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How should a government finance redistribution policies?

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Abstract

In OECD countries, redistribution policies are provided for young and old generations. Taxation of many kinds to finance the redistribution policy exists, just as redistribution policies of many kinds exist. Our paper sets a model with heterogeneous labor productivity for households and sectors of two types: a skilled sector and an unskilled sector. The model elucidates how the government should collect tax revenue for redistribution policies. Results show that the labor income tax can always shrink income inequality. However, the consumption tax increases wage inequality between skilled and unskilled sector. It is not always sufficient to shrink income inequality after redistribution, even if skilled workers increase. A corporate tax shrinks income inequality if intertemporal consumption is substitutive. Results show that the redistribution policy effects depend on how the government collects tax revenue.

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1. Introduction

Our paper presents an examination of how a government should collect tax revenues to provide a redistribution policy to shrink income inequality. Taxation of many kinds is used to support redistribution policies: labor income taxation, consumption taxation, corporate tax, capital income tax, and so on. Similarly, redistribution policies of many kinds exist: pensions are provided for older people; income transfers are made according to need or means testing; etc.

The government should provide a redistribution policy to reduce income inequality. However, the government should collect tax revenue to provide redistribution. The explanation provided in this paper shows that differences in taxation bring about different effects on income growth and income inequality and affect the manner in which the government should finance tax revenue.

A redistribution policy is necessary to shrink income inequality. In Japan, the Gini coefficient is at a constant level in spite of its aging society with fewer children. Although aging can magnify income inequality, the Gini coefficient is kept at a constant level thanks to redistribution policy.

In an aging society with fewer children, we must consider intergenerational inequality. Given fewer people of younger generations, the tax burden per young person continues to increase and intergenerational inequality expands. Consequently, redistribution policies become unsustainable. Therefore, a consumption tax levied for all generations must be considered in aging societies such as those existing in many OECD countries. Corporate tax rates are expected to decrease worldwide because governments are striving to make national firms competitive.

Our paper presents the following results. A redistribution policy financed by labor income tax can always shrink income inequality: output (gross domestic product) does not change. Although the consumption tax and the corporate tax increase the output (GDP), the income inequality might be exacerbated, depending on the parametric condition. Considering social welfare, labor income taxation should be used as a redistribution policy, not a consumption tax or capital income tax.

As described by the OECD (2014a), the consumption tax share reached 31% of average tax revenue as a percentage of total taxation in OECD countries at 2012. Especially, the consumption tax share to total tax revenue continues increasing in an aging society. However, the corporate income tax share is nearly constant. The corporate

tax rate is tending to decrease worldwide.[1] In spite of the decreased incorporate tax rate, the corporate tax share does not decrease. We can regard the reason as low corporate taxes increasing firm profits. Consequently, the tax revenue increases.

Many governments worldwide aim to decrease income inequality. However, OECD (2014b, 2014c) shows the OECD average Gini coefficient raised from 0.29 in the 1980s to 0.32 at 2011–2012. Figure 1 shows the Gini coefficient and poverty rate in OECD countries.
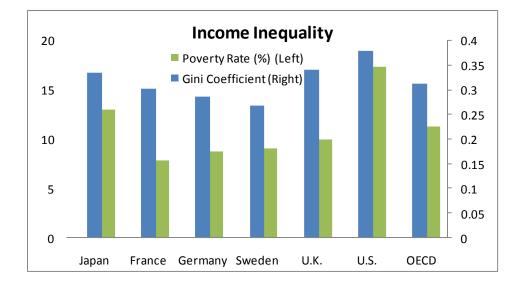

Fig. 1 Income Inequality in OECD Countries (data: OECD (2014b))[2].

Some means of taxation must exist. Consumption taxes, which are levied on the sale of goods and services, are widely used in OECD countries. The consumption tax rates in EU countries are higher than in other countries. As shown by Carroll and Viard (2012), the consumption tax presents no disincentive for working, compared with labor income taxation.[3] Moreover, consumption is observable and governments can find the ability to pay the tax and can levy a tax burden appropriately in terms of equality. The optimal consumption tax was examined first by Ramsey (1927), who presents the Ramsey rule that the consumption tax must be levied based on the elasticity of consumption goods.

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[1] For instance, the corporate income tax in Japan reached 43.3% in the past. However, the corporate tax in Japan in recent years is 25.5% (The tax rate includes only the national tax. data: Ministry of Finance Japan).

[2] Data are for 2010.

[3] The consumption tax can have a distortive effect on labor and leisure decisions because the consumption tax decreases the real wage rate, thereby reducing consumption.

