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Network externalities between carriers or machines:

How they work in the smartphone industry

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Network externalities between carriers or machines: how they work in the smartphone industry *

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Abstract

In this paper, we consider a duopoly model where two firms sell two differentiated products and there is a network externality between either carriers or machines. We derive the equilibria of these games and illustrate the effects of a change in quality on the equilibrium quantity of each good. Furthermore, we compare fully compatible and incompatible equilibrium outcomes and discover some insights on relations between them. Such insights were not found in earlier studies that considered only the network externality between carriers.

Keywords: Smartphone market, Multi-product firm, Duopoly, Cannibalization, Network externality

JEL Classification Numbers: D21, D43, L13, L15

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1 Introduction

Over the last decade, mobile phones have spread rapidly in many developed countries. In the market for traditional mobile phones, there is just one network externality (network effect), as has been recognized since the seminal work of Katz and Shapiro (1985).[1]

In addition to these standard mobile phones, smartphones, for example, the *iPhone* from Apple, have recently increased their share and importance in our daily lives. Figure 1, for example, illustrates the market for smartphones in Japan.

Insert Figure 1 here.

One notable property of the smartphone market that differs from the market for standard mobile phones is that it contains the following two externalities.

First, there is a network externality between carriers that has been considered in the existing literature, such as Katz and Shapiro (1985) and Chen and Chen (2011). According to this externality, a consumer who purchases a product or service from a certain firm gains a network benefit when other consumers purchase the same or different product or service from the same firm. In Japan, for example, there are three major carriers, NTT DoCoMo, KDDI, and Softbank, all of which provide some special services that are mutually beneficial for their respective customers.

Second, we should recognize the existence of another important network externality between distinct types of smartphones supplied *to different carriers* by the same producer of smartphone devices.[2]In the real world, for instance, a customer of a carrier who has

[1]In Belleflamme and Peitz (2011, p.549), network effects has been formally defined as follows: "A product is said to exhibit network effects if each user's utility is increasing in the number of other users of that product or products compatible with it."

[2]In Kitamura (2013), I define this externality as follows: "A consumer who purchases a product from a certain firm gains a network benefit when other consumers purchase the same product from the same or different firm."

Apple's *iPhone* gains a network benefit when the number of *iPhone* users increases, even when these users are customers of other carriers. This network benefit takes the form of enhancement of reputation about the *iPhone*, or an increase in complementary goods, such as application software for the *iPhone*. Thus, even if consumers who use the *iPhone* do not use the same carrier, all consumers gain a network benefit from the increase in the number of *iPhone* users. To the best of our knowledge, this externality has received no attention in the previous studies that consider network externality.

In order to analyze such a market, one has to consider the idea of cannibalization. Cannibalization means that a company reduces the sales of one of its products by introducing a similar, competing product in the same market. Although Katz and Shapiro (1985) and Chen and Chen (2011) analyze the oligopolistic market in which each firm supplies a single product, considering the real economy, there are oligopolistic markets in which each firm produces and sells multiple products that are differentiated vertically in the same market. From each consumer's point of view, the quality of technology that each firm uses to produce its goods is different. Therefore, each consumer places different values on the high-quality goods of each firm. An example of this type of market is the "beer-like" beverage market that emerged in Japan in 1994. This market is composed of beer and *happoshu* or *low-malt beer*. *(*Happoshu) or low-malt beer is a tax category of Japanese liquor that most often refers to a beer-like beverage with less than 67% malt content. In the Japanese alcoholic-beverage tax system, lower tax is imposed on low-malt beer than "beer" with more than 67% malt content. Consequently, the market price of the former is lower than that of the latter. Therefore, leading makers such as Kirin, Asahi, and Sapporo Breweries sell beer and low-malt beer brands in the same beer-like beverage market. This market is not only horizontally but also vertically differentiated. Similarly, multi-product firms (abbreviated as "MPFs" hereafter) exist in the smartphone market. For example, in Japan, both KDDI and Softbank supply Apple's *iPhone* and Google's

Android smartphone. Although only Softbank supplied the *iPhone* initially, KDDI has also adopted it recently. I illustrate the smartphone market with MPFs in Figure 2.

Insert Figure 2 here.

Haruvy and Prasad (1998), a study closely related to mine, analyzed a market in which a monopolist sells a high-end and low-end version of the same product. The authors find some conditions under which producing both goods is optimal in the market with network externality. However, although each firm produces two differentiated goods, the two goods are sold in different markets, each with different types of consumers. In our model, we assume that both goods are supplied to the *same* market.

Furthermore, the *iPhone* is made by only Apple (that is, vertical integration), but *Google's* smartphones are made by many different producers. That is, *Google* only supplies the *Android* platform, and when the platform is updated, each producer must fix the programming of their product to apply the new platform programming. So, *Android* smartphones have more bugs, as compared with Apple's *iPhone*. Therefore, even in the smartphone market, there may exist vertical differentiation in quality.[3] Thus, in the real world, there may be many MPFs that differentiate their goods not only horizontally but also in quality, in the same market.

