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Population Aging, Fiscal Sustainability and PAYG Pension Reform

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Abstract

This paper examines how pay-as-you-go (PAYG) pension reform from a defined-benefit scheme to a defined-contribution scheme affects fiscal sustainability and economic growth in an overlapping generations model with endogenous growth. We show that in economies in which the old-age dependency ratio is high and the size of pension benefits under a defined-benefit scheme is large, such a pension reform mitigates the negative effect of population aging on fiscal sustainability and economic growth. However, we also show that this type of pension reform entails an intergenerational conflict of interest between current and future generations. Population aging might exacerbate the extent of this conflict.

Keywords: Population aging, PAYG pensions, Defined-benefit schemes, Defined-contribution schemes, Fiscal sustainability

JEL classification: D91, H55, O41

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1 Introduction

During the last half century, the demographic structure of all developed nations changed dramatically. Due to the decline in both fertility and mortality rates, the relative share of elderly persons in society increased rapidly, and this trend is predicted to persist for years to come.[1] Such rapid population aging and the resulting increases in social security expenses place enormous pressure on government financing in these nations. Japan is now confronting the most severe such situation in the world. According to the OECD economic outlook for 2015, Japan's gross government debt reached 230% of GDP on a gross basis, the highest among all OECD countries. Oguro (2014) argues that these recent increases in the government debt-GDP ratio in Japan are attributable primarily to the rapidly swelling social security costs due to population aging.[2] Chronic fiscal deficits and social security financing are closely related . [3] At present, many OECD countries face a host of challenges to achieving fiscal sustainability.

Faced with these challenges, many developed countries have implemented a series of social security reforms since the 1990s. In particular, countries such as Japan, Denmark, and Italy have abandoned the traditional practice of continuously increasing contributions to maintain pension benefits for the elderly. Instead, they employ a demographically modified indexation program. Most countries that employ pay-as-you-go (PAYG) pension systems maintain elderly living standards by adjusting pension benefits to maintain the ratio of pensions to average wages of working generations at a constant value (i.e., wage indexation). Under this type of defined-benefit PAYG pension scheme, declining birth rates and aging populations have necessitated increases in the pension contributions of younger generations. However, because of the rapid pace at which populations are aging, the conventional approach (i.e., increasing contributions to maintain pension benefit levels) imposes an excessive burden on younger and future generations through a hike in the social security tax rate or the issuance of government debt. Under a de-

[1]For example, in Japan, the share of the population aged 65 and older (i.e., the old-age dependency ratio) in 1990 was 12%. By 2005, the elderly share of the population had risen to 20%. Government projections by the National Institute of Population and Social Security indicate that this figure will reach 40% by 2060.

[2]Social security payments in Japan are partly funded by the state budget. Therefore, government deficits and social security financing are closely related. For example, the sum of social security spending for fiscal 2013, 110 trillion yen, consists of approximately 60 trillion yen in social insurance premiums and approximately 10 trillion yen in revenue from asset management, and the shortfall of approximately 40 trillion yen is to be paid with public funds from local and central governments.

[3]An IMF report on the Japanese economy (2009) argues that fiscal and social security reform is an critical issue in Japan and suggests that Japan should increase its consumption tax rate to 15% to secure a stable source of revenues to ensure that its social security obligations are met.

mographically modified indexation program, however, if the number of workers supporting the pension system decreases, then the wage indexation rate for pension benefits also decreases, even if the average wages of the working generations increase. Hence, pensions for the elderly are adjusted automatically based on changes in demographic conditions. This introduction of a demographically modified indexation program paves the way for a cap on future contribution increases or a fixed-contribution PAYG pension scheme. For example, the 2004 pension reform in Japan shifted away from the standard practice of increasing employee pension contributions to guarantee a 59% benefit level and instead capped future contributions at 18.3% (i.e., a fixed-contribution program). In addition, Japan introduced a demographically modified indexation program to ensure that the size of pension benefits was consistent with the new contribution cap.[4] At present, many other OECD countries are introducing automatic pension benefit adjustment mechanisms and modified indexation systems to prevent unrestricted future contribution or government debt increases.[5] From a theoretical perspective, these recent PAYG pension reforms signify a transition from defined-benefit schemes to defined-contribution schemes.

Motivated by these recent policy experiences in several OECD countries, this paper examines PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme affects fiscal sustainability and economic growth in an overlapping generations (OLG) model with endogenous growth and public debt. In the present analysis, the government employs a defined-benefit PAYG pension scheme with wage indexation and finances it through wage income tax and debt issuance. The government also conforms to the constant budget deficit-GDP ratio rule. Under these fiscal and social security policies, there can be two long-run equilibria, one locally stable and one locally unstable, and the locally unstable equilibrium determines the upper limit of the initial public debt-capital ratio below (resp. above) which a fiscal policy is sustainable and the economy converges to the stable equilibrium (resp. a fiscal policy is no longer sustainable). We consider a fiscal policy sustainable if there exists an initial value of the public debt-capital ratio such that the ratio converges to some finite value in the long run. We also consider a fiscal policy more sustainable if the range of the initial public debt-capital ratio for which the public debt-capital ratio converges to some finite level becomes wider due to the changes in fiscal policy parameters or demographic conditions.

Under the framework explained above, this paper shows that a decline in pop-

[4]The new pension level established by the 2004 pension reform was 50%. For further information on the 2004 pension reform in Japan, see, for example, Komamura (2007).

[5]For example, Denmark, Greece, Hungary, Italy, Korea and Turkey have each linked future increases in pension ages to changes in life expectancy. In addition, Sweden introduced a defined-contribution PAYG pension scheme in 1999. See OECD (2013) for additional details.

ulation growth rate lowers the upper limit of the initial public debt-capital ratio below which a fiscal policy is sustainable. We also show that in economies in which the old-age dependency ratio is high, a decline in population growth rate negatively affects economic growth. These results imply that population aging caused by a decline in the population growth rate negatively affects both fiscal sustainability and economic growth. This negative effect of population aging on fiscal sustainability has yet to be shown explicitly in the existing literature.

Moreover, this paper shows that in economies in which the old-age dependency ratio is high and the size of pension benefits under a defined-benefit scheme is large, PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme increases the upper limit of the initial public debt-capital ratio below which a fiscal policy is sustainable and positively affects economic growth. These results imply that reforming a PAYG pension system from a defined-benefit scheme to a defined-contribution scheme may mitigate the negative effects of population aging on fiscal sustainability and economic growth. However, we also show that this type of PAYG pension reform entails an intergenerational conflict of interest between current and future generations. Population aging might exacerbate the extent of this conflict. These results imply that such a PAYG pension reform becomes less feasible in a rapidly aging society.

The present paper relates to the literature that examines the conditions for fiscal sustainability in the context of OLG growth models (e.g., Chalk, 2000; Rankin and Roffia, 2003; Bräuninger, 2005; Yakita, 2008; Arai, 2011; Rankin, 2014). This literature defines fiscal sustainability in terms of the existence of a steady-state equilibrium for a given fiscal policy. Among them, Bräuninger (2005) and Yakita (2008) are closely related to our contribution because these studies employ an endogenous growth framework. [6] Using an AK-type endogenous growth model, Bräuninger (2005) demonstrates that under a fiscal rule according to which the public spending-GDP ratio and budget deficit-GDP ratio are constant, there is a stable steady-state growth path as long as the initial public debt-capital ratio is below a certain threshold. Using an endogenous growth model with public capital, Yakita (2008) demonstrates that under the golden rule of public finance for public investment with a constant budget deficit-GDP ratio, there is a stable steady-state growth path as long as the initial public debt-capital ratio is below a certain threshold, which is increasing in the stock of public capital. However, these studies do not explicitly consider public pensions and are therefore unable to capture the effects of PAYG pension reform on fiscal sustainability. Therefore, this paper extends Bräuninger (2005) by explicitly incorporating a PAYG pension

[6] Arai (2011) also employs a Barro (1990) type endogenous growth model with productive government expenditure and shows that if the public spending-GDP ratio is small, an increase in the public spending-GDP ratio makes fiscal policy more sustainable.

system and examines the effects of population aging and PAYG pension reforms on fiscal sustainability and economic growth. To the best of our knowledge, these issues have yet to be examined extensively in the literature.

This paper also relates to the vast body of literature on public pensions and population aging in the context of OLG models (e.g., Zhang et al. 2001, 2003, 2005; Ono, 2003; Fanti and Gori, 2012; Artige et al, 2014; Tabata, 2015). However, most of these studies do not consider pension policy together with public debt. Moreover, there are few analytical studies that examine the effect of a transition from a defined-benefit scheme to a defined-contribution scheme on fiscal sustainability and economic growth, despite that these types of PAYG pension reforms have recently become common among OECD countries. To the best of our knowledge, Ono (2003) is an exceptional analytical study that analyzes a public pension financed by public debt. However, Ono (2003) employs a neoclassical growth framework and does not focus on economic growth or fiscal sustainability issues. To the best of our knowledge, Artige et al. (2014) and Tabata (2015) are exceptional studies that compare the macroeconomic performance of a defined-benefit PAYG pension scheme with that of a defined-contribution PAYG pension scheme in an aging society. In particular, Tabata (2015) is closely related to our contribution because it examines how reforming a PAYG pension system from a defined-benefit scheme to a defined-contribution scheme affects economic growth in an endogenous growth model. However, neither Artige et al. (2014) nor Tabata (2015) consider pension policy together with public debt and are therefore unable to capture the effects of PAYG pension reforms on fiscal sustainability. Therefore, this paper extends Tabata (2015) by explicitly incorporating public debt and examines how reforming a PAYG pension system from a defined-benefit scheme to a defined-contribution scheme affects fiscal sustainability and economic growth in an OLG model with endogenous growth.

This paper is organized as follows. Section 2 establishes the basic model. Section 3 describes the dynamic property of the economy and defines the concept of fiscal sustainability. Section 4 examines how reforming a PAYG pension system from a defined-benefit scheme to a defined-contribution scheme affects the relationship between population aging and fiscal sustainability. Section 5 analyzes how such a pension reform influences the prediction of the effect of population aging on economic growth. Section 6 discusses the welfare implications of PAYG pension reform. Section 7 concludes the paper.

2 The model

We consider a two-period OLG model of endogenous economic growth with a PAYG pension financed by a wage income tax and debt issuance.

2.1 Individuals

In each period, N_t individuals are born, and the population grows at the constant rate $n \in (-1, \infty)$. Individuals born in period t are called by generation t. Individuals live for a maximum of two periods (youth and old age). In youth, each individual is endowed with one unit of labor and inelastically supplies it. An individual dies at the beginning of old age with probability $1 - \pi$ and lives through old age with probability $\pi \in (0, 1]$. In each period t, there exist only two generations: the active working young and the retired old. Therefore, the old-age dependency ratio (i.e., the ratio of old dependents to the young working population) in period t is given by $\frac{\pi N_{t-1}}{N_t} = \frac{\pi}{1+n}$. Thus, population aging (i.e., an increase in the old-age dependency ratio) is triggered if there is either a decrease in the population growth rate n or an increase in the old-age survival probability π. In the following analysis, to avoid repetitive explanations, we focus primarily on the analysis of population aging caused by a decline in the population growth rate n. Appendix A provides some numerical simulation results for the case in which population aging is caused by an increase in the old-age survival probability rate π. Then, we confirm that the analogous predictions hold in both cases.

Each individual derives utility from his or her own consumption in both youth and old age. Thus, the lifetime expected utility of an agent in generation t is expressed as

$$u_t = \ln c_t^y + \beta\pi \ln c_{t+1}^o, \tag{1}$$

where c_t^y is consumption in youth and c_{t+1}^o is consumption in old age. As in Bräuninger (2005) and Yakita (2008), the instantaneous utility function is assumed to be logarithmic for the sake of tractability.

In youth, each individual inelastically supplies one unit of labor, earns wage income w_t on which the wage income tax is levied at rate τ_t, and allocates his or her disposable income $(1 - \tau_t)w_t$ to current consumption c_t^y and saving s_t. Following Blanchard (1985), we assume the existence of actuarially fair insurance for the sake of simplicity. The insurance company promises each individual a payment $\frac{R_{t+1}}{\pi} s_t$ in exchange for which the individual's estate s_t accrues to the company, where π is the average survival probability and R_{t+1} is the gross rate of interest. In old age, survivors retire and consume their returns on private savings $\frac{R_{t+1}}{\pi} s_t$ and pension benefits P_{t+1}. Thus, the budget constraints of an agent from generation t are

$$c_t^y + s_t = (1 - \tau_t)w_t, \tag{2}$$

$$c_{t+1}^o = \frac{R_{t+1}}{\pi} s_t + P_{t+1}. \tag{3}$$

By maximizing (1), subject to (2) and (3), we obtain

$$s_t = \frac{\beta\pi}{1+\beta\pi}\left[(1-\tau_t)w_t - \frac{P_{t+1}}{\beta R_{t+1}}\right]. \tag{4}$$

This saving equation states that a higher old-age survival probability π implies higher savings, whereas both higher pension benefits P_{t+1} and higher wage income tax rates τ_t imply lower savings.

