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Loss Aversion, Stochastic Compensation, and Team Incentives

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Loss Aversion, Stochastic Compensation, and Team Incentives*

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Abstract

We investigate moral-hazard problems with limited liability where agents have expectation-based reference-dependent preferences. We show that stochastic compensation for low performance can be optimal. Because of loss aversion, the agents have first-order risk aversion to wage uncertainty. This causes the agents to work harder when their low performance is stochastically compensated. We also examine team incentives for credibly employing such stochastic compensation. In an optimal contract, low- and high-performance agents are equally rewarded if most agents achieve high performance. Team incentives can be optimal even when there are only two agents and the degree of loss aversion is not large.

JEL Classification Numbers: D03, D86, M12, M52

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1 Introduction

How a principal encourages an agent to work hard is one of the main themes in organizational economics. The main insight of the literature on moral-hazard problems is that the principal pays a high wage to the agent if and only if his performance is high.¹ In practice, however, firms often pay bonuses to low-performance agents as well as high-performance ones. Recent empirical studies on executive compensation show that CEOs are often paid for luck as well as their performance (Bertrand and Mullainathan 2001; Garvey and Milbourn 2006). To explain why firms sometimes reward both low- and high-performance agents, we focus on a prominent behavioral aspect, *loss aversion*: people are more sensitive to losses than to same-sized gains. We investigate a moral-hazard model with limited liability in which an agent exhibits expectation-based loss aversion à la Kőszegi and Rabin (2006, 2007). Because the agent has first-order risk aversion to wage uncertainty, the principal can reduce the agent's feeling of losses if she stochastically compensates for his low performance. This generates a new trade-off on the agent's incentive compatibility constraint, which leads to stochastic compensation as an optimal contract: an agent is always rewarded when his performance is high, and he is stochastically rewarded when his performance is low.

The agent's utility consists of intrinsic *consumption utility* and psychological *gain-loss utility* defined as the difference between his realized outcome and his reference point. The agent is loss averse in both the wage dimension and the effort-cost dimension, and his reference point is determined by his recent expectations regarding his wage and effort cost. For example, suppose that an agent expects to work hard and to receive either \$0 or \$30 with equal probabilities. Suppose that he actually works hard. In the wage dimension, his expected gain-loss utility consists of a weighted average of the following four cases with equal weights. There is no gain-loss in two cases: he expects to receive \$0 and actually receives \$0, and he expects to receive \$30 and actually receives \$30. The agent feels a loss of \$30 in the case where he expects to receive \$30 but actually receives \$0. Similarly, the agent feels a gain of \$30 in the case where he expects to receive \$0 but actually receives \$30. Because the agent is loss averse, his feeling of a \$30 loss looms larger than that of a \$30 gain. Therefore, his expected gain-loss utility for wage is negative and represents his aversion to wage uncertainty. In the effort-cost dimension, he feels neither gains nor losses because he expects to work hard and actually works hard.

To determine the agent's reference points endogenously, we assume that the agent's

¹We use male pronouns to refer to the agent and female pronouns to refer to the principal.

reference points are determined by his rational expectations and that his expectations are updated according to his chosen action before the outcome is realized. Because the agent correctly anticipates that his belief will be adapted to his chosen action, he takes this into account when he chooses his action. This solution concept is called the *choice-acclimating personal equilibrium* (CPE), which is introduced by Kőszegi and Rabin (2007). CPE is plausible when the action is determined long before the outcome is realized, and hence the agent’s belief is acclimated before he observes the actual outcome.

We first analyze a single-agent moral-hazard model with limited liability and derive the optimal contract. We highlight a key trade-off between the *standard-incentive effect* in the intrinsic utility and the *loss-reducing effect* in the gain-loss utility on the agent’s incentive compatibility constraint. On the one hand, the standard-incentive effect leads an agent to work less under stochastic compensation than under non-stochastic compensation because rewarding for the agent’s low performance weakens his incentive to work hard. On the other hand, the loss-reducing effect can lead the agent to work more under stochastic compensation because (i) the principal can reduce the agent’s expected loss when he works hard and (ii) she can increase his expected loss when he works less. Hence, regarding the gain-loss utility, the agent works more by stochastic compensation for his low performance. As a result, stochastic compensation is optimal when the loss-reducing effect outweighs the standard-incentive effect. Stochastic compensation is more likely to be adopted when the probability of success in the project is small. We also show that the stochastic compensation is optimal even when we additionally impose an individual rationality constraint as well as the limited liability constraint. Specifically, if stochastic compensation is optimal when we do not impose the individual rationality constraint, then—irrespective of the level of the agent’s reservation utility—stochastic compensation is still optimal even when we impose such a constraint. Importantly, the principal only partially compensates for the agent’s low performance in the optimal contract. This is because if she compensates for his low performance almost surely, then the standard-incentive effect dominates the loss-reducing effect.