The remainder of this paper is organized in the following manner. Section 2 presents the model. Sections 3 and 4 prove and discuss the main results. Section 5 provides the conclusion.

[3]Another example of vertical differentiation in this industry is confirmed by the following outcome of Geekbench (the first URL is for the iPhone and the second for Android smartphones). This shows that the iPhone and Android smartphones differ in quality.

URL: http://browser.primatelabs.com/geekbench2/1030202

URL: http://browser.primatelabs.com/android-benchmarks

2 The model

In this section, I analyze a duopolistic market in the smartphone service industry with two kinds of network externalities. To pay attention to the externality between machines or devices, I omit carriers' phone services charges, because carriers in Japan charge their customers a fixed communication services fee, including an installment plan for the smartphone.[4]

Suppose there are two firms, ($i = 1, 2$), each producing two goods (good H and good L) that differ in terms of quality, where 1, 2 imply Firm 1 and Firm 2 in the duopoly case, respectively. Let V_H and V_L denote the quality level of the two goods. Then, the maximum amount consumers are willing to pay for each good is assumed to be $V_H > V_L > 0$. Further, we assume $V_H = (1 + \mu)V_L$, where μ represents the difference in quality between the two goods. For simplicity, we normalize the quality of the low-quality good as $V_L = 1$. Good $\alpha(= H, L)$ is assumed to be homogeneous for any consumer. For simplicity, suppose that each firm has no production and fixed costs. Under these assumptions, each firm's profit is defined as follows:

$$\pi_i = p_{iH}x_{iH} + p_{iL}x_{iL} \qquad i = 1, 2, \tag{1}$$

where $p_{i\alpha}$ is the price of good α sold by firm i, and $x_{i\alpha}$ is the firm's output. Each firm chooses the quantity to supply that maximizes this profit function in Cournot fashion.

Now, we describe the consumers' behavior in our model.

Following the standard specification in the literature–for example, Katz and Shapiro (1985)–we assume that there is a continuum of consumers that is characterized by a taste parameter θ that is uniformly distributed between $-\infty$ and $r > 0$ with density 1.

[4]Customers who use a smartphone from any carrier can practically use the call service free of charge by using the software, "Line."

It is assumed that a consumer of type $\theta \in (-\infty, r], r > 0$ obtains a net surplus from one unit of good α of firm i at price $p_{i\alpha}$. Furthermore, we assume that there exists *a network externality between carriers* or *a network externality between machines*. The former implies that a consumer who purchases a product or service from a certain firm gains a network benefit when other consumers purchase the same or different product from the same firm. We define the latter externality as follows: A consumer who purchases a product from a certain firm gains a network benefit when other consumers purchase the same product, regardless of its carrier.

Then, the surplus of the consumer θ who buys good α $(= H, L)$ from firm i $(= 1, 2)$ is given by[5]

$$U_{i\alpha}(\theta) = V_\alpha \theta + \nu V_\alpha g^e_{i\alpha} - p_{i\alpha}, \qquad i = 1, 2, \quad \alpha = H, L, \tag{2}$$

where ν represents the strength of the network externality. $g^e_{i\alpha}$ is the expectation of network benefit that a consumer obtains by purchasing one unit of good α from firm i. More precisely, we assume that the function $g^e_{i\alpha}(\cdot)$ is linear and define $g^e_{i\alpha}$ as follows in the two cases of network externality:

- Network externality between carriers

$$\begin{aligned} g^e_{i\alpha} &\equiv g_{i\alpha}(x^e_{iH}, x^e_{iL}, x^e_{jH}, x^e_{jL}, \phi_c) \\ &= x^e_{iH} + x^e_{iL} + \phi_c(x^e_{jH} + x^e_{jL}) \\ &= X^e_i + \phi_c X^e_j, \quad i,\ j = 1, 2, i \neq j, \alpha = H, L. \end{aligned} \tag{3}$$

Here, $X^e_i = x^e_{iH} + x^e_{iL}$ and ϕ_c is the degree of compatibility between carriers.

[5]This surplus is modeled similarly to Baake and Boom (2001).

- Network externality between machines or devices

$$\begin{aligned} g_{i\alpha}^e &\equiv g_{i\alpha}(x_{iH}^e, x_{iL}^e, x_{jH}^e, x_{jL}^e, \phi_m) \\ &= x_{1\alpha}^e + x_{2\alpha}^e + \phi_m(x_{1\beta}^e + x_{2\beta}^e) \\ &= X_\alpha^e + \phi_m X_\beta^e, \quad \alpha,\ \beta = H, L, \alpha \neq \beta, i = 1, 2. \end{aligned} \tag{4}$$

Here, $X_\alpha^e = x_{1\alpha}^e + x_{2\alpha}^e$ and ϕ_m is the degree of compatibility between machines.