2.2 Production

Each firm has constant returns-to-scale technology, and the aggregate production function is expressed as

$$Y_t = \theta K_t^{\alpha}(A_t N_t)^{1-\alpha}, \tag{5}$$

where θ, Y_t, K_t, and N_t denote the fixed technology parameter and the aggregate levels of output, physical capital, and labor input, respectively. A_t represents labor productivity, which is assumed to be driven by a positive spillover from the size of the aggregate capital stock to the productivity of workers in the manner suggested by Romer (1986). To ensure the existence of a balanced growth path, as in Grossman and Yanagawa (1993), we assume that A_t takes the following form as $A_t \equiv \frac{K_t}{N_t}$. We assume that capital fully depreciates within one period. Solving a representative firm's problem, we obtain the optimal conditions for the firm as follows

$$R_t = \alpha\theta\left(\frac{K_t}{A_t N_t}\right)^{\alpha-1} = \alpha\theta = R, \tag{6}$$

$$w_t = (1-\alpha)\theta A_t\left(\frac{K_t}{A_t N_t}\right)^{\alpha} = (1-\alpha)\theta\frac{K_t}{N_t} = \bar{w}k_t, \tag{7}$$

where $R \equiv \alpha\theta$, $\bar{w} \equiv (1-\alpha)\theta$ and $k_t \equiv K_t/N_t$. The gross interest rate is constant over time whereas the wage rate per labor is proportional to the capital per worker. By using $A_t \equiv \frac{K_t}{N_t}$ and $\bar{w}$, GDP per worker is rewritten as

$$y_t \equiv \frac{Y_t}{N_t} = \theta k = \frac{\bar{w}}{1-\alpha}k_t, \tag{8}$$

that is, (5) and $A_t \equiv \frac{K_t}{N_t}$ generate an AK-type production function.

2.3 Government

2.3.1 Budget constraints

The government pursues a social security policy based on a PAYG pension system. As there are πN_{t-1} old agents who receive pension benefits P_t, government spending in period t is given by $P_t \pi N_{t-1}$. The government finances its spending through a wage income tax and the issuance of public debt. Thus, the government's budget constraint in period t is

$$B_{t+1} = (1 + r)B_t + P_t \pi N_{t-1} - \tau_t w_t N_t, \tag{9}$$

where B_t is the stock of public debt in period t and $\tau_t w_t N_t$ is the revenue from a wage income tax. Because the risk-free gross interest rate is equated to the gross capital interest rate through arbitrage (i.e., $1 + r_t = R$), r_t is also constant at R (i.e., $r_t = r$).

2.3.2 Fiscal rule regarding newly issued debt

The government conforms to two distinct fiscal policy rules regarding newly issued debt and pension payments. Let us first explain the fiscal rule regarding newly issued debt. The government borrows a specified fraction μ of national income Y_t and attempts to hold the budget deficit-GDP ratio constant at μ:

$$B_{t+1} - B_t = \mu Y_t. \tag{10}$$

This type of fiscal rule is employed in the EU and other OECD countries.[7] Existing studies such as Bräuninger (2005) and Yakita (2008) also employ an analogous assumption. We follow this tradition for the sake of simplicity in the following analysis.

2.3.3 Fiscal rule regarding pension payments

Next, let us explain the fiscal rule regarding pension payments. To formulate the transition from a defined-benefit scheme to a defined-contribution scheme in the simplest manner, as in Tabata (2015), the pension payment for old agents in generation $t - 1$, P_t, is determined by the following payment rule:

$$P_t = \left[\psi \frac{N_t}{\pi N_{t-1}} \eta + \phi(1 - \eta)\right] w_t = \left[\psi \frac{1 + n}{\pi} \eta + \phi(1 - \eta)\right] w_t. \tag{11}$$

Equation (11) indicates that the pension payment for old agents in generation $t - 1$, P_t, adjusted according to changes in the current generation's wages w_t (i.e.,

[7] Japanese law is supposed to limit the issuance of deficit-covering bonds. The government is allowed to borrow only to invest and not to fund current spending (i.e., golden rule of public finance). A special deficit-financing bill must be submitted for each fiscal year to issue deficit-covering bonds. Because this paper does not consider public investment that explicitly benefits future generations, in our context, Japanese fiscal law technically holds that the budget deficit-GDP ratio μ must be zero. However, this is far from reality. At present, Japanese government intends to bring the primary balance into the black by 2020.

wage indexation). The term $\psi\frac{1+n}{\pi}\eta + \phi(1-\eta)$ denotes the replacement ratio for the current generation's wages w_t. The replacement ratio is defined as a convex combination of a demographic adjustment component (i.e., $\psi\frac{1+n}{\pi}$) and a fixed coefficient component (i.e., ϕ). The index $\eta \in [0,1]$ measures the weight assigned to the demographic adjustment component. A demographic adjustment component (i.e., $\psi\frac{1+n}{\pi}$) characterizes a demographically modified indexation in which the wage indexation rate for pension benefits automatically decreases with population aging. In addition, the index $\psi \in [0,1)$ measures the size of pension payments in the demographic adjustment component, whereas $\phi \in [0,1)$ measures the size of pension payments in the fixed coefficient component.

Given (11), suppose that $\eta = 0$, the per capita pension benefit of generation $t-1$, is given simply by $P_t = \phi w_t$ and that the wage replacement ratio is constant ϕ irrespective of the old-age dependency ratio $\frac{\pi}{1+n}$. In this case, the PAYG pension scheme is a pure defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$. Next, suppose that $\eta = 1$, the per capita pension benefit of generation $t-1$, is fully adjusted in response to changes in demographic conditions and satisfies the following equality condition: $\psi w_t N_t = P_t \pi N_{t-1}$. As explained in Appendix B, this equality condition is the well-known formulation of the government budget constraint under a pure defined-contribution scheme. Therefore, in this case, the PAYG pension scheme is a pure defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$. Finally, suppose that $\eta \in (0,1)$. In this case, the PAYG pension scheme is a convex combination of the two aforementioned schemes. We denote this type of pension system as a mixed-payment scheme. Under a mixed-payment scheme, as the value of η approaches 1, the weight assigned to the defined-contribution component increases. Therefore, the transition from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0,1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0,1)$ is described simply by the increase in the value of η from 0 to 1.

As explained in the Introduction, as many OECD countries employ demographically modified indexation programs, their PAYG pension systems possess both defined-benefit and defined-contribution components. Our simple specification enables us to capture this combination of attributes in a reduced-form manner. Of course, our mixed-payment scheme is rather abstract and cannot capture the complex structures of recent PAYG pension systems in their entirety. Nevertheless, this simple framework enables us to capture several realistic features of recent pension reforms.

By differentiating (11) with respect to n and η, we obtain

$$\frac{\partial P_t}{\partial n} \begin{cases} > 0, & \text{for } \eta \in (0,1], \\ = 0, & \text{for } \eta = 0, \end{cases} \tag{12}$$

$$\frac{\partial P_t}{\partial \eta} \begin{cases} \leq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ > 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{13}$$

Equation (12) indicates that when a PAYG pension system is financed by a pure defined-contribution scheme or a mixed-payment scheme (i.e., $\eta \in (0, 1]$), a decline in the population growth rate implies a lower per capita pension benefit. However, when a PAYG pension system is financed by a pure defined-benefit scheme (i.e., $\eta = 0$), the per capita pension benefit is unaffected by the population growth rate.

Further, Equation (13) indicates that assigning a larger weight η to the defined-contribution component implies a lower (resp. higher) per capita pension benefit when (i) the old-age dependency ratio $\frac{\pi}{1+n}$ is relatively high (resp. low); (ii) the fixed contribution rate ψ under a defined-contribution scheme is relatively small (resp. large); or (iii) the fixed replacement ratio ϕ under a defined-benefit scheme is sufficiently large (resp. small) to satisfy $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$).

2.3.4 Determination of wage income tax rate

Finally, let us explain the endogenous determination of the wage income tax rate τ_t. Due to the budget deficit rule (10) and the pension payment rule (11), the level of government spending and of newly issued debt have already been determined, and thus, the wage income tax rate τ_t has to be adjusted to meet the government budget constraint (9). Hence, by substituting (7), (10) and (11) into (9) and dividing it by $w_t N_t$, we obtain

$$\tau_t = \left[\frac{r}{w} x_t + \tilde{\tau}(\eta, n) - \frac{\mu}{1-\alpha} \right] \equiv \tau(x_t; \tilde{\tau}(\eta, n)), \tag{14}$$

where

$$\tilde{\tau}(\eta, n) \equiv \psi\eta + \phi \frac{\pi}{1+n}(1-\eta), \tag{15}$$

$x_t \equiv \frac{b_t}{k_t}$ and $b_t \equiv \frac{B_t}{N_t}$. To focus our analysis on the case in which the wage income tax is nonnegative for any positive value of the public debt-capital ratio x_t, we assume that the following parameter conditions hold.

$$\tilde{\tau}(\eta, n) \geq \frac{\mu}{1-\alpha}. \tag{A1}$$

Here, suppose there is no newly issued public debt (i.e., $\mu = 0$) or any stock of public debt (i.e., $x_t = 0$); hence, the relationship $\tau_t = \tilde{\tau}(\eta, n)$ holds. Therefore, (A1) is more likely to hold when the wage tax rate in the no-public-debt case $\tilde{\tau}(\eta, n)$ is sufficiently high or the budget deficit-GDP ratio μ is sufficiently low.

By differentiating (14) with respect to n and η, we obtain

$$\frac{\partial \tau}{\partial n} = \frac{\partial \tilde{\tau}(\eta, n)}{\partial n} \begin{cases} < 0, & \text{for } \eta \in [0, 1), \\ = 0, & \text{for } \eta = 1, \end{cases} \tag{16}$$

$$\frac{\partial \tau}{\partial \eta} = \frac{\partial \tilde{\tau}(\eta, n)}{\partial \eta} \begin{cases} \leq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ > 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{17}$$

Equation (16) indicates that when a PAYG pension system is financed by a pure defined-benefit scheme or mixed-payment scheme (i.e., $\eta \in [0, 1)$), a decline in the population growth rate implies a higher wage income tax rate. However, when a PAYG pension system is financed by a pure-contribution scheme (i.e., $\eta = 1$), the wage income tax rate is unaffected by the population growth rate.

Further, Equation (17) indicates that assigning a larger weight η to the defined-contribution component implies a lower (resp. higher) wage income tax rate when (i) the old-age dependency ratio $\frac{\pi}{1+n}$ is relatively high (resp. low); (ii) the fixed contribution rate ψ under a defined-contribution scheme is relatively small (resp. large); or (iii) the fixed replacement ratio ϕ under a defined-benefit scheme is sufficiently large (resp. small) to satisfy $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{p}{1+n} < \frac{\psi}{\phi}$).

3 General equilibrium and fiscal sustainability

This section characterizes the competitive equilibrium allocation of the economy. Further, we explicitly define the concept of fiscal sustainability.

3.1 Market clearing and dynamics

First, let us consider the market equilibrium conditions. Noting that an arbitrage opportunity exists between a real asset and bond, we can set the market clearing condition for capital as $K_{t+1} + B_{t+1} = s_t N_t$, which expresses the equality of the total savings by young agents in generation t, $s_t N_t$, to the sum of the stocks of aggregate physical capital and aggregate public debt. Dividing both sides by N_t leads to

$$(1 + n)(k_{t+1} + b_{t+1}) = s_t. \tag{18}$$

The commodity market clears automatically according to Walras's law.

