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On endogenous Stackelberg leadership

The case of horizontally differentiated duopoly and asymmetric network compatibility effects

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On endogenous Stackelberg leadership: The case of horizontally differentiated duopoly and asymmetric net work compatibility effects

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Abstract

Introducing product compatibility associated with network externalities (hereafter, network compatibility effects) into a horizontally differentiated duopoly model, we consider how network compatibility effects and the level of product substitutability affect endogenous timing decisions in the cases of quantity- and price-setting competition. In particular, we demonstrate the following. First, given asymmetric network compatibility effects between the products of the firms, there is Stackelberg equilibrium where the firm providing a product with a larger network compatibility effect than some certain level of product substitutability emerges as a leader (follower), whereas the firm providing a product with a smaller network compatibility effect than some certain level of product substitutability emerges as a follower (leader) in the case of quantity (price)-setting competition. Second, the Stackelberg equilibrium is Pareto-superior for both firms compared with other equilibria. However, with alternative formulation determining network size, with respect to the endogenous Stackelberg leader−follower relationship, the revers holds.

Keywords: Stackelberg equilibrium; Nash equilibrium; leader-follower; product compatibility; network externality; product substitutability; fulfilled expectations; horizontally differentiated products

JEL classification: D21, D43, D62, L15

1. Introduction

In the field of industrial organization, many studies have analyzed the choice of firm roles in market or timing decisions with respect to strategic variables including price, quantity, and other firm activities (e.g., R&D, advertising). With the first- and second-mover advantage, when comparing the Stackelberg and Cournot−Nash equilibria, we find that firms prefer to be leaders (followers) if the strategic relationships between them indicate that they are substitutes (complements) with respect to the relevant strategic variables, i.e., price and quantity. Equivalently, the same is true if negative (positive) slopes of the reaction functions are found in the relevant strategic variable space. For example, Gal-Or (1985) demonstrates that firms are willing (unwilling) to commit first when the reaction functions are downward (upward) sloping. In this case, the firms have a first (second)-mover advantage. Furthermore, Dowrick (1986) considers the conditions where firms agree upon the choice of role of leader and follower in the Stackelberg duopoly model and demonstrates that each firm prefers to be a leader when the slope of the reaction functions is downward. In contrast, each firm prefers that the other firm will be the leader when the slope of the reaction functions is upward.

Hamilton and Slutsky (1990) construct the extended endogenous timing game of an observable delay framework and consider the endogenous timing of simultaneous versus sequential moves. That is, the players determine both the timing and the action. If players choose their actions at different times, then the player choosing a later time observes the action taken by the initiating player, giving rise to a sequential-play subgame. Thus, Stackelberg equilibrium holds. If the players instead choose actions at the same time, then a simultaneous-play subgame takes place. Thus, Nash equilibrium holds. As shown below, we use the framework in Hamilton and Slutsky (1990), to consider the endogenous timing decisions governing quantities and prices, and demonstrate that Stackelberg equilibrium holds given asymmetric network compatibility effects between firms.

In terms of related analysis, Robson (1990) analyzes a model of price-setting duopoly including the endogenous choice of strategic timing by introducing timing costs into an extended game. Elsewhere, van Dame and Hurkens (1999) consider a linear quantity-setting duopoly including asymmetric costs between the firms and analyze a two-stage game where each firm can either commit to a quantity in stage 1 or wait until stage 2. They show that based on the risk dominance considerations in Harsanyi and Selten (1988), the low-cost firm will emerge as the endogenous Stackelberg leader, whereas commitment is riskier for the high-cost firm (see also Amir and Grillo, 1999). Similarly, van Dame and Hurkens (2004) address a linear price-setting duopoly and show that the low-cost firm will emerge as the endogenous price leader (see also Amir and Stepanova, 2006).

In other work, Li (2014) examines price leadership in a vertically differentiated product market and shows that the high-quality firm acting as price leader risks dominating the other equilibrium when the low-quality firm leads. Finally, using a horizontally differentiated duopoly model with linear demand and asymmetric constant marginal costs, Yang et al. (2009) compare price and quantity competition under endogenous timing and demonstrate that endogenous timing in the price-setting duopoly leads to two sequential move games in which one firm moves first and the other firm moves second, i.e., two Stackelberg equilibria. Furthermore, they show that endogenous timing in the quantity-setting duopoly leads to a simultaneous-move game, in which both firms move first, i.e., a Cournot−Nash equilibrium.

In the abovementioned studies, the strategic space is assumed to be exogenously given, i.e., a quantity- and price-setting duopoly. However, Singh and Vives (1984) consider the endogenous choice of strategic variables, i.e., prices and/or quantities, as well as the distribution of roles in duopoly games. That is, provided the products are substitutes (complements), it is a dominant strategy for each firm to choose the quantity (price) contract (see also Boyer and Moreaux, 1987a, 1987b).