For simplicity, we assume that the parameter of the degree of compatibility in both cases, $\phi_\delta \in \{0, 1\}(\delta = c, m)$ takes just 0 or 1. Thus, when the value of each parameter is 0 (1), it implies that consumers are incompatible (compatible) in each case.

We do not explicitly model the process through which consumers' expectations are formed. However, we impose the requirement that in equilibrium, consumers' expectations are fulfilled. That is, we assume the following *fulfilled expectations Cournot equilibrium*: when consumers form rational expectations, in equilibrium, the consumers' expected quantity is equal to actual quantity. Each firm chooses its output level under the following assumptions:

(a) Consumers' expectations about the size of networks are given.

(b) The actual output level of the other firm is fixed.

Assumption (b) is the standard Cournot assumption. Assumption (a) implies that in this model, the firms are unable to commit themselves, so that only the output levels of the *fulfilled expectations Cournot equilibrium* are credible announcements.

Furthermore, we assume that consumers must make their purchase decisions before the actual network sizes are known. Thus, the timing of the game is as follows.

1st Stage: Consumers form expectations about the size of the network with which each firm is associated.
2nd Stage: The firms play an output game, taking consumers' expectations as given. This game generates a set of prices. Consumers then make their purchase decisions by comparing their reservation prices with the prices set by the two firms ($i = 1, 2$).

Each consumer determines to buy nothing, or one unit of the good α, from firm i to maximize his/her surplus.

Before deriving the inverse demand of each good, we assume that for an arbitrary type-θ_α consumer,

$$U_{1\alpha}(\theta_\alpha) = U_{2\alpha}(\theta_\alpha), \quad \alpha = H, L. \tag{5}$$

This assumption states that the net surplus from buying the good from firm 1 or firm 2 must be equal, as long as the two firms produce the good with the same quality and have positive sales. From (2) and (5), we obtain

$$\begin{aligned} V_\alpha \hat{\theta}_\alpha + \nu V_\alpha g^e_{1\alpha} - p_{1\alpha} &= V_\alpha \hat{\theta}_\alpha + \nu V_\alpha g^e_{2\alpha} - p_{2\alpha} \\ \Longleftrightarrow \quad p_{1\alpha} - \nu V_\alpha g^e_{1\alpha} &= p_{2\alpha} - \nu V_\alpha g^e_{2\alpha}. \end{aligned} \tag{6}$$

Here, $p_{1\alpha} - \nu V_\alpha g^e_{1\alpha} = p_{2\alpha} - \nu V_\alpha g^e_{2\alpha}$ is *the expected hedonic price* of brand α, that is, the price adjusted for the network size. This hedonic price is used by Katz and Shapiro (1985). Thus, I may let

$$p_\alpha \equiv p_{1\alpha} - \nu V_\alpha g^e_{1\alpha} = p_{2\alpha} - \nu V_\alpha g^e_{2\alpha}, \quad \alpha = H, L. \tag{7}$$

I assume that there exists a consumer who is indifferent between the two goods of the same firm. This consumer's type is denoted by $\hat{\theta}_i$. Then, we have

$$U_{iH}\left(\hat{\theta}_i\right) = U_{iL}\left(\hat{\theta}_i\right) > 0 \tag{8}$$

$$\Longleftrightarrow \ (1+\mu)\hat{\theta}_i + \nu(1+\mu)g^e_{iH} - p_{iH} = \hat{\theta}_i + \nu g^e_{iL} - p_{iL}$$

$$\Longleftrightarrow \ \hat{\theta}_i = \frac{1}{\mu}\{p_{iH} - p_{iL} - (\nu(1+\mu)g^e_{iH} - \nu g^e_{iL})\} \quad i = 1,2. \tag{9}$$

Equations (7) and (9) yield

$$\hat{\theta}_1 = \frac{1}{\mu}\{p_{1H} - p_{1L} - (\nu(1+\mu)g^e_{1H} - \nu g^e_{1L})\} = \frac{1}{H-L}\{p_{2H} - p_{2L} - (\nu(1+\mu)g^e_{2H} - \nu g^e_{2L})\} = \hat{\theta}_2,$$

and therefore,

$$\hat{\theta}_1 = \hat{\theta}_2.$$

So I may let

$$\hat{\theta} \equiv \hat{\theta}_i \quad i = 1,2. \tag{10}$$

Furthermore, as in the preceding chapter, we suppose that there exists a type of consumer $\underline{\theta}_L$, who is indifferent between purchasing good L and purchasing nothing. Then, the following equation holds:

$$U_{iL}\left(\underline{\theta}_L\right) = U_{2L}\left(\underline{\theta}_L\right) = 0$$

$$\Leftrightarrow \ \underline{\theta}_L = p_{iL} - \nu g^e_{iL}. \tag{11}$$

Then, from (2), (8), (11) and the increasing function of $U_{iL}(\cdot)$, we see that

$$U_{iH}(\hat{\theta}) = U_{iL}(\hat{\theta}) > U_{1L}(\underline{\theta}_L) = U_{2L}(\underline{\theta}_L) = 0.$$

So, equivalently we have

$$\hat{\theta} > \underline{\theta}_L. \tag{12}$$

Thus, as in the previous chapter, I obtain the next lemma.