Next, let us consider the dynamic properties of the economy, which are summarized in Figure 1. First, we consider the dynamics of the public debt-labor ratio b_t. Dividing both sides of (10) by N_t and substituting (8) into it, we obtain the dynamic evolution of the public debt-labor ratio b_t as follows:

$$\frac{b_{t+1}}{b_t} = \frac{1}{1+n} bg(x_t;\mu), \tag{19}$$

where

$$bg(x_t;\mu) \equiv 1 + \frac{\mu \bar{w}}{1-\alpha}\frac{1}{x_t}.$$

From (19), we can confirm that the relationships $\frac{\partial bg(x_t;\mu)}{\partial x_t} < 0$, $\lim_{x_t \to 0} bg(x_t;\mu) = \infty$, $\lim_{x_t \to \infty} bg(x_t;\mu) = 1$ hold. In addition, $bg(x_t;\mu)$ satisfies the following property: $\frac{\partial bg(x_t;\mu)}{\partial \mu} > 0$. This implies that a larger budget deficit-GDP ratio μ shifts the $bg(x_t;\mu)$ curve upward.

We then consider the dynamics of the capital-labor ratio k_t. Using (15), the per capita pension benefit P_t in (11) is rewritten as $P_t = \frac{1+n}{\pi}\tilde{\tau}(\eta,n)w_t$. By substituting (4), (6), (7), (14), (19) and $P_t = \frac{1+n}{\pi}\tilde{\tau}(\eta,n)w_t$ into (18) and rearranging it, we obtain the dynamic evolution of the capital-labor ratio k_t as follows:

$$\frac{k_{t+1}}{k_t} = \frac{1}{1+n} kg(x_t;\tilde{\tau}(\eta,n),\mu), \tag{20}$$

where

$$kg(x_t;\tilde{\tau}(\eta,n),\mu) \equiv \frac{\left\{\beta\pi[1-\tilde{\tau}(\eta,n)] - \frac{\mu}{1-\alpha}\right\}\frac{\bar{w}}{1+\beta\pi} - (1+\frac{\beta\pi}{1+\beta\pi}r)x_t}{1+\frac{1}{1+\beta\pi}\frac{\bar{w}}{R}\tilde{\tau}(\eta,n)}.$$

From (20), we can confirm that the relationships $\frac{\partial kg(x_t;\tilde{\tau}(\eta,n),\mu)}{\partial x_t} < 0$, $\lim_{x_t \to 0} kg(x_t;\tilde{\tau}(\eta,n),\mu) < \infty$, $\lim_{x_t \to \infty} kg(x_t;\tilde{\tau}(\eta,n),\mu) = -\infty$ hold. In addition, $kg(x_t;\tilde{\tau}(\eta,n),\mu)$ satisfies the following property: $\frac{\partial kg(x_t;\tilde{\tau}(\eta,n),\mu)}{\partial \mu} < 0$. This implies that a larger budget deficit-GDP ratio μ shifts the $kg(x_t;\tilde{\tau}(\eta,n),\mu)$ curve downward.

The dynamics of this economy are completely described by the two difference equations of (19) and (20). Because the evolution of the capital-labor ratio k_t and the public debt-labor ratio b_t depends upon the public debt-capital ratio x_t, the evolution of x_t is given by the following:

$$\frac{x_{t+1}}{x_t} = \frac{\frac{b_{t+1}}{b_t}}{\frac{k_{t+1}}{k_t}} = \frac{bg(x_t;\mu)}{kg(x_t;\tilde{\tau}(\eta,n),\mu)}. \tag{21}$$

Here, note that the following inequality holds.

$$x_{t+1} \geq x_t \Leftrightarrow bg(x_t; \mu) \geq kg(x_t; \tilde{\tau}(\eta, n), \mu). \quad (22)$$

The steady-state equilibrium is then characterized by the conditions that $x_{t+1} = x_t = x$ or:

$$bg(x; \mu) = kg(x; \tilde{\tau}(\eta, n), \mu). \quad (23)$$

From (19), (20) and (23), in the steady-state equilibrium, the state variable k_t and b_t grow at the same rate.

Figure 1 depicts the relationship between $bg(x_t; \mu)$ and $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ where there exist two steady state equilibria E_1 and E_2. Steady state E_1 is characterized by a lower public debt-capital ratio x_1^*, whereas steady state E_2 is characterized by a higher public debt-capital ratio x_2^*. Given the value of x_t, $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ is decreasing in μ, whereas $bg(x_t; \mu)$ is increasing in μ. Therefore, the case in which two steady-state equilibria exist is likely to occur when the budget deficit-GDP ratio μ is sufficiently small.[8] From Figure 1, the inequality $kg(x_t; \tilde{\tau}(\eta, n), \mu) > bg(x_t; \mu)$ holds when $x_1^* < x_t < x_2^*$. In this region, because the capital-labor ratio k_t grows faster than the public debt-labor ratio b_t, the public debt-capital ratio x_t decreases monotonically (i.e., $\frac{x_{t+1}}{x_t} < 1$). However, when $x_t < x_1^*$ or $x_t > x_2^*$, the inequality $kg(x_t; \tilde{\tau}(\eta, n), \mu) < bg(x_t; \mu)$ holds. In these regions, the public debt-labor ratio b_t grows faster than the capital-labor ratio k_t, and the public debt-capital ratio x_t increases monotonically (i.e., $\frac{x_{t+1}}{x_t} > 1$). Therefore, if the initial public debt-capital ratio x_0 is below the critical level x_2^*, the public debt-capital ratio gradually converges to the steady state x_1^*. Thus, steady state E_1 is characterized by a lower public debt-capital ratio x_1^* and locally stable. However, if the initial public debt-capital ratio x_0 is above the critical level x_2^*, the public debt-capital ratio x_t increases monotonically, and the level of the capital-labor ratio k_t decreases from (20) and will go to zero within a finite number of periods.[9] Once the level of the capital-labor ratio k_t reaches zero, there is no commodity in the economy, and hence the value of government debt vanishes. Thus, such a situation can be regarded as government bankruptcy.

Figure 2 illustrates the case in which no steady-state equilibrium exists. This case is likely to occur when the budget deficit-GDP ratio μ is sufficiently large. In this case, as shown in Figure 2, the inequality $bg(x_t; \mu) > kg(x_t; \tilde{\tau}(\eta, n), \mu)$ holds for all $x_t > 0$. Thus, regardless of the initial public debt-capital ratio $x_0 > 0$, the public debt-capital ratio x_t increases monotonically (i.e., $\frac{x_{t+1}}{x_t} > 1$), and the level of the capital-labor ratio k_t decreases from (20) and goes to zero within a finite

[8]See Appendix C details on the the parameter conditions that ensure the existence of the steady-state equilibria.

[9]See Appendix D for details on the stability property of the steady-state equilibria.

number of periods. Hence, the economy always follows the path to government bankruptcy.

Hereafter, for clarity of exposition, we focus our analysis on the case in which there exist two steady-state equilibria given the demographic conditions (π, n) and fiscal policy parameters (η, ψ, ϕ, μ), as in the case of Figure 1.

3.2 Definition of fiscal sustainability

Before concluding this section, to clarify the following discussion, we explicitly define the concept of fiscal sustainability as employed in our paper. Based upon Arai (2011), we define the following two terms regarding the sustainability of fiscal policy.

Definition 1 (a sustainable fiscal policy) *A fiscal policy is sustainable if there exists an initial value of the public debt-capital ratio such that the ratio converges to some finite value in the long run.*

Definition 2 (a more sustainable fiscal policy) *A fiscal policy becomes more sustainable if the range of the initial public debt-capital ratio for which the public debt-capital ratio converges to some finite level widens due to changes in the fiscal policy parameters* (η, ψ, ϕ, μ) *or demographic conditions* (π, n).

In our paper, as described in Figure 1, the steady-state value of x_2^* determines the upper limit of the initial public debt-capital ratio below which the public debt-capital ratio converges to a finite value of x_1^*. This means that a higher value of x_2^* leads to a wider range of the initial public debt-capital ratio for which the public debt-capital ratio converges to the finite value of x_1^*. Therefore, we can state that a fiscal policy becomes more (resp. less) sustainable if the value of x_2^* increases (resp. decreases) due to changes in the fiscal policy parameters (η, ψ, ϕ, μ) or demographic conditions (π, n).

4 Fiscal sustainability

In this section, we examine how population aging caused by a decline in the population growth rate n influences the sustainability of a fiscal policy. We also consider how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$ affects theoretical predictions of the effect of population aging on fiscal sustainability.

4.1 Population aging and fiscal sustainability

Let us first examine how population aging caused by a decline in the population growth rate n influences the sustainability of a fiscal policy. From (19), a decline in the population growth rate n has no effect on the $bg(x_t;\mu)$ curve. However, given $\frac{\partial kg}{\partial \tilde{\tau}} < 0$ and (16), by differentiating $kg(x_t;\tilde{\tau}(\eta,n),\mu)$ in (20) with respect to n, we obtain

$$\frac{\partial kg(x_t;\tilde{\tau}(\eta,n),\mu)}{\partial n} = \underbrace{\frac{\partial kg}{\partial \tilde{\tau}}}_{(-)} \frac{\partial \tilde{\tau}(\eta,n)}{\partial n} \begin{cases} > 0, & \text{for } \eta \in [0,1), \\ = 0, & \text{for } \eta = 1. \end{cases} \tag{24}$$

As described in Figure 3, Equation (24) indicates that when a PAYG pension system is financed by a pure defined-benefit scheme or a mixed-payment scheme (i.e., $\eta \in [0,1)$), a decline in the population growth rate shifts the $kg(x_t;\tilde{\tau}(\eta,n),\mu)$ curve downward and thus increases the value of x_1^* and decreases the value of x_2^*. However, when a PAYG pension system is financed by a pure defined-contribution scheme (i.e., $\eta = 1$), a decline in the population growth rate n has no effect on the $kg(x_t;\tilde{\tau}(\eta,n),\mu)$ curve, and thus, both x_1^* and x_2^* remain unchanged. In fact, by differentiating (23) with respect to n, we obtain the following results:[10]

$$\frac{\partial x_1^*(\eta,n,\mu)}{\partial n} \begin{cases} < 0, & \text{for } \eta \in [0,1), \\ = 0, & \text{for } \eta = 1, \end{cases} \tag{25}$$

$$\frac{\partial x_2^*(\eta,n,\mu)}{\partial n} \begin{cases} > 0, & \text{for } \eta \in [0,1), \\ = 0, & \text{for } \eta = 1. \end{cases} \tag{26}$$

Equation (26) indicates that when a PAYG pension system is financed by a pure defined-benefit scheme or a mixed-payment scheme (i.e., $\eta \in [0,1)$), a decline in the population growth rate decreases the value of x_2^*, reduces the range of the initial public debt-capital ratio for which x_t converges to x_1^* and thus makes a fiscal policy less sustainable. In particular, an excessive decline in the population growth rate n shifts the $kg(x_t;\tilde{\tau}(\eta,n),\mu)$ curve downward substantially, which may eliminate any steady-state equilibria as described in Figure 2. In this case, a fiscal policy is no longer sustainable for any positive initial public debt-capital ratio. However, Equation (26) also indicates that when PAYG pension is financed by a pure defined-contribution scheme (i.e., $\eta = 1$), the value of x_2^* is unaffected by the population growth rate n. Therefore, a decline in the population growth rate does not necessarily make a fiscal policy less sustainable.

[10]From (23), the steady state value of x_i^* ($i = 1,2$) depends upon the values of η, n and μ. To stress these relationships, we denote x_i^* as $x_i^*(\eta,n,\mu)$.

These results indicate that population aging caused by a decline in the population growth rate may negatively affect the sustainability of a fiscal policy, but the extent of its negative effect depends substantially on the type of PAYG pension scheme. The mechanism underlying these theoretical results is explained as follows. Under a pure defined-benefit scheme or a mixed-payment scheme (i.e., $\eta \in [0, 1)$), from (14), a decline in the population growth rate n increases the old-age dependency ratio $\frac{\pi}{1+n}$, which positively affects the wage income tax rate τ_t. This higher wage income tax rate τ_t leads to a lower rate of saving by young individuals, which negatively affects capital accumulation and thus shifts the $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ curve downward. We denote this negative effect of a decline in n on capital accumulation the "tax burden effect". Under a pure defined-contribution scheme (i.e., $\eta = 1$), however, the wage income tax rate is unaffected by the value of the old-age dependency ratio. Therefore, this "tax burden effect" does not exist, and thus, the $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ curve is not affected by a decline in the population growth rate.

