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Necessarily welfare-improving privatization

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Abstract

This paper examines the welfare effect of privatization in a mixed oligopoly model where multiple oligopolistic industries compete for a common factor. We find that privatization necessarily improves welfare in a benchmark case with symmetric costs across all oligopolistic industries. Moreover, we show that a production subsidy necessarily reduces welfare regardless of the level of privatization.

Keywords: Mixed oligopoly, privatization, subsidization, general equilibrium.

JEL classifications: L13, L32, L33.

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1 Introduction

Privatization of state-owned firms has been as controversial as the other liberalization policies, e.g. competition and trade policies. Reason Foundation (2015) reports the latest cases of successful privatization in the United States in 2014.[1] Moreover, 'Many emerging economies have launched ambitious efforts to privatize their infrastructure industries' (Jiang et al., 2015, p. 294) in order to provide multinational enterprises with investment opportunities.

These facts motivate a large literature on the effects of privatization mainly in the context of a mixed oligopoly.[2] This literature begins with de Fraja and Delbono (1989) who show that moving from full nationalization to full privatization improves welfare. By allowing for the intermediate case between full nationalization and full privatization, Matsumura (1998) finds that partial privatization is optimal (welfare-maximizing). In addition to the effects of privatization alone, White (1996) examines the interplay between privatization and subsidization, demonstrating that privatization has no effect on the optimal production subsidy when the government subsidizes both the state-owned and private firms.[3]

This strand of literature leads to a number of useful implications, but they rest on a partial equilibrium model. The purpose of this paper is to examine the welfare effects of privatization and subsidization by taking into account the general equilibrium effects through the factor market. To this end, we combine Matsumura's (1998) approach with the model of Dixit and Grossman (1986) in which multiple oligopolistic industries compete for a common factor. In this setting with identical production costs across all oligopolistic industries, we establish two results. First, any privatization improves welfare,

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[1]According to this report, 'The U.S. Department of Agriculture announced a plan to privatize poultry inspection in 2014,' and 'The General Services Administration announced in January it is closing its warehouses ⋯ and will no longer buy, ship or store office supplies, tools and other common-use retail items in favor of accepting agency orders.'

[2]For an extensive literature survey, see Matsumura and Shimizu (2010) and Matsumura and Tomaru (2012). Cato and Matsumura (2015) provide a further review by paying special attention to the open economy case.

[3]This 'irrelevance result' or 'Privatization Neutrality Theorem' has been challenged by many works, which are surveyed in Matsumura and Okumura (2013) in detail.

and hence full privatization turns to be optimal. Second, any subsidization reduces welfare, and thus zero subsidy is optimal irrespective of the degree of privatization. Both of these results are hopefully useful in the sense that they convince us the importance of general equilibrium considerations.

This paper is organized as follows. Section 2 presents a model, Section 3 states the main results, and Section 4 concludes. Technical arguments are summarized in Appendix.

2   Model

The model follows Dixit and Grossman (1986). There are $n + 1, n \geq 1$ duopolistic industries. In industry 1, one state-owned firm (firm 1) and one private firm (firm 2) play a Cournot game while the other industries are a standard Cournot duopoly by private firms. In industry i, each firm employs one unit of capital and a_i units of labor to produce one unit of output. In addition, the government subsidizes both firms in industry 1. Then, letting x_1^i and x_2^i denote output of firms 1 and 2 in industry i, the profit of each firm of industry 1 is defined by

$$\pi_1^1 \equiv p\left(x_1^1 + x_2^1\right) x_1^1 - (a_1 + r - s)x_1^1$$
$$\pi_2^1 \equiv p\left(x_1^1 + x_2^1\right) x_2^1 - (a_1 + r - s)x_2^1,$$

where $p(\cdot)$ is an inverse demand function with $p'(\cdot) < 0$, r is capital rental, and s is a per-unit production subsidy.[4] Similarly, the profit of duopolists in private sectors $i = 2, \cdots, n + 1$ is defined by

$$\pi_1^i \equiv p\left(x_1^i + x_2^i\right) x_1^i - (a_i + r)x_1^i$$
$$\pi_2^i \equiv p\left(x_1^i + x_2^i\right) x_2^i - (a_i + r)x_2^i.$$

Following Matsumura (1998), the state-owned firm in industry 1 is assumed to maximize the weighted sum of profit and welfare. In order to

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[4]Note that the wage rate is normalized to one by implicitly assuming that one unit of labor produces one unit of the numeraire good.