Further, Tremblay and Tremblay (2011) develop a model in which both the timing of play and the strategic choice variables, i.e., quantity and price, are endogenous. They show that the dynamic Cournot−Bertrand outcome can be a subgame perfect Nash equilibrium, in which the firm choosing quantity (price) moves first (second).[1]

Similar issues have also received attention in the context of a vertically differentiated duopoly model with a fixed convex cost function for quality. For example, Aoki and Prusa (1997) and Aoki (2003) demonstrate that firms select distinctive qualities and that a firm producing a high-quality product earns larger profits than a firm producing a low-quality product, regardless of the competition mode. In this case, the leader (follower) in a sequential Stackelberg game must decide to produce a high (low) -quality product. However, because both firms prefer to commit to the production of a high-quality product, they both choose to move first.

Lambertini (1996, 1999) considers endogenous timing with a vertically differentiated Bertrand duopoly and demonstrates that if firms endogenously decide the timing of quality choices, only simultaneous-move equilibria can arise. Jinji (2004) examines this same issue in the context of a vertically differentiated Cournot duopoly and establishes that the outcomes of the endogenous timing game depend on whether

[1] Tremblay et al. (2013) show that the degree of product differentiation is sufficiently large for the Cournot−Bertrand equilibrium to be stable.

firms are able to choose their relative position in the quality space before they determine the timing of their quality choice. In other words, if firms cannot select their relative position, in line with the result in Lambertini (1999), only simultaneous-move equilibria persist. In this case, the firms have an incentive to move first because the first mover can earn larger profits than the second mover. Alternatively, if both firms can choose their relative position, only sequential-move equilibria emerge. In this case, the firm choosing to produce the low (high)-quality product decides to be the first (second) mover. In a more recent study, Lambertini and Tampieri (2012) determine that a firm producing a low (high)-quality product takes the leader's (follower's) role in a vertically differentiated quantity-setting duopoly model (see also Lambertini and Tedeschi, 2007).

We have surveyed the related literature published up to now that addresses the distribution of roles in market competition. Furthermore, we refer to the literature considering the same issue in the case of the precommitment of the strategic variables, such as process (cost reducing) and product (quality improving) R&D, and advertising. For example, Amir et al. (2000) consider the endogenous timing of process R&D with technology spillovers by applying the frameworks in D'Aspremont and Jacquemin (1988) and De Bondt and Henriques (1995).[2] In particular, these studies assume that a spillover effect arises because a rival firm's R&D stimulates the availability of technological knowledge, i.e., incoming spillovers. They then demonstrate the existence of a unique equilibrium in the assignment of the leader and follower roles in which one firm that is better at absorbing knowledge spillovers leads and the other firm follows.

In contrast, Atallah (2005) assumes that a spillover effect arises from the leakage of technological information from a rival firm's R&D. In this case, the result is the

[2] See also Tesoriere (2008), and Vandekerckhove and De Bondt (2008).

opposite of that presented by Amir et al. (2000) and others. In other words, the first mover is a firm that suffers only a small leakage of technological knowledge from its own process R&D. Like Atallah (2005), Toshimitsu (2012) considers the distribution of roles in product R&D (or advertising) investment competition in the presence of demand spillovers and shows that the firm producing that product with low (high)-demand spillovers will emerge as the leader (follower), irrespective of the competition mode.

To sum up, the nature of the reaction functions determine the first- or second-mover advantage, the endogenous distribution of these roles, and the leader-follower relationship. Put differently, these are strategic relationships (i.e., strategic substitutes or complements) between firms. In turn, these strategic relationships depend on the properties (i.e., substitutes or complements) between products or the strategic variables (i.e., quantity or price), as addressed in Boyer and Moreaux (1987a, 1987b). Further, for endogenous leadership to hold in a duopolistic game, there are requirements for certain asymmetric characteristics between the firms themselves, the attributes of their products, and their strategic variables. For example, extending the strategic taxonomy of Fudenberg and Tirole (1984), Tombak (2006) examines a "strategic asymmetry" two-stage game where one firm regards its rival's second-stage strategic variable as a strategic complement, while the other firm regards its rival's second-stage strategic variable as a strategic substitute.

In this paper, by focusing on product compatibility and network externalities, i.e., the network compatibility effects, as the characteristics of the products, we reconsider the distribution of roles in a horizontally differentiated duopoly. As for the asymmetric strategy space (i.e., price vs. quantity) in Tremblay et al. (2011) and the asymmetric product quality (i.e., low vs. high quality) in Lambertini and Tampieri (2012), we assume that the degree of network compatibility effects is exogenously given and asymmetric. Under these circumstances, we examine endogenous Stackelberg leadership in network product and service industries. That is, even though the products provided by competing firms are substitutes, at least in terms of horizontal product differentiation such as brand names, it is possible for the strategic relationships between the firms to be complements. This corresponds well with the properties of products associated with network externalities, including internet services in the ITC industry and application software (e.g., word processing, spreadsheet, and database products).

Using this approach, we identify with which of these characteristics a firm (or product) will emerge as either a leader or a follower, based on the framework of endogenous timing decisions, i.e., the extended game with observable delay as developed by Hamilton and Slutsky (1990). Furthermore, we reconsider that the results derived depend on both the competition mode and the formulation of network size.