4.2 PAYG pension reform and fiscal sustainability

Next, let us examine how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution with a fixed contribution rate $\psi \in [0, 1)$ affects the sustainability of a fiscal policy. In this paper, this type of pension reform is described by assigning an increased weight η to the defined-contribution component. Given $\frac{\partial kg}{\partial \tilde{\tau}} < 0$ and (17), by differentiating $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ with respect to η, we obtain

$$\frac{\partial kg(x_t; \tilde{\tau}(\eta, n), \mu)}{\partial \eta} = \underbrace{\frac{\partial kg}{\partial \tilde{\tau}}}_{(-)} \frac{\partial \tilde{\tau}(\eta, n)}{\partial \eta} \begin{cases} \geq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ < 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{27}$$

As described in Figures 4 and 5, Equation (27) indicates that when the condition $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) holds, an increase in the weight η assigned to the defined-contribution component shifts the $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ curve upward (resp. downward) and thus decreases (resp. increases) the value of x_1^* and increases (resp. decreases) the value of x_2^*. By differentiating (23) with respect to η, we obtain the following results:

$$\frac{\partial x_1^*(\eta, n, \mu)}{\partial \eta} \begin{cases} \leq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ > 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}, \end{cases} \tag{28}$$

$$\frac{\partial x_2^*(\eta, n, \mu)}{\partial \eta} \begin{cases} \geq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ < 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{29}$$

Equation (29) indicates that when the condition $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) holds, an increase in the weight η assigned to the defined-contribution component increases (resp. decreases) the value of x_2^*, expands (resp. contracts) the range of the initial public debt-capital ratio for which x_t converges to x_1^* and thus makes a fiscal policy more (resp. less) sustainable. Therefore, the transition from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$ positively (resp. negatively) affects the sustainability of a fiscal policy when (i) the old-age dependency ratio $\frac{\pi}{1+n}$ is relatively high (resp. low); (ii) the fixed contribution rate ψ under a defined-contribution scheme is relatively small (resp. large); or (iii) the fixed replacement ratio ϕ under a defined-benefit scheme is sufficiently large (resp. small) to satisfy $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$).

The mechanism underlying these theoretical results is explained as follows. Based on (13) and (17), when the condition $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) holds, an increase in the weight η assigned to the defined-contribution component negatively (resp. positively) affects both per capita pension benefits and the wage income tax rate. According to (20), the lower (resp. higher) wage income tax rate τ_t and per capita pension benefits P_{t+1} motivate young individuals to save more (resp. less) for their old age, which positively (resp. negatively) affects capital accumulation and thus shifts the $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ curve upward (resp. downward).

Now, let us consider how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$ affects theoretical predictions of the effect of population aging on fiscal sustainability. Figure 6 depicts numerical examples of the relationship between the population growth rate n and the steady-state value of the debt-capital ratio x_2^* under alternative weights η on the defined-contribution component (i.e., $\eta = 0$, 0.3, 0.6 and 1). The parameters used in the baseline simulations are as follows: $\alpha = 0.3$, $\beta = 1$, $\pi = 0.9$, $1 + n = 0.8$, $\phi = 0.3$, $\psi = 0.3$, $\mu = 0.01$ and $\theta = 14.71$.[11] Note that the objective of these numerical examples is not to calibrate our simple model to actual data but to supplement the qualitative results. The quantitative results obtained in this paper

[11] One period in this model is assumed to be approximately 30 years. Based on estimates of the labor income share, we set the value of α at 0.3. The period life tables in Japan indicate that in 2015, the probability of a man surviving to age 65 conditional on his reaching age 15 is 0.88. Thus, the benchmark value of the old-age survival probability π is set to 0.9. In addition, to achieve a TFR of 1.6 per couple, the value of the gross population growth rate $1 + n$ is set to 0.8. To achieve a 30 % benefit level under a pure defined-benefit scheme, the value of the fixed replacement ratio ϕ is set to 0.3. Analogously, to achieve a 30% fixed contribution rate under a pure defined-contribution scheme, the value of the fixed contribution rate ψ is set to 0.3. Moreover, we set the value of μ at 0.01 and the value of β at 1 to ensure the existence of two steady-state equilibria. Finally, to achieve a 3% per capita GDP growth rate at benchmark values, the value of θ was adjusted to 14.71.

should be interpreted with caution. Consistent with the results of (26), when $\eta \in [0, 1)$ (i.e., under a pure defined-benefit scheme or a mixed-payment scheme), the steady-state value of x_2^* decreases monotonically as the population growth rate declines. However, when $\eta = 1$ (i.e., under a pure defined-contribution scheme), the steady-state value of x_2^* remains the same irrespective of the value of the weight η assigned to the defined-contribution component.[12]

Figure 6 also shows how the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) affects the relationship between the population growth rate n and the steady-state value of x_2^*. As shown in (29), this type of PAYG pension reform positively (resp. negatively) affects the steady-state value of x_2^* when $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) or $1 + n < \frac{\pi\phi}{\psi}$ (resp. $1 + n > \frac{\pi\phi}{\psi}$). From (14), given the value of x_t, the overall wage income tax rates remain the same regardless of the value of η when $1+n = \frac{\pi\phi}{\psi}$. Therefore, based on (23), the steady-state value of x_2^* remains the same regardless of the value of η when $1 + n = \frac{\pi\phi}{\psi}$, as shown in Figure 6. However, (14) also means that when $1 + n < \frac{\pi\phi}{\psi}$ (resp. $1 + n > \frac{\pi\phi}{\psi}$), the overall wage income tax rate decreases (resp. increases) with the value of η, which shifts the $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ curve upward (resp. downward). Therefore, from (23), the steady-state value of x_2^* increases (resp. decreases) with the value of η.

These results suggest that in economies in which the population growth rate is relatively low and the size of pensions under a defined-benefit scheme is sufficiently large to satisfy $1+n < \frac{\pi\phi}{\psi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) mitigates or overcomes the negative effect of population aging (caused by a decline in the population growth rate) on fiscal sustainability. The intuitions are explained as follows. Under (14) and (15), giving a larger weight η to the defined-contribution component decreases the impact of changes in the old-age dependency ratio on the wage income tax rate, which alleviates the negative effect of a decline in n on fiscal sustainability due to the tax burden effect. Therefore, in economies in which the population growth rate is relatively low and the size of pensions under a defined-benefit scheme is sufficiently large to satisfy $1 + n < \frac{\phi}{\psi}$, the transition from a defined-benefit scheme with a fixed replacement ratio ϕ to a defined-contribution scheme with a fixed contribution rate ψ mitigates or even overcomes the negative effect of population aging (caused by a decline in the population growth rate) on fiscal sustainability.

[12]Note that if the population growth rate is excessively low, the steady-state value of x_2^* cannot be defined when $\eta \in [0, 1)$ in our simulation because there are no steady-state equilibria, as shown in Figure 2. In our baseline simulation, for example, if $\eta = 0$, the steady-state value of x_2^* cannot be defined when $1 + n < 0.7$.

5 Economic growth

In this section, we examine how population aging caused by a decline in the population growth rate n affects economic growth. We also consider how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$ affects theoretical predictions of the effect of population aging on economic growth.

5.1 Population aging and economic growth

Let us first examine how population aging caused by a decline in the population growth rate n affects economic growth. For simplicity, we focus our analysis on the case in which the economy is in a stable steady-state equilibrium E_1 in which the state variables k_t and b_t grow at the same rate. In this case, from (8) and (20), the gross per capita rate of output growth is described by

$$\frac{y_{t+1}}{y_t}|_{x_t=x_1^*} = \frac{k_{t+1}}{k_t}|_{x_t=x_1^*} = \frac{kg(x_1^*(\eta, n, \mu); \tilde{\tau}(\eta, n), \mu)}{1+n} \equiv G^*(x_1^*(\eta, n, \mu), \tilde{\tau}(\eta, n), n, \mu). \tag{30}$$

Then, given (16), (25), $\frac{\partial G^*}{\partial \tilde{\tau}} < 0$, $\frac{\partial G^*}{\partial x_1^*} < 0$ and $\frac{\partial G^*}{\partial n} < 0$, by differentiating (30) with respect to n, we obtain

$$\frac{dG^*(x_1^*(\eta, n, \mu), \tilde{\tau}(\eta, n), n)}{dn} = \underbrace{\frac{\partial G^*}{\partial \tilde{\tau}}}_{(-)} \frac{\partial \tilde{\tau}(\eta, n)}{\partial n} + \underbrace{\frac{\partial G^*}{\partial x_1^*}}_{(-)} \frac{\partial x_1^*(\eta, n, \mu)}{\partial n} + \underbrace{\frac{\partial G^*}{\partial n}}_{(-)}. \tag{31}$$

where

$$\underbrace{\frac{\partial G^*}{\partial \tilde{\tau}}}_{(-)} \frac{\partial \tilde{\tau}(\eta, n)}{\partial n} \begin{cases} > 0, & \text{for } \eta \in [0, 1), \\ = 0, & \text{for } \eta = 1, \end{cases}$$

$$\underbrace{\frac{\partial G^*}{\partial x_1^*}}_{(-)} \frac{\partial x_1^*(\eta, n, \mu)}{\partial n} \begin{cases} > 0, & \text{for } \eta \in [0, 1), \\ = 0, & \text{for } \eta = 1. \end{cases}$$

Equation (31) indicates that when a PAYG pension system is financed by a pure defined-benefit scheme or a mixed-payment scheme (i.e., $\eta \in [0, 1)$), a decline in the population growth rate n has two negative effects and one positive effect on capital accumulation. The first negative effect is the "tax burden effect," which is reflected in the first and the second terms on the right-hand side (RHS) of (31). Equation (14) implies that a decline in the population growth rate n positively affects the wage income tax rate τ_t in the following two ways. First, a decline in n increases the dependency ratio $\frac{\pi}{1+n}$, which positively affects the wage income tax rate τ_t. Second, a decline in n increases the debt-capital ratio x_1^*, as shown in (25), which positively affects the wage income tax rate τ_t. These two positive effects of a decline in n on τ_t lead to a lower rate of saving by young individuals, which negatively affects capital accumulation. The second negative effect is the "substitution effect" from physical capital accumulation to public debt-holding, which is reflected in the second term on the RHS of (31). As shown in (25), a decline in the population growth rate n increases the steady-state value of the public debt-capital ratio x_1^*, which implies a shift in savings from physical capital accumulation to public debt-holding. Thus, this substitution effect negatively affects capital accumulation. The one positive effect is the "anti-dilution effect," which is reflected in the third term on the RHS of (31). From (30), a decline in the population growth rate n increases the ratio of old to young in the population, which mitigates the dilution of savings into a larger workforce. Thus, this anti-dilution effect positively affects capital accumulation. However, Equation (31) also indicates that when a PAYG pension system is financed by a pure defined-contribution scheme (i.e., $\eta = 1$), the two aforementioned negative effects of a decline in the population growth rate n on capital accumulation disappear and only the positive "anti-dilution effect" exists. Therefore, when a PAYG pension system is financed by a pure defined-contribution scheme (i.e., $\eta = 1$), a decline in the population growth rate n always increases the rate of per capita output growth.

Unfortunately, as the steady-state value of x_1^* is determined by a somewhat complicated form of the implicit function shown in (23), it is difficult to further investigate this issue analytically. Therefore, in the next subsection, we provide some numerical examples. However, to clarify the following discussion, in the next subsection, we first examine how reforming a PAYG pension system from a pure defined-benefit scheme (i.e., $\eta = 0$) to a pure defined-contribution scheme (i.e., $\eta = 1$) affects economic growth.

5.2 PAYG pension reform and economic growth

Let us then examine how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution with a fixed contribution rate $\psi \in [0, 1)$ affects economic growth. For simplicity, we again focus our analysis on the case in which the economy is in the stable steady-state equilibrium E_1.

Given (17),(28), $\frac{\partial G^*}{\partial \tilde{\tau}} < 0$ and $\frac{\partial G^*}{\partial x_1^*} < 0$, by differentiating (30) with respect to η, we obtain

$$\frac{dG^*(x_1^*(\eta, n, \mu), \tilde{\tau}(\eta, n), n, \mu)}{d\eta} = \underbrace{\frac{\partial G^*}{\partial \tilde{\tau}}}_{(-)} \frac{\partial \tilde{\tau}(\eta, n)}{\partial \eta} + \underbrace{\frac{\partial G^*}{\partial x_1^*}}_{(-)} \frac{\partial x_1^*(\eta, n, \mu)}{\partial \eta} \begin{cases} \geq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ < 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}. \end{cases} \tag{32}$$

where

$$\underbrace{\frac{\partial G^*}{\partial \tilde{\tau}}}_{(-)} \frac{\partial \tilde{\tau}(\eta, n)}{\partial \eta} \begin{cases} \geq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ < 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}, \end{cases}$$

$$\underbrace{\frac{\partial G^*}{\partial x_1^*}}_{(-)} \frac{\partial x_1^*(\eta, n, \mu)}{\partial \eta} \begin{cases} \geq 0, & \text{for } \frac{\pi}{1+n} \geq \frac{\psi}{\phi}, \\ < 0, & \text{for } \frac{\pi}{1+n} < \frac{\psi}{\phi}. \end{cases}$$

Equation (32) indicates that an increase in the weight η assigned to the defined-contribution component positively (resp. negatively) affects economic growth when (i) the old-age dependency ratio $\frac{\pi}{1+n}$ is relatively high (resp. low); (ii) the fixed contribution rate ψ under a defined-contribution scheme is relatively small (resp. large); or (iii) the fixed replacement ratio ϕ under a defined-benefit scheme is sufficiently large (resp. small) to satisfy $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$).

