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Cost Reduction can Decrease Profit and Welfare in a Monopoly

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Cost Reduction can Decrease Profit and Welfare in a Monopoly *

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Abstract

This paper develops a monopoly model in which two vertically differentiated goods are supplied and involve a within-product network externality. Within this model, I examine how the cost of the high-quality good affects the firm's profit and welfare, demonstrating a surprising result that both the profit and welfare are U-shaped in the cost and thus, in particular, a decrease in the marginal cost can reduce the monopoly profit. I show that the assumptions of the fulfilled expectations equilibrium and multi-product monopoly lead to this counter intuitive possibility. Furthermore, changes in production costs and in quality yield cannibalization such that the consumption of one good increases while that of the other decreases.

Keywords: Multi-product firm, Monopoly, Cannibalization, Network externality

JEL Classification Numbers: D21, D42, L12, L15

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1 Introduction

The majority of smartphone carriers sells both high- and low-quality smartphones.[1] In this industry, network externalities exist both across the products supplied by the same firm and within products (i.e., all consumers of a good gain as the number of users who purchase the same smartphone device increases). The existence of these network externalities motivate one to explore the market with (i) a within-product network externality and (ii) multi-product firms.[2] However, to my knowledge, no study has examined the positive and normative consequences of such a market.

Incorporating a within-product network externality into a multi-product monopoly model, this paper examines firm and consumer behavior and the resulting market configurations.[3] First, I find that under certain conditions, cannibalization arises namely an increase in the number of consumers of one good occurs at the expense of those of other goods sold by same firm (Copulsky, 1976).[4] Second, I show a counterintuitive result that a decrease in the marginal cost of a high-quality good can *reduce* the firm profit. More precisely, the profit becomes U-shaped in the marginal cost of the high-quality good. Third, the relationship between welfare and the marginal cost becomes U-shaped.[5] Two assumptions play a key role in these striking results. The first is the fulfilled expectations equilibrium, according to which " consumers' expectations about the sizes of the networks are given" (Katz and Shapiro, 1985, pp. 427–428). In this case, the firm cannot commit itself and thus is unable to transfer the network sizes optimally in response to the change

[1] An example of vertical differentiation between iPhone and Android smartphones in found in Geekbench (see http://browser.primatelabs.com/geekbench2/1030202 and http://browser.primatelabs.com/android-benchmarks).

[2] Kitamura (2013) defines this externality as follows: "A consumer who purchases a product from a certain firm gains a network benefit when other consumers purchase the same product from the same or different firm."

[3] I use a monopoly model to isolate the implication of a within-product network externality and a multi-product firm and to stress that the result holds even in the absence of strategic interactions among oligopolistic firms. The oligopoly case will be left to future research.

[4] The relevance of cannibalization has been established empirically. For instance, Ghose et al. (2006) and Smith and Telang (2008) find that 16% of used books, 24% of used CDs, and 86% of used DVDs directly cannibalize new product sales at Amazon.com.

[5] While in Lahiri and Ono(1988), they find that under cournot *oligopoly* a marginal cost reduction in a firm with a sufficiently low share decreases welfare, in this paper, under *monopoly* I show the similar result caused by two key assumptions: fulfilled expectations equilibrium and multi-product firm.

in the marginal cost.[6] The second important assumption is that of a multi-product firm. After making this assumption, the cost reduction leads to cannibalization so that the transition of network within firm affects profit and welfare.

The profit U-shaped in the marginal cost implies that a cost reduction, either through innovation or through an R&D subsidy, can decrease the firm profit. Here, let us consider a monopoly firm supplying two vertically differentiated products, namely a high-quality good and a low-quality good. Then, the firm's U-shaped profit in relation to the production cost suggests that such a cost reduction will decrease the monopoly profit if the production cost of the high-quality good is high and the degree of the cost reduction is small. The reason is that the equilibrium concept of fulfilled expectations means the firm cannot optimally transfer the network of the low-quality good to that of the high-quality good when the marginal cost of the latter decreases. Thus, the positive effect on the firm's profit from the high-quality good does not dominate the negative effect from the low-quality good in spite of the cost reduction. More importantly, a drastic cost reduction is needed to increase the profit. The result is that a small R&D subsidy is detrimental rather than beneficial.

There is a large literature on network externalities and multi-product firms. Katz and Shapiro (1985) are the first to formulate a duopoly model with a network externality across both firms' products.[7] Baake and Boom (2001) and Chen and Chen (2011) respectively consider an oligopoly and a duopoly model of vertical product differentiation with a network externality in which firms decide their degree of product compatibility. However, each firm only supplies one product, not multiple products. In this paper, the degree of compatibility is exogenous but a single firm produces two types of products. By contrast, Haruvy and Prasad (1998) analyze a market in which a monopolist sells high- and low-end versions of the same product and derive the conditions under which producing both goods is optimal with a network externality. On the other hand,

[6]This equilibrium concept, proposed by Katz and Shapiro (1985), has been used in the literature on network industries (e.g., Barrett and Yang, 2001; Hahn, 2003). In contrast to Katz and Shapiro (1985), whose main result holds irrespective of whether consumers form an expectation before the output decision, my result crucially depends on the assumption that consumers form an expectation *before* the output decision.

