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Abstract

We consider a duopoly model in which firms with different costs supply two vertically differentiated products in the same market. We show that the efficient firm produces more of the high-quality good and the inefficient one produces more of the low-quality good in equilibrium. We also find that a change in the quality superiority of goods and relative cost efficiency ratios leads to cannibalization from one good to the other and characterize graphically the product line strategies of firms through the two ratios.

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1 Introduction

A real economy has oligopolistic markets in which firms produce and sell multiple, vertically differentiated products in the same market. Such markets present more cases of *cannibalization.* Cannibalization occurs *in a market* when a firm increases the output of one of its products by reducing the output of a similar product competing in the same market.

The objective of this study is to examine cannibalization in a market from the strategic viewpoint of a multi-product firm supplying two goods differentiated in quality.

We do not consider new entries to the market and the choice of quality level, as in Johnson and Myatt (2003), but a duopoly in which each firm produces and supplies two vertically differentiated goods, that is, a high-quality and a low-quality good, in the market. We then explore the condition under which both firms specialize in, or else one of them specializes in, the high-quality or low-quality good. To understand how cannibalization affects the product line strategies of firms, we consider two ratio indicators: (1) quality superiority ratio of the high-quality good relative to the low-quality good, and (2) the relative marginal cost efficiency ratio of the high-quality good between the two firms.

We find that cannibalization is a product line control strategy characterized by the quality superiority of the high-quality good relative to the low-quality good and the relative cost efficiency of the efficient firm. By limiting our study to at most two vertically differentiated goods that each firm supplies to the same market, we characterize the product line strategies of firms through cannibalization graphically in the plane of the two ratio indicators.

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2 The Model and Derivation of an Equilibrium

Suppose there are two firms, $i = 1, 2$, and each firm produces two goods (good H and good L) differing in terms of quality, where 1 and 2 imply firms 1 and 2, respectively, in the duopoly. Let V_H and V_L denote the quality level of the two goods. We assume that the maximum amount consumers are willing to pay for each good is $V_H = \mu V_L = \mu > V_L = 1$. Thus, for simplicity, we normalize the quality of the low-quality good as $V_L = 1$ and assume that the quality of the high-quality good is μ fold that of the low-quality good. Good α $(= H, L)$ is assumed to be homogeneous for any consumer.

Consumer preferences are standard a la Mussa–Rosen. Following the standard specification in the literature, for example, Katz and Shapiro (1985), we assume a continuum of consumers characterized by taste parameter θ, which is uniformly distributed between 0 and $r(> 0)$, with density 1. We further assume that a consumer of type $\theta \in [0, r]$,for $r > 0$, obtains a net surplus from one unit of good α from firm i at price $p_{i\alpha}$. Thus, the utility (net benefit) of consumer θ who buys good α $(= H, L)$ from firm i $(= 1, 2)$ can be given by

$$U_{i\alpha}(\theta) = V_\alpha \theta - p_{i\alpha} \qquad i =, 1, 2 \quad \alpha = H, L. \tag{1}$$

Each consumer decides to buy either nothing or one unit of good α from firm i to maximize his/her surplus.

From the following three usual assumptions about consumers, we derive the demand for good H as $Q_H = r - \widehat{\theta}$ and that for good L as $Q_L = \widehat{\theta} - \underline{\theta}_{iL}$, where $Q_\alpha = q_{i\alpha} + q_{j\alpha}, \alpha = H, L, i \neq j, i, j = 1, 2$. The demand function is similar to that derived in Bonanno (1986), but it varies from Bonnano in that both firms supply two vertically differentiated products in the same market. For details of the derivation, see Kitamura and Shinkai (2013).