The mechanism underlying these theoretical results is explained as follows. Based on (13) and (17), when the condition $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) holds, an increase in the weight η assigned to the defined-contribution component negatively (resp. positively) affects both per capita pension benefits and the wage income tax rate. According to (20), the lower (resp. higher) wage income tax rate τ_t and per capita pension benefits P_{t+1} motivate young individuals to save more (resp. less) for their old age, which positively (resp. negatively) affects capital accumulation and economic growth. This effect is reflected in the first term on the RHS of (32). Moreover, from (28), an increase in the weight η assigned to the defined-contribution component induces a shift in savings from public debt-holding to physical capital accumulation (resp. from physical capital accumulation to public debt-holding), which also positively (resp. negatively) affects capital accumulation and economic growth. This effect is reflected in the second term on the RHS of (32).[13]

Now, let us consider how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution

[13]In our simple model, as inferred from (30), any policy that reduces the size of the PAYG

scheme with a fixed contribution rate $\psi \in [0, 1)$ affects theoretical predictions of the effect of population aging on economic growth. Figure 7 provides numerical examples of the relationship between population growth n and the rate of per capita output growth under alternative weightings η on the defined-contribution component (i.e., $\eta = 0$, 0.3, 0.6 and 1). Here, note that the rate of per capita output growth in period t is given by $g_t = (\frac{k_{t+1}}{k_t})^{\frac{1}{30}} - 1$ because one period in this model is assumed to be approximately 30 years. Consistent with the results of (31), when $\eta = 1$ (i.e., under a pure defined-contribution scheme), the rate of per capita output growth increases monotonically as the population growth rate declines. Moreover, when $\eta \in [0, 1)$ (i.e., under a pure defined-benefit scheme or a mixed-payment scheme), there is a hump-shaped relationship between the population growth rate and the rate of per capita output growth.

This hump-shaped relationship between the population growth rate and the rate of per capita output growth is intuitively explained as follows. As discussed in Section 5-1, when a PAYG pension system is financed by a pure defined-benefit scheme or a mixed-payment scheme (i.e., $\eta \in [0, 1)$), a decline in the population growth rate n has two negative effects (i.e., the "tax burden effect" and "substitution effect") and one positive effect (i.e., the "anti-dilution effect") on capital accumulation. Based on (23) and (30), it is straightforward to expect that the effect of decreasing n is likely to be negative under a pure defined-benefit scheme or a mixed-payment scheme (i.e., $\eta \in [0, 1)$), when n is extremely small. Because the ratio of old to young people is large, the positive anti-dilution effect becomes negligible. However, because the wage income tax rate is high due to the high dependency ratio and the high public debt-capital ratio, any increase in the wage income tax burden stemming from a decline in the population growth rate n causes a considerable negative impact on capital accumulation. Therefore, when n is extremely small, the negative tax burden effect and substitution effect are likely to dominate the positive anti-dilution effect. Conversely, the effect of decreasing n is likely to be positive when n is extremely large. Because the ratio of old to young in the population is small, the positive anti-dilution effect is exacerbated. However, because the wage income tax rate is low due to the low dependency ratio and the low public debt-capital ratio, the negative tax burden effect and substitution effect become negligible. Therefore, when n is extremely large, the positive anti-dilution effect is likely to dominate the negative tax burden effect and substitution

pension leads to higher rates of capital accumulation and economic growth. Therefore, ceteris paribus, a reduction in the fixed contribution rate ψ or fixed replacement ratio ϕ positively affects economic growth. Unlike these straightforward pension reduction policies, the effect of a transition from a defined-benefit scheme to a defined-contribution scheme on the overall size of the PAYG pension and economic growth depends on the extent of population aging. These features suggest that this type of PAYG pension reform may have a non-trivial impact on the relationship between population aging and economic growth.

effect.

Figure 7 also shows how the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) affects the relationship between the population growth rate and the rate of per capita output growth. As shown in (32), this type of PAYG pension reform positively (resp. negatively) affects economic growth when $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) or $1 + n < \frac{\pi\phi}{\psi}$ (resp. $1 + n > \frac{\pi\phi}{\psi}$). As explained in Section 4-2, when $1 + n = \frac{\pi\phi}{\psi}$, the both steady-state value of x_1^* and the wage income tax rate remain the same regardless of the value of η. Therefore, when $1 + n = \frac{\pi\phi}{\psi}$, based on (30), the rate of per capita output growth also remains unchanged regardless of the value of η, as shown in Figure 7. However, when $1 + n < \frac{\pi\phi}{\psi}$ (resp. $1 + n > \frac{\pi\phi}{\psi}$), as (17) and (28) hold, both the value of x_1^* and the wage income tax rates decrease (resp. increase) with the value of η. Therefore, the rate of per capita output growth increases (resp. decreases) with the value of η.

These results suggest that in economies in which the population growth rate is relatively low and the size of pensions under a defined-benefit scheme is sufficiently large to satisfy $1+n < \frac{\pi\phi}{\psi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) mitigates or overcomes the negative growth effect of population aging caused by a decline in the population growth rate.[14] The intuitions are explained as follows. Under (14), assigning a larger weight η to the defined-contribution component decreases the impact of changes in the old-age dependency ratio on the wage income tax rate, which alleviates the negative growth effect of a decline in n due to the tax burden effect. Moreover, from (28), a larger weight η assigned the defined-contribution component induces a shift in savings from public debt-holding to physical capital accumulation, which weakens the negative growth effect of a decline in n due to the substi-

[14] In economies in which the population growth rate is relatively high and the size of pensions under a defined-benefit scheme is sufficiently small to satisfy $1 + n > \frac{\pi\phi}{\psi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) exerts competing influences on the growth effect of population aging caused by a decline in the population growth rate. Based on (14), assigning a larger weight η to the defined-contribution component decreases the impact of a change in the old-age dependency ratio on the social security tax rate, which alleviates the negative growth effect of a decline in n due to the tax burden effect. However, from (28), a larger weight η of the defined-contribution component induces a shift in savings from physical capital accumulation to public debt-holding, which strengthens the negative growth effect of a decline in n due to the substitution effect. Furthermore, when $1 + n > \frac{\pi\phi}{\psi}$, (13) and (17) indicate that an increase in the weight η assigned to the defined-contribution component increases the size of PAYG pensions and decreases savings, which weakens the positive growth effect of a decline in n due to the anti-dilution effect.

tution effect. Furthermore, when $1+n < \frac{\pi\phi}{\psi}$, (13) and (17) indicate that an increase in the weight η assigned to the defined-contribution component decreases the size of PAYG pensions, and increases savings, which strengthens the positive growth effect of a decline in n due to the anti-dilution effect. These factors mitigate the negative growth effect of population aging caused by a decline in the population growth rate. Therefore, in economies in which the population growth rate is relatively low and the size of pensions under a defined-benefit scheme is sufficiently large to satisfy $1 + n < \frac{\pi\phi}{\psi}$, the transition from a defined-benefit scheme with a fixed replacement ratio ϕ to a defined-contribution scheme with a fixed contribution rate ψ mitigates or even overcomes the negative growth effect of population aging caused by a decline in the population growth rate.

The prediction that reforming a PAYG pension system from a defined-benefit scheme to a defined-contribution scheme mitigates the negative growth effect of population aging has already been recognized in Artige et al. (2014) and Tabata (2015). With explicit consideration of public debt financing, this paper confirms that the analogous predictions hold when the government conforms to a constant budget deficit-GDP ratio rule.

6 PAYG pension reform and welfare

In this section, we examine how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution with a fixed contribution rate $\psi \in [0, 1)$ affects the welfare level of each generation. Unfortunately, because it is difficult to investigate this issue analytically, this section only provides a numerical example.

We consider the following policy experiment. Initially, the economy is in the stable steady-state equilibrium in which the old-age dependency ratio is sufficiently high to satisfy $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ and the government employs a pure defined-benefit scheme (i.e., $\eta = 0$). Then, in period 5, the government reforms the PAYG pension system from a defined-benefit scheme to a defined-contribution scheme. More concretely, in period 5, the weight η assigned to the defined-contribution component is increased from 0 to η^*, where $\eta^* = 0.3, 0.6, 1$, and remains constant at η^* for all $t \geq 5$ (i.e., $\eta_t = \eta^*$ for all $t \geq 5$).

Figures 8-1 to 8-3 show how the increased weight η assigned to the defined-contribution component in period $t \geq 5$ influences the evolution of the wage income tax rate τ_t, the rate of per capita output growth g_t and the public debt-capital ratio x_t, respectively. The solid line depicts the case without PAYG pension reform, whereas the broken, dot-dash and dotted lines reflect the case in which the weight η is increased from 0 to 0.3, 0.6 and 1, respectively. Here, note that the rate of per capita output growth in period t is given by $g_t = (\frac{k_{t+1}}{k_t})^{\frac{1}{30}} - 1$.

Because we consider the case in which the old-age dependency ratio is sufficiently high to satisfy $\frac{\pi}{1+n} > \frac{\psi}{\phi}$, from (13) and (17), such a policy experiment negatively affects the per capita pension benefit P_t in period $t \geq 5$ and the wage income tax rate in period $t \geq 5$. Thus, as shown in Figure 8-1, the wage income tax rate begins to decrease in period 5. Moreover, the decline in the per capita pension benefit in period $t \geq 5$ and the resulting decline in the wage income tax rate in period $t \geq 5$ motivate individuals in generation $t \geq 4$ to save more for their old age, which enhances the speed of capital accumulation in period $t \geq 4$.[15] Thus, as shown in Figure 8-2, the rate of per capita output growth begins to increase in period 4. Furthermore, from (21), the higher growth rate of the capital-labor ratio in period $t \geq 4$ negatively affects the public debt-capital ratio in period $t \geq 5$. Thus, as shown in Figure 8-3, the public debt-capital ratio begins to decrease in period 5. This decline in the public debt-capital ratio in period $t \geq 5$ induces a further decline in the wage income tax rate in period $t \geq 5$. Thus, as shown in Figures 8-2 and 8-3, the rate of per capita output growth increases over time, whereas the public debt-capital ratio decreases over time, and each of them gradually converges to its own new steady-state value.

Figure 8-4 depicts the net lifetime utility gain or loss of agents belonging to generations 1–7 (seven generations) and describes how the lifetime utility levels of the agents are affected by the increased weight η assigned to the defined-contribution component in period $t \geq 5$. The solid line depicts the case without PAYG pension reform, whereas the broken, dot-dash and dotted lines reflect the welfare differences between $\eta = 0.3$, 0.6, 1 and 0, respectively. As shown in Figure 8-4, the increased weight η assigned to the defined-contribution component in period $t \geq 5$ reduces the lifetime utility levels of agents in generation 4, while it increases the lifetime utility levels of agents in generation $t \geq 5$. This implies that reforming the PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution with a fixed contribution rate $\psi \in [0, 1)$ deteriorates the welfare of the initial old agents in period 5, while it improves the welfare of other, subsequent generations.