Even when the principal would like to adopt stochastic compensation, how she can credibly commit to such a scheme is an important issue in practice. By applying our insight to multi-agent moral-hazard problems, we show that the principal adopts a team-based incentive scheme even when each agent’s probability of success in a project is independent. In the optimal contract, the principal rewards both high- and low-performance agents equally if most agents accomplish their projects; otherwise she rewards only high-performance agents.

Furthermore, such a team-based incentive scheme—based on joint performance evaluation—arises even when the principal hires only two agents and the degree of loss aversion is not large. Our result on team incentives helps explain why sometimes low-performance employees are rewarded as well as high-performance ones, especially when a company makes high profits.

The study most closely related to ours is Herweg, Müller and Weinschenk (2010), who analyze a single-agent moral-hazard model where the agent is loss averse. Their main finding is that the optimal contract is a binary bonus scheme even under a rich performance measure. They also show that even when the principal faces an implementation problem under non-stochastic wage schemes as first pointed out by Daido and Itoh (2007), the principal can still induce the agent to take the desired action if she compensates for the agent’s low performance. By imposing the individual rationality constraint but not imposing the limited liability constraint, Herweg, Müller and Weinschenk (2010) show that the principal wants to compensate for the agent’s low performance almost surely if the agent is sufficiently loss averse; otherwise, she never compensates for the agent’s low performance. Although their result is prominent, two sensitive issues arise. First, in their model the optimal compensation probability for low performance approaches one, and hence the optimal contract is not well-defined. Second, the optimal wage received by the agent for low performance goes to negative infinity. In contrast, by imposing the limited liability constraint we shed light on a new trade-off between the standard-incentive effect and the loss-reducing effect on the incentive compatibility constraint. With focusing on the trade-off, we extensively analyze the properties of stochastic compensation and derive a new insight for team incentives.

Some recent literature also applies expectation-based reference-dependent preferences to moral-hazard problems.² Daido and Itoh (2007) examine a single-agent model with limited liability and study the Pygmalion and Galatea effects as self-fulfilling prophecies. Gill and Stone (2010) analyze a rank-order tournament under agent loss aversion.³ Macera (2011) extends Kőszegi and Rabin’s (2009) dynamic loss aversion model to a repeated moral-hazard situation and studies the intertemporal allocation of incentives.

Recent empirical and experimental research finds the importance of expectation-based reference-dependent preferences. Crawford and Meng (2011) estimate cab drivers’ labor

²For the literature which applies expectation-based reference-dependent preferences to other problems, see Heidhues and Kőszegi (2008), Lange and Ratan (2010) and Herweg and Mierendorff (2013).

³In Gill and Stone (2011), they analyze a team production problem in which the output is equally shared among agents. They examine how the agents’ reference points, which are different from CPE, affect the agents’ equilibrium efforts.

supply decisions based on the model of Kőszegi and Rabin (2006), and reconcile the findings between Camerer, Babcock, Loewenstein, and Thaler (1997) and Farber (2005, 2008). Fehr and Goette (2007) conduct a randomized field experiment among bike messengers, and find that loss aversion explains well the labor-supply decisions of the bike messengers. Abeler, Falk, Goette, and Huffman (2011) design a real-effort experiment in which the subjects choose how long they work on a simple repetitive task. They confirm the validity of expectation-based reference-dependent preferences; the higher the subjects' expectations are, the longer they work and the more they earn. Gill and Prowse (2012) conduct real-effort sequential-move tournament experiments. Their results are consistent with the theoretical prediction of expectation-based reference-dependent preferences under CPE.

The rest of this paper is organized as follows. Section 2 sets up the model and explains the concept of CPE. Section 3 analyzes the optimal wage schemes in a single-agent model. Section 4 examines the multi-agent moral-hazard problems and shows that a principal may adopt team-based incentive schemes. Section 5 concludes. All proofs are provided in the Appendix.

2 The Model

Suppose the following moral-hazard model in which a risk-neutral principal hires an agent. The agent makes a binary-effort decision $a \in \{0, 1\}$ at the cost of $d \cdot a$ where $d > 0$. Actions $a = 1$ and $a = 0$ mean that the agent works and shirks, respectively. The performance of the agent is either high or low, which is denoted by $Q \in \{H, L\}$. The probability of realizing $Q = H$ is q_1 if $a = 1$ and q_0 if $a = 0$, where $0 \leq q_0 < q_1 < 1$. Let $\Delta_q \equiv q_1 - q_0$. The agent is subject to limited liability. To highlight the main results in a simple manner, we now assume away the agent's individual rationality constraint; we analyze the model with the individual rationality constraint in Section 3.4.