The Ramsey rule is extended by Corlett and Hague (1953), Diamond and Mirrlees (1971a, 1971b), and others.[4] Bargain and Donni (2012) consider the targets for individuals within households and derive the optimal indirect taxation rule.

As surveyed by Chari and Kehoe (1999), Kocherlakota (2010), and others, Chamley (1986) and Judd (1985) derive that an optimal capital income tax rate is zero. These results show that the taxation must not be levied for accumulation factors. Therefore, if the labor income is determined by the accumulation factor such as human capital, then the labor income taxation must be zero to maximize social welfare as derived by Jones, Manuelli, and Rossi (1997). Chen and Lu (2013) report income taxation of two types and derive that the transition from capital income taxation to labor income taxation reduces welfare in the long run. Andorokovich, Daly and Naqib (1992) examine tax reform effects on welfare, which changes from a single tax instrument to hybrid with income taxation and the consumption tax. Correia (2010) derives that consumption with redistribution can raise the welfare level, depending on the distribution of wealth in households. The modified model brings about non-zero outcomes, derived by Correia (1996), who considers non-taxable factors of production.

The income-growth-maximizing tax system is considered. Endogenous economic growth considering public investment is examined by Barro (1990), Futagami, Morita, and Shibata (1993) and by others. Typically, these studies set the model financed by income taxation for public investment to pull up the income growth rate. Watanabe, Miyake, and Yasuoka (2014) examine the case of a consumption tax. The consumption tax brings about higher income growth than the case of income taxation. Gaube (2005) derives optimal taxation in the public investment model with heterogeneous skilled labor. As shown in the field of optimal taxation problems specifically by Bertola, Foellmi and Zweimüller (2006), and Petrucci (2002), capital income taxation reduces capital accumulation and decreases income growth.

Caselli (1999), Meckl and Zink (2004), and Miyake, Muro, Nakamura, and Yasuoka (2009) set a model with income inequality by which individual ability is distributed uniformly. Using the model, they examine the manner and degree to which a technology shock affects income inequality. Aronsson, Sjögren and Dalin (2009) set a redistribution model with individual ability type and derive non-zero capital income taxation. Werning (2007) also shows that individual skill differences have a role in determining the optimal tax rate for redistribution. Kim (1989) considers the heterogeneous labor supply and

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[4] Our paper refers to surveys of optimal taxation by Boadway (2012), Kocherlakota (2010), and Salanié (2011).

derives that the optimal taxation depends on the elasticities of labor supply. Boadway, Marchand, Pestieau and Racionero (2002) and Fleurbaey and Maniquet (2007) derive different tax burdens among different individual ability and preferences for leisure. As a redistribution form, an education subsidy can be considered. Shindo and Yanagihara (2011) derives that an education subsidy as a redistributive policy increases welfare.

The remainder of this paper has the following structure. Section 2 sets the model with redistribution policy. Section 3 presents derivation of the equilibrium. We examine how the difference of taxation to provide the redistribution policy affects income inequality and an income level in section 4. Section 5 explains analyses of social welfare. The final section provides concluding and remarks.

2. Model

This model economy has three agents: households, firms, and government. The population size is set as unity. The population size does not grow over time.

2.1 Household

The individuals in the household live in two periods: a young period and old period. They consume in each period. c_{1t} and c_{2t+1} respectively denote consumption in the young period and that in the old period. On consumption in each period is levied a consumption tax at rate τ_c.

Individuals in the young period supply their labor inelastically to gain labor income. $\overline{w}_t$ denotes the wage rate. The government collects tax revenue for labor income at tax rate τ to provide for a lump-sum redistribution T_t. Defining $1 + r_{t+1}$ as an interest rate in t+1 period, the household lifetime budget constraint is shown as

$$(1 + \tau_c)c_{1t} + \frac{(1 + \tau_c)c_{2t+1}}{1 + r_{t+1}} = (1 - \tau)\overline{w}_t + T_t. \tag{1}$$

The household in our economy model cares about consumption in young and old periods. Our paper assumes the following log utility function shown below.

$$u_t = \frac{c_{1t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{c_{2t+1}^{1-\theta}}{1-\theta}, \quad 0 < \theta, 0 < \rho \tag{2}$$

If $\theta = 1$, then this function form becomes a log utility function: $u_t = \ln c_{1t} + \frac{1}{1+\rho}\ln c_{2t+1}$.

Also, ρ denotes the discount rate for the utility in the old period.