These results are intuitively explained as follows. The increased weight η assigned to the defined-contribution component in period $t \geq 5$ negatively affects the per capita pension benefit in period $t \geq 5$, which reduces the lifetime utility level of agents in generation $t \geq 4$. We denote this negative effect of an increase in η on the welfare of agents in generation $t \geq 4$ as the "pension reduction effect." Because agents in generation 4 (the initial old agents in period 5) are only affected by this negative "pension reduction effect," the lifetime utility level of agents in

[15]We consider the case in which the PAYG pension reform in period $t \geq 5$ is expected by the agents in generation 4 in their young age. Even if we consider the case in which the PAYG pension reform is unexpected by agents in generation 4, the main implication of our welfare analysis is not significantly changed.

generation 4 is deteriorated by the increased weight η assigned to the defined-contribution component in period $t \geq 5$. However, the increased weight η assigned to the defined-contribution component in period $t \geq 5$ reduces the wage income tax rate in period $t \geq 5$, directly or indirectly, through the decline in the public debt-capital ratio in period $t \geq 5$, which positively affects the lifetime utility level of agents in generation $t \geq 5$. We denote this positive effect of an increase in η on the welfare of agents in generation $t \geq 5$ as the "tax reduction effect." Moreover, this decline in wage income tax rate in period $t \geq 5$ leads to a higher capital-labor ratio in period $t \geq 5$, which also positively affects the lifetime utility level of agents in generation $t \geq 5$. We denote this positive effect of an increase in η on the welfare of agents in generation $t \geq 5$ as the "capital accumulation effect." Generation 5 and subsequent generations are affected not only by the negative "pension reduction effect" but also the positive "tax reduction effect" and "capital accumulation effect." Therefore, in our example, these two positive welfare effects dominate the one negative welfare effect, and thus the lifetime utility level of agents in generation $t \geq 5$ is improved by the increased weight η assigned to the defined-contribution component in period $t \geq 5$. Furthermore, as the increased weight η assigned to the defined-contribution component in period $t \geq 5$ increases the savings level of all generations born after period 4, over time, the impact of increased saving by successive generations increases. Thus, the positive "capital accumulation effect" increases over time. Therefore, the welfare gain due to the increased weight η in period $t \geq 5$ increases for future generations.

Finally, Figure 9 depicts how population aging caused by a decline in the population growth rate influences the intergenerational conflict between current and future generations over PAYG pension reform policy. The solid line reflects the case without PAYG pension reform, whereas the broken, dot-dash and dotted lines illustrate the welfare differences between $\eta = 1$ and 0, when $1 + n = 0.85$, 0.8 and 0.75, respectively. As shown in Figure 9, as the population growth rate decreases, the net welfare losses of generation 4 tend to be large while the net welfare gains of generation 5 and subsequent generations become large. Consequently, population aging caused by a decline in the population growth rate exacerbates the extent of intergenerational conflict over PAYG pension reform policy.

This result is intuitively explained as follows. In our model, when the condition $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ holds, the positive "capital accumulation effect" increases in the decline in the population growth rate. This result is explicitly shown in Figure 7. Therefore, given the value of η (i.e., the extent of PAYG pension reform), as the population growth rate decreases, the positive "capital accumulation effect" increases, and thus, the net welfare gains of generation 5 and subsequent generations increase. However, from (12), the per capita pension benefit decreases with the decline in the population growth rate. Therefore, given the value of η (i.e., the extent of PAYG pension reform), as the population growth rate decreases, the negative “pension reduction effect” becomes more serious, and thus, the net welfare losses of generation 4 increase. These effects lead to an increase in the net welfare losses of generation 4 and an increase in the net welfare gains of generation $t \geq 5$.

These results imply that reforming a PAYG pension system from a defined-benefit scheme to a defined-contribution scheme becomes more politically difficult in a rapidly aging society. Thus, as in the conventional discussion of social security reform, a Pareto-improving policy must be designed to enhance its political feasibility. Considering such a Pareto-improving policy will be one of the promising directions for future research.

7 Concluding Remarks

This paper examined how reforming a PAYG pension system from a defined-benefit scheme to a defined-contribution scheme affects fiscal sustainability and economic growth in an OLG model with endogenous growth. We showed that in economies in which the old-age dependency ratio is high and the size of pension benefits under a defined-benefit scheme is large, such a pension reform mitigates the negative effect of population aging on fiscal sustainability and economic growth. However, we also showed that this type of pension reform entails an intergenerational conflict of interest between current and future generations. Population aging might exacerbate the extent of this conflict.

Appendix A: Population aging caused by an increase in the old-age survival probability rate π

In this section, we examine how population aging caused by an increase in the old-age survival probability rate π affects fiscal sustainability and economic growth. We also consider how reforming a PAYG pension system from a defined-benefit scheme with a fixed replacement ratio $\phi \in [0, 1)$ to a defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$ affects theoretical predictions of the effect of population aging (caused by an increase in the old-age survival probability rate π) on fiscal sustainability and economic growth.

Fiscal sustainability

Figure 10 provides numerical examples of the relationship between the old-age survival probability rate π and the steady-state value of the debt-capital ratio x_2^* under alternative weightings η assigned to the defined-contribution component (i.e., $\eta = 0$, 0.3, 0.6 and 1). The parameters used in the baseline simulations are the same in Figures 6 and 7. As inferred from (23), when the value of η is relatively low, there is a hump-shaped relationship between the old-age survival probability rate and the steady-state value of x_2^*. This result implies that the effect of population aging caused by an increase in π on fiscal sustainability is positive when the old-age survival probability rate π is sufficiently low, but it could be negative when the old-age survival probability rate π is sufficiently high. Figure 10 also shows how the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) affects the relationship between the old-age survival probability rate π and the steady-state value of x_2^*. As shown in (29), this type of PAYG pension reform positively (resp. negatively) affects the steady-state value of x_2^* when $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) or $\pi > \frac{\psi(1+n)}{\phi}$ (resp. $\pi < \frac{\psi(1+n)}{\phi}$). From (14), given the value of x_t, the overall wage income tax rates remain the same regardless of the value of η when $\pi = \frac{\psi(1+n)}{\phi}$. Therefore, based on (23), the steady-state value of x_2^* remains the same regardless of the value of η when $\pi = \frac{\psi(1+n)}{\phi}$, as shown in Figure 10. However, (14) also means that when $\pi > \frac{\psi(1+n)}{\phi}$ (resp. $\pi < \frac{\psi(1+n)}{\phi}$), the wage income tax rate decreases (resp. increases) with the value of η, which shifts the $kg(x_t; \tilde{\tau}(\eta, n), \mu)$ curve upward (resp. downward). Therefore, from (23), the steady-state value of x_2^* increases (resp. decreases) with the value of η. These results suggest that in economies in which the old-age survival probability rate is relatively high and the size of pensions under a defined-benefit scheme is sufficiently large to satisfy $\pi > \frac{\psi(1+n)}{\phi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) mitigates or overcomes the negative effect of population aging (caused by an increase in the old-age survival probability rate) on fiscal sustainability.

Economic growth

As in Section 5, we focus our analysis on the case in which the economy is in a stable steady-state equilibrium E_1. Figure 11 provides numerical examples of the relationship between the old-age survival probability rate π and the rate of per capita output growth under alternative weightings η assigned to the defined-contribution component (i.e., $\eta = 0$, 0.3, 0.6 and 1). The parameters used in the baseline simulations are the same in Figures 6 and 7. As inferred from (30), when the value of η is relatively low, there is a hump-shaped relationship between the old-age survival probability rate and the rate of per capita output growth. This result implies that the growth effect of population aging caused by an increase in the old age survival rate π is positive when the old-age survival probability rate π is sufficiently low, but it could be negative when the old-age survival probability rate π is sufficiently high. Figure 11 also shows how the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) affects the relationship between the old-age survival probability rate and the rate of per capita output growth. As shown in (32), this type of PAYG pension reform positively (resp. negatively) affects economic growth when $\frac{\pi}{1+n} > \frac{\psi}{\phi}$ (resp. $\frac{\pi}{1+n} < \frac{\psi}{\phi}$) or $\pi > \frac{\psi(1+n)}{\phi}$ (resp. $\pi < \frac{\psi(1+n)}{\phi}$). As explained in Section 4-2, when $\pi = \frac{\psi(1+n)}{\phi}$, the both steady-state value of x_1^* and the wage income tax rates remain the same regardless of the value of η. Therefore, when $\pi = \frac{\psi(1+n)}{\phi}$, based on (30), the per capita output growth rate also remains the same regardless of the value of η, as shown in Figure 11. However, when $\pi > \frac{\psi(1+n)}{\phi}$ (resp. $\pi < \frac{\psi(1+n)}{\phi}$), as (17) and (28) hold, both the value of x_1^* and the wage income tax rate decrease (resp. increase) with the value of η. Therefore, the rate of per capita output growth increases (resp. decreases) with the value of η. These results suggest that in economies in which the old-age dependency ratio is relatively high and the size of pensions under a defined-benefit scheme is sufficiently large to satisfy $\pi > \frac{\psi(1+n)}{\phi}$, the transition from a pure defined-benefit scheme with a fixed replacement ratio ϕ (i.e., $\eta = 0$) to a pure defined-contribution scheme with a fixed contribution rate ψ (i.e., $\eta = 1$) mitigates or overcomes the negative growth effect of population aging caused by a increase in the old-age survival probability rate.

Appendix B: PAYG pension system characterized by a pure defined- contribution scheme with a fixed contribution rate $\psi \in [0, 1)$

Under a pure defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$, all PAYG pension payments must be financed by contributions. Therefore, the PAYG pension system must satisfy the following balanced-budget constraint:

$$\psi w_t N_t = P_t \pi N_{t-1}. \tag{33}$$

Thus, we can confirm that the relationship $P_t = \psi \frac{1+n}{\pi} w_t$ holds, which corresponds to the value of P_t in (11) when $\eta = 1$.

Further, the budget constraints of integrated government (including the PAYG pension system) in period t is given as

$$B_{t+1} = (1 + r)B_t + P_t \pi N_{t-1} - (\hat{\tau}_t + \psi) w_t N_t, \tag{34}$$

where $\hat{\tau}_t$ denotes the wage income tax imposed to finance interest payments on debt minus newly issued debt (i.e., $rB_t - (B_{t+1} - B_t)$). By substituting (7),(8),(10) and (33) into (34), we obtain

$$\hat{\tau}_t + \psi = \frac{r}{\bar{w}}x_t + \psi - \frac{\mu}{1-\alpha}, \tag{35}$$

which corresponds to the value of τ_t in (14) when $\eta = 1$. Therefore, when $\eta = 1$, the PAYG pension scheme is characterized by a pure defined-contribution scheme with a fixed contribution rate $\psi \in [0, 1)$.

Appendix C: Existence of steady-state equilibria

In this appendix, we briefly consider the parameter conditions that ensure the existence of steady-state equilibria. Rewriting (23) as a quadratic functional form, we obtain

$$\Gamma(x; \tilde{\tau}(\eta, n), \mu) \equiv \gamma_1 x^2 - \gamma_2(\tilde{\tau}(\eta, n), \mu)x + \gamma_3(\tilde{\tau}(\eta, n), \mu) = 0, \tag{36}$$

where

$$\gamma_1 \equiv 1 + \frac{\beta\pi}{1+\beta\pi}r > 0,$$

$$\gamma_2(\tilde{\tau}(\eta, n), \mu) \equiv \left\{\beta\pi[1 - \tilde{\tau}(\eta, n)] - \frac{\mu}{1-\alpha}\right\}\frac{\bar{w}}{1+\beta\pi} - \left(1 + \frac{1}{1+\beta\pi}\frac{\bar{w}}{R}\tilde{\tau}(\eta, n)\right),$$

$$\gamma_3(\tilde{\tau}(\eta, n), \mu) \equiv \left(1 + \frac{1}{1+\beta\pi}\frac{\bar{w}}{R}\tilde{\tau}(\eta, n)\right)\frac{\mu}{1-\alpha}\bar{w} > 0.$$

In addition, the inequalities $\frac{\partial\gamma_2(\tilde{\tau}(\eta,n),\mu)}{\partial\mu} < 0$ and $\frac{\partial\gamma_3(\tilde{\tau}(\eta,n),\mu)}{\partial\mu} > 0$ hold. As inferred from Figure 1, as we confine our attention to the case in which the gross rate of per capita output growth is positive (i.e., $kg(x_t; \tilde{\tau}(\eta, n), \mu) > 0$), the steady-state equilibria exist if and only if the quadratic equation (36) has real roots or the discriminant of the quadratic equation (36), $D = [\gamma_2(\tilde{\tau}(\eta, n), \mu)]^2 - 4\gamma_1\gamma_3(\tilde{\tau}(\eta, n), \mu) \equiv D(\tilde{\tau}(\eta, n), \mu)$, is non-negative. From $D(\tilde{\tau}(\eta, n), \mu)$, we can confirm that the following relations hold.

$$\lim_{\mu\to 0} D(\tilde{\tau}(\eta, n), \mu) = [\gamma_2(\tilde{\tau}(\eta, n), 0)]^2 > 0,$$

$$\frac{\partial D(\tilde{\tau}(\eta, n), \mu)}{\partial\mu} = 2\gamma_2\frac{\partial\gamma_2(\tilde{\tau}(\eta, n), \mu)}{\partial\mu} - 4\gamma_1\frac{\partial\gamma_3(\tilde{\tau}(\eta, n), \mu)}{\partial\mu} < 0 \;\text{ if }\; \gamma_2(\tilde{\tau}(\eta, n), \mu) \geq 0.$$

These results imply that, assuming that the relationship $\gamma_2(\tilde{\tau}(\eta, n), \mu) \geq 0$ holds, the condition $D(\tilde{\tau}(\eta, n), \mu) > 0$ holds for sufficiently small values of μ. The parameter conditions that ensure $\gamma_2(\tilde{\tau}(\eta, n), \mu) \geq 0$ are given by

$$1 \leq \frac{\left\{\beta\pi[1 - \tilde{\tau}(\eta, n)] - \frac{\mu}{1-\alpha}\right\}\frac{\bar{w}}{1+\beta\pi}}{1 + \frac{1}{1+\beta\pi}\frac{\bar{w}}{R}\tilde{\tau}(\eta, n)} \equiv kg(0; \tilde{\tau}(\eta, n), \mu)$$

From (20), these parameter conditions imply that net growth rate of aggregate capital is positive when there is no given stock of public debt. In this paper, we implicitly confine our analysis to the case in which the parameter condition $kg(0;\tilde{\tau}(\eta,n),\mu) \geq 1$ holds.