[7]For more extensive surveys, see Katz and Shapiro (1994) and Shy (2001).

Desai(2001) considers two segments duopoly markets for high-quality and low-quality goods represented by Hotelling type model without network externality. He examines whether the cannibalization problem affects a firm's price and quality decision. However, in their models, the two goods are sold in different markets, each with different types of consumers. Instead, I assume that both goods are supplied to the *same* market.

This paper is organized as follows. Section 2 presents the model and Section 3 derives the main results. Then, Section 4 shows the comparative statics. Section 5 concludes, and Appendix provides the proofs of the results in the main text.

2 The Model

This section presents the model. While I basically follow Katz and Shapiro (1985), who consider an oligopolistic network industry, I modify their model in two ways. First, I assume a monopoly to eliminate the strategic effect between the firms. Second, this single firm produces two vertically differentiated goods which may involve a network externality. In what follows, I describe the market equilibrium after characterizing the behavior of the firm and consumers.

I begin by considering the firm's behavior. Suppose a monopolistic firm producing two goods (H and L) that differ in their quality, and let V_H and V_L ($V_H > V_L$) denote the quality of each good. For simplicity, I assume that $V_H = (1 + \mu)V_L$, where $\mu > 0$ measures the degree of quality difference, and that the quality of good L is normalized to one (i.e., $V_L = 1$). The marginal cost of producing each good is given by c_H and c_L, respectively, which satisfy $c_H > c_L = 0$. Then, the firm's profit is defined by

$$(p_H - c_H)x_H + p_L x_L, \tag{1}$$

where x_α and p_α, for $\alpha = H, L$, are the output and price of good α, respectively. The monopolist chooses outputs to maximize (1).

To derive the inverse demand functions, I now describe the behavior of consumers. Following Katz and Shapiro (1985), consider a continuum of consumers characterized by a taste parameter θ that is uniformly distributed in $[-R, r]$, $R, r > 0$ with density one.[8]

[8] I assume that R is large enough to avoid a corner solution.

By purchasing one unit of good α, consumer $\theta \in [-R, r]$ obtains a net surplus[9]

$$U_\alpha(\theta) = V_\alpha\theta + \nu V_\alpha g^e_\alpha - p_\alpha, \qquad \alpha = H, L, \tag{2}$$

where the first term in the right-hand side is the intrinsic utility of consuming the good and the second term represents a network externality. Parameter $\nu > 0$ measures the degree of the network externality and g^e_α is the expectation over the network benefit, which takes the form

$$g^e_\alpha \equiv g_\alpha(x^e_\alpha) = x^e_\alpha, \quad \alpha = H, L. \tag{3}$$

Where, x^e_α is the expectation of output level of good α. Therefore, Eq. (3) represents the within-product externality.

Based on these preparations, I now derive the inverse demand functions. When consumer $\hat{\theta}$ is indifferent between purchasing good H and good L, it must hold that

$$\begin{aligned} U_H\left(\hat{\theta}\right) &= U_L\left(\hat{\theta}\right) > 0 \\ \iff (1+\mu)\hat{\theta} + \nu(1+\mu)g^e_H - p_H &= \hat{\theta} + \nu g^e_L - p_L. \end{aligned}$$

Thus, the index of this consumer is obtained as

$$\hat{\theta} = \frac{1}{\mu}\{p_H - p_L - \nu((1+\mu)g^e_H - g^e_L)\}. \tag{4}$$

Furthermore, there should be a consumer $\underline{\theta}_L$ who is indifferent between purchasing good L and nothing. The index of such a consumer satisfies

$$U_L\left(\underline{\theta}_L\right) = 0,$$

and, hence, is obtained as

$$\underline{\theta}_L = p_L - \nu g^e_L. \tag{5}$$

Then, from (2), (4), and (5), and given that $U_L(\cdot)$ is increasing in θ, I have

[9]Baake and Boom (2001) adopt a similar expression for the consumer surplus.

$$U_H(\hat{\theta}) = U_L(\hat{\theta}) > U_L(\underline{\theta}_L) = 0,$$

which is equivalent to

$$\hat{\theta} > \underline{\theta}_L. \tag{6}$$

The following lemma follows from this result.[10]

Lemma 1. *Any consumer $\theta \in (-R, \underline{\theta}_L)$ buys nothing, while consumer $\theta \in (\ \underline{\theta}_L, \widehat{\theta})$ ($\theta \in [\widehat{\theta}, r]$) buys good L (good H).*