Wage payment schemes from the principal to the agent can depend on the performance of the agent and the outcome of a lottery. To analyze the possibility of stochastic compensation, suppose that before setting the wage levels the principal can use and commit to any kind of a finite-outcome lottery.⁴ Let $n \in \{1, \dots, N\}$ be the outcome of the reduced lottery with associated probability $p_n > 0$; for any given two lotteries, we can construct a new lottery. The wage vector for the agent can be expressed as $\mathbf{w} \equiv (w_{H1}, \dots, w_{HN}; w_{L1}, \dots, w_{LN})$ where $w_{Qn} \geq 0$ is the wage when the performance of the agent is Q and the outcome of the lottery

⁴Section 4 analyzes a multi-agent model where the principal can adopt a team-based incentive scheme in order to credibly commit to such stochastic compensation.

is n . The agent's expected wage under \mathbf{w} is represented by

$$\pi(a, \mathbf{w}) = \sum_{n=1}^N p_n [q_a w_{Hn} + (1 - q_a) w_{Ln}].$$

A key assumption of our model is that the agent's overall utility consists of intrinsic consumption utility and psychological gain-loss utility. We assume that each agent has expectation-based reference-dependent preferences à la Kőszegi and Rabin (2006, 2007). In our model, the agent has two consumption dimensions: effort cost and wage. For each consumption dimension, the agent feels psychological gain-loss by comparing his realized outcome with his reference outcomes. We assume that the agent has the same gain-loss function for each consumption dimension. For deterministic reference point cases, let the agent's reference point for his effort and his wage be \hat{a} and \hat{w} , respectively. If he actually exerts effort a and receives wage w , his overall utility is

$$w - ad + \mu(w - \hat{w}) + \mu(-ad + \hat{a}d),$$

where $\mu(\cdot)$ is a gain-loss function that satisfies the assumptions introduced by Bowman et al. (1999), which correspond to Kahneman and Tversky's (1979) value function. In what follows, we assume that $\mu(\cdot)$ is piecewise linear in order to focus on the effect of loss aversion. Then, when the consumption is x and reference point is r , we can simply define the gain-loss function as

$$\mu(x - r) = \begin{cases} \eta(x - r) & \text{if } x - r \geq 0, \\ \eta\lambda(x - r) & \text{if } x - r < 0. \end{cases}$$

where $\eta \geq 0$ represents the weight on the gain-loss payoff, and $\lambda \geq 1$ is the degree of the loss aversion.

Following Kőszegi and Rabin (2006, 2007), we assume that the reference point is determined by rational beliefs about outcomes and that the reference point itself is stochastic if the outcome is stochastic. The agent feels gain-loss by comparing each possible outcome with every reference point. For example, suppose that an agent with $\eta > 0$ and $\lambda > 1$ had been expecting to receive \$100, \$150, or \$200 with equal probabilities. If he actually receives \$150, then he feels a gain of \$50 relative to \$100, no gain-loss relative to \$150 and a psychological loss of \$50 relative to \$200. Because the loss of \$50 looms larger than the gain of \$50, his gain-loss utility is negative in this case.

Note that the agent's expected intrinsic utility is $\pi(a, \mathbf{w}) - ad$. To define the expected gain-loss utility formally, let \hat{a} be the agent's reference point for his own effort decision. That is, \hat{a} represents the agent's belief of the action he will choose. Similarly, denote \hat{w} as the

agent's reference wage based on \hat{a} . Then, the agent's expected gain-loss utility in the wage dimension is

$$\begin{aligned} \pi(a, \mathbf{w}|\hat{a}, \hat{\mathbf{w}}) \equiv & \sum_{n=1}^N \sum_{m=1}^N p_n p_m [q_a q_{\hat{a}} \mu(w_{Hn} - \hat{w}_{Hm}) + (1 - q_a) q_{\hat{a}} \mu(w_{Ln} - \hat{w}_{Hm}) \\ & + q_a (1 - q_{\hat{a}}) \mu(w_{Hn} - \hat{w}_{Lm}) + (1 - q_a) (1 - q_{\hat{a}}) \mu(w_{Ln} - \hat{w}_{Lm})], \quad (1) \end{aligned}$$

and the agent's expected gain-loss utility in the effort-cost dimension is $\mu(-ad + \hat{a}d)$. To explain Equation (1) clearly, suppose that the agent expects that the outcome of the lottery is m , but the actual outcome is n with probability $p_n p_m$. When the agent expects to succeed with probability $q_{\hat{a}}$, he compares his reference wage \hat{w}_{Hm} to his actual wage for success w_{Hn} (resp. for failure w_{Ln}) with probability $q_a q_{\hat{a}}$ (resp. $(1 - q_a) q_{\hat{a}}$). Conversely, when the agent expects to fail with probability $1 - q_{\hat{a}}$, he compares his reference wage \hat{w}_{Lm} to his actual wage w_{Hn} (resp. w_{Ln}) with probability $q_a (1 - q_{\hat{a}})$ (resp. $(1 - q_a) (1 - q_{\hat{a}})$). The agent's expected overall utility is denoted by

$$U(a, \mathbf{w}|\hat{a}, \hat{\mathbf{w}}) \equiv \pi(a, \mathbf{w}) - ad + \pi(a, \mathbf{w}|\hat{a}, \hat{\mathbf{w}}) + \mu(-ad + \hat{a}d).$$