Lemma 1. *Any consumer* $\theta \in (-\infty, \underline{\theta}_L)$ *buys nothing, consumer* $\theta \in (\underline{\theta}_L, \widehat{\theta})$ $(\theta \in [\widehat{\theta}, r])$ buys *good* L (*good* H), *respectively.*

2.1 Derivation of Equilibrium

From Lemma 1, we obtain the following system of equations:

$$\begin{cases} r - \hat{\theta} = X_H \\ r - \underline{\theta}_L = X_H + X_L \equiv x_{1H} + x_{2H} + x_{1L} + x_{2L}, \end{cases} \tag{13}$$

where $X_\alpha = x_{1\alpha} + x_{2\alpha}, \alpha = H, L$.

Substituting (**??**) and (11) into these equations and solving them for p_{iH} and p_{iL}, the inverse demand functions are obtained as

$$\begin{cases} p_{iH} = (1+\mu)(r + \nu g^e_{iH} - X_H) - X_L \\ p_{iL} = r - X_H - X_L + \nu g^e_{iL}. \end{cases} \tag{14}$$

To maximize the profit function, each firm determines each quantity q_{iH} and q_{iL}, given consumers' expectations,

$$\max_{q_{iH},q_{iL}} \pi_i.$$

Here, $\pi_i = \{(1+\mu)(r+\nu g^e_{iH} - X_H) - X_L\}x_{iH} + (r - X_H - X_L + \nu g^e_{iL})x_{iL}$ from (1).

The first-order conditions for profit maximization are

$$\begin{cases} \frac{\partial \pi_i}{\partial x_{iH}} = -(1+\mu)x_{iH} + (1+\mu)(r+\nu g^e_{iH} - X_H) - X_L - x_{iL} = 0 \\ \frac{\partial \pi_i}{\partial x_{iL}} = -x_{iH} - x_{iL} + r + \nu g^e_{1L} - X_H - X_L = 0, \qquad i = 1, 2. \end{cases} \tag{15}$$

Furthermore, to guarantee positive quantities and downward-sloping demand in all situations, we assume that

$$0 < \nu < 1 \quad and \quad 0 < \mu < \frac{2\nu}{3-2\nu}. \tag{16}$$

From the first-order condition (15), we have the following reaction functions for $x_{iH}$and x_{iL}:[6]

$$x_{iH} = -\frac{3-\nu(1+\mu)\frac{\partial g_{iH}}{\partial x_{iL}}}{(1+\mu)(3-\nu\frac{\partial g_{iH}}{\partial x_{iH}})}x_{iL} + \frac{r}{3-\nu\frac{\partial g_{iH}}{\partial x_{iH}}}, \tag{17}$$

$$x_{iL} = -\frac{3-\nu\frac{\partial g_{iL}}{\partial x_{iH}}}{3-\frac{\nu\partial g_{iL}}{\partial x_{iL}}}x_{iH} + \frac{r}{3-\nu\frac{\partial g_{iL}}{\partial x_{iL}}}\ . \tag{18}$$

- Case 1 (Network externality between carriers)

  In this case of a network externality between carriers, we consider two extreme settings: $\phi_c = 0$ and $\phi_c = 1$.

  - Case of full compatibility ($\phi_c = 1$)

[6] Then, we solve these reaction functions given by $q_{1\alpha} = q_{2\alpha}$.

From (3) and the assumption of fulfilled expectations, in equilibrium we have

$$g^e_{i\alpha} = X^e_i + X^e_j,\ \ i, j = 1, 2, i \neq j. \tag{19}$$

Thus, from the first-order conditions (15),

$$\begin{cases} x^{*FC}_{iH} = \frac{r}{3-2\nu} \\ x^{*FC}_{iL} = 0. \end{cases} \tag{20}$$

Then, the equilibrium price is determined as follows:

$$p^{FC}_H = \frac{r(1+\mu)}{3-2\nu}. \tag{21}$$

– Case of incompatibility ($\phi_c = 0$)

From (3) and the assumption of fulfilled expectations, in equilibrium we have

$$g^e_{i\alpha} = X^e_i,\ \ i, = 1, 2, . \tag{22}$$

Thus, from the first-order conditions (15),

$$\begin{cases} x^{*IC}_{iH} = \frac{r}{3-\nu} \\ x^{*IC}_{iL} = 0. \end{cases} \tag{23}$$

This leads to the following equilibrium price:

$$p^{IC}_H = \frac{r(1+\mu)}{3-\nu}. \tag{24}$$

- Case 2 (Network externality between machines or devices)

  As with Case 1, the following two settings can be considered.