From Lemma 1, the market-clearing conditions of goods H and L are

$$r - \hat{\theta} = x_H, \quad r - \underline{\theta}_L = x_H + x_L.$$

Substituting (4) and (5) into these equations and solving for p_H and p_L yields the inverse demand functions:

$$p_H = (1+\mu)(r + \nu g_H^e - x_H) - x_L, \quad p_L = r + \nu g_L^e - x_H - x_L.$$

Thus, the profit in (1) can be rewritten as

$$\{(1+\mu)(r + \nu g_H^e - x_H) - x_L - c_H\}x_H + \{r + \nu g_L^e - x_H - x_L\}x_L. \tag{7}$$

Having described the behavior of the firm and consumers, I now derive the market equilibrium. For this purpose, I employ Katz and Shapiro's (1985) concept of the fulfilled expectations equilibrium, which requires that consumers' expected quantities equal the actual outputs. In addition, the firm chooses the outputs *after* taking consumers' expectations about the network size as given. From (7), the first-order conditions for profit maximization are

$$\begin{aligned} -(1+\mu)x_H + (1+\mu)(r + \nu g_H^e - x_H) - x_L - x_L - c_H = 0, \\ -x_H - x_L + r + \nu g_L^e - x_H - x_L = 0. \end{aligned} \tag{8}$$

[10] See the Appendix for the proof.

In addition, to guarantee positive outputs in equilibrium, I make two additional assumptions:

$$0 < \nu < \frac{2(1+\mu-\sqrt{1+\mu})}{1+\mu}, \tag{9}$$

and

$$\underline{c}_H < c_H < \overline{c}_H, \tag{10}$$

where $\underline{c}_H = \nu(1+\mu)r/2$ and $\overline{c}_H = (2\mu-\nu-\nu\mu)r/(2-\nu)$.

The equilibrium outcomes are obtained from $g^e_\alpha = x^e_\alpha = x_\alpha$ and (8):

$$\begin{cases} -(1+\mu)x_H + (1+\mu)(r+\nu g^e_H - x_H) - 2x_L - c_H = 0 \\ -2x_H - 2x_L + r + \nu g^e_L = 0 \\ g^e_H = x_H \\ g^e_L = x_L. \end{cases}$$

Then, the equilibrium outputs and prices are

$$x^*_H = \frac{(2-\nu)\{(1+\mu)r - c_H\} - 2r}{Z}, x^*_L = \frac{-(1+\mu)\nu r + 2c_H}{Z}, \tag{11}$$

and

$$p^*_H = \frac{r(1+\mu)(2\mu-2\nu-\mu\nu)+\{(1+\mu)\nu^2-3(1+\mu)\nu+2\mu\}c_H}{Z}, p^*_L = \frac{2r(\mu-\nu-\mu\nu)+\nu c_H}{Z}, \tag{12}$$

where $Z = (1+\mu)(2-\nu)^2 - 4 > 0$ by (9). These outcomes lead to the equilibrium profit:

$$\begin{aligned} \pi^* = \tfrac{1}{Z^2}\Big[&\{\mu(2-\nu)^2+\nu^2\}c_H^2 - 2r\{\mu^2(2-\nu)^2 + 2\mu^2 + \mu\nu(3\nu-4)\}c_H \\ &+r^2(1+\mu)(\mu^2(\nu-2)^2 + 4\nu^2 + \mu\nu(-8+5\nu)\Big]. \end{aligned} \tag{13}$$

This completes the description of the model.

3 U-Shaped Profit

Based on the results in the previous section, this section demonstrates that the firm profit is U-shaped in the marginal cost of the high-quality good. The proof of the results are left in Appendix.

3.1 Output

First, I consider the effects of an increase in the marginal cost of producing the high-quality good on each quantity, as described in the following proposition.

Proposition 1. *An increase (decrease) in c_H leads to cannibalization, such that it reduces (raises) the output of the high-quality good and raises (reduces) the output of the low-quality good.*

This proposition is a natural result, since the firm would like to produce a relatively efficient product.[11]

3.2 Profit

Next, I address the effect on the firm profit, which can be stated in

Proposition 2. *Suppose a within-product network externality exists. Then, the firm profit is U-shaped in c_H.*

This is illustrated in Figure 1.

Insert Figure 1 here.

This result implies that a small cost reduction can decrease the monopoly profit. When c_H is high enough, the firm does not moderate cost reduction. In other words, the firm does not accept an innovation or subsidy unless it is able to drastically reduce c_H. This proposition suggests that if c_H is sufficiently high, a decrease in it *reduces* the firm's profit.

As emphasized in the Introduction, the assumption that consumers form their expectations *before* the output decision is crucial to the above result.[12] To see why, let

[11]The same property is confirmed in Kitamura and Shinkai (2013), who consider a duopoly market without a network externality.