define the public firm's objective, we now compute welfare. Denoting by y output of the numeraire good, the market-clearing conditions for labor and capital are given by

$$y + \sum_{i=1}^{n+1} a_i \left(x_1^i + x_2^i \right) \equiv y + \sum_{i=1}^{n+1} a_i X^i = l \tag{1}$$

$$\sum_{i=1}^{n+1} \left(x_1^i + x_2^i \right) = \sum_{i=1}^{n+1} X^i = k, \tag{2}$$

where $X^i \equiv x_1^i + x_2^i$ is total output in industry i, and l and k are the endowment of labor and capital, respectively. These conditions and some rearrangements lead to national income I:

$$
I \equiv \underbrace{\sum_{i=1}^{n+1} \left(\pi_1^i + \pi_2^i \right)}_{\text{aggregate profits in oligopolistic industries}} + \underbrace{y - y}_{\text{profit in the numeraire industry}}
$$
$$
+ \underbrace{l + rk}_{\text{factor income}} \underbrace{- s \left(x_1^1 + x_2^1 \right)}_{\text{subsidy payment}}
$$
$$
= \sum_{i=1}^{n+1} \left[p \left(X^i \right) X^i - a_i X^i \right] + l.
$$

Consumer surplus CS is

$$CS \equiv \sum_{i=1}^{n+1} \left[\int_0^{X^i} p(Q)dQ - p \left(X^i \right) X^i \right].$$

Summing CS and I up, welfare W is obtained as

$$W \equiv CS + I = \sum_{i=1}^{n+1} \left[\int_0^{X^i} p(Q)dQ - a_i X^i \right] + l. \tag{3}$$

As mentioned earlier, the state-owned firm (firm 1) in industry 1 chooses x_1^1 to maximize the weighted sum of its own profit and welfare $\theta \pi_1^1 + (1-\theta)W$ where $\theta \in [0,1]$ represents the degree of privatization; $\theta = 0$ (resp. $\theta = 1$) corresponds full nationalization (resp. privatization). Then, the first-order conditions for objective maximization are derived as follows.

$$\theta \left[x_1^1 p' \left(X^1 \right) - r + s \right] + p \left(X^1 \right) - a_1 = 0 \tag{4}$$

4

$$x_2^1 p'\left(X^1\right) + p\left(X^1\right) - a_1 - r + s = 0 \tag{5}$$

$$x_1^i p'\left(X^i\right) + p\left(X^i\right) - a_i - r = 0 \tag{6}$$

$$x_2^i p'\left(X^i\right) + p\left(X^i\right) - a_i - r = 0, \tag{7}$$

where (4) is the first-order condition of the state-owned firm in industry 1, (5) is the counterpart of the private firm in industry 1, and (6) and (7) are the profit-maximization conditions in industry $i, i = 2, \cdots, n+1$. Our model consists of Eqs. (2), (4), (5), (6) and (7), and determines $x_1^1, x_2^1, x_1^i, x_2^i$ and r.

3   Results

Using the above model, this section examines the welfare effects of privatization (an increase in θ) and production subsidy.[5] To simplify analysis, let us multiply (5) by θ and sum the resulting equation and (4). Then, we have

$$\theta\left[X^1 p'\left(X^1\right) + p\left(X^1\right) - a_1 - 2r + 2s\right] + p\left(X^1\right) - a_1 = 0. \tag{8}$$

Summing (6) and (7) up yields

$$X^i p'\left(X^i\right) + 2p\left(X^i\right) - 2a_i - 2r = 0. \tag{9}$$

Thus, the model reduces to Eqs. (2), (8) and (9), which determines X^1, X^i and r. As in the literature that assumes a Cournot duopoly, we make:

Assumption: $x_1^i p''\left(X^i\right) + p'\left(X^i\right) < 0$ *and* $x_2^i p''\left(X^i\right) + p'\left(X^i\right) < 0$

This assumption, which is familiar in the literature, requires the inverse demand function not to be too convex. Then, totally differentiating the above $n+2$-system, we have:[6]

Lemma. $\dfrac{\partial X^1}{\partial \theta} < 0, \dfrac{\partial X^i}{\partial \theta} > 0, i = 2, \cdots, n+1$ and $\dfrac{\partial r}{\partial \theta} < 0.$

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[5]The simplest case of two oligopolistic industries is in Appendix.
[6]Main results are proved in Appendix.