Here, $\widehat{\theta}$, the threshold between the demand for products H and L, is given by

$$\widehat{\theta} = (p_H - p_L)/(\mu - 1). \tag{2}$$

We then obtain the inverse demand functions as follows:

$$\begin{cases} p_H = \mu(r - Q_H) - Q_L \\ p_L = r - Q_H - Q_L. \end{cases} \tag{3}$$

Furthermore, assume that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L = 0$, where $c_{i\alpha}$ is firm i's marginal and average cost of good α. This implies that a high-quality good incurs a higher cost of production than a low-quality good. Here, without loss of generality, we assume that $c_{2H} > c_{1H} = 1 > c_{iL} = 0$, implying that firm 1 is more efficient than firm 2. Under these assumptions, each firm's profit can be defined as follows:

$$\pi_i = (p_{iH} - c_{iH})q_{iH} + p_{iL}q_{iL} \qquad i = 1, 2, \tag{4}$$

where $p_{i\alpha}$ is the price of good α sold by firm i and $q_{i\alpha}$ is the firm's output of good α. Each firm chooses a quantity of supply that maximizes its profit function in Cournot fashion.

By solving the system of usual first-order condition equations of q_{iH} and q_{iL}, $i = 1, 2$, we obtain the following Nash equilibrium quantities:

$$\begin{cases} q_{1H}^* = (r - (2 - c_{2H})/(\mu - 1))/3, \quad q_{1L}^* = (2 - c_{2H})/(3(\mu - 1)) \\ q_{2H}^* = (r - (2c_{2H} - 1)/(\mu - 1))/3, \quad q_{2L}^* = (2c_{2H} - 1)/(3(\mu - 1)). \end{cases} \tag{5}$$

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We find that the second-order condition holds for both firms. From (5), the nonnegative output condition, and the assumption that $c_{2H} > 1$, we find that

$$q_{1L}^* \gtreqless 0 \Leftrightarrow 2 \gtreqless c_{2H}$$

$$q_{2H}^* > (=)0 \Leftrightarrow \mu > (\leq)(2c_{2H} - 1 + r)/r$$

$$q_{1H}^* > (=)0 \Leftrightarrow \mu > (\leq)(2 + r - c_{2H})/r \tag{6}$$

hold.

From (3) and (5), we obtain the following equilibrium prices of the goods:

$$p_H^* = (\mu r + c_{2H} + 1)/3, \ p_L^* = r/3. \tag{7}$$

We also obtain the equilibrium profit of firm i:

$$\pi_1^* = \{\mu r^2 - 2r(2 - c_{2H}) + (2 - c_{2H})^2/(\mu - 1)\}/9, \tag{8}$$

$$\pi_2^* = \{\mu r^2 - 2r(2c_{2H} - 1) + (2c_{2H} - 1)^2/(\mu - 1)\}/9. \tag{9}$$

Thus, we can easily establish the following proposition.

Proposition 1 *There exists a versioning strategy equilibrium in which both firms produce both types of goods. In the equilibrium, $q_{1H}^* > q_{2H}^*$, $q_{2L}^* > q_{1L}^*$, and $\pi_1^* > \pi_2^*$.*

Proof: From the equilibrium outcomes of (5), (8), and (9), we have

$$q_{1H}^* - q_{2H}^* = q_{2L}^* - q_{1L}^* > 0, \tag{10}$$

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and

$$\pi_1^* - \pi_2^* > 0 \Leftrightarrow \mu > (2r + 1 + c_{2H})/(2r).$$

However, for $q_{2H}^* > 0$, $\mu > (2c_{2H} + r - 1)/r$ must hold. Nevertheless, we can show that $(2c_{2H} + r - 1)/r > (2r + 1 + c_{2H})/(2r)$ if $1 < c_{2H}$; therefore, if $\mu > (2c_{2H} + r - 1)/r$, then $\mu > (2r + 1 + c_{2H})/(2r)$ and the result follows.∎

The proposition implies that the efficient firm 1 (inefficient firm 2) produces more of the high-quality good H (low-quality good L) than the inefficient firm 2 (efficient firm 1). It also asserts that the efficient firm 1 earns more than the inefficient firm 2 because of the cost efficiency of firm 1 over firm 2 for the high-quality good H under the non-negative assumption of output (6) in equilibrium.

Next, we examine the conditions under which cannibalization occurs from one product to another in equilibrium. Note that "a product cannibalizes a similar product" when a firm increases its output of the product by reducing that of a similar product supplied in the same market.