The optimal allocations to maximize the utility function (2) subject to the budget constraint (1) are reduced as shown below.

$$c_{1t} = \frac{1}{1 + \tau_c} \frac{(1 - \tau)\overline{w}_t + T_t}{1 + \frac{(1 + r_{t+1})^{\frac{1}{\theta} - 1}}{(1 + \rho)^{\frac{1}{\theta}}}} \tag{3}$$

$$c_{2t+1} = \left(\frac{1 + r_{t+1}}{1 + \rho}\right)^{\frac{1}{\theta}} c_{1t} \tag{4}$$

Defining the household saving as $s_t = (1 - \tau)\overline{w}_t + T_t - (1 + \tau_c)c_{1t}$, the saving is given by this equation and (3) as presented below.

$$s_t = \frac{(1 - \tau)\overline{w}_t + T_t}{1 + \frac{(1 + \rho)^{\frac{1}{\theta}}}{(1 + r_{t+1})^{\frac{1}{\theta} - 1}}} \tag{5}$$

If a log utility function is assumed, then the household saving is independent of an interest rate because $\theta = 1$.

2.2 Firm

Two sectors exist in the firm: one for a skilled sector and the other for an unskilled sector. Both sectors produce the same final goods.

Our paper assumes heterogeneity of labor productivity. Labor productivity a is distributed uniformly between 0 and $\overline{a}$ $(0 < \overline{a})$. In 2.2.3, young people decide the sector in which they work.

2.2.1 Skilled Sector

The final goods in the skilled sector are inputted by capital stock K_t and labor N_t using the following technology.

$$Y_t = AK_t^\varepsilon N_t^{1-\varepsilon}, \ 0 < \varepsilon < 1, 0 < A \tag{5}$$

Y_t denotes the final good produced by the skilled sector. We consider a profit maximizing problem. The profit function of the skilled sector is shown as

$$\pi_t = (1 - \tau_R)(AK_t^\varepsilon N_t^{1-\varepsilon} - w_t N_t) - (1 + r_t)K_t. \tag{6}$$

τ_R denotes the corporate tax rate. w_t represents the wage rate per effective labor. The corporate tax is levied on the profit: the firm revenue minus costs including the labor wage.[5] This paper assumes that the capital stock is fully depreciated in a single period. With a perfectly competitive market, the profit maximization problem that the firms

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[5] This setting is that the capital cost is not allowed to subtract the cost in a corporate tax system. This setting is the same as that with Doi (2010). Doi (2010) considers that the rental cost is not the cost to subtract from corporate profit. Then, this corporate tax form affects the demand for labor and capital. We refer to another corporate tax form in the following section.

decides the demand of labor N_t and capital stock K_t to maximize the profit function (6) reduces the following wage rate and interest rate in the skilled sector.

$$w_t = A(1 - \varepsilon)\frac{K_t^\varepsilon}{N_t^\varepsilon} \tag{7}$$

$$1 + r_t = (1 - \tau_R)A\varepsilon\frac{K_t^{\varepsilon-1}}{N_t^{\varepsilon-1}} \tag{8}$$

2.2.2 Unskilled Sector

Unskilled sector inputs only labor and produces the final goods as the following technology.

$$Z_t = BL_t, \ \ 0 < B \tag{9}$$

Therein, Z_t denotes the final goods produced by the unskilled sector. Then, the wage rate in unskilled sector is defined as w_t^c. Considering the profit function as $\pi_t = BL_t - w_t^c L_t$, the profit maximizing condition reduces as

$$w^c = B \ . \tag{10}$$

The wage rate in the unskilled sector is constant over time.

2.2.3 Labor Mobility

Younger people supply labor either to the skilled sector or to the unskilled sector. If individuals work in the skilled sector, then they can obtain the wage $\overline{w}_t = aw_t$, depending on their ability a. However, labor in the unskilled sector obtains the wage given by (10) in spite of their ability $\overline{w}_t = B$. Therefore, if they have little ability to work in the skilled sector, they had better work in the unskilled sector because of the higher wage rate.

Based on the points presented above, individuals can hold $aw_t > B$ work at the skilled sector. Individuals having the ability to hold $aw_t < B$ work in the unskilled sector. Defining a_t^* to hold the following equation, we have

$$a_t^* w_t = B, \tag{11}$$

Considering uniform distribution of ability a, the share of working in skilled sector is $\frac{\overline{a}-a^*}{\overline{a}}$. The share of working in the unskilled sector is $\frac{a^*}{\overline{a}}$. Then, the effective labor supply in the skilled sector is $N_t = \int_{a^*}^{\overline{a}} a\frac{1}{\overline{a}}da$. Therefore, the wage rate and the interest rate in the skilled sector are written as follows.