The intuition behind this result is as follows. From Eqs. (4) and (5), marginal revenue of the semi-public firm (firm 1) in industry 1 decreases, but that of the private firm (firm 2) is unchanged as a result of an increase in θ. This leads firm 1 to produce less, and firm 2 to produce more, but total output decreases because firm 1's output reduction dominates firm 2's output expansion. This reduction in X^1 puts upward pressure on r, and the decline in r increases outputs in the other oligopolistic industries.

Differentiating (3) with respect to θ and using the above lemma, we arrive at:

Proposition 1. *If all oligopolistic industries have the identical technology, privatization necessarily improves welfare, and hence full privatization is optimal.*

It is easily inferred from (3) that welfare effects of privatization are unclear if a_i differs across industries. However, if all the oligopolistic industries have the same production cost, privatization becomes necessarily beneficial. The reason is as follows. As shown in Appendix, it holds that $X^1 > X^i, i = 2\cdots, n+1$, that is, the market distortion associated with imperfect competition is stronger in industry i than in industry 1. Privatization-led increase in X^i and decrease in X^1 plays a role in mitigating this stronger distortion in industry i, and thus enhances welfare.

Noting that the effects of privatization (an increase in θ) and subsidization (an increase in s) have the opposite sign, we find:

Proposition 2. *If all oligopolistic industries have the identical technology, privatization has no effect on the optimal subsidy, which is always zero.*

Subsidizing industry 1 increases its output, from which the capital rental rises. This rise in r increases marginal cost of the other oligopolistic industries, and decreases their output. As a result of these effects, subsidization

negatively affects welfare because it makes market distortion in industry i stronger than before. To sum, subsidization to industry 1 crowds the other oligopolistic industries out, thereby yielding welfare losses.

It follows from the above observation that the benevolent government chooses zero subsidy, which is not affected by the degree of privatization. In this sense, Proposition 2 may be a special case of the 'irrelevance result' or the 'Privatization Neutrality Theorem.'

4   Conclusion

We have reconsidered two results concerning privatization of a semi-public firm by highlighting general equilibrium effects in the factor market. In our model, it is shown that if all the oligopolistic industries have the same technology, (i) privatization is necessarily improves welfare, and (ii) the optimal production subsidy is zero and is not affected by the degree of privatization. The general equilibrium effects mentioned earlier play a crucial role behind these results.

Our results may contribute to the literature on privatization and mixed oligopoly, but they admittedly rest on a number of simplifying assumptions. Among others, we have assumed a quasi-linear preference by strictly following Dixit and Grossman (1986). However, if one considers fully general equilibrium effects, the assumption of a quasi-linear preference must be dropped so that the income effect is taken into account. Neary (2009), in the context of international trade, develops such a model, and his model has been increasingly applied in a variety of fields.[7] It is our future research agenda to extend our results to such a richer model.

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[7]See Colacicco (2014) for the comprehensive survey on Neary's (2009) approach.

Appendix

Proof of Lemma

Totally differentiating the $n+2$-dimensional system of (2), (8) and (9), the effects of an increase in θ on $X^1, X^i, i = 2, \cdots, n+1$ become

$$\frac{\partial X^1}{\partial \theta} = \frac{2A}{\Delta}\left(\sum_{i \neq j \neq \cdots \neq k} \underbrace{B_i B_j \cdots B_k}_{n-1}\right) \tag{10}$$

$$\frac{\partial X^i}{\partial \theta} = -\frac{2A}{\Delta} \cdot \prod_{\substack{j \neq 1, i}}^{n+1} B_j \tag{11}$$

$$\frac{\partial r}{\partial \theta} = -\frac{A}{\Delta} \cdot \prod_{i=2}^{n+1} B_i, \tag{12}$$

where Δ is the determinant of the coefficient matrix of the totally-differentiated system, and is positive (resp. negative) if the number of oligopolistic industries $n+1$ is odd (resp. even). Concretely, Δ and the other notations are defined by

$$\Delta \equiv 2\left\{\left[\theta\left(X^1 p_1'' + p_1'\right) + p_1'\right] \sum_{i \neq j \neq \cdots \neq k} \underbrace{B_i B_j \cdots B_k}_{n-1} + \theta \prod_{i=2}^{n+1} B_i\right\}$$

$$A \equiv -\left(X^1 p_1' + p_1 - a_1 - 2r + 2s\right) > 0$$

$$B_i \equiv X^i p_i'' + 3p_i' < 0.$$

These sign patterns lead to

$$\frac{\partial X^1}{\partial \theta} < 0, \quad \frac{\partial X^i}{\partial \theta} > 0, \quad \frac{\partial r}{\partial \theta} < 0.$$