Appendix D: Stability of steady-state equilibria

In this appendix, we investigate the stability of steady states E_1 and E_2 and show that E_1 is locally stable and E_2 is unstable, or $\left|\frac{dx_{t+1}}{dx_t}\right| < 1$ holds at x_1^* and $\left|\frac{dx_{t+1}}{dx_t}\right| > 1$ holds at x_2^*.

By differentiating (21) with respect to x_t, we obtain

$$\frac{dx_{t+1}}{dx_t} = \frac{bg}{kg} + \frac{x_t}{kg}\frac{\partial bg}{\partial x_t} - \frac{bg}{kg}\frac{x_t}{kg}\frac{\partial kg}{\partial x_t}.$$

Because $bg = kg$ holds in the steady-state equilibria, at the steady-state value x^* ($x^* = x_1^*, x_2^*$), the differential simplifies to

$$\left.\frac{dx_{t+1}}{dx_t}\right|_{x_{t+1}=x_t=x^*} = 1 + \frac{x^*}{kg}\left(\left.\frac{\partial bg}{\partial x_t}\right|_{x_t=x^*} - \left.\frac{\partial kg}{\partial x_t}\right|_{x_t=x^*}\right). \tag{37}$$

From Figure 1, the inequalities $\frac{\partial bg}{\partial x} < \frac{\partial kg}{\partial x} < 0$ ($\frac{\partial kg}{\partial x} < \frac{\partial bg}{\partial x} < 0$) hold at x_1^* (x_2^*). Therefore, $\frac{dx_{t+1}}{dx_t} < 1$ holds at x_1^*, and $\frac{dx_{t+1}}{dx_t} > 1$ holds at x_2^*. Then, by differentiating $bg(x_t;\mu)$ with respect to x_t and evaluating at the steady states, we obtain

$$\left.\frac{\partial bg}{\partial x_t}\right|_{x_t=x^*} = -\frac{\mu\bar{w}}{1-\alpha}\frac{1}{(x^*)^2}.$$

In addition, rearranging (23) yields

$$-\frac{\mu\bar{w}}{1-\alpha}\frac{1}{(x^*)^2} = \frac{1}{x^*}\left[1 - kg(x^*;\tilde{\tau}(\eta,n),\mu)\right].$$

By substituting these two equations into (37), we obtain

$$\left.\frac{dx_{t+1}}{dx_t}\right|_{x_{t+1}=x_t=x^*} = \frac{x^*}{kg}\left(\frac{1}{x^*} - \left.\frac{\partial kg}{\partial x_t}\right|_{x_t=x^*}\right) > 0, \because \frac{\partial kg}{\partial x_t} < 0.$$

Therefore, $\left|\frac{dx_{t+1}}{dx_t}\right| < 1$ holds at x_1^*, and $\left|\frac{dx_{t+1}}{dx_t}\right| > 1$ holds at x_2^*. Hence, E_1 is locally stable, whereas E_2 is unstable.

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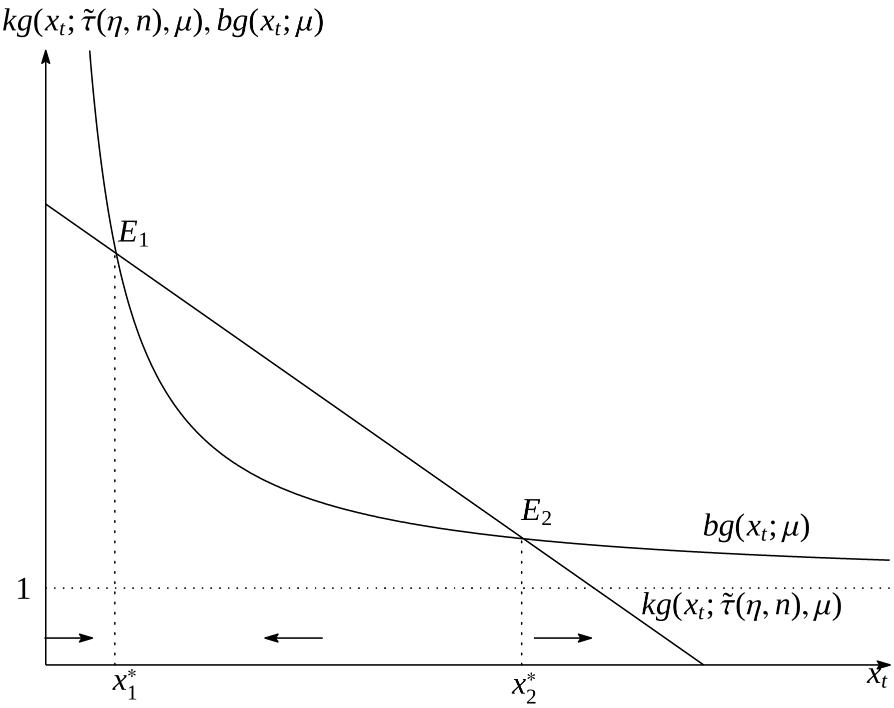

Figure 1: Two steady states in the case where μ is small

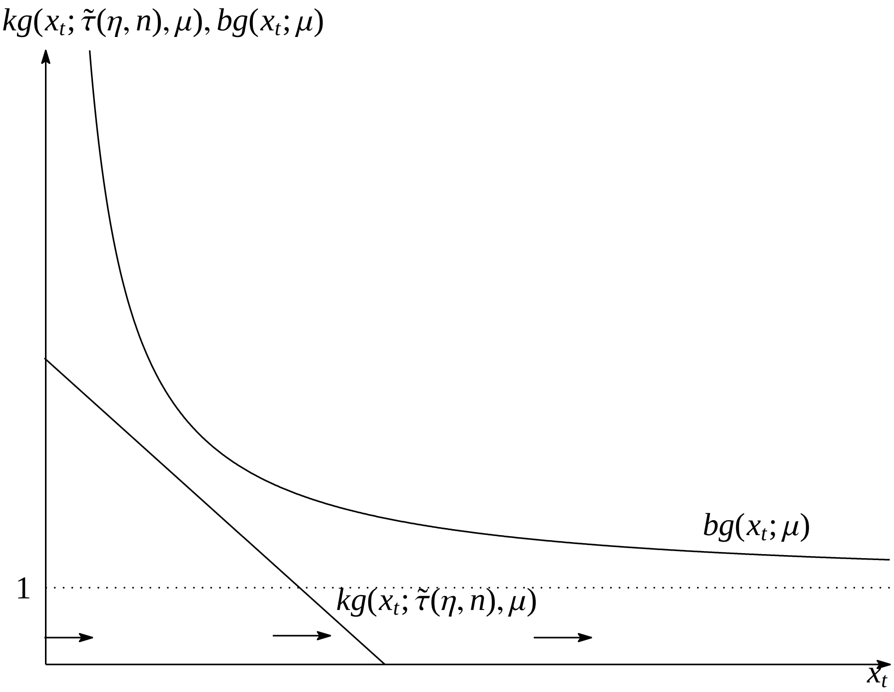

Figure 2: No steady state in the case where μ is large

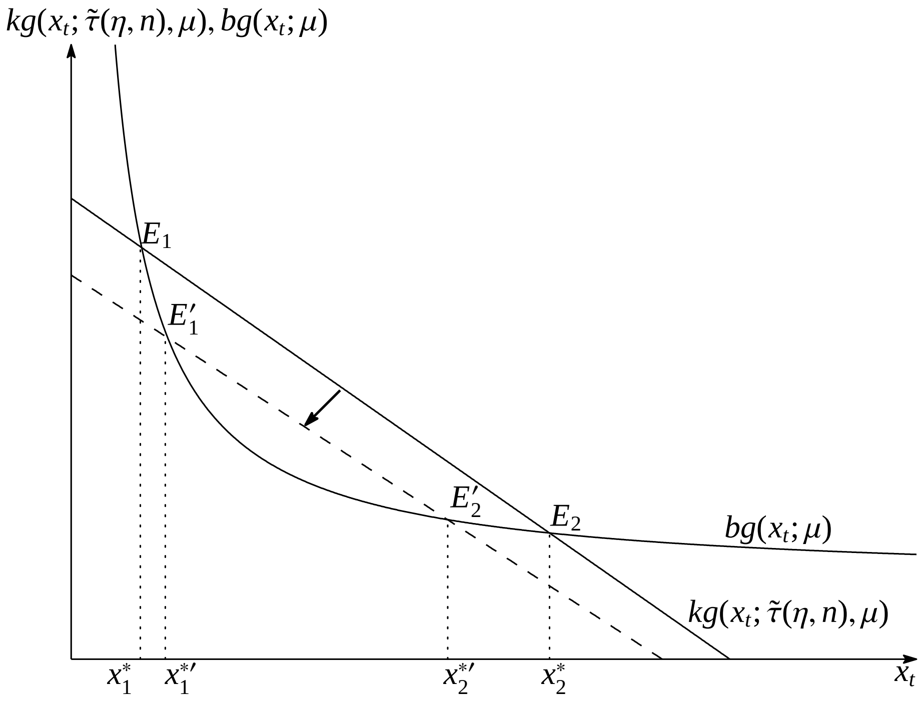

Figure 3: The effect of an decline in n.

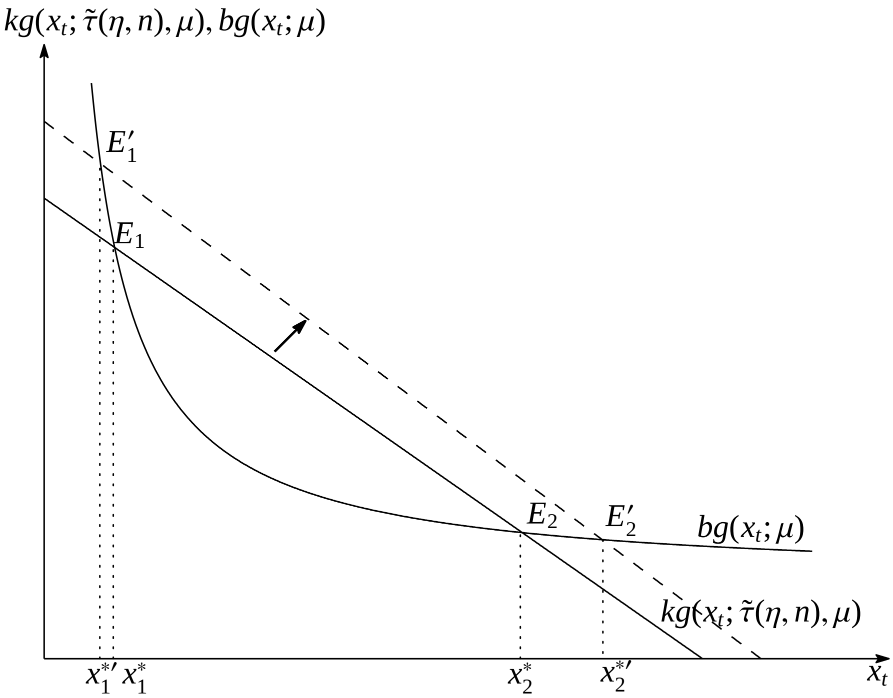

Figure 4: The effect of the PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme when $\frac{\pi}{1+n} > \frac{\psi}{\phi}$.