$$w_t = A(1 - \varepsilon)\left[\frac{\overline{a}^2 - \overline{a}_t^{*2}}{2\overline{a}}\right]^{-\varepsilon} K_t^\varepsilon \tag{12}$$

$$1 + r_t = (1 - \tau_R)A\varepsilon K_t^{\varepsilon-1}\left[\frac{\overline{a}^2 - \overline{a}_t^{*2}}{2\overline{a}}\right]^{1-\varepsilon} \tag{13}$$

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2.3 Government

The government collects tax revenues to provide the lump-sum redistribution T_t. The government budget constraint is shown below.

$$T_t = \tau_c \left[\int_0^{\bar{a}} \frac{c(a)_{1t}}{\bar{a}} da + \int_0^{\bar{a}} \frac{c(a)_{2t}}{\bar{a}} da \right] + \tau \left[\int_{a^*}^{\bar{a}} \frac{a w_t}{\bar{a}} da + \int_0^{a^*} \frac{B}{\bar{a}} da \right] + \tau_R \left(A K_t^\theta N_t^{1-\theta} - w_t N_t \right) \quad (14)$$

The first term of the right-hand-side shows the consumption tax revenue. $c_{1t}(a)$ and $c_{2t}(a)$ shows that the consumption level depends on ability a. The second term shows revenues from labor income. The final term shows corporate tax revenues.

3. Equilibrium

The equilibrium of this paper is specified as the capital stock dynamics. Considering household savings as $s_t = (1 - \tau) w_t - c_{1t}$, the savings of skilled labor s_t^s and unskilled labor s_t^u are given as

$$s_t^s = \frac{1}{\alpha_{t+1}} \int_{a_t^*}^{\bar{a}} \left((1 - \tau) a w_t + T_t \right) \frac{1}{\bar{a}} da \quad (15)$$

$$s_t^u = \frac{1}{\alpha_{t+1}} \int_0^{a_t^*} \left((1 - \tau) B + T_t \right) \frac{1}{\bar{a}} da \quad (16)$$

where

$$\alpha_{t+1} = 1 + \frac{(1+\rho)^{\frac{1}{\theta}}}{(1+r_{t+1})^{\frac{1}{\theta}-1}} \quad . \quad (17)$$

The dynamics of capital stock $K_{t+1} = s_t^s + s_t^u$ is shown as follows.

$$K_{t+1} = \frac{1}{\alpha_{t+1}} \left(\frac{1-\tau}{\bar{a}} \left(\frac{(\bar{a}^2 - a_t^{*2}) w_t}{2} + B a_t^* \right) + T_t \right). \quad (18)$$

Given K_t, the share of working in skilled sector a_t^* (11), the wage rate w_t (12) and the interest rate $1 + r_t$ (13) are determined. K_{t+1} is given as (18). Consumption c_{1t}, c_{2t} are given respectively by (3) and (4). The redistribution benefit is given as (14).

The local stability condition at the steady state is $-1 < \frac{dK_{t+1}}{dK_t} < 1$. $\frac{dK_{t+1}}{dK_t}$ without a redistribution policy derived as[6]

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[6] See Appendix for a detailed proof.

$$\frac{dK_{t+1}}{dK_t} = \frac{1}{\alpha\bar{a}} \frac{\dfrac{(\bar{a}^2 - a^{*2})\varepsilon w}{2K} \dfrac{1}{1 + \dfrac{2\varepsilon a^*}{\bar{a}^2 - a^{*2}}}}{1 + \dfrac{1-\theta}{\theta}\dfrac{\alpha-1}{\alpha}(1-\varepsilon)\left(1 - \dfrac{1}{\dfrac{\bar{a}^2 - a^{*2}}{2\varepsilon a^{*2}} + 1}\right)}, \tag{19}$$

where $\frac{\alpha-1}{\alpha} > 0$ is positive because of (17). The steady state is defined by that endogenous variables x_t is constant over time as x. With $\theta = 1$, which is a log utility function and the case in which that the saving is unaffected by the interest rate, we always obtain $\frac{dK_{t+1}}{dK_t} > 0$.

However, if $\theta \neq 1$, then the denominator of (19) might be negative. Defining θ^* to hold $1 + \frac{1-\theta}{\theta}\frac{\alpha-1}{\alpha}(1-\varepsilon)\left(1 - \frac{1}{\frac{\bar{a}^2-a^{*2}}{2\varepsilon a^{*2}}+1}\right) = 0$, we obtain $\frac{dK_{t+1}}{dK_t} < 0$ with $\theta > \theta^*$, which is not substitutive between young and old consumption.

4. Redistribution Policy

In this section, we examine how the redistribution policy affects the capital stock, wage rate, the share of skilled labor, and other outcomes. Our paper presents consideration of taxation of labor income, corporate profits, and consumption, and presents an examination of whether different financing of revenues to provide a lump-sum redistribution brings about a substantial difference in income levels and income inequality, or not.