We derive the optimal wage schemes according to the equilibrium concept defined by Kőszegi and Rabin (2007): the *choice-acclimating personal equilibrium* (CPE). Under CPE, the agent's reference point is acclimated to the action taken by him. This is plausible when the action is determined long before the outcome and payment occur, and hence he updates his belief to the action he took before the outcome is realized. Because the agent knows that his belief will change on the basis of his chosen action before the outcome and payment occur, he takes this change into account when he decides what action to take. Therefore, each agent's action itself determines his reference point under CPE, and the condition to choose working under CPE is represented by

$$U(1, \mathbf{w}|1, \mathbf{w}) \geq U(0, \mathbf{w}|0, \mathbf{w}). \quad (\text{CPE-IC})$$

(CPE-IC) is the incentive compatibility constraint under CPE: the agent's overall utility when his reference action is 1 and he actually chooses 1 is greater than or equal to that when his reference is 0 and he actually chooses 0.

The timing is as follows:

1. The principal picks up a lottery that can be tied to a wage payment scheme.
2. The principal offers a wage payment scheme subject to the limited liability.

3. The agent chooses his action.
4. The performance signal and the outcome of the lottery are realized, and the wage is paid.

3 The Optimal Wage Scheme

This section analyzes the optimal wage schemes in the model described in Section 2. Note that if the principal wants to implement low effort, then setting $w_{Qn} = 0$ for all Q and n is obviously the optimal wage scheme even under agent loss aversion. Therefore, throughout this paper, we assume that the project is so valuable that the principal wants to make the agent work. In what follows, we set $w_{Hn} \geq w_{Hn'}$ for any $n \geq n'$ without loss of generality.

3.1 The Optimal Contract without Loss Aversion

First, we examine a benchmark case in which an agent is not loss-averse before analyzing the optimal wage scheme under agent loss aversion. Given a lottery, the principal's problem is to minimize her expected payment given that the agent works as follows:

$$\min_{\mathbf{w}} \sum_{n=1}^N p_n [q_1 w_{Hn} + (1 - q_1) w_{Ln}]$$

subject to

$$\sum_{n=1}^N p_n (w_{Hn} - w_{Ln}) \geq \frac{d}{\Delta_q}, \tag{IC}$$

$$w_{Qn} \geq 0 \text{ for all } Q, n, \tag{LL}$$

where (IC) is the incentive compatibility constraint to induce the agents to exert high effort, and (LL) is the limited liability constraint. Because the left hand side of (IC) is decreasing in w_{Ln} , the optimal contract scheme satisfies $w_{Ln} = 0$ for all n . To minimize the expected payment, the principal reduces w_{Hn} to hold (IC) with equality. As a result, any wage scheme that satisfies $w_{Ln} = 0$ for any n and (IC) with equality is optimal.

To confirm the robustness of the result, we also examine an alternative case where an agent is risk averse. Suppose that the agent has a utility function $m(\cdot)$ in the wage dimension such that $m(0) = 0$, $m(\cdot)$ is twice differentiable, $m'(\cdot) > 0$ and $m''(\cdot) < 0$. Then, (IC) is

replaced by:

$$\sum_{n=1}^N p_n (m(w_{Hn}) - m(w_{Ln})) \geq \frac{d}{\Delta_q}. \quad (\text{IC}')$$

Note that the left hand side of (IC') is still decreasing in w_{Ln} . Because $m(\cdot)$ is strictly concave, w_{Hn} is constant across n in the optimal contract. Hence, the unique optimal wage scheme specifies that $w_{Hn} = m^{-1}\left(\frac{d}{\Delta_q}\right) > 0$ and $w_{Ln} = 0$ for all n . These results are summarized as follows:

Proposition 1. Suppose an agent is not loss averse ($\lambda = 1$). Then, a non-stochastic compensation scheme is optimal.

Proposition 1 demonstrates that a non-stochastic compensation scheme ($N = 1$) is always optimal under standard concave utility.