2. Endogenous leader−follower relationship and network compatibility effects

2.1 Cournot−Nash equilibrium and strategic relationships

We consider quantity competition in a horizontally differentiated product market associated with product compatibility and network externalities. Based on the framework in Economides (1996), we assume a linear inverse demand function for product i as follows:

$$p_i = A - (q_i + \theta q_j) + f(S_i^e), \quad i, j = 1,2, i \neq j, \tag{1}$$

where A is the intrinsic market size, q_i (q_j) is the quantity of firm *i* (*j*), and $\theta \in (0,1)$ represents the degree of product substitutability. The network externality function is given by $f(S_i^e)$, where S_i^e is the expected network size of firm *i*. Based on the concept of a fulfilled expectations equilibrium presented by Katz and Shapiro (1985), we assume that $S_i^e = S_i$, where S_i is the real network size of firm *i*. Following Chen and Chen (2011), we assume a linear network externality function; $f(S_i) = aS_i$, where $a \in (0,1)$ denotes the network externality parameter for network size. Furthermore, using equation (3.15) in Shy (2001, p. 62), the network size of firm *i* is given by:

$$S_i = q_i + \alpha_i q_j, \quad i, j = 1,2, i \neq j, \tag{2}$$

where $\alpha_i \in [0,1]$, $i = 1,2$, is the degree of product *i*'s compatibility with product *j*. Equation (2) implies that firm *i* will provide a compatible product which the rival firm's product *j* can operate. If $\alpha_i = 1$ (0), $i = 1,2$, product *i* operates (does not operate) perfectly with product *j*. Given equation (2), q_i $(\alpha_i q_j)$, $i, j = 1,2, i \neq j$, is the *own (incoming) effect* on network size.

Based on equations (1) and (2), the inverse demand function for firm *i* is given by:

$$p_i = A - (1-a)q_i - (\theta - a\alpha_i)q_j, \quad i, j = 1,2, i \neq j. \tag{3}$$

Regarding equation (3), we assume that the own-price effect exceeds the cross-price effect: i.e., $\left|\frac{dP_i}{dq_i}\right| > \left|\frac{dP_i}{dq_j}\right|$, $i, j = 1,2, i \neq j$. Thus, it follows that $1 - a > |\theta - a\alpha_i|$, $i = 1,2$.

Although the products are horizontally differentiated in terms of their intrinsic attributes, i.e., $1 > \theta > 0$, the relationship between the products will be complementary (substitutionary) in terms of their operational properties if the degree of product compatibility with the network externality is larger (smaller) than that of their product substitutability, i.e., $a\alpha_i > (<)\theta,\ i = 1,2.$ As shown below, the nature of the products determines the strategic relationships between the firms and the external effects on their profits. Hereafter, we denote product compatibility with a network externality as the network compatibility effect, i.e., $a\alpha_i$. To simplify the analysis, we assume that production costs are zero. Thus, the profit function is $\pi_i = p_i q_i,\ i = 1,2.$

In view of equation (3), we derive the reaction function for firm *i* as follows.

$$q_i = \frac{A}{2(1-a)} - \frac{\theta - a\alpha_i}{2(1-a)} q_j, \quad i, j = 1,2, i \neq j. \tag{4}$$

Given equation (4), the strategic relationship between the firms depends on the degree of product substitutability and network compatibility effect:

$$\frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_i,\ i, j = 1,2, i \neq j. \tag{5}$$

Equation (5) implies that a strategic substitutionary (complementarity) relationship between the firms arises if the degree of product substitutability is larger (smaller) than that of the network compatibility effect. In particular, even though the two products are substitutable, a relationship of strategic complementarity arises under quantity competition when $a\alpha_i > \theta$.

Using the first-order condition to maximize profit, the profit function is represented by $\pi_i = p_i q_i = (1-a)(q_i)^2,\ i = 1,2.$ Thus, we derive the external effect of an increase in firm *j* on the profit of firm *i* as follows:

$$\frac{\partial \pi_i}{\partial q_j} = 2(1-a) q_i \frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_i,\ i, j = 1,2, i \neq j. \tag{6}$$

For the following analysis, we assume asymmetric compatibility between the firms:

Assumption 1 $1 \geq \alpha_1 > \alpha_2 \geq 0$.

Given equation (4), we derive the following Cournot–Nash equilibrium:

$$q_i^{\,N} = \frac{A\{2(1-a)-(\theta - a\alpha_i)\}}{D}, \quad i = 1,2, \tag{7}$$

where $D \equiv 4(1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$ and $2(1-a)-(\theta - a\alpha_i) > 0, \quad i = 1,2.$ Both of these conditions are satisfied because the own-price effect exceeds the cross-price effect. Given equation (7), superscript N denotes the Cournot−Nash equilibrium.