4.1 Labor Income Tax

In this subsection, we assess a redistribution policy with income taxation. Then, the government budget constraint is shown as

$$T_t = \frac{\tau}{\bar{a}}\left(\frac{(\bar{a}^2 - a_t^{*2})w_t}{2} + Ba_t^*\right) . \tag{20}$$

Considering (18) and (20), the dynamics equation of K_t is given as shown below.

$$K_{t+1} = \frac{1}{\alpha_{t+1}}\frac{1}{\bar{a}}\left(\frac{(\bar{a}^2 - a_t^{*2})w_t}{2} + Ba_t^*\right) \tag{21}$$

This equation is equal to the dynamics equation without a redistribution policy and labor taxation. Therefore, the capital stock, wage rate in the skilled sector, interest rate, and the share of skilled labor in the steady state do not change. The inequality between the skilled sector and the unskilled sector, representing wage inequality, does not change

because $\frac{w^s}{w^u} = \frac{w}{B}$ is constant.

We define $\frac{(1-\tau)(\bar{a}-a^*)}{2} + T$ as the average income of skilled labor. Then, the income inequality between average skilled labor and average unskilled labor is given as $Q \equiv \frac{\frac{(1-\tau)(\bar{a}-a^*)}{2}+T}{(1-\tau)B+T}$. This ratio is regarded as household disposable income inequality. Because of income tax rate τ and redistribution income T, this income inequality shrinks. Therefore, the following proposition is established.

Proposition 1

A redistribution policy with labor income taxation can reduce household disposable income inequality between skilled labor and unskilled labor without changing the capital stock, wage rate, interest rate or the share of skilled workers.

We regard the result as intuitive. Regarding redistribution within the same generation, that is younger people, the aggregate saving does not change. Therefore, the level of capital stock does not change. This tax instrument can always shrink the income inequality.

4.2 Consumption Tax

This subsection presents an examination of how the redistribution policy financed by consumption tax affects inequality.

If the government uses the consumption tax to provide a lump-sum redistribution, then the government budget constraint in the steady state is shown as

$$T = \tau_c \left[\int_0^{\bar{a}} \frac{c_1(a)}{\bar{a}} da + \int_0^{\bar{a}} \frac{c_2(a)}{\bar{a}} da \right] . \tag{22}$$

From total differentiation of (22) by T and τ_c at the steady state in $\tau_c = 0$, the following equation is obtained as

$$\frac{dT}{d\tau_c} = \varphi, \tag{23}$$

where $\varphi = \int_0^{\bar{a}} \frac{c_1(a)}{\bar{a}} da + \int_0^{\bar{a}} \frac{c_2(a)}{\bar{a}} da$. Total differentiation of (18) by K, T at the steady state, we obtain $\frac{dK}{d\tau_c}$ as

$$\frac{dK}{dT} = \frac{\bar{a}}{\frac{(\bar{a}^2-a^{*2})\varepsilon w}{2K} \frac{1}{1+\frac{2\varepsilon a^{*2}}{\bar{a}^2-a^{*2}}} \left(\frac{dK_t}{dK_{t+1}}-1\right)} .. \tag{24}$$

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The sign of $\frac{dK}{dT}$ is ambiguous. If the log utility function is assumed $(\theta = 1)$, then this sign is always positive because the denominator is positive, given by the local stable condition. However, our paper assumes CRRA utility function which includes a log utility function as a more general utility function. Therefore, the sign of the denominator is positive or negative. If $\theta < \theta^*$, then we obtain $\frac{dK}{dT} > 0$. The redistribution policy increases the capital stock. An increase in capital stock increases the wage rate in skilled labor as follows because of (11) and (12):[7]

$$\frac{dw}{dK} = \frac{\varepsilon w}{K} \frac{1}{1 + \frac{2\varepsilon a^{*2}}{\bar{a}^2 - a^{*2}}} > 0. \tag{25}$$

An increase in capital stock increases the wage rate in skilled sector raises because of (24). Then the wage inequality $\frac{w^s}{w^u} = \frac{w}{B}$ is magnified by this redistribution policy. The share of skilled labor also increases. Total differentiation of (11) by a^*, w at the steady state reduces to the following.

$$\frac{da^*}{dw} = -\frac{a^*}{w} \tag{26}$$

The redistribution policy increases the wage rate and then pulls up the share of skilled labor. However, this redistribution policy raises labor income inequality among skilled laborers because of a decrease in a^*; i.e. the variance of ability a in skilled labor increases.