  - Case of full compatibility ($\phi_m = 1$)

    In equilibrium, we obtain

$$g_{i\alpha}^e = X_H^e + X_L^e, \ . \tag{25}$$

    Thus, from the first-order conditions (15),

$$\begin{cases} x_{iH}^{*FM} = \frac{r}{3-2\nu} \\ x_{iL}^{*FM} = 0. \end{cases} \tag{26}$$

    Thus, the equilibrium price of good H is the same as in equation (21), that is,

$$p_H^{FM} = \frac{r(1+\mu)}{3-2\nu} \tag{27}$$

  - Case of incompatibility ($\phi_m = 0$)

    Similarly, we have

$$g_{i\alpha}^e = X_\alpha^e, \ \alpha, = H, L. \tag{28}$$

    Thus, from the first-order conditions (15),

$$\begin{cases} x_{iH}^{*IM} = \frac{r\{(3-2\nu)\mu-2\nu)\}}{(2\nu-3)^2\mu+4\nu(\nu-3)} \\ x_{iL}^{*IM} = \frac{-2(1+\mu)\nu r}{(2\nu-3)^2\mu+4\nu(\nu-3)}. \end{cases} \tag{29}$$

From equation (29), we can immediately find that

$$x_{iH}^{IM} < x_{iL}^{IM} \tag{30}$$

Now, we consider the point elasticity of firm demand ϵ_α in this case as follows:

$$\epsilon_\alpha = -\frac{\partial x_{i\alpha}}{\partial p_\alpha}\frac{p_\alpha}{x_{i\alpha}}, \qquad \alpha = H, L. \tag{31}$$

Then, from equation (29), we have ϵ_H and ϵ_L as follows:

$$\epsilon_H = \frac{3\mu - 4\nu - 2\mu\nu}{(3\mu - 2\nu - 2\mu\nu)(1-\nu)} > \frac{-(3\mu - 4\nu - 4\mu\nu)}{2\nu(1+\mu)(1-\nu)} = \epsilon_L. \tag{32}$$

Therefore, in equilibrium, each firm produces more low-quality goods L than high-quality goods H, because the elasticity of demand of good L, that is, ϵ_L, is less than that of good H, that is, ϵ_H. The equilibrium prices are

$$\begin{cases} p_H^{IM} = \frac{r(1+\mu)(3\mu-4\nu-2\mu\nu)}{(2\nu-3)^2\mu+4\nu(\nu-3)} \\ p_L^{IM} = \frac{r(3\mu-4\nu-4\mu\nu)}{(2\nu-3)^2\mu+4\nu(\nu-3)}. \end{cases} \tag{33}$$

Furthermore, the effects of an increase in the quality of the high-quality good on each quantity can be confirmed as follows:

$$\begin{cases} \frac{\partial x_{iH}^{*IM}}{\partial \mu} = \frac{-6r\nu(3-2\nu)}{\{(2\nu-3)^2\mu+4\nu(\nu-3)\}^2} < 0, \\ \frac{\partial x_{iL}^{*IM}}{\partial \mu} = \frac{18r\nu}{\{(2\nu-3)^2\mu+4\nu(\nu-3)\}^2} > 0. \end{cases} \tag{34}$$

Proposition 1 *Suppose there is a network externality, not between carriers, but machines or devices. Then, an increase in the quality difference between two goods leads* *to a decrease in the quantity of high-quality goods and an increase in that of low-quality goods.*

From the reaction functions (17) and (18), in this case $\frac{\partial g_{i\alpha}}{\partial x_{i\beta}} = 0$. This makes the slope of the reaction functions steeper. Thus, if there is only the network externality between machines and the two goods are incompatible, the competition between the two differentiated goods is very fierce. However, an increase in the quality difference between the two goods (μ) makes the slope of the reaction function (17) steeper and increases the x_{iL}-intercept of one in the $x_{iH} - x_{iL}$ plane (the reaction function $x_{iL}(x_{iH})$ is unchanged). Consequently, the increase in μ makes the intersection points of the reaction functions move toward the upper left in the $x_{iH} - x_{iL}$ plane. Thus, the equilibrium output of the high-quality good decreases and more of the low-quality good is produced. This is an example of cannibalization, where the low-quality good L drives the high-quality good H out of the market. That is, an increase in the quality difference between the two goods gives rise to relaxing competition in these goods. It also has a positive effect on the equilibrium output of the low-quality goods; however, this change in the output of the low-quality good leads to lower production of the high-quality good.

3 Relationships between equilibria

3.1 Comparison between Compatible and Incompatible Equilibria

In this section, in each case, we compare two equilibrium outcomes: fully compatible and incompatible equilibria.