From (5), we have

$$q_{2H}^* - q_{2L}^* \gtreqless 0 \Leftrightarrow \mu \gtreqless (2(2c_{2H} - 1) + r)/r, \tag{11}$$

$$q_{1H}^* - q_{1L}^* \gtreqless 0 \Leftrightarrow \mu \gtreqless (2(2 - c_{2H}) + r)/r, \tag{12}$$

and

$$q_{2H}^* - q_{1L}^* = q_{1H}^* - q_{2L}^* \gtreqless 0 \iff \mu \gtreqless (c_{2H} + 1 + r)/r. \tag{13}$$

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From (6), we also find that

$$(c_{2H}+1+r)/r > (2c_{2H}-1+r)/r \text{ and } (2(2-c_{2H})+r)/r \gtreqless (2c_{2H}-1+r)/r \iff 5/4 \gtreqless c_{2H}.$$

(14)

Thus, from (5), we immediately obtain the following proposition.

Proposition 2 *In the duopoly equilibrium derived as above, the following inequalities hold for the outputs of the high-quality and low-quality goods of each firm*:

$$0 \; < \; q_{2H}^* < q_{1L}^* < q_{1H}^* \leq q_{2L}^* \; for \; (2c_{2H}-1+r)/r < \mu \leq (c_{2H}+1+r)/r \; and \; 1 < c_{2H} < 2 \; (I)$$

$$0 \; < \; q_{1L}^* < q_{2H}^* < q_{2L}^* < q_{1H}^* \; for \; (2c_{2H}-1+r)/r < \mu < (2(2-c_{2H})+r)/r \; and \; 1 < c_{2H} < 2 \; (II)$$

$$0 \; < \; q_{1L}^* < q_{2L}^* \leq q_{2H}^* < q_{1H}^* \; for \; (2(2c_{2H}-1)+r)/r < \mu \; and \; 1 < c_{2H} < 2 \; (III),$$

where the Greek numbers in the equations represent the area number in Figure 1.

Figure 1 summarizes the result of proposition 2 in the $c_{2H} - \mu$ plane.

(Insert Figure 1 here)

Note that we assume $c_{2H} > c_{1H} = 1$ and $V_H = \mu V_L = \mu > V_L = 1$. Thus, the horizontal and vertical axes variables in Figure 1 imply the relative cost ratio c_{2H} and quality value ratio μ. At any point (c_{2H}, μ) in Areas I, II, and III in Figure 1, both firms supply high- and low-quality goods. Thus, as the quality value ratio μ is sufficiently high and the relative cost ratio c_{2H} sufficiently low in these areas, the inefficient firm produces far more of the low-quality good with no production cost than it does of the

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high-quality good, which has a higher positive cost. In contrast, the efficient firm produces moderately more of the high-quality good H than it does of the low-quality good L, since its production cost for good H is lower than that of its rival firm. However, its marginal revenue from good H is not high, because its quality superiority μ is not very large. As the point (c_{2H}, μ) moves from Area I to Areas II and III in Figure 1, cannibalization proceeds from the low-quality good to the high-quality good in both firms. Such cannibalization is stronger for the efficient firm than for the inefficient one.

This result is consistent with Calzada and Valletti (2012), where the optimal strategy for a film studio is to introduce versioning if their goods are not close substitutes for each other. Thus, when the quality superiority of the high-quality good H is large compared to good L to some extent, we can conclude that they are not close substitutes for each other. Then, the result in the above proposition confirms that it would be better for both firms to supply both goods in the market, that is, to obey the "versioning strategy" in Calzada and Valletti (2012).

In contrast, we examine the case in which the quality speriority μ is not large and the relative cost efficiency c_{2H} is strong. We examine the relationship between the equilibrium outputs carefully in (14) and (5), to obtain the following proposition.