Now, we consider the household disposable income inequality. Then, the effects of the redistribution policy on the inequality are

$$\frac{dQ}{dT} = Q\left(\frac{\bar{a}}{(\bar{a} + a^*)w} \frac{dw}{dT} - \left(\frac{1}{B} - \frac{1}{\frac{\bar{a} + a^*}{2}w}\right)\right). \tag{27}$$

If $\frac{dw}{dT} < \frac{1}{\bar{a}}\left(\frac{(\bar{a} + a^*)w}{B} - 2\right)$, that is, if an increase in wage rate is small, then the household disposable income inequality between the two groups can be reduced by virtue of the redistribution policy. This condition holds for the case in which an increase in the wage rate is low level.

Considering (11) and (13), it is apparent that that the redistribution policy affects an interest rate in the steady state as shown below.

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[7] Total differentiation of (11) and (12) reduces to $wda^* + a^*dw = 0$ and $dw = \frac{\varepsilon w}{K}dK - \varepsilon\left(\frac{\bar{a}^2 - a^{*2}}{2\bar{a}}\right)^{-1}\left(-\frac{a^*}{\bar{a}}\right)wda^*$. Considering these two equations, we obtain (24).

$$\frac{dr}{dT} = -\frac{(1-\varepsilon)(1+r)}{K}\left(1 - \frac{1}{\frac{\bar{a}^2 - a^{*2}}{2\varepsilon a^{*2}} + 1}\right)\frac{dK}{dT} \qquad (28)$$

As long as $\theta < \theta^*$, an interest rate decreases because of the redistribution policy. Then, the following proposition is established.

Proposition 2

With $\theta < \theta^*$, the redistribution policy financed by consumption tax raises the capital stock, wage rate, share of skilled labor, wage inequality between skilled labor and unskilled labor, and wage inequality among skilled laborers. It decreases the interest rate. If $\frac{dw}{dT} < \frac{1}{\bar{a}}\left(\frac{(\bar{a}+a^*)w}{B} - 2\right)$, then the household disposable income inequality between skilled labor and unskilled labor shrinks.

In the case of $\theta < \theta^*$, a redistribution policy raises the capital stock. This case might be a usual case. The consumption tax brings about income transfer from older people to younger people. Household income of young people increases and saving increases. Then, an increase in investment raises the capital stock. An increase in the capital stock raises productivity in skilled labor and the wage rate in skilled labor. Then, labor mobility to skilled labor increases. Because of these effects, income inequality does not shrink.

If $\theta > \theta^*$, then we obtain the opposite result. This case is complementary between consumption in the young period and that in the old period. $\frac{dK}{dT}<0$ brings about an increase in the interest rate. Saving might decrease because an increase in the interest rate decreases the necessary saving to maintain the consumption level in the old period. The decrease in the capital stock decreases the wage rate for skilled labor. Actually, the share of skilled labor and $\frac{dQ}{dT}$ is always negative. Income inequality, regarded as wage inequality between skilled labor and unskilled labor, household disposable income inequality and wage inequality among skilled labor decreases. Then, the following proposition is established.

Proposition 3

With $\theta > \theta^*$, the redistribution policy financed by consumption tax reduces the capital stock, wage rate, share of skilled labor, wage inequality between skilled labor and unskilled labor, wage inequality among skilled laborers, and the household disposable income between skilled labor and unskilled labor. It also increases the interest rate.

This result is unobtainable at the log utility function $\theta = 1$. The analyses presented herein demonstrate that the inter-temporal substitution of consumption should be considered in providing redistribution policy.

4.3 Corporate Tax

This section presents assessment of a redistribution policy with a corporate tax. Then, the government budget constraint is shown below.

$$T = \tau_R \varepsilon A K_t^\varepsilon N_t^{1-\varepsilon} = \tau_R(1 + r)K \tag{29}$$

From total differentiation of (29) by T and τ_R at the steady state in $\tau_R = 0$, the following equation is obtained.

$$dT = (1 + r)Kd\tau_R \tag{30}$$

From total differentiation of (11), (12), (13), (17), (18) and considering (19) and (30), we obtain the sign of $\frac{dK}{dT}$ as

$$\frac{dK}{dT} = \frac{\bar{a}\left(1 - \dfrac{1 - \theta}{\theta}\dfrac{\alpha - 1}{(1 + r)}\right)}{\dfrac{(\bar{a}^2 - a^{*2})\varepsilon w}{2K}\dfrac{1}{1 + \dfrac{2\varepsilon a^{*2}}{\bar{a}^2 - a^{*2}}}\left(\dfrac{dK_t}{dK_{t+1}} - 1\right)}. \tag{31}$$