- Case 1 (Network externality between carriers)

From (20) and (23), we have the following relations:

$$\begin{cases} x_{iH}^{FC} > x_{iH}^{IC} \\ x_{iL}^{FC} = x_{iL}^{IC} = 0 \end{cases} \tag{35}$$

and

$$p_H^{FC} > p_H^{IC}. \tag{36}$$

Furthermore, we also have

$$\pi_i^{FC} > \pi_i^{IC}. \tag{37}$$

Thus, we have the following proposition:

Proposition 2 *When only a network externality between carriers exists, the prices of both goods and the profit of each firm are higher in the compatible than in the incompatible case.*

This proposition is very natural and similar to results in Katz and Shapiro (1985). Furthermore, we can easily show that social welfare—defined as the sum of consumer surplus and producer surplus—is also higher when the two firms are in the compatible than in the incompatible case.

- Case 2 (Network externality between machines)

  Similarly, from (26) and (29), we have the following relations:

$$\begin{cases} x_{iH}^{FM} > x_{iH}^{IM} \\ x_{iL}^{IM} > x_{iL}^{FM} = 0 \end{cases} \tag{38}$$

and

$$p_H^{FM} > p_H^{IM}, \tag{39}$$

$$\pi_i^{FM} > \pi_i^{IM}. \tag{40}$$

We obtain the following proposition.

Proposition 3 *When only a network externality between machines exists, the price of the high-quality good is higher in the compatible case, as compared to the incompatible case. Furthermore, the profit of each firm is always higher when the two goods are in the compatible than in the incompatible case.*

The quantity of the low-quality good decreases when there is a change from the incompatible to the fully compatible case under two goods. In spite of this, the relationship between profits under fully compatible and incompatible equilibria is the same as in Katz and Shapiro's (1985) model (Case 1).

3.2 Comparison between Case 1 and Case 2

Finally, we explore the differences in case 1 and case 2 with respect to quantity, price, and profit. From the previous subsection, we find the following relations:

$$\begin{cases} x_{iH}^{FM} = x_{iH}^{FC} > x_{iH}^{IC} > x_{iH}^{IM} \\ x_{iL}^{IM} > x_{iL}^{FM} = x_{iL}^{FC} = x_{iL}^{IC} = 0 \end{cases} \tag{41}$$

and

$$p_H^{FM} = p_H^{FC} > p_H^{IC} > p_H^{IM}, \tag{42}$$

$$\begin{cases} \pi_i^{FM} = \pi_i^{FM} > \pi_i^{IC} > \pi_i^{IM} & if \quad \mu < \hat{\mu} \\ \pi_i^{FM} = \pi_i^{FM} > \pi_i^{IM} > \pi_i^{IC} & if \quad \hat{\mu} < \mu, \end{cases} \tag{43}$$

where $\hat{\mu}$ satisfies $(\nu - 2)(2\nu - 3)^2\hat{\mu} + 4\nu(1-\nu)(3-\nu) = 0$.[7]

Proposition 4 *Making the carriers or machines compatible always increases the quantity, price of the high-quality goods, and profits of both firms. Furthermore, each firm sells more and sets a higher price for high-quality goods when the two carriers are incompatible, than when the two machines are incompatible. However, if the difference in the quantity of the two goods is large enough, the firms earn more when the two machines are incompatible, than when the two carriers are incompatible.*

This implies that it is better for both firms to compete in a market in which there exists only a network externality between machines and even produce the low-quality good, if the difference in the quality of the two goods is too large. This is because if μ is large enough, then both firms are better-off producing the low-quality good from proposition 1; they benefit by selling more of the low-quality good when the two goods are incompatible. Thus, in this case, the two firms sell more of their low-quality goods and decrease the output of high-quality goods; that is, cannibalization occurs.[8]

4 Welfare Analysis

In this section, we compare social welfare in each case. The social surplus in the equilibrium derived in the preceding section is given by

[7] We can easily confirm that $\hat{\mu}$ satisfies both positive quantities and the downward-sloping demand condition (16) as follows: $0 < \hat{\mu} < 2\nu/(3-2\nu)$.

[8] We showed that cannibalization occurs in the market without the network externality when the firm has some production cost, in Kitamura and Shinkai (2014). However, in the market with network externality between machines, cannibalization occurs even when there is no production cost.

$$W^* = \int_{\underline{\theta}_L}^{\hat{\theta}^*} (\theta + g^*_{iL}) d\theta + \int_{\hat{\theta}^*}^{r} \{(1+\mu)\theta + g^*_{iH}\} d\theta. \tag{44}$$

- Case 1 (Network externality between carriers) The social welfare in equilibrium of the full compatibility case ($\phi_c = 1$) and incompatible case ($\phi_c = 0$) is, respectively,

$$\begin{aligned} W^{FC} &= \frac{4(1+\mu)r^2}{(3-2\nu)^2}, \\ W^{IC} &= \frac{4(1+\mu)r^2}{(3-\nu)^2}. \end{aligned} \tag{45}$$