[12]This assumption implies that a monopolist's announcement of its planned level of output has no effect on consumer expectations.

us drop this assumption. That is, I compare this case with the case in which the firm can control both its output and the expected network size; it maximizes the profit with taking $g_\alpha^e = x_\alpha$ into consideration. Then, I have the following lemma.

Lemma 2. *In the monopoly model with the fulfilled expectations equilibrium derived above, if c_H increases, then marginal changes in the equilibrium quantities of good H and L are less than when the firm can control the expected network size.*

I have assumed that the firm takes the expected network size as given (i.e., it cannot control the expected network size). However, the expected network size must coincide with the actual network size in equilibrium. In other words, the monopolist choose outputs to maximize the profit without recognizing that the expected network size is equal to the actual network size. This lack of information leads the firm to either under-produce or over-produce compared with the case in which the firm can control the expected network size. Indeed, in the present model, if c_H increases, then the monopolist produces more of good H and less of good L compared with the case in which it can control the expected network size. To check this result, let us compute the first-order conditions when the firm can control the expected network size:

$$(1+\mu)(\nu\frac{\partial g_H}{\partial x_H} - 1)x_H + (p_H - c_H) - x_L = 0, \ (\nu\frac{\partial g_L}{\partial x_L} - 1)x_L + p_L - x_H = 0.$$

By contrast, if the firm cannot control the expected network size, the corresponding conditions are

$$-(1+\mu)x_H + (p_H - c_H) - x_L = 0, \ -x_H + p_L - x_L = 0.$$

When the monopolist can control the expected network size, an increase in output affects the network externality as represented by $\partial g_\alpha / \partial x_\alpha = 1$. This difference in the first-order conditions results in Lemma 2. In fact, when the firm can control the expected network size, the equilibrium outputs are as follows:

$$x_H^{*C} = \frac{(1-\nu)\{(1+\mu)r - c_H\} - r}{2(1+\mu)(1-\nu)^2 - 2}, \ x_L^{*C} = \frac{-r(1+\mu)\nu r + c_H}{2(1+\mu)(1-\nu)^2 - 2},$$

where superscript $*C$ indicates the case in which the firm can control the expected network size. Then, I can show that

$$\frac{\partial x_H^{*C}}{\partial c_H} - \frac{\partial x_H^*}{\partial c_H} < 0, \quad \frac{\partial x_L^{*C}}{\partial c_H} - \frac{\partial x_L^*}{\partial c_H} > 0.$$

The intuition behind Proposition 2 is explained from Proposition 1 and Lemma 2. According to these, a decrease in c_H increases the output of good H and decreases that of good L. However, these changes are not as drastic as in the case when the firm can control the expected network size. Thus, the firm cannot aggressively transfer the network of good L to that of good H in spite of the decrease in c_H, and the positive effect on the profit from good H is not able to dominate the negative effect of good L. This finding is impossible, however, if the firm can control the expected network size.

Indeed, we can observe this fact more plausibly as follows. I consider the effect of an increase in c_H on the profit from producing each individual good: $\pi^* = \pi_H^* + \pi_L^* \equiv (p_H^* - c_H)x_H^* + p_L^* x_L^*$. Using this decomposition of profits, I have the following lemma.[13]

Lemma 3. *π_H^* is monotonically decreasing in c_H, and π_L^* is monotonically increasing in c_H.*

Figure 2 illustrates this lemma.

Insert Figure 2 here.

Given this lemma and Figure 2, when c_H decreases by a sufficiently large amount, the negative effect on π_L^* (i.e., $\frac{\partial \pi_L^*}{\partial c_H}$) dominates the positive effect on π_H^* (i.e., $\frac{\partial \pi_H^*}{\partial c_H}$). Accordingly, if c_H is initially high, a decrease in c_H reduces the overall profit. The opposite holds when c_H is low enough.

Remark 1. One natural question regarding to Proposition 2 is whether the profit continues to be U-shaped in c_H even if the two goods are compatible. To answer it, I modify the form of network externality (3) as follows:

$$g_\alpha^e \equiv g_\alpha(x_H^e, x_L^e, \phi) = x_\alpha^e + \phi x_\beta^e \quad \alpha,\ \beta = H, L, \alpha \neq \beta,\ 0 < \phi \leq 1,$$

[13] Note that the lemma requires the existence of positive equilibrium outputs: (9) and (10).

where ϕ is a parameter that measures the degree of compatibility between the two goods. The following proposition gives an affirmative answer to the above question.

Proposition 3. *Suppose that a within-product network externality and partial compatibility ($\phi < 1$) exist between the two differentiated goods. Then, the firm's profit is U-shaped in c_H.*

This proposition implies that the firm's profit can decrease when c_H decreases except for the case of $\phi = 1$ as long as a within-product network externality exists.