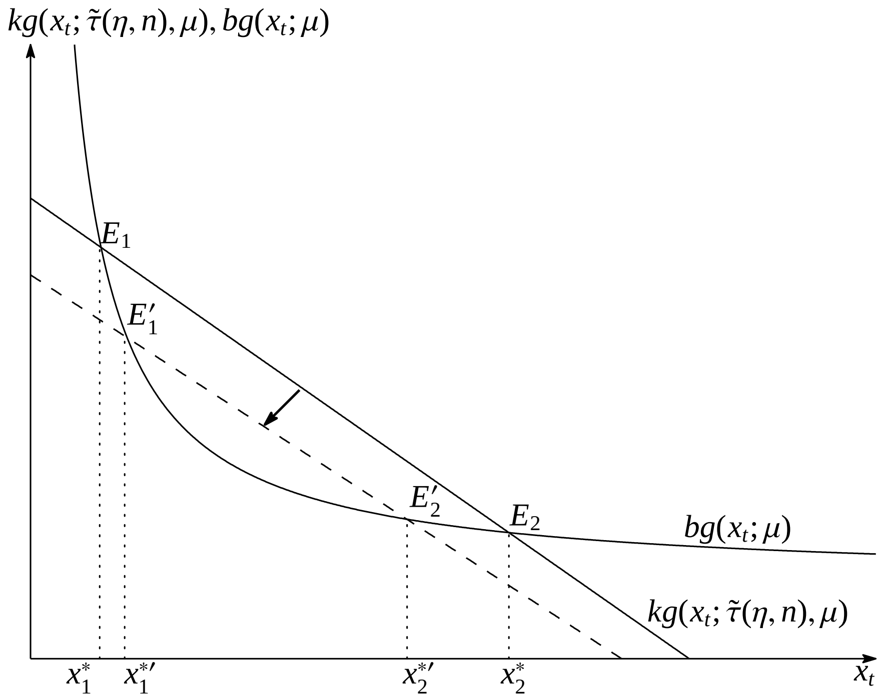

Figure 5: The effect of the PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme when $\frac{\pi}{1+n} < \frac{\psi}{\phi}$.

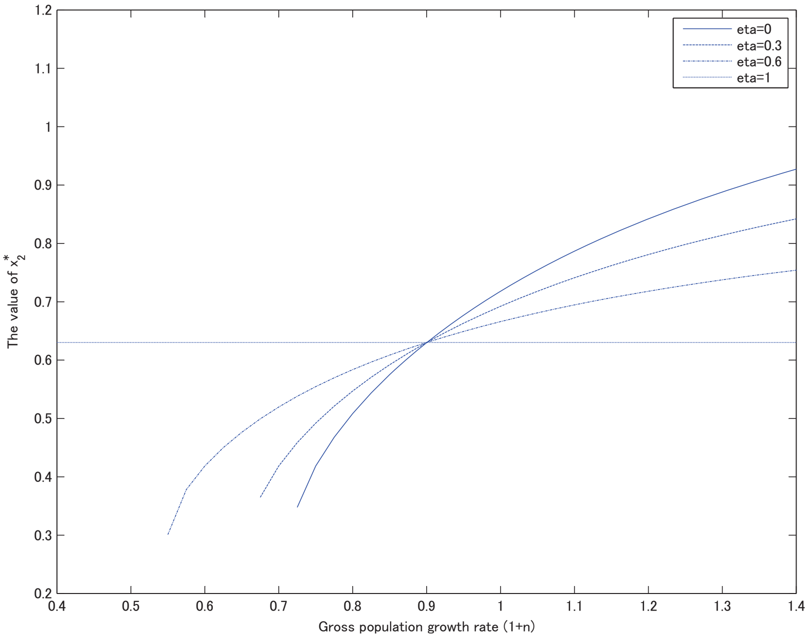

Figure 6: The effect of the PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme on the relation between population growth rate and the value of x_2^*.

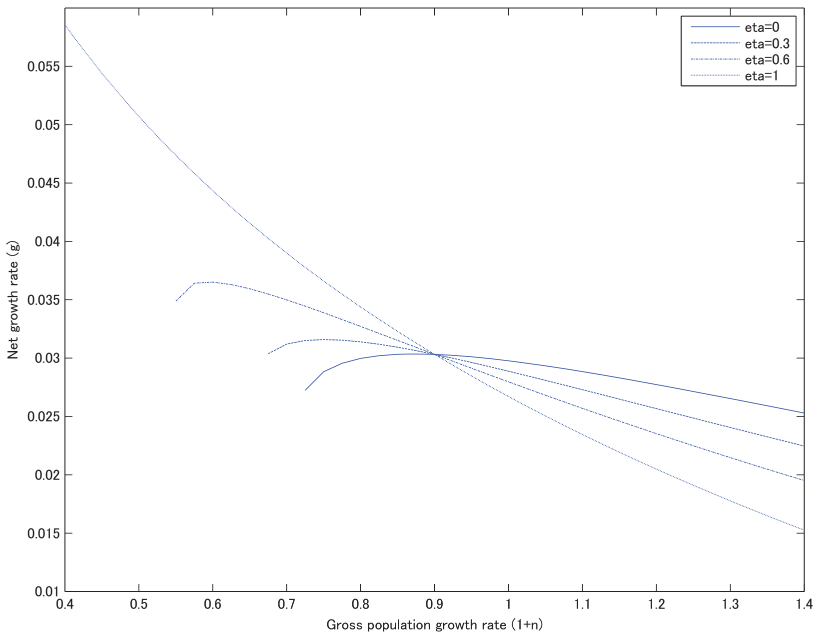

Figure 7: The effect of the PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme on the relation between population growth rate and economic growth.

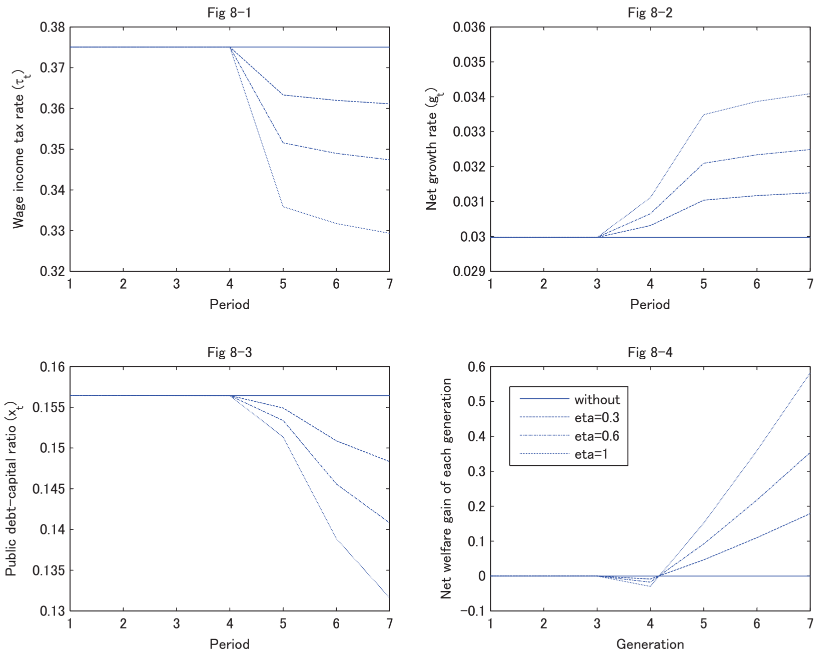

Figure 8: The effect of the PAYG pension reform on the dynamic evolutions of τ_t, g_t and x_t.

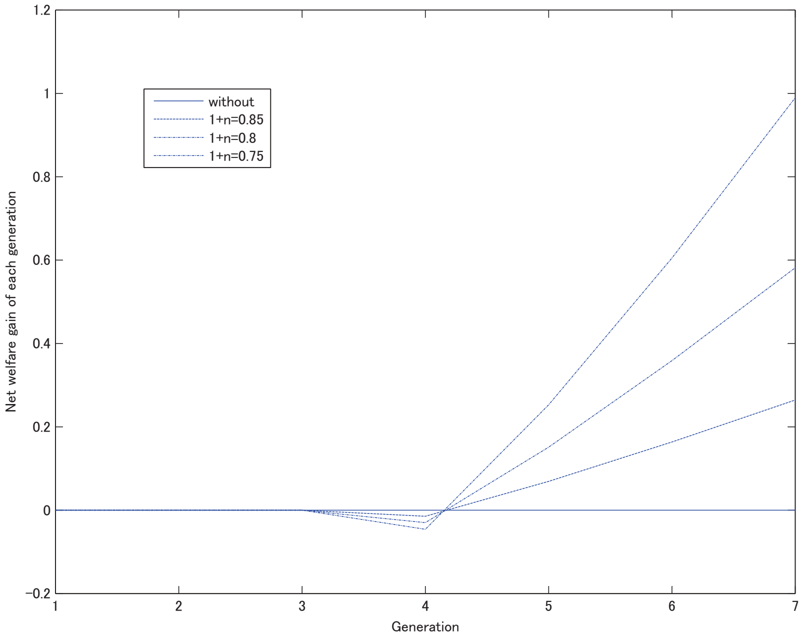

Figure 9: Population aging and intergenerational conflicts.

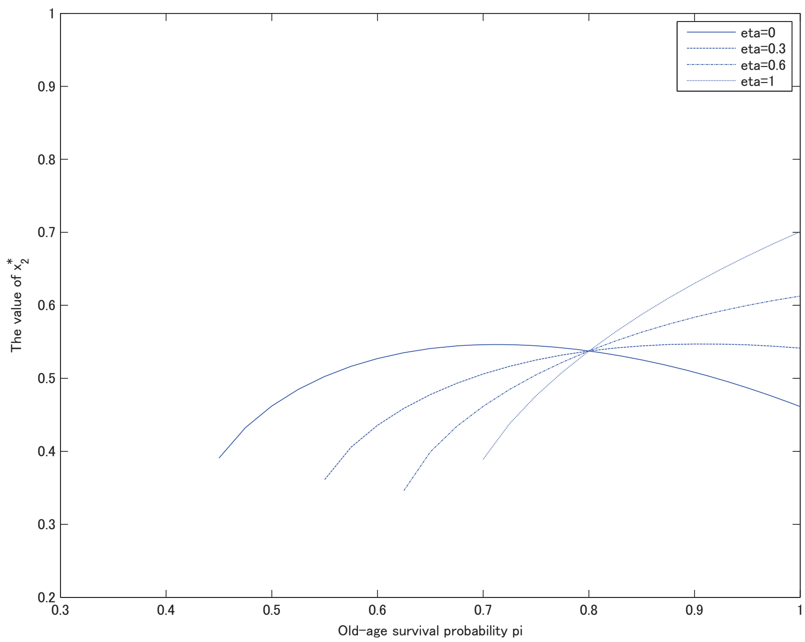

Figure 10: The effect of the PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme on the relation between old-age survival probability rate and the value of x_2^*.

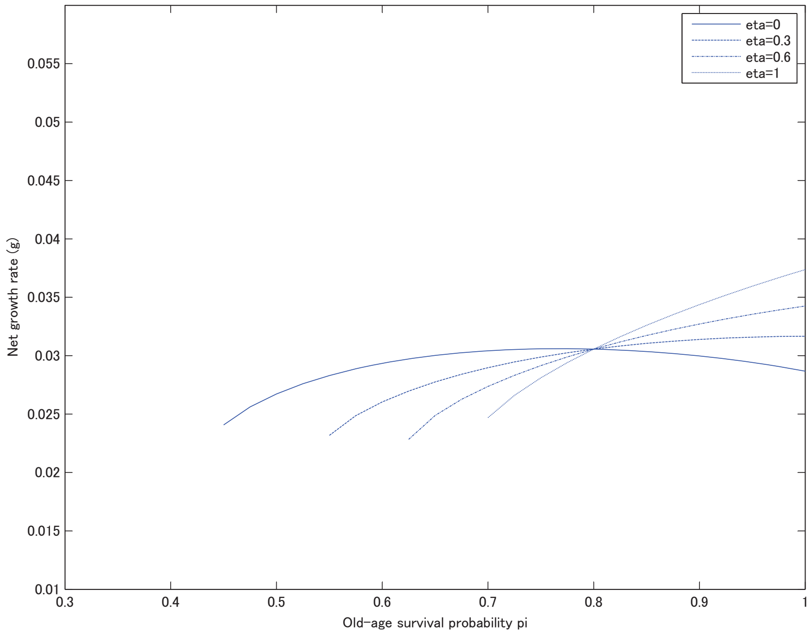

Figure 11: The effect of the PAYG pension reform from a defined-benefit scheme to a defined-contribution scheme on the relation between old-age survival probability rate and economic growth.