From (46), when there is only the network externality between carriers, achieving complete compatibility improves social welfare, that is,

$$W^{FC} > W^{IC} \tag{46}$$

- Case 2 (Network externality between machines) The social welfare in equilibrium of the full compatibility case ($\phi_m = 1$) and incompatible case ($\phi_m = 0$) is, respectively,

$$W^{FM} = \frac{4(1+\mu)r^2}{(3-2\nu)^2}, \tag{47}$$

$$\begin{aligned} W^{IM} = \; & \frac{2\mu r^2(2v-1)(1+u)}{\{(2\nu-3)^2\mu + 4\nu(\nu-3)\}^2}\{2(3-2\nu)^3\mu^3 \\ & 2(3-2\nu)(16\nu^2 - 32\nu + 9)\mu^2 \\ & (-80\nu^3 + 236\nu^2 - 132\nu - 9)\mu \\ & -4\nu(8\nu^2 - 21\nu + 3)\}. \end{aligned} \tag{48}$$

Similar to the previous case 1, in this case too, compatibility between two devices is better from the social surplus viewpoint. That is,

$$W^{FM} > W^{IM} \tag{49}$$

Proposition 5 *Complete compatibility is always socially optimal, regardless of which network externality—between carriers or machines—exists.*

This result is similar to Katz and Shapiro (1985). That is, if the firm does not have any production cost or costs for compatibility, the social welfare in full compatibility equilibrium always exceeds that in incompatibility equilibrium. In case 1, both firms produce only high-quality goods, in spite of carrier compatibility. On the other hand, in case 2, each firm sells both goods when two devices are incompatible. This proposition implies that only high-quality goods survive when the market is socially optimal.

Finally, for social welfare, we compare case 1 and case 2. By the equations (46) and (48), we find that

$$\begin{cases} W^{FC} = W^{FM} > W^{IC} > W^{IM} & \textit{if } \nu \textit{ and } \mu \textit{ are not too high} \\ W^{FC} = W^{FM} > W^{IM} > W^{IC} & \textit{if } \nu \textit{ and } \mu \textit{ are too high.} \end{cases} \tag{50}$$

This is shown in Figure 3.(Figure 3 represents $W^{IC} - W^{IM}$.)

Insert Figure 3 here.

5 Concluding Remarks

Extending Katz and Shapiro's (1985) model, this paper has theoretically analyzed firm behavior and the resulting market configuration in the smartphone industry.

In section 2, we constructed a duopoly model where two firms sell two differentiated products and there is a network externality between either carriers or machines. We then derived proposition 1 that highlights the effects of a change in the quality of goods on the quantity of each good. Here, we also mentioned cannibalization. In section 3, we tried to compare fully compatible and incompatible equilibria. The equilibrium output of the low-quality good is produced only if there is a network externality between machines and the two machines are incompatible. Furthermore, we find that in both cases, there is only a network externality between carriers and machines; and the quantity of the high-quality good, prices of the two goods, and profit of each firm are higher when the carriers or goods are compatible rather than incompatible. Finally, in this section, we considered the differences in the two kinds of network externalities. Then, we showed that as long as the difference in the quantity of the the two goods is too large, the two firms make more profit when the two machines are incompatible, than when both carriers are incompatible.

In section 4, we conducted welfare analysis and found that, in both cases, complete compatibility is always socially optimal. Furthermore, if the difference in the quantity of two goods and the strength of the network externality are too large, social welfare in the incompatible case where only the network externality between machines exists exceeds that in the case of the network externality between carriers only.

However, we have so far focused on only the downstream market. Therefore, we will extend this analysis to include the upstream market. Furthermore, in this study, we considered a duopoly model without production cost. Thus, future studies must analyze the case where firms have some production costs, including the costs of making carriers or machines compatible.

Appendix: Lemma 1

Proof: By equation (2) and (8), for arbitrary type $\theta > \hat{\theta}_i$, From (2) and (12), we also have, for arbitrary type $\theta \in (\underline{\theta}_L, \widehat{\theta})$,

$$\begin{aligned} U_{iL}(\widehat{\theta}) - U_{iL}(\underline{\theta}_L) &= \hat{\theta} + \nu g^e_{iL} - p_L - (\underline{\theta}_L + \nu g^e_{iL} - p_L) \\ &= \hat{\theta} - \underline{\theta}_L > 0. \end{aligned}$$

$$\begin{aligned} U_{iH}(\theta) - U_{iL}(\theta) &= (1+\mu)\theta + \nu(1+\mu)g^e_{iH} - p_{iH} - \theta - \nu g^e_{iL} + p_{iL} \\ &= \mu\theta - \{p_{iH} - p_{iL} - (\nu(1+\mu)g^e_{iH} - \nu g^e_{iL})\} \\ &> \mu\hat{\theta}_i - \{p_{iH} - p_{iL} - (\nu(1+\mu)g^e_{iH} - \nu g^e_{iL})\} \\ &= 0. \end{aligned}$$

From (2) and (12), we also have, for arbitrary type $\theta \in (\underline{\theta}_L, \widehat{\theta})$,

$$\begin{aligned} U_{iL}(\widehat{\theta}) - U_{iL}(\underline{\theta}_L) &= \hat{\theta} + \nu g^e_{iL} - p_{iL} - (\underline{\theta}_L + \nu g^e_{iL} - p_{iL}) \\ &= \hat{\theta} - \underline{\theta}_L > 0. \end{aligned}$$

■

References

[1] Baake, P. and Boom, A. (2001), "Vertical product differentiation, network externalities, and compatibility decisions," *International Journal of Industrial Organization*, 19, pp.267-284.