A corporate tax directly decreases capital income because of (8). This negative effect on capital income prevents investment in the capital stock. Defining θ^{**} such that $1 = \frac{1-\theta}{\theta}\frac{\alpha-1}{(1+r)}$, if $\theta < \theta^{**} < \theta^*$, we obtain $\frac{dK}{dT} < 0$.[8] Then, the wage rate of skilled labor and the share of skilled labor decrease because of (25) and (26). Considering (27), the redistribution policy brings about $\frac{dQ}{dT} < 0$, the household disposable income inequality shrinks. Totally differentiating (8), (11), (12), (13), (17), (18) and (29) by r, w, K, a^*, T, τ_R at $\tau_R = 0$, we obtain

$$\frac{dr}{d\tau_R} = -\frac{(1 - \varepsilon)(1 + r)}{K}\left(1 - \frac{1}{\dfrac{\bar{a}^2 - a^{*2}}{2\varepsilon a^{*2}} + 1}\right)\frac{dK}{dT} - (1 + r). \tag{32}$$

The second term of the right-hand-side shows the direct effect of corporate taxation on an interest rate, which is a negative effect on the interest rate. However, a decrease

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[8] The numerator of (31) is negative in $\frac{1-\theta}{\theta} > \frac{(1+r)}{\alpha-1}$. The sign of $\frac{dK_{t+1}}{dK_t}$ is negative in $\frac{1-\theta}{\theta} > -\frac{1}{\frac{\alpha-1}{\alpha}(1-\varepsilon)\left(1-\frac{1}{\frac{\bar{a}^2-a^{*2}}{2\varepsilon a^{*2}}+1}\right)}$. This inequality reduces to $\theta^{**} < \theta^*$.

in capital stock pulls up the marginal productivity of capital stock, which is shown by the first term on the right-hand-side. The results in $\theta < \theta^{**} < \theta^*$ are the same as those in $\theta^{**} < \theta^* < \theta$. Then, the following proposition is established.

Proposition 4
With $\theta < \theta^{**} < \theta^*$ or $\theta^{**} < \theta^* < \theta$, the redistribution policy financed by a corporate tax decreases the capital stock at the steady state. Then, the wage rate of skilled labor, wage inequality between skilled labor and unskilled labor, the share of skilled labor, and household disposable income inequality all decrease. The effect on the interest rate is ambiguous.

Actually, $\theta < \theta^{**} < \theta^*$ shows the case with a positive sign in denominator of (31) and negative sign in numerator of (31). This case shows that an increase in capital stock brings more capital stock; the negative effect of a corporate tax reduces the capital stock. We regard this case as an ordinary case. However, in $\theta^{**} < \theta^* < \theta$, the sign in the denominator of (31) is negative and the sign in numerator of (31) is positive, which is not an ordinary case. This case shows that an increase in capital stock reduces the investment for capital stock in the subsequent period. Then, even if the effect of a corporate tax on the capital stock is small and the household saving increases, this increase reduces the steady state capital stock because of the negative sign of (19).

If $\theta^{**} < \theta < \theta^*$, then both the numerator and denominator of (31) are positive. We can obtain $\frac{dK}{dT} > 0$. Then, we can obtain the same case with the case of consumption tax as shown by the following proposition.

Proposition 5
With $\theta^{**} < \theta < \theta^*$, the capital stock in the steady state increases. Then, the wage rate of skilled labor, the share of skilled labor, and wage inequality between skilled labor and unskilled labor all increase. Household lifetime income inequality shrinks in $\frac{dw}{dT} < \frac{1}{\bar{a}}\left(\frac{(\bar{a}+a^*)w}{B} - 2\right)$. The interest rate decreases.

This case shows the effect of corporate tax on capital accumulation when an increase in capital stock raises more investment for capital accumulation, which is the ordinary case.

Our paper sets the corporate tax on profits, or revenues minus labor costs. If we consider the corporate tax as

$$\pi_t = (1 - \tau_R)\big(AK_t^{\theta}N_t^{1-\theta} - w_tN_t - (1 + r_t)K_t\big), \qquad (33)$$

where the corporate tax revenue is zero because the product function has constant returns to scale in a perfectly competitive market.

This corporate tax is the same with taxation for an interest rate. If taxation for interest income is considered, then the result is the same as those described above. Considering τ_r as the capital income tax rate, (8) changes to

$$1 + (1 - \tau_r)r_t = A\varepsilon \frac{K_t^{\varepsilon-1}}{N_t^{\varepsilon-1}}. \qquad (34)$$

Capital income taxation affects the interest rate, which has the same effect as corporate taxation.