Taking equations (5) and (6) into account, we categorize the Cournot–Nash equilibrium across three cases. In case (i) ((ii)), a strong (weak) network compatibility effect arises: $\theta < (>) a\alpha_i, \quad i = 1,2,$ When a strong (weak) network compatibility effect arises, according to the strategic complementarity (substitutionary) relationship under quantity competition, the reaction curves for both firms are upward (downward) sloping. See Figure 1 (2). In case (iii), where asymmetric network compatibility effects arise, i.e., $a\alpha_1 > \theta > a\alpha_2,$ the reaction curve of firm *1* (*2*) is upward (downward) sloping.[3] See Figure 3.

2.2 Endogenous leader−follower decision and a natural Stackelberg situation

Applying the framework of endogenous timing decisions, i.e., the extended game

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[3] It is characterized by a game with strategic heterogeneity (see Monaco and Sabarwal, 2015). One player exhibits a strategic complement and the other a strategic substitute.

with observable delay developed by Hamilton and Slutsky (1990), we demonstrate an endogenous leader−follower decision and derive a natural Stackelberg situation.

We assume firm j (i) is a leader (follower). Considering equation (4), we derive the Stackelberg equilibrium under quantity competition as follows:

$$q_j^{\ L} = \frac{A\{2(1-a)-(\theta - a\alpha_j)\}}{D-(\theta - a\alpha_1)(\theta - a\alpha_2)}, \tag{8}$$

$$q_i^{\ F} = \frac{A\{D-2(1-a)(\theta - a\alpha_i)\}}{2(1-a)\{D-(\theta - a\alpha_1)(\theta - a\alpha_2)\}}, \tag{9}$$

where $i, j = 1,2, i \neq j.$ We can similarly obtain the outcomes in the case of the opposite roles, i.e., firm j (i) is a follower (leader). Given equations (8) and (9), superscript *L (F)* denotes the leader (follower) in the Stackelberg equilibrium.

Using equations (7), (8), and (9), and comparing the quantities in the Nash equilibrium and in the Stackelberg equilibrium, we derive the following relationships:

$$q_j^{\ L} > (<) q_j^{\ N} \Leftrightarrow (\theta - a\alpha_i)(\theta - a\alpha_j) > (<) 0, \tag{10}$$

$$q_j^{\ F} > (<) q_j^{\ N} \Leftrightarrow \theta - a\alpha_i < (>) 0, \tag{11}$$

$$q_j^{\ L} > (<) q_j^{\ F} \Leftrightarrow (\theta - a\alpha_i)(\theta - a\alpha_j) > (<) 0, \tag{12}$$

where $i, j = 1,2, i \neq j.$

We now compare the profits in the Nash and Stackelberg equilibria. Given *Assumption 1*, and considering equations (6), (10), (11), and (12), we derive the following results as in *Lemma 1*.

Lemma 1

(i) If $1 > a\alpha_1 > a\alpha_2 > \theta,$ *then* $q_i^{\ L} > q_i^{\ F} > q_i^{\ N}$ *and* $\pi_i^{\ F} > \pi_i^{\ L} > \pi_i^{\ N},$ $i = 1,2.$

(ii) If $\theta > a\alpha_1 > a\alpha_2 \geq 0$, *then* $q_i^L > q_i^N > q_i^F$ *and* $\pi_i^L > \pi_i^N > \pi_i^F$, $i = 1,2$.

(iii) If $1 > a\alpha_1 > \theta > a\alpha_2 \geq 0$, *then* $q_1^N > q_1^F > q_1^L$, $q_2^F > q_2^N > q_2^L$, $\pi_1^L > \pi_1^N > \pi_1^F$, *and* $\pi_2^F > \pi_2^L > \pi_2^N$.

In *Lemma 1 (i)*, where the degree of the network compatibility effect is larger than that of product substitutability, both firms prefer being the follower to being the leader and to playing a simultaneous-move game. In particular, both firms have a second-mover advantage. Thus, considering Theorem V (A. ii) in Hamilton and Slutsky (1990) and Lemma 1 in Yang, et al. (2009), there are multiple equilibria in the extended game with observable delay, i.e., two Stackelberg equilibria. See S_1 and S_2 in Figure 1.

In *Lemma 1 (ii)*, if the degree of the network compatibility effect is smaller than that of product substitutability, both firms prefer being the leader to being the follower and to playing a simultaneous-move game. In this case, both firms have a first-mover advantage. Thus, considering Theorem V (A. i) in Hamilton and Slutsky (1990) and Lemma 2 in Yang, et al. (2009), there is a unique simultaneous-move game equilibrium in the extended game with observable delay, i.e., a Cournot−Nash equilibrium. See N in Figure 2.

In *Lemma 1 (iii)*, there is asymmetry between the firms with respect to the degree of the network compatibility effect. That is, if the degree of the network compatibility effect for firm *1* (*2*) is larger (smaller) than that of the product substitutability, firm *1* prefers being the leader to being the follower and to playing a simultaneous-move game, whereas firm *2* prefers being the follower to being the leader and to playing a simultaneous-move game. Thus, considering Theorem V (B) in Hamilton and Slutsky (1990), the sequential move game equilibrium in the extended game with observable delay is unique, i.e., a Stackelberg equilibrium. See S_1 in Figure 3.