3.2 The Optimal Contract with Loss Aversion

Now we examine the optimal wage schemes under agent loss-aversion. We first show that if the principal can use only non-stochastic compensation ($N = 1$), then high effort may not be implementable. In this case, $\pi(a, \mathbf{w} | a, \mathbf{w}) = -q_a(1 - q_a)\eta(\lambda - 1)|w_H - w_L|$. Note that $-q_1(1 - q_1) + q_0(1 - q_0) = -\Delta_q(1 - q_0 - q_1)$. Hence, the principal's problem is represented by

$$\begin{aligned} \min_{w_H, w_L \geq 0} \quad & q_1 w_H + (1 - q_1) w_L \\ \text{s.t.} \quad & w_H - w_L - (1 - q_0 - q_1)\eta(\lambda - 1)|w_H - w_L| \geq \frac{d}{\Delta_q}, \end{aligned} \quad (2)$$

where (2) is the CPE-IC condition in this case. Notice that if $(1 - q_0 - q_1)\eta(\lambda - 1) \geq 1$, then (2) is never satisfied.

Proposition 2. Suppose $(1 - q_0 - q_1)\eta(\lambda - 1) \geq 1$. Then, high effort ($a = 1$) is not implementable by any non-stochastic compensation scheme.

The implementation problem in Proposition 2 is first pointed out by Daido and Itoh (2007) and is examined by Herweg, Müller and Weinschenk (2010). Intuitively, when the degree of loss aversion is not small and the probability of success in the project is small, the agent is more likely to suffer from losses because it is difficult to achieve his project even if

he works hard. In order to make the agent work hard, the principal needs to alleviate his feeling of losses; but she cannot do so without employing stochastic compensation.⁵

Even when high effort is not implementable by non-stochastic incentive schemes, the principal can induce the agent to work hard by stochastically compensating for the agent's failure. In the following analysis, we assume away negative-bonus wage schemes:

Assumption 1. $w_{Hn} \geq w_{Ln}$ for all n .

Assumption 1 holds for most wage schemes in practice. Notice that Assumption 1 is justified if each agent can secretly dispose his output: the agent's performance would deteriorate if he prefers reporting low performance to reporting high performance.⁶

The next lemma shows that (i) the principal can always implement high effort, (ii) the agent receives a positive wage whenever he accomplishes his project, and (iii) the principal offers a binary payment scheme generically.

Lemma 1. (i) High effort ($a = 1$) is implementable.

(ii) In the optimal wage scheme, $w_{Hn} = w > 0$ for all n .

(iii) A binary payment scheme is optimal: $w_{Ln} \in \{w, w_L\}$ for all n . Generically, it is the unique optimal wage scheme.

Lemma 1 (i) shows that, as Herweg, Müller and Weinschenk (2010) point out, the principal can always implement high effort by stochastic compensation. In addition, Lemma 1 (ii) shows that the principal sets a positive constant wage when the agent accomplishes the project. Intuitively, because the agent dislikes wage uncertainty, the principal can always encourage him to work hard by paying a positive constant wage when the agent succeeds. Lemma 1 (iii) comes from the fact that the principal's problem becomes a linear programming problem owing to the assumptions of linear consumption utility and piecewise-linear gain-loss utility. Then, the problem has a unique solution at an extreme point of the constraint set generically.⁷ Note that even when the agent's performance is low, he may stochastically receive the positive wage. In addition, the proof of Lemma 1 (iii) shows that $w_L = 0$ holds in the optimal contract when we do not impose the individual rationality constraint. In-

⁵Relatedly, Spiegel (2012) shows that if a principal does not face moral-hazard problems (without the incentive compatibility constraint), in the optimal contract she randomizes the transfer to an agent in order to relax the individual rationality constraint.

⁶See Innes (1990) and Matthews (2001) for the detailed discussion.

⁷This method is developed by Herweg, Müller and Weinschenk (2010). In Section 3.5, we discuss the case of which the agent's consumption utility is concave.

tuitively, if $w_L > 0$ then the principal can reduce w and w_L by the same amount without violating constraints.⁸

Let α denote the probability of which the agent gets a positive wage when his performance is low: $\alpha \equiv \text{Prob}(w_{Q_n} = w | Q = L)$. The principal compensates for the agent's failure with this probability. We refer to α as the compensation rate. By Lemma 1, the principal's problem can be reduced to choose a wage $w > 0$ and a compensation probability $\alpha \in [0, 1]$ so as to minimize her expected payment. In this setting, $\alpha = 0$ means that the agent's wage depends only on his own outcome. On the other hand, $\alpha = 1$ means full compensation: the principal offers a flat-wage contract.