[2] Belleflamme, P. and Peitz, M. (2011), *Industrial Organization Markets and Strategies*, Cambridge: Cambridge University Press.

[3] Chen, H-C. and Chen, C-C. (2011), "Compatibility under differentiated duopoly with network externalities," *Journal of Industry, Competition and Trade*, 11, pp.43-55.

[4] Cournot, A. (1838), *Recherches sur les Principes Mathematiques de la Theorie de la Richesse*, Paris: Calmann-Levy (new edition 1974).

[5] Haruvy, E. and Prasad, A. (1998), "Optimal product strategies in the presence of network externalities," *Information Economics and Policy*, 10, pp.489-499.

[6] Katz, M. and Shapiro, C. (1985), "Network externalities, competition, and compatibility," *American Economic Review*, 75(3), pp.424-440.

[7] Kitamura, R. and Shinkai, T. (2014), "Cannibalization may allow a cost-inefficient firm to earn more than a cost-efficient firm in a duopoly with two vertically differentiated goods," *Discussion Paper Series No.113, School of Economics, Kwansei Gakuin University*, Nishinomiya, 16 pages.

[8] Kitamura, R. (2013), "A theoretical analysis of the smart phone industry," *Master's Thesis in Economics, Graduate School of Economics,* (*unpublished*) Kwansei Gakuin University, Nishinomiya, 67 pages.

[9] Kitamura, R. and Shinkai, T. (2013), "The economics of cannibalization: A duopoly in which firms supply two vertically differentiated products," *Discussion Paper Series No.100, School of Economics, Kwansei Gakuin University*, Nishinomiya, 14 pages.

Figure 1

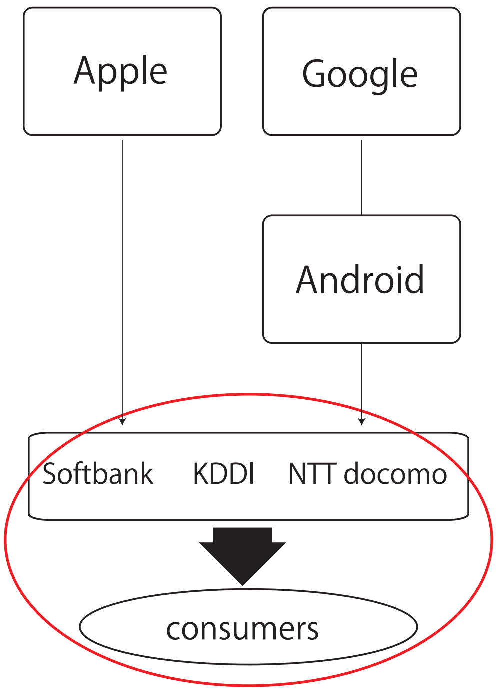

Figure 2

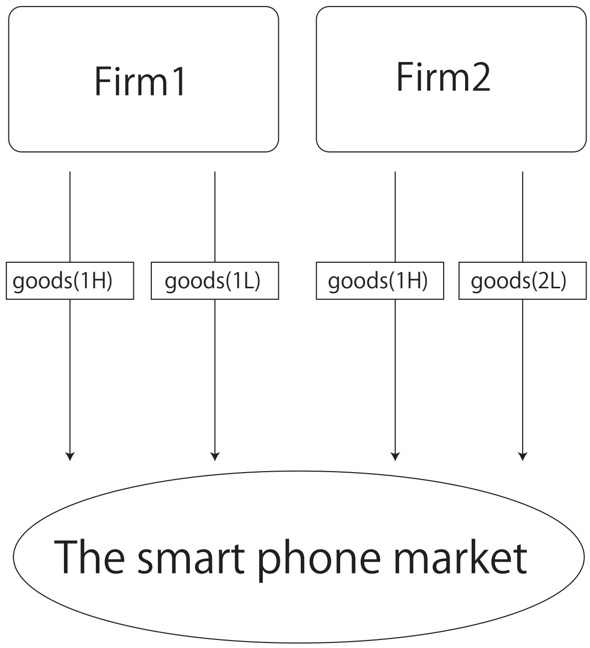
Firm1
Firm2
goods(1H)
goods(1L)
goods(1H)
goods(2L)
The smart phone market

Figure 3