Here, we use the definition of a Natural Stackelberg Situation (NSS) presented by Albaek (1990) as follows.

Definition

In a NSS, one firm prefers being the leader to being the follower and to playing a simultaneous-move game, and the other firm prefers being the follower to being the leader and to playing a simultaneous-move game.

Furthermore, we assume the necessary condition for a NSS to hold as follows.

Assumption 2 $a > \theta$.

Otherwise, i.e., $\theta > a$, the strategic relationship under quantity competition is always substitutionary, irrespective of the product compatibility level (i.e., α_i). Based on *Lemma 1 (ii)*, there is a unique Cournot−Nash equilibrium. Therefore, we summarize the analysis above as *Proposition 1*.

Proposition 1

There is a NSS where the firm providing the product with a larger (smaller) network compatibility effect than a certain product substitutability level is the leader (follower) under quantity competition.

Because the degree of the network compatibility effect is larger (smaller) than that of product substitutability, a(n) increase (decrease) in the output of the rival firm increases the profit of the relevant firm. In this case, the firm providing the product where the degree of the network compatibility effect is larger than that of product substitutability has a strong incentive to choose being a first mover so as to commit a small amount of output, i.e., $q_1^N > q_1^F > q_1^L$. On the other hand, the firm providing the product where the degree of the network compatibility effect is smaller than that of the product substitutability prefers being a second mover to a first mover because the small output of the rival firm increases its profit. As a result, the natural Stackelberg situation under quantity competition holds where this equilibrium is Pareto-superior for both firms when compared with the other equilibria.

3. Price competition and an alternative formulation of network size

3. 1 Bertrand−Nash equilibrium and strategic relationships

Taking equation (3) into account, we derive the direct demand function of firm i as follows.

$$q_i = \frac{\{(1-a)-(\theta - a\alpha_i)\}A - (1-a)p_i + (\theta - a\alpha_i)p_j}{(1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2)}, \quad i, j = 1,2, i \neq j. \qquad (13)$$

Based on equation (13), the reaction function for firm i is given by

$$p_i = \frac{\{(1-a)-(\theta - a\alpha_i)\}A}{2(1-a)} + \frac{\theta - a\alpha_i}{2(1-a)} p_j, \quad i, j = 1,2, i \neq j. \qquad (14)$$

Thus, the strategic relationship between the firms depends on the degrees of product substitutability and the network compatibility effect as follows:

$$\frac{\partial p_i}{\partial p_j} > (<) 0 \Leftrightarrow \theta > (<) a\alpha_i,\ i, j = 1,2, i \neq j. \tag{15}$$

Equation (15) implies that a strategic complementarity (substitutionary) relationship between the firms holds if the degree of product substitutability is higher (lower) than that of the network compatibility effect. In particular, even though the two products are substitutable, a relationship of strategic substitutionary is sustained under price competition when $\theta < a\alpha_i$.

Furthermore, we derive the external effect of an increase the price of firm *j* on the profit of firm *i* as follows:

$$\frac{\partial \pi_i}{\partial p_j} > (<) 0 \Leftrightarrow \theta > (<) a\alpha_i,\ i, j = 1,2, i \neq j. \tag{16}$$

Given equation (14), we derive the following Bertrand–Nash equilibrium:

$$p_i^{BN} = \frac{A\{2(1-a)^2 - (1-a)(\theta - a\alpha_i) - (\theta - a\alpha_1)(\theta - a\alpha_2)\}}{D},\quad i = 1,2, \tag{17}$$

where $2(1-a)^2 - (1-a)(\theta - a\alpha_i) - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$ $i = 1,2$. This condition is satisfied because the own-price effect exceeds the cross-price effect. Given equation (17), superscript *BN* denotes the Bertrand−Nash equilibrium.

Taking equations (15) and (16) into account, we categorize the Bertrand–Nash equilibrium into three cases. In case (i) ((ii)), where a strong (weak) network compatibility effect, i.e., $\theta < (>) a\alpha_i$, $i = 1,2$, holds, according to the strategic substitutionary (complementarity) relationship under price competition, the reaction curves of both firms are downward (upward) sloping. See Figure 2 (1). The strategic relationships in these two cases lie opposite to those under the quantity competition case. In case (iii), where the asymmetric network compatibility effect, i.e., $a\alpha_1 > \theta > a\alpha_2$, arises, the reaction curve of firm *1* (*2*) is downward (upward) sloping. See Figure 4.