The principal's problem becomes:

$$\begin{aligned} & \min_{w>0, \alpha \in [0,1]} [q_1 + \alpha(1 - q_1)]w \\ \text{s.t. } & \underbrace{(1 - \alpha)}_{\text{(SI)}} w + \underbrace{(1 - \alpha)[1 - (1 - \alpha)(2 - q_0 - q_1)]\eta(\lambda - 1)}_{\text{(LR)}} w \geq \frac{d}{\Delta_q}. \end{aligned} \quad (\text{CPE-IC}')$$

(CPE-IC') exhibits a sharp trade-off between *standard-incentive effect* (SI) and the *loss-reducing effect* (LR). (SI) is derived from the intrinsic utility: increasing the compensation rate α reduces the incentive to work hard. (LR) is derived from the gain-loss utility by comparing a positive wage of w with a base wage of zero. This means increasing the compensation rate α encourages the agent to work hard because it reduces wage uncertainty when working, provided α is not too large.⁹ Notice that (SI) does not depend both on η and λ , while the effect of (LR) increases as η or λ increases. Hence, in contrast to the concave utility case, if the agent is loss averse, he is more likely to work hard when his low performance is stochastically compensated ($\alpha > 0$) than when low performance always leads to a low wage ($\alpha = 0$). Furthermore, full compensation ($\alpha = 1$) is never optimal because it does not satisfy (CPE-IC'). Also, if $\alpha = 0$, then (CPE-IC') is equivalent to (2) with $w_L = 0$.

Because (CPE-IC') holds with equality in the optimal contract, the optimal amount of w is determined as a function of α :

$$w(\alpha) \equiv \frac{d}{\Delta_q(1 - \alpha)\{1 + [1 - (1 - \alpha)(2 - q_1 - q_0)]\eta(\lambda - 1)\}}.$$

By plugging $w(\alpha)$ into the objective function and solving it, we characterize the optimal wage schemes as follows:

⁸Note that w_L in the optimal contract may be positive if we additionally impose the individual rationality constraint. See Section 3.4.

⁹Note that (LR) is increasing in α if $\alpha < \frac{3-2q_1-2q_0}{4-2q_1-2q_0}$.

Proposition 3. The optimal wage scheme under loss aversion is to pay a wage $w(\alpha^*) > 0$ with probability one when an agent's performance is high and with probability α^* when his performance is low. The optimal compensation rate α^* is determined by:

$$\alpha^* = \begin{cases} 0 & \text{if } \frac{1}{\eta(\lambda-1)} \geq (2 - q_0 - q_1)(1 + q_1) - 1, \\ \frac{1}{1-q_1} \left(\sqrt{1 - \left[1 + \frac{1}{\eta(\lambda-1)}\right] \cdot \frac{1-q_1}{2-q_0-q_1}} - q_1 \right) & \text{if } \frac{1}{\eta(\lambda-1)} < (2 - q_0 - q_1)(1 + q_1) - 1. \end{cases}$$

Proposition 3 shows that even when the agent fails in his project the principal may pay the same amount of wage as the agent succeeds. If the loss-reducing effect outweighs the standard-incentive effect in (CPE-IC'), then the stochastic compensation becomes optimal. The optimal compensation probability is increasing in $\eta(\lambda - 1)$: the principal is more likely to adopt the stochastic compensation as the agent is more loss averse. Also, the principal increases the compensation rate, as it is harder for the agent to accomplish his project when he shirks (q_0 decreases).

Note that two of our results—that binary payment schemes are optimal and that compensating for the failure can be optimal if λ is not small—come from the insights of Herweg, Müller and Weinschenk (2010). They show that without the limited liability constraint and with the individual rationality constraint, the optimal compensation rate α^* is either zero (no compensation for the failure) or arbitrarily close to one (full compensation for the failure). The logic of their result is that the principal can decrease the agent's expected loss by increasing the compensation rate and decreasing the base wage, without violating both the incentive compatibility and individual rationality constraints. Although their logic is important, two sensitive issues arise. First, the optimal compensation rate in their model approaches one, and hence the optimal contract is not well-defined. Second, more importantly, a base wage goes to negative infinity in their optimal contract. In contrast, we impose the limited liability constraints to focus on the effect of loss aversion on (CPE-IC'), and shed light on the trade-off between the standard-incentive effect and loss-reducing effect. By this trade-off, the principal adopts a stochastic compensation even when the individual rationality constraint does not bind.¹⁰ This is because stochastic compensation for the failure encourages the loss-averse agent to work hard even though it makes the agents more likely to shirk if they are loss-neutral.

¹⁰Herweg, Müller and Weinschenk (2010) mention that if the agent is subjected to limited liability, then the optimal compensation probability may be well defined.