The effects of subsidization can be obtained just by replacing A in (10), (11) and (12) with -2θ, and hence

$$\frac{\partial X^1}{\partial s} = -\frac{4\theta}{\Delta}\left(\sum_{i \neq j \neq \cdots \neq k} \underbrace{B_i B_j \cdots B_k}_{n-1}\right) > 0 \tag{13}$$

$$\frac{\partial X^i}{\partial s} = \frac{4\theta}{\Delta} \cdot \prod_{\substack{j \neq 1, i}}^{n+1} B_j < 0 \tag{14}$$

$$\frac{\partial r}{\partial s} = \frac{2\theta}{\Delta} \cdot \prod_{i=2}^{n+1} B_i > 0. \tag{15}$$

Proof of Proposition 1

Differentiating (3) with respect to θ, the welfare effect of privatization is computed as

$$
\begin{aligned}
\frac{\partial W}{\partial \theta} &= \sum_{i=1}^{n+1} \left[p\left(X^i\right) - a_i \right] \frac{\partial X^i}{\partial \theta} \\
&= \frac{2A}{\Delta} \left\{ \left[p\left(X^1\right) - a_1 \right] \sum_{i \neq j \neq \cdots \neq k} \underbrace{B_i B_j \cdots B_k}_{n-1} - \left[p\left(X^i\right) - a_i \right] \prod_{j \neq i, j=2}^{n+1} B_j \right\}.
\end{aligned}
$$

(16)

Although the sign of (16) is ambiguous, it can be shown to be positive if all the oligopolistic industries have the same technology, i.e. $a_1 = a_2 = \cdots = a_{n+1}$. In this case, (16) simplifies to

$$
\frac{\partial W}{\partial \theta} = \frac{2A}{\Delta} \left\{ \sum_{i=2}^{n+1} \left[p\left(X^1\right) - p\left(X^i\right) \right] \prod_{j \neq i, j=2}^{n+1} B_j \right\}.
$$

Since comparing (8) and (9) allows us to know that $X^1 > X^i$ and $p\left(X^1\right) < p\left(X^i\right)$, $\partial W / \partial \theta$ becomes necessarily positive.

Proof of Proposition 2

Differentiating (3) with respect to s and setting $a_1 = a_2 = \cdots = a_{n+1}$, we get

$$
\frac{\partial W}{\partial s} = -\frac{4\theta}{\Delta} \left\{ \sum_{i=2}^{n+1} \left[p\left(X^1\right) - p\left(X^i\right) \right] \prod_{j \neq i, j=2}^{n+1} B_j \right\} < 0,
$$

by making an argument parallel with the effect of privatization. As a result, it is optimal to choose zero subsidy, and the degree of privatization has no effect on the optimal subsidy.

Special case of two oligopolistic sectors

In the simplest case of two oligopolistic industries, Δ is negative and the comparative statics outcomes are obtained as

$$
\frac{\partial X^1}{\partial \theta} = \frac{2A}{\Delta} < 0, \quad \frac{\partial X^2}{\partial \theta} = -\frac{2A}{\Delta} > 0, \quad \frac{\partial r}{\partial \theta} = -\frac{\left(X^2 p_2'' + 3 p_2'\right) A}{\Delta} < 0
$$

$$\frac{\partial X^1}{\partial s} = -\frac{4\theta}{\Delta} > 0, \quad \frac{\partial X^2}{\partial s} = \frac{4\theta}{\Delta} < 0, \quad \frac{\partial r}{\partial s} = \frac{2\theta\left(X^2 p_2'' + 3p_2'\right)}{\Delta} > 0.$$

The welfare effect of privatization and production subsidy is then

$$\begin{aligned}
\frac{\partial W}{\partial \theta} &= \frac{2A}{\Delta}\left[p\left(X^1\right) - a_1 - p\left(X^2\right) + a_2\right] \\
\frac{\partial W}{\partial s} &= -\frac{4\theta}{\Delta}\left[p\left(X^1\right) - a_1 - p\left(X^2\right) + a_2\right].
\end{aligned}$$

While the sign of these equations is generally ambiguous, $\partial W/\partial \theta > 0$ and $\partial W/\partial s < 0$ in the special case with $a_1 = a_2$.

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