3.2 Endogenous leader−follower decision and a NSS

By following the same procedure as in quantity-setting competition, we derive the leader's price in the Stackelberg equilibrium:

$$p_j^{BL} = \frac{A\{2(1-a)^2 - (1-a)(\theta - a\alpha_j) - (\theta - a\alpha_1)(\theta - a\alpha_2)\}}{D - (\theta - a\alpha_1)(\theta - a\alpha_2)}. \tag{18}$$

Substituting equation (18) into equation (14), we obtain the follower's price:

$$p_i^{BF} = \frac{A[(1-a)H - (\theta - a\alpha_i)G]}{2(1-a)\{D - (\theta - a\alpha_1)(\theta - a\alpha_2)\}}, \tag{19}$$

where $H \equiv D - 2(\theta - a\alpha_1)(\theta - a\alpha_2) > 0$ and $G \equiv 2(1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2) > 0$. Given equations (18) and (19), superscript *BL* (*BF*) denotes a leader (follower) in a Stackelberg equilibrium.[4]

Using equations (15), (17), (18), and (19), with respect to the prices in the Nash equilibrium and in the Stackelberg equilibrium, we derive the following:

$$p_j^{BL} > (<) p_j^{BN} \Leftrightarrow (\theta - a\alpha_i)(\theta - a\alpha_j) > (<) 0, \tag{20}$$

$$p_j^{BF} > (<) p_j^{BN} \Leftrightarrow \theta - a\alpha_i > (<) 0, \tag{21}$$

$$p_j^{BL} > (<) p_j^{BF} \Leftrightarrow (\theta - a\alpha_i)(\theta - a\alpha_j) > (<) 0, \tag{22}$$

where $i, j = 1,2, i \neq j$.

We also compare the profits in the Nash and Stackelberg equilibria. Under

[4] Because $H - G = 2\{(1-a)^2 - (\theta - a\alpha_1)(\theta - a\alpha_2)\} > 0$ and $1 - a > \theta - a\alpha_i$ hold, it follows that $p_i^{BF} > 0$.

Assumption 1, and equations (16), (20), (21), and (22), with respect to prices and profits, we derive the following outcomes as in *Lemma 2* (see Figures 1, 2, and 4).

Lemma 2

(i) If $1 > a\alpha_1 > a\alpha_2 > \theta$, *then* $p_i^{BL} > p_i^{BN} > p_i^{BF}$ *and* $\pi_i^{BL} > \pi_i^{BN} > \pi_i^{BF}$, $i = 1,2$.

(ii) If $\theta > a\alpha_1 > a\alpha_2 \geq 0$, *then* $p_i^{BL} > p_i^{BF} > p_i^{BN}$ *and* $\pi_i^{BF} > \pi_i^{BL} > \pi_i^{BN}$, $i = 1,2$.

(iii) If $1 > a\alpha_1 > \theta > a\alpha_2 \geq 0$, *then* $p_1^{BF} > p_1^{BN} > p_1^{BL}$, $p_2^{BN} > p_2^{BF} > p_2^{BL}$, $\pi_1^{BF} > \pi_1^{BL} > \pi_1^{BN}$, *and* $\pi_2^{BL} > \pi_2^{BN} > \pi_2^{BF}$.

In *Lemma 2 (i)*, where the degree of the network compatibility effect is larger than that of product substitutability, both firms prefer being the leader to being the follower and to playing a simultaneous-move game. That is, both firms have a first-mover advantage. Thus, considering Theorem V (A. i) in Hamilton and Slutsky (1990) and Lemma 2 in Yang et al. (2009), there is a unique simultaneous move equilibrium in the extended game with observable delay, i.e., a Bertrand−Nash equilibrium. See N in Figure 2.

In *Lemma 2 (ii)*, where the degree of the network compatibility effect is smaller than that of product substitutability, both firms prefer being the follower to being the leader and to playing a simultaneous-move game. That is, both firms have a second-mover advantage. Thus, considering Theorem V (A. ii) in Hamilton and Slutsky (1990) and Lemma 1 in Yang et al. (2009), there are multiple equilibria in the extended game with observable delay, i.e., two Stackelberg equilibria. See S_1 and S_2 in Figure 1.

In *Lemma 2 (iii)*, there is asymmetry between the firms in terms of with the degree of the network compatibility effect. That is, when the degree of the network compatibility effect of firm *1* (*2*) is larger (smaller) than that of product substitutability, firm *1* prefers being the follower to being the leader and to playing a simultaneous-move game, whereas firm *2* prefers being the follower to being the leader and to playing a simultaneous-move game. Thus, considering Theorem V (B) in Hamilton and Slutsky (1990), the sequential move equilibrium in the extended game with observable delay is unique, i.e., a Stackelberg equilibrium. See S_2 in Figure 4.

Therefore, given *Assumption 2*, we summarize the above analysis as *Proposition 2*.

Proposition 2

There is a NSS where the firm providing the product with a strong (weak) network compatibility effect is the follower (leader) under price competition.

This result in *Proposition 2* is contrary to that in *Proposition 1*. That is, the firm having weak (strong) network compatibility is the Stackelberg price leader (follower). If the network compatibility effect is smaller (larger) than a certain level of product substitutability, a(n) increase (decrease) in the price of the rival firm increases the profit of the firm. In this case, the firm providing the product with a weak network compatibility effect has an incentive to choose to be the first mover and to commit the lowest price, i.e., $p_2^{BN} > p_2^{BF} > p_2^{BL}$. On the other hand, the firm providing the product with a strong network compatibility effect prefers being the second mover to the first mover because the low price of the rival firm, i.e., p_2^{BL}, increases its profit. As a result, the natural Stackelberg situation under price competition arises. This equilibrium is

Pareto-superior for both firms compared with the other equilibria.