Proposition 3 *From the duopoly equilibrium derived above, we find that*

$$q_{1L}^* = 0 < q_{2L}^* < q_{2H}^* < q_{1H}^* \leq \text{ for } 1 < \mu < (2 - c_{2H} + r)/r \text{ and } 2 < c_{2H} \text{ (VI)}$$

$$q_{1L}^* = 0 < q_{2H}^* < q_{2L}^* < q_{1H}^* \text{ for } (2c_{2H} - 1 + r)/r < \mu < (2(2 - c_{2H}) + r)/r \text{ and } 2 < c_{2H} \text{ (V)}$$

$$q_{2H}^* = q_{1L}^* = 0 < q_{1H}^* < q_{2L}^* \text{ for } 1 < \mu < (c_{2H} + 1 + r)/r \text{ and } 2 < c_{2H} \text{ (IV)}$$

$$q_{2H}^* = 0 < q_{1L}^* < q_{1H}^* < q_{2L}^* \text{ for } 1 < (2(2c_{2H} - 1) + r)/r < \mu < (2c_{2H} - 1 + r)/r,$$

$$5/4 < c_{2H} < 2 \text{ (VII)}$$

$$q_{2H}^* = 0 < q_{1H}^* < q_{1L}^* < q_{2L}^* \text{ for } 1 < (2 + r - c_{2H})/r < \mu < (c_{2H} + 1 + r)/r,$$

$$\mu < (2(2c_{2H} - 1) + r)/r \text{ and } 1 < c_{2H} < 2 \text{ (VIII)}$$

$$q_{2H}^* = q_{1H}^* = 0 < q_{1L}^* < q_{2L}^* \text{ for } 1 < \mu < (2 + r - c_{2H})/r \text{ and } 1 < c_{2H} < 2 \text{ (IX)},$$

where the Greek numbers in the equations represent the area numbers in Figure 1. Figure 1 summarizes the results of proposition 3 in the $c_{2H} - \mu$ plane.

From (5), when the quality superiority μ is small and the relative cost efficiency c_{2H} are large, the efficient firm never supplies its low-quality good, and so in equilibrium, the market at first becomes a three-goods market. At any point (c_{2H}, μ) in Areas V and VI, the market is filled with large quantities of the high-quality good H supplied by both firms, but has relatively small quantities of the low-quality good L supplied by firm 2. As the quality superiority μ reduces further, the inefficient firm 2 stops producing the high-quality good H and specializes in the low-quality good. Thus, in Area IV, the efficient firm 1 specializes in supplying the high-quality good and the inefficient firm 2 specializes in supplying the low-quality good and the market becomes a vertically differentiated two-goods market.

On the other hand, when the relative cost efficiency c_{2H} is sufficiently small, if the

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relative quality superiority μ also reduces to nearly one and the goods become close substitutes to each other, the best strategy for both firms is to stop production of its high-quality good H and specialize in the low-quality good L. In Area IX, the market becomes a two low-quality goods market!