3.3 The Effect of Loss Aversion on the Principal's Payment

This subsection discusses how the degree of loss aversion affects the principal's expected payment in the optimal contract. Notice that if $\frac{1}{\eta(\lambda-1)} \geq (2 - q_0 - q_1)(1 + q_1) - 1$, then the optimal compensation rate is $\alpha^* = 0$, and hence the principal's expected payment is:

$$W_0 \equiv \frac{q_1 d}{\Delta_q [1 - (1 - q_1 - q_0)\eta(\lambda - 1)]}.$$

If $q_1 + q_0 < 1$, the expected payment increases as the degree of loss aversion increases. More interestingly, if $q_1 + q_0 > 1$, the expected payment decreases as the degree of loss aversion increases so that the principal may decide to hire a loss-averse agent rather than a loss-neutral agent. The comparative statics of the latter case sharply contrasts with that of the concave-utility case. The intuition is simple: if the probability of success is high, then a loss-averse agent works harder than a loss-neutral agent because a loss-averse agent has a much stronger incentive to minimize his expected loss.

Similarly, if $\frac{1}{\eta(\lambda-1)} < (2 - q_0 - q_1)(1 + q_1) - 1$, then by the envelope theorem the principal's expected payment is decreasing in $\eta(\lambda - 1)$ if and only if:

$$1 - (1 - \alpha^*)(2 - q_1 - q_0) > 0.$$

As in the case $\alpha^* = 0$, if the probabilities of success in the project are not small, then the expected payment can decrease as the degree of loss aversion increases.

3.4 The Optimal Wage Scheme with the Individual Rationality Constraint

This subsection examines the robustness of Proposition 3 by additionally imposing the individual rationality (IR) constraint. Suppose that in addition to the above setting, each agent does not accept the contract if his overall utility is less than his reservation utility $\bar{u} \in \mathbb{R}$. In the Appendix, we show that Lemma 1 holds even when the IR constraint is binding. Particularly, a binary payment scheme is still optimal in the optimal contract because we can reduce the principal's problem to a linear programming. However, w_L may be not equal to zero in this case. The CPE constraint also binds in the optimal contract; otherwise, the principal offers a flat-wage contract. As a result, IR constraint can be written as:

$$[q_1 + \alpha(1 - q_1)] [1 - (1 - \alpha)(1 - q_1)\eta(\lambda - 1)] w(\alpha) + w_L \geq d. \quad (\text{IR})$$

In the proof, we show that there exists $\alpha \in [0, 1)$ that satisfies (IR) with equality. Define the value of α that satisfies (IR) with equality by α^{IR} if α^{IR} is unique. If it is not unique,

then define the value of α that minimizes the principal's expected payment among such values by α^{IR} . In the proof, we constructively show how to find this value. We characterize the optimal wage schemes as follows:

Proposition 4. In the optimal wage scheme, a principal pays $w(\alpha^{**}) > 0$ with probability one when an agent's performance is high, whereas she pays $w(\alpha^{**}) > 0$ with probability α^{**} and $w_L < w(\alpha^{**})$ with probability $1 - \alpha^{**}$ when his performance is low. The optimal compensation rate α^{**} is determined by $\alpha^{**} = \max\{\alpha^*, \alpha^{IR}\}$. Also, $w_L = 0$ if $\alpha^{**} > 0$.

Proposition 4 shows that even when we take the IR constraint into account, the main properties of the optimal contract in Proposition 3 are still valid: the agent always receives a high wage if his performance is high, whereas he stochastically receives the high wage if his performance is low. This result contrasts with that of Herweg, Müller and Weinschenk (2010): when $\eta(\lambda - 1) > 1$, the agent receives a high wage almost surely even if his performance is low; otherwise, the agent never receives a high wage if his performance is low. In addition, we show that when the principal uses a stochastic compensation scheme ($\alpha^{**} > 0$), the LL constraint always binds ($w_L = 0$) irrespective of the level of the agent's reservation wage. This result highlights the importance of taking the LL constraint into account under agent loss aversion.

When $\bar{u} = 0$, we obtain the following result:

Corollary 1. Suppose $\bar{u} = 0$. Then, $w_L = 0$ for all parameters. The optimal compensation rate α^{**} is determined by

$$\alpha^{**} = \max\left\{\alpha^*, 1 - \frac{1}{(1 - q_0)\eta(\lambda - 1)}\right\}.$$

Figure 1 describes the optimal compensation rates α^{**} when $q_0 = 0.1$. When the degree of loss aversion is modest such that $\eta(\lambda - 1) = 1$, the IR constraint never binds. Then, the trade-off between the standard-incentive effect and the loss-reducing effect on the CPE-IC constraint determines the optimal compensation rate. The principal adopts a stochastic compensation scheme if q_1 takes middle values (the thick line in Figure 2). As the degree of loss aversion increases, however, the IR constraint is more likely to bind in the optimal contract. When $\eta(\lambda - 1) = 1.5$, the principal adopts a stochastic compensation scheme for any q_1 , and the IR constraint binds in the optimal contract if and only if $q_1 \geq 0.55$ (the thin line in Figure 1). When the degree of loss aversion becomes large such that $\eta(\lambda - 1) = 2$,