Furthermore, without the network compatibility effect, i.e., $a\alpha_i = 0, i = 1,2,$ it follows that the follower's price is lower than the leader's price, similar to the equilibrium price in the standard Stackelberg competition. However, it follows in our model that $p_1^{BF} > p_2^{BL}$ because $a\alpha_1 > \theta > a\alpha_2$. That is, the follower can set a higher price than the leader's price because the follower's product is sufficiently compatible with the leader's product.

3.3 Alternative formulation of network size

With respect to the formulation of network size, Chen and Chen (2011) assume:

$$S_i = q_i + \alpha_j q_j, \quad i, j = 1,2, i \neq j, \tag{23}$$

where $\alpha_j q_j$ implies the *spillover (or leakage) effect* from firm *j* on the network size of firm *i*. That is, firm *j* provides a product that a user of product *i* can operate and that works in product *i*. Here, we also assume $a\alpha_1 > \theta > a\alpha_2$.

Under quantity competition, we change equations (5) and (6) as follows:

$$\frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_j, \tag{24}$$

$$\frac{\partial \pi_i}{\partial q_j} < (>) 0 \Leftrightarrow \theta > (<) a\alpha_j, \tag{25}$$

where $i, j = 1,2, i \neq j$. Thus, the strategic relationships and the external effect depend on the level of product substitutability and the rival firm's network compatibility effect. That is, if the rival firm provides the product with the network compatibility effect larger (smaller) than a certain level of product substitutability, the reaction curve for the competing firm is upward (downward) sloping and the external effect on profit is positive (negative). Therefore, the firm providing the product with a strong (weak) network compatibility effect will emerge as the follower (leader) under quantity-setting competition. This result lies opposite to *Proposition 1*.

Similarly for price-setting competition, we obtain the following:

$$\frac{\partial p_i}{\partial p_j} > (<) 0 \Leftrightarrow \theta > (<) a\alpha_j, \tag{26}$$

$$\frac{\partial \pi_i}{\partial p_j} > (<) 0 \Leftrightarrow \theta > (<) a\alpha_j, \tag{27}$$

where $i, j = 1,2, i \neq j$. If the rival firm provides the product with the network compatibility effects larger (smaller) than some level of product substitutability, then the reaction curve of the firm is downward (upward) sloping and the external effect on the profit is negative (positive). Therefore, the firm providing the product with a strong (weak) network compatibility effect will emerge as the leader (follower) under price-setting competition. This result lies opposite to *Proposition 2*.

4. Conclusion

In the model in this paper, the properties of a firm-specific product, i.e., product substitutability and network compatibility, determine the strategic relationships and the external effects. In particular, we have demonstrated that, in a quantity (price)-setting duopoly, if the network compatibility effect is smaller than a certain level of product substitutability, the strategic relationship of the rival firm regarding the firm is substitutionary (complementarity) and the external effect on the rival firm's profit is negative (positive). In general, if the strategic relationship of the rival firm regarding the firm is one of substitutionary (complementarity) and the external effect on the rival firm's profit is negative (positive), then the firm chooses to be the leader (follower) within the framework of an endogenous timing decision game. Based on this result, we showed that given asymmetric network compatibility effects between the firms' products, the firm providing the product with a larger (smaller) network compatibility effect than a certain level of product substitutability is the Stackelberg leader under quantity (price) competition. In other words, we propose that a natural Stackelberg situation holds in a game with strategic heterogeneity.

We appreciate that our model depends on specific assumptions, e.g., linearity of the functions. However, by focusing on the properties of the products associated with network externalities and compatibility, we have illustrated which effects of these properties determine the endogenous distribution of roles, i.e., Stackelberg leadership.

Further, although we assumed exogenously determined product compatibility, we should consider the level of product compatibility as a firm's strategic variable. Thus, we should examine Stackelberg leadership in the context of endogenous product compatibility choice (see, e.g., Toshimitsu, 2014). For similar reasons, although we assume that the asymmetric network compatibility effects are also exogenously given, we should consider how asymmetric network compatibility effects arise between firms.