If $\phi = 1$, then $g^e_\alpha = x^e_H + x^e_L$ ($\alpha = H, L$). Because the two goods are fully compatible, this case corresponds to the case analyzed by Katz and Shapiro (1985), that is there is the within-*firm* network externality. Then, we find that the firm's profit is a monotonically decreasing function of c_H. However, the case of fully compatible goods is a special situation,[14] because I consider the within-product network externality, and fully compatible products do not have individual networks. This result implies that the within-product network externality offers different equilibrium outcomes and properties to the within-firm network externality established in Katz and Shapiro (1985).

Remark 2. Thus far, I have assumed that a monopolist's announcement of its planned level of output has no effect on consumer expectations. Then, another natural question is whether the profit continues to be U-shaped in c_H even when its announcement of output level partially affects consumer expectations. In order to address it, I modify the form of network externaity (3) as follows:

$$g^e_\alpha \equiv g_\alpha(x^e_\alpha, x_\alpha, \epsilon) = \epsilon x_\alpha + (1-\epsilon)x^e_\alpha \quad \alpha = H, L,\ 0 \leq \epsilon \leq 1.$$

In this formulation, the monopolist's announcement of its output level has ϵx_α influence on consumer expectations. With this generalization, I can obtain:

[14]See the Appendix for a special case, that is, $\frac{\partial \pi^*}{\partial c_H}|_{c_H = \bar{c}_{H\phi}} = 0$ only if $\phi = 1$.

Proposition 4. *Suppose that a within-network externality exists between the two differentiated goods and the monopolist's announcement of its planned level of output partially affects ($\epsilon < 1$) consumer expectations. Then, the firm's profit is U-shaped in c_H.*

Thus, the firm's profit is U-shaped in so far as its announcement of outputs imperfectly (that is when $0 \leq \epsilon < 1$) effects on consumer expectations.

When $\epsilon = 1$, $g^e_\alpha = x_\alpha (\alpha = H, L)$. As mentioned in Lemma 2, this implies that the monopolist can perfectly control the expected network size. Then, it chooses the output levels to maximize the profit with understanding that the consumer expectations are equal to the actual network size. Thus in the same way as reasons of Proposition 2, the firm's profit is monotonically decreasing in c_H only when $\epsilon = 1$.

4 Further Discussion

In this section, I address two issues that are important but have not been discussed in the last section. One is related to social welfare, while the other is the effect of μ.

4.1 Welfare

First, I examine the welfare effect of a change in c_H. Noting that welfare is equal to the sum of the consumer surplus and the firm's profit, it is defined by

$$\begin{aligned} W^* &\equiv \int_{\underline{\theta}_L}^{\hat{\theta}^*} (\theta + \nu g^*_L) d\theta + \int_{\hat{\theta}^*}^{r} (1+\mu)(\theta + \nu g^*_H) d\theta - c_H x^*_H \\ &= \frac{(1+\mu)r^2}{2} + \nu(1+\mu)(r - \hat{\theta}^*) g^*_H + \nu(\hat{\theta}^* - \underline{\theta}^*_L) g^*_L - \frac{(\underline{\theta}^*_L)^2}{2} - \frac{\mu(\hat{\theta}^*)^2}{2} - c_H x^*_H \\ &= \frac{(1+\mu)r^2}{2} + \nu(1+\mu) x^*_H g^*_H + \nu x^*_L g^*_L - \frac{(r - x^*_H - x^*_L)^2}{2} - \frac{\mu(r - x^*_H)^2}{2} - c_H x^*_H, \end{aligned}$$

where superscript * indicates the equilibrium outcome. Lengthy manipulations allow me to have a notable relationship $W^* = 3\pi^*/2$. Hence, the following result is immediately obtained.

Proposition 5. *Suppose that a within-product network externality exists. Then, social welfare is U-shaped in* c_H.

This proposition is natural since the consumer surplus is larger when c_H takes an extremely large or small value and only one side of the network is larger than it is when c_H takes an intermediate value and each network size is small.[15] Recalling Remark 1 and discussion after Proposition 3, I immediately find that welfare with fully compatible products ($\phi = 1$) is a monotonically decreasing function of c_H because, in that case, the network size of each product is always the sum of the network sizes of both products.

Proposition 2 and 5 imply that a drastic cost reduction is needed to increase the profit and welfare when the production cost of the high-quality good is high. Then, as a mentioned in Section 1, these suggest that if the production subsidy is insufficient, subsization can reduce both the firm's profit and welfare.

4.2 Effect of μ on Outputs

Throughout this paper, I have focused on the effect of c_H. Finally, I consider the effect of an increase in the quality of the high-quality good μ on each quantity, as stated in the following proposition.

Proposition 6. *An increase in* μ *leads to an increase (decrease) in* $x_H^*(x_L^*)$.