3   Conclusion

In this study, we considered and proposed a duopoly cannibalization model in which two firms produce and sell at most two distinct products that are differentiated vertically in the same market. We showed that in the market equilibrium, the efficient firm produces more of the high-quality good and the inefficient one more of the low-quality good. When the relative quality superiority of the high-quality good is small (large), cannibalization is stronger in firm 2 (firm 1) than in firm 1 (firm 2).
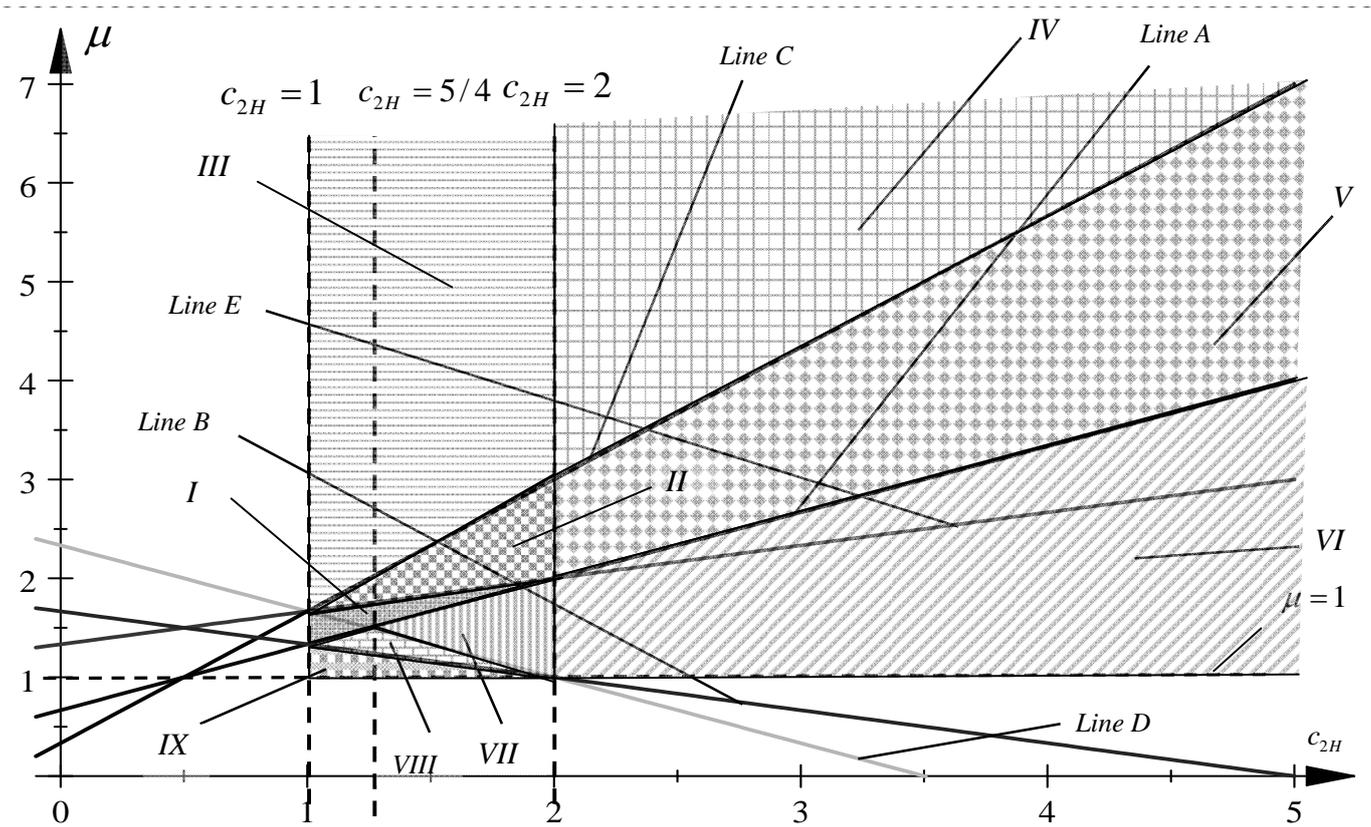
Furthermore, we showed that a change in the ratios of quality superiority and relative cost efficiency leads to cannibalization and that it crucially affects the decision making of a firm's product line.

4   Acknowledgement

References

[1] Bonanno, G. (1986), "Vertical Differentiation with Cournot Competition," *Economic Notes*, 15, No.2, pp.68-91.

[2] Calzada, J. and Valletti, T. (2012), "Intertemporal Movie Distribution: Versioning when Customers Can Buy Both Versions," *Marketing Science,* 31, No.4, pp.649-667.

[3] Johnson, J. P. and Myatt, D. (2003), "Multiproduct Quality Competition: Fighting Brands and Product Line Pruning," *American Economic Review*, 93, No.3, pp.3748-3774.

[4] Katz, M. and Shapiro, C. (1985), "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75, No.3, pp.424-440.

[5] Kitamura, R. and Shinkai, T. (2013), "The Economics of Cannibalization: A Duopoly in which Firms Supply Two Vertically Differentiated Products, " *presented paper in the EARIE 2013, Annual Conference of European Association for Research Industrial Economics, Evora, Portugal August 30-September 1*, pp.1-22 (August, 2013).

[6] Motta, M. (1993), "Endogenous Quality Choice: Price vs. Quantity Competition," *Journal of Industrial Economics*, 41, No.2, pp.113-131.

Line A: $\mu = (2c_{2H} - 1 + r)/r$, Line B: $\mu = (2 + r - c_{2H})/r$, Line C: $\mu = (2(2c_{2H} - 1) + r)/r$

Line D: $\mu = (2(2 - c_{2H}) + r)/r$, Line E: $\mu = (c_{2H} + 1 + r)/r$

Figure 1   Classification of Product Line Strategy in $c_{2H} - \mu$ Plane ($r = 3$)