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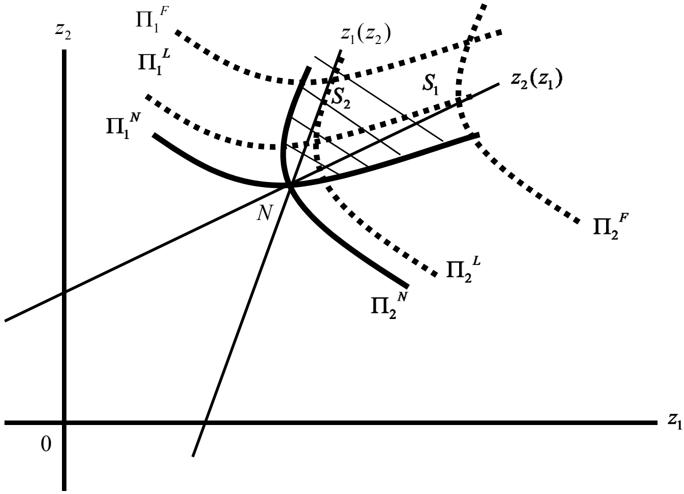

Figure 1

Strong network compatibility effects: $a\alpha_1 > a\alpha_2 > \theta$

$z_i = q_i$ or p_i, $i = 1,2.$

The shaded area represents the Pareto-superior sets.

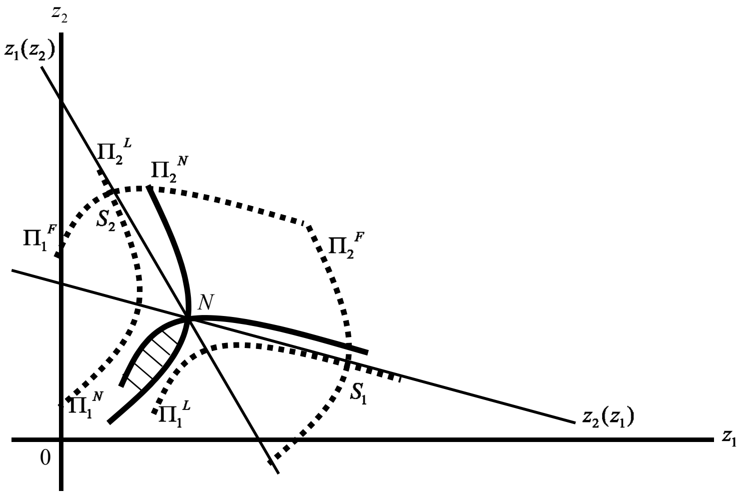

Figure 2

Weak network compatibility effects: $\theta > a\alpha_1 > a\alpha_2$

$z_i = q_i$ or p_i, $i = 1,2.$

The shaded area represents the Pareto-superior sets.

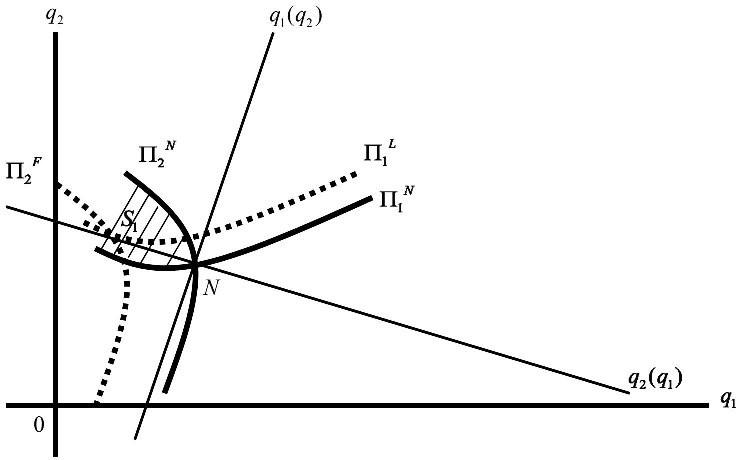

Figure 3

Quantity-setting duopoly with asymmetric network compatibility effects: $a\alpha_1 > \theta > a\alpha_2$

S_1 is a natural Stackelberg situation where firm *1* (*2*) is the leader (follower).

The shaded area represents the Pareto-superior sets.

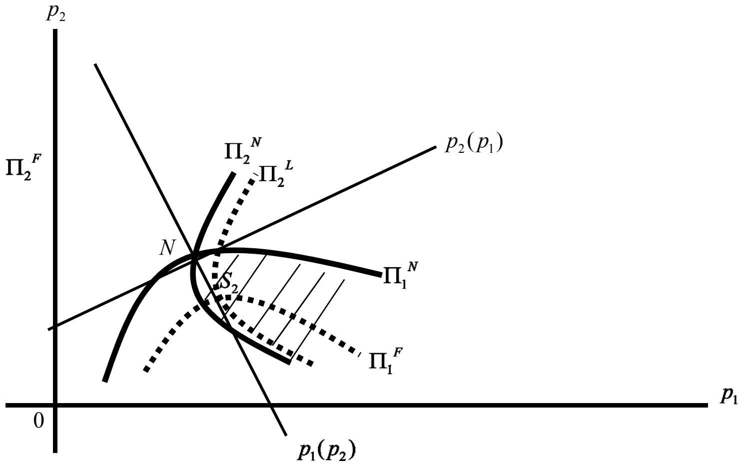

Figure 4

Price-setting duopoly with asymmetric network compatibility effects: $a\alpha_1 > \theta > a\alpha_2$

S_2 is a natural Stackelberg situation where firm *2* (*1*) is the leader (follower).

The shaded area represents the Pareto-superior sets.