This proposition is also interesting because cannibalization occurs as a result of not only c_H but also μ.[16] That is, an increase in μ has a contrasting effect in the sense that it raises (reduces) $x_H(x_L)$. The intuition for this proposition is as follows. A larger difference in the quality of the two goods implies that the high-quality good is superior to the low-quality good, which has a positive effect on the utility of the consumer. Thus, when the quality difference of the two goods becomes large , the monopolist has an incentive to increase x_H. In such a case, cannibalization occurs as it raises x_H while

[15] The consumer surplus is also U-shaped in c_H.

[16] In Proposition 1, the change in the parameter of supply side c_H causes cannibalization, while in Proposition 6, that of demand side μ leads to cannibalization.

x_L decreases. Conversely, when the difference in the quality of the two goods decreases, the consumer does not value the high-quality good over the low-quality good. Thus, the monopolist will expand x_L since it is costly to produce x_H. In this case, cannibalization occurs such that the firm produces more of good L and less of good H. For example, the iPad Mini cannibalized sales of the larger iPad.[17]

5 Concluding Remarks

Highlighting a within-product network externality, this paper has theoretically analyzed multi-product monopoly behavior and the resulting market configurations. In particular, I focused on a monopoly model where a single firm sells two differentiated products (low- and high-quality goods) in a market with a within-goods network externality.

The notable result is that the firm profit is U-shaped in the production cost of the high-quality good. This result implies that the firm profit may decrease in spite of a cost reduction. Then, I have shown that two assumptions, the fulfilled expectations equilibrium and multi-product monopoly, yield the counterintuitive result. Moreover, I addressed the two cases in which (i) the two goods are partially and fully compatible and (ii) a firm's announcement of its output partially and perfectly affects consumer expectations, and established that when a within-product network externality exists, the firm profit is U-shaped except for two polar cases in which the two goods are completely compatible and in which a firm's announcement perfectly influences on consumer expectations. In addition, I analyzed the effect of a change in the production cost of the high-quality good on welfare, finding that welfare is also U-shaped in the cost.

Furthermore, I highlighted that changes in the production cost and in the quality of the high-quality good affect the quantities. Moreover, by using the example of cannibalization, I found that an increase (decrease) in the production cost of the high-quality good and a decrease (increase) in its quality bring about cannibalization, such that the firm raises (reduces) the output of the high-quality good while it reduces (raises) the output of the low-quality good.

In this paper, I exclusively focused on a monopoly model without choosing product

[17]See the internet articles by Keizer (2012) and Seward (2013).

compatibility, but future studies should aim to analyze a model when the firm can choose a compatible product with a fixed cost of making its products compatible.

Appendix

Proof of Lemma 1

According to Eqs. (2) and (4), for arbitrary $\theta > \hat{\theta}_i$, from (2) and (6), we have

$$\begin{aligned} U_L(\widehat{\theta}) - U_L(\underline{\theta}_L) &= \hat{\theta} + \nu g_L^e - p_L - (\underline{\theta}_L + \nu g_L^e - p_L) \\ &= \hat{\theta} - \underline{\theta}_L > 0, \end{aligned}$$

for arbitrary type $\theta \in (\underline{\theta}_L, \widehat{\theta})$. Then,

$$\begin{aligned} U_H(\theta) - U_L(\theta) &= (1+\mu)\theta + \nu(1+\mu)g_H^e - p_H - \theta - \nu g_L^e + p_L \\ &= \mu\theta - \{p_H - p_L - (\nu(1+\mu)g_H^e - \nu g_L^e)\} \\ &> \mu\hat{\theta} - \{p_H - p_L - (\nu(1+\mu)g_H^e - \nu g_L^e)\} \\ &= 0. \end{aligned}$$

From (2) and (6), we have

$$\begin{aligned} U_L(\widehat{\theta}) - U_L(\underline{\theta}_L) &= \hat{\theta} + \nu g_L^e - p_L - (\underline{\theta}_L + \nu g_L^e - p_L) \\ &= \hat{\theta} - \underline{\theta}_L > 0, \end{aligned}$$

for arbitrary type $\theta \in (\underline{\theta}_L, \widehat{\theta})$.

Proof of Proposition 1

From equilibrium outcome (11), we have $\partial x_H^*/\partial c_H < 0$ and $\partial x_L^*/\partial c_H > 0$.