5. Welfare

This section presents consideration of the welfare level. We define the social welfare function as the following equation:

$$W_t = \int_0^{\bar{a}} \frac{c(a)_{1t}^{1-\theta}}{1-\theta}\frac{1}{\bar{a}}da + \frac{1}{1+\rho}\int_0^{\bar{a}} \frac{c(a)_{2t+1}^{1-\theta}}{1-\theta}\frac{1}{\bar{a}}da. \qquad (35)$$

Consumption levels of $c(a)_{1t}$ and $c(a)_{2t+1}$ are given respectively by (3) and (4). In a steady state, the redistribution policy financed by labor income taxation always pulls up the social welfare given by (35). In labor income taxation, the capital stock does not change and income inequality as unskilled labor and household disposable income decreases. Moreover, thanks to the decrease in marginal utility, the income transfer from the rich to the poor increases the level of social welfare.

However, the consumption tax and corporate tax do not always increase social welfare. If the redistribution policy financed by the consumption tax and corporate tax increases the wage rate and the share of skilled labor because of an increase in capital stock, then the level of social welfare increases. However, an increase in capital stock reduces the interest rate and might decrease consumption in the old period. Therefore, social welfare does not always increase.

6. Conclusion

Our paper presents an examination of how differences of taxation affect both income inequality and the income level. As a result, financing by labor income taxation can

always shrink income inequality. The redistribution policy with a consumption tax increases wage inequality. Income inequality after redistribution within the group and between groups might be magnified. However, the corporate tax and capital income tax reduce the capital stock and wage inequality in the case in which an intertemporal consumption is substitutive.

Aghion (2002) shows that inequality within a group increases and that inequality between groups decreases as the skill premium puzzle. A technology shock is a useful tool to explain this skill premium puzzle. However, our paper presents derivation of the fact that income inequality is brought about by other reasons such as taxation. From the view of optimal taxation studies, our paper shows that labor income should be used for redistribution policies to raise social welfare, not a consumption tax or capital income tax.

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Appendix

A.1 Derivation of $\frac{dK_{t+1}}{dK_t}$

Total differentiation of (11)–(13), (17) and (18) at the steady state derives the following equations.

$$da_t^* = -\frac{a^*}{w} dw_t \tag{A.1}$$

$$\frac{\varepsilon w}{K} dK_t = \left(1 + \frac{2\varepsilon a^{*2}}{\bar{a}^2 - a^{*2}}\right) dw_t \tag{A.2}$$

$$dr_{t+1} = -\frac{(1-\varepsilon)(1+r)}{K} dK_{t+1} + \frac{2a^*(1+r)(1-\varepsilon)}{\bar{a}^2 - a^{*2}} da_{t+1}^* \tag{A.3}$$

$$d\alpha_{t+1} = \frac{\theta - 1}{\theta}\left(\frac{1+\rho}{1+r}\right)^{\frac{1}{\theta}} dr_{t+1} \tag{A.4}$$

$$\bar{a}\alpha dK_{t+1} = \frac{\bar{a}^2 - a^{*2}}{2} dw_t - \bar{a}K \frac{\theta - 1}{\theta}\left(\frac{1+\rho}{1+r}\right)^{\frac{1}{\theta}} dr_{t+1} \tag{A.5}$$

Putting (A.1)–(A.4) into (A.5), we obtain the stable condition $\frac{dK_{t+1}}{dK_t}$.

A.2 Existence of steady state equilibrium

This appendix presents the existence of a steady state equilibrium with numerical examples. The steady state equilibrium is given by the following equations.

$$\bar{a}\left(1 + (1+\rho)^{\frac{1}{\theta}}(1+r)^{1-\frac{1}{\theta}}\right)K = \frac{\bar{a}^2}{2}w + \frac{B}{2}a^* \tag{A.6}$$

$$w = A(1-\varepsilon)\left(\frac{\bar{a}^2 - a^{*2}}{2\bar{a}}\right)^{-\varepsilon} K^\varepsilon \tag{A.7}$$

$$1 + r = A\varepsilon\left(\frac{\bar{a}^2 - a^{*2}}{2\bar{a}}\right)^{1-\varepsilon} K^{\varepsilon-1} \tag{A.8}$$

$$a^* = \frac{B}{w} \tag{A.9}$$

We set parameters $\bar{a} = 1, \rho = 0.1, \varepsilon = 1$, $A = 1, B = 0.1$ for an exemplary case. We obtain K=0.104854 at $\theta = 1.5$, K=0.098634 at $\theta = 1$, and K=0.088894 at $\theta = 0.5$. These results demonstrate the existence of the steady state equilibrium.