table 2 and Figure 4 show these three patterns of response probabilities. The average response probability is kept equal for every case. Case 2 shows an increasing pattern in response probabilities throughout the season due to mouth-to-mouth communication and other factors. Case 3, in contrast, shows a pattern that is high at the beginning of a season due to massive promotion and publicity in the mass media, but then becomes low toward the end of the season. Case 4 might be generated if the firm concentrates its promotional activities at the beginning and end of a season, or response probabilities are increased by mouth-to-mouth communication toward the end of the season. We will compare the optimal price paths for those three cases against the optimal path for Case 1 (i.e., constant response probabilities).

Table 2 Relative Patterns of Response Probabilities (ϕ_t)

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Case 1	1	1	1	1	1	1
Case 2	0.5	0.7	0.9	1.1	1.3	1.5
Case 3	1.5	1.3	1.1	0.9	0.7	0.5
Case 4	1.6	1	0.4	0.4	1	1.6

Figure 4 Relative Patterns of Response Probabilities



Normal distributions are assumed for both reservation prices and reservation times. Optimal period-by-period prices are computed (indicated as a) and three parameter models are fitted (indicated as b) for each case. The results are in Tables 3 and 4 and Figures 3 and 4.

Table 3 Comparison of Optimal Price Path (period-by-period)

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Value of Objective Function
case 1-a	3.77	3.07	2.47	1.94	1.52	1.25	1.63673
case 2-a	4.46	3.50	2.71	2.06	1.58	1.27	1.24322
case 3-a	3.41	2.73	2.22	1.79	1.44	1.22	2.15756
case 4-a	3.04	2.32	2.13	1.95	1.54	1.26	2.00642

Table 4 Comparison of Optimal Price Path (3-parameter mode)

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Estimated Parameter Values			Value of Objective Function
							α	β	γ	
case 1-b	3.77	3.04	2.48	1.98	1.51	1.06	3.766	0.728	0.816	1.63481
case 2-b	4.49	3.44	2.73	2.12	1.56	1.03	4.491	1.065	0.739	1.23772
case 3-b	3.41	2.72	2.24	1.81	1.42	1.05	3.413	0.691	0.764	2.15680
case 4-b	3.04	2.33	2.00	1.74	1.52	1.33	3.036	0.710	0.545	2.00148

Figure 5 Comparison of Optimal Paths (period-by-period)

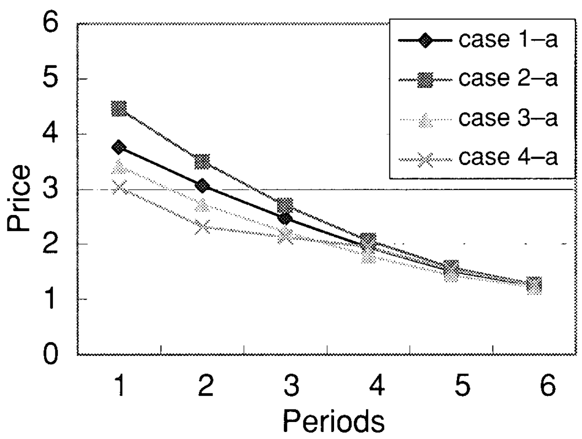
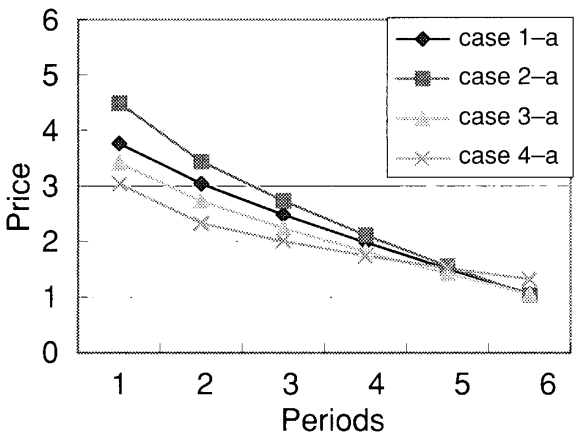


Figure 6 Comparison of Optimal Paths (3-parameter model)



From Figures 5 and 6 we can see that the over-time pattern variations in response probabilities affect optimal price paths. For Case 2, the demand elasticity at the beginning of the season is relatively low because of low response probabilities. The prices are higher than those of Case 1 because of this. In comparison, high response probabilities of Cases 3 and 4 give higher price elasticity at the beginning of the season. This is the reason that the prices are lower than in Case 1.

The last columns of Tables 3 and 4 show the values of objective function (3) for the respective optimal paths. Case 3 gives the highest profit and Case 2 the lowest. Those results suggest a high-promotion, low-price strategy at the beginning of a season for fashion apparel items. It may not be profitable to concentrate promotional activities only in the last half of a season. Also much mouth-to-mouth communication cannot be expected if the prices are high at the beginning of the season.

FUTURE TASKS

In this paper we examined optimal pricing patterns for fashion apparel items on the bases of relatively simple assumptions. In order to verify those conclusions on the optimality of different cases, however, we will have to introduce explicit assumptions on the relationships between response probabilities and levels of promotional activities and mouth-to-mouth communication, and incorporate promotional costs in the objective function before we compute optimal price paths. The following are the list of points which are not considered in this paper and yet important enough to warrant future investigation.

(1) In order to see the applicability of this theory to retail management, it will be necessary to obtain empirical supports. The joint distributions of reservation prices and reservation times may be determined to an extent by consumer surveys prior to a season. By using such empirical distributions and computing optimal prices for some items, one may investigate if the sales quantities over a season follow the patterns predicted by the model.

(2) From a practical point of view, it is unreasonable to assume that a retail decision maker always knows the reservation price-time joint distributions for all items in his/her stock, because normally a decision maker must deal with many items. In order to help such a decision maker, it is more important to develop some heuristics (e.g., Bitran et. Al 1998) which allow him/her to modify future price patterns on the basis of the past prices and quantities sold.

(3) A further task, left for future research, is the explicit recognition of variable reservation price-time joint distributions over time. Especially, a fashion item might make "a big break" in a season and invalidate prior assumptions on the reservation distributions and thus destroy all planning on prices. The heuristics mentioned above may be the best way to counteract such drastic changes in prior assumptions.

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