Proof of Proposition 2

$$\begin{cases} \frac{\partial^2\pi^*}{\partial c_H^2} = \frac{2\{\mu(2-\nu)^2+\nu^2\}}{Z^2} > 0 \\ \\ \frac{\partial\pi^*}{\partial c_H}|_{c_H=\underline{c}_H} = \frac{-r\{\mu(2-\nu)-\nu\}}{(2-\nu)Z} < 0, \quad \frac{\partial\pi^*}{\partial c_H}|_{c_H=\bar{c}_H} = \frac{2r\nu}{(2-\nu)Z} > 0. \end{cases}$$

Proof of Lemma 3

The individual profits from producing goods H and L are given by

$$\pi_H^* = \frac{\{c_H(2-\nu)+r\{\mu(-2+\nu)+\nu\}\}\{c_H\{\mu(2-\nu)-\nu\}+r(1+\mu)\{\mu(-2+\nu)+2\nu\}\}}{Z^2}$$
$$\pi_L^* = \frac{\{-2c_H+r(1+\mu)\nu\}\{-c_H\nu+2r\{\mu(-1+\nu)+\nu\}\}}{Z^2},$$

respectively, so that

$$\begin{cases} \frac{\partial\pi_H^*}{\partial c_H}|_{c_H=\bar{c}_H} = \frac{-r}{Z} < 0, \quad \frac{\partial^2\pi_H^*}{\partial c_H^2} = \frac{2(2-\nu)(2\mu-\nu-\nu\mu)}{Z^2} > 0 \\ \frac{\partial\pi_L^*}{\partial c_H}|_{c_H=\underline{c}_H} = rZ > 0, \quad \frac{\partial^2\pi_L^*}{\partial c_H^2} = \frac{4\nu}{Z} > 0. \end{cases}$$

Proof of Proposition 3

The equilibrium outcomes for $0 < \phi \leq 1$ are obtained as follows.

$$\begin{cases} x_H^* = \frac{(2-\nu)\{r(1+\mu)-c_H\}-r\{2-\phi(1+\mu)\nu\}}{Z_\phi}, \; x_L^* = \frac{(1+\mu)(2-\nu)-\{r(1+\mu)-c_H\}(2-\phi\nu)}{Z_\phi} \\ \\ p_H^* = \frac{r(1+\mu)(2(\phi-1)\nu+\mu\{2-(1-\phi)\nu\}+c_H\{(1-\phi)\nu(-3+\nu+\phi\nu)-\nu\{-2+(3-2\phi)\nu-(1-\phi^2)\nu^2\}\}}{Z_\phi} \\ \\ p_L^* = \frac{2r\{(\phi-1)\nu+\mu\{1+\phi-1)\nu\}\}+(1-\phi)\nu c_H\}}{Z_\phi} \\ \\ \pi^* = \frac{1}{Z_\phi^2}\Big[\{\mu(2-\nu)^2+(1-\phi)^2\nu^2\}c_H^2 + 2r\{-2(1-\phi)^2\nu^2+\mu^2(-2+\nu)\{2-(1-\phi)\nu\}+\mu\nu(1-\phi)\{4+(2\phi-3)\nu\}\}c_H \\ \quad +r^2(1+\mu)\{4(1-\phi)^2\nu^2+\mu^2\{2-(1-\phi)\nu\}^2-\mu\nu(1-\phi)\{8-5(1-\phi)\nu\}\}\Big], \end{cases}$$

where $Z_\phi = \nu(1-\phi)(\phi\nu+\nu-4)+\mu\{4-2(2-\phi)\nu+(1-\phi^2)\nu^2\} > 0$. Furthermore, $\underline{c}_{H\phi} < c_H < \bar{c}_{H\phi}$ where $\underline{c}_{H\phi} = (1+\mu)(1-\phi)r\nu/(2-\phi\nu)$ and $\bar{c}_{H\phi} = \{r\{(1+\mu)(2-\nu) - \{2-\phi(1+\mu)\nu\}\}/(2-\nu)$.

Then,

$$\begin{cases} \frac{\partial \pi^*}{\partial c_H}|_{c_H=\underline{c}_{H\phi}} = \frac{-r\{\mu(2-\nu)-(1-\phi)\nu\}}{(2-\phi\nu)Z_\phi} < 0 \\ \frac{\partial \pi^*}{\partial c_H}|_{c_H=\bar{c}_{H\phi}} = \frac{2(1-\phi)r\nu}{(2-\nu)Z_\phi} \geq 0. \end{cases}$$

Thus, the firm profit is U-shaped in c_H except for the case of $\phi = 1$.

Proof of Proposition 4

The equilibrium outcomes for $0 \leq \epsilon \leq 1$ are given as follows:

$$\begin{cases} x_H^* = \frac{(2-\nu-\nu\epsilon)\{(1+\mu)r-c_H\}-2r}{Z_\epsilon}, \ x_L^* = \frac{2c_H-\nu(1+\mu)(1+\epsilon)r}{Z_\epsilon} \\ p_H^* = \frac{c_H\{-3\nu+\nu\epsilon(-1+\nu)+\nu^2+\mu(-1+\nu)(-2+\nu+\epsilon\nu)\}+r(1+\mu)\{(1+\epsilon)\nu(-2+\epsilon\nu)+\mu\{2-(1+3\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\}}{Z_\epsilon} \\ p_L^* = \frac{-c_H(-1+\epsilon)\nu+r\{(1+\epsilon)(-2+\epsilon\nu)\nu+\mu\{2-2(1+\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\}}{Z_\epsilon} \\ \pi^* = \frac{1}{Z_\epsilon^2}\Big[\{\mu(-1+\epsilon\nu)(-2+\nu+\epsilon\nu)^2+\nu\{\nu+\epsilon^2(5-2\nu)\nu-\epsilon^3\nu^2-\epsilon(8-6\nu+\nu^2)\}\}c_H^2 \\ \quad +2r\{(1+\epsilon)^2\nu^2(-2+\epsilon\nu)+\mu^2(-1+\epsilon\nu)(-2+\nu+\epsilon\nu)^2+\mu\nu\{4-3\nu+2\epsilon^3\nu^2+\epsilon^2\nu(-7+4\nu)+2\epsilon(4-5\nu+\nu^2)\}\}c_H \\ \quad +r^2(1+\mu)\{(-1-\epsilon)\nu\{(1+\epsilon)(-2+\epsilon\nu)\nu+\mu\{2-2(1+\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\} \\ \quad -\{(1+\epsilon)\nu+\mu(-2+\nu+\epsilon\nu)\}\{(1+\epsilon)(-2+\epsilon\nu)\nu+\mu\{2-(1+3\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}\}\}\Big], \end{cases}$$

where $Z_\epsilon = (1+\mu)(2-\nu-\nu\epsilon)^2-4 > 0$ if and only if $0 < \nu < 2(1+\mu-\sqrt{1+\mu})/(1+\mu)(1+\epsilon)$. Furthermore, $\underline{c}_{H\epsilon} < c_H < \bar{c}_{H\epsilon}$ where $\underline{c}_{H\epsilon} = \nu(1+\mu)(1+\epsilon)r/2$ and $\bar{c}_{H\epsilon} = r\{2\mu-\nu(1+\mu)(1+\epsilon)\}/(2-\nu-\nu\epsilon)$.

Then,

$$\begin{cases} \frac{\partial \pi^*}{\partial c_H}|_{c_H=\underline{c}_{H\epsilon}} = \frac{-r\{\nu\{-1+\epsilon(\nu-3)+\nu\epsilon^2\}+\mu\{2-(1+3\epsilon)\nu+\epsilon(1+\epsilon)\nu^2\}}{Z_\epsilon} < 0 \\ \frac{\partial \pi^*}{\partial c_H}|_{c_H=\bar{c}_{H\phi}} = \frac{2(1-\epsilon)r\nu}{(2-\nu-\nu\epsilon)Z_\epsilon} \geq 0. \end{cases}$$

Thus, the firm profit is U-shaped in c_H except for the case of $\epsilon = 1$.

Proof of Proposition 6

Straightforward manipulations give

$$\frac{\partial x_H^*}{\partial \mu} = \frac{(2-\nu)\{(2-\nu)^2 c_H - 2\nu r\}}{Z^2} > 0, \ \frac{\partial x_L^*}{\partial \mu} = \frac{-2\{(2-\nu)^2 c_H - 2\nu r\}}{Z^2} < 0.$$

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Figure 1 $(r = 1, \nu = 1/2, \mu = 1)$

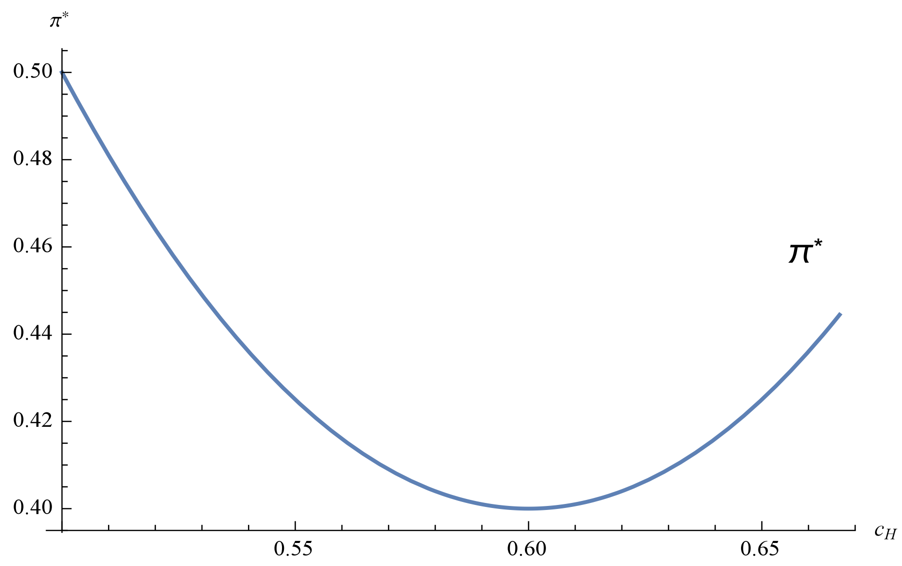

Figure 2 $(r = 1, \nu = 1/2, \mu = 1)$