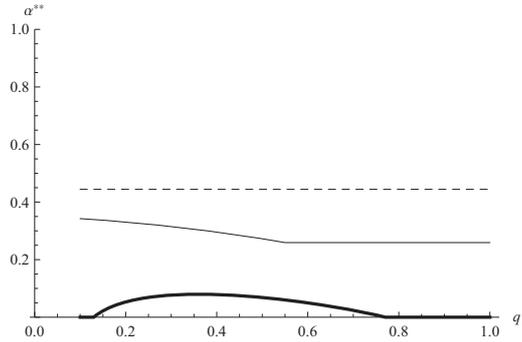


Figure 1: The optimal compensation rates when $\bar{u} = 0$ and $q_0 = 0.1$. Thick line when $\eta(\lambda - 1) = 1$, Thin line when $\eta(\lambda - 1) = 1.5$, Dashed line when $\eta(\lambda - 1) = 2$.

the IR constraint always binds, and hence the optimal compensation rate does not depend on q_1 (the dashed line in Figure 1). In summary, the IR constraint becomes relevant for determining the optimal wage scheme if the degree of loss aversion is large.

Finally, Proposition 4 has an implication on executive compensation. As Murphy (1999) summarizes, most stock options do not adjust for market-wide common shocks, and there is little empirical evidence of relative performance evaluation in executive compensation. Bertrand and Mullainathan (2001) find that executives are often paid for luck.¹¹ Importantly, Garvey and Milbourn (2006) highlight that executives are rewarded for good luck but do not suffer from bad luck. Proposition 4 implies if an executive is loss averse, then his wage is not sensitive to his performance when the profits are high; on the other hand, the executive's wage depends on his performance when the profits are low. This result gives an insight into why stock options are widely used as executive compensation even though the stock options do not remove industry-wide shocks.

3.5 Loss Aversion and Concave Consumption Utility

This subsection briefly discusses the case in which the agent has concave consumption utility as well as gain-loss utility. Suppose that the agent is both risk and loss averse. Even in this case, stochastic compensation can still reduce the agent's expected losses and encourage him to work hard by stochastically compensating for his failure. However, the optimal wage scheme may not be binary under concave consumption utility. When the agent has linear consumption utility, the principal pays the same amount of wage for low performance

¹¹Oyer (2004) shows such "pay for luck" can be an optimal contract when a manager's reservation utility is market-sensitive.

stochastically as that for high performance in order to reduce the agent’s expected loss. When the agent has concave consumption utility, such a wage scheme may be too costly for the principal. In this case, the wage when the agent fails in the project and when the principal compensates for it may be less than that when he succeeds in the project. If the degree of loss aversion is more crucial than the concavity of the intrinsic utility, the principal still adopts a binary payment scheme to reduce the agent’s expected loss in gain-loss utility.¹²

4 Multi-Agent Moral Hazard and Team Incentives

This section examines team-based incentive schemes as an application of stochastic compensation. We first discuss when the principal ties her employees’ wages to the company’s profits. We then analyze the optimal wage schemes when the principal can hire only two agents and ties an agent’s wage to the other agent’s performance.

4.1 The Many-Agent Moral-Hazard Model

So far, we have assumed that the principal can credibly commit to any lottery. How the principal can commit to such stochastic compensation, however, is an important issue in practice. For example, if the lottery used for the stochastic compensation is not verifiable, then the principal cannot commit to pay according to the result of the lottery. This casts doubt on the credibility of stochastic compensation schemes. Indeed, such stochastic compensation does not seem prevalent in practice.¹³

Even in this case, the principal can credibly commit to team-based incentive schemes: an agent’s wage depends not only on his own performance but also others’ performance. Suppose that the principal hires sufficiently many identical agents. Then, by the same derivation as in Proposition 3, in an optimal contract, the principal pays a high wage to both high- and low-performance agents if most agents (more than a fraction α^* of all agents) accomplish their projects; otherwise, she pays the high wage only to high-performance agents. This may help explain why firms often use team-based incentive schemes (Chiappori and Salanié 2003; Lazear and Oyer 2012). In particular, our results can explain why companies sometime pay high wages not only to high-performance employees but also to low-performance employees when the companies earn high profits.

¹²Herweg, Müller and Weinschenk (2010) extensively discuss this issue.

¹³Herweg, Müller and Weinschenk (2010) also doubt the plausibility of a stochastic compensation: “[r]estricting the principal to offer nonstochastic wage payments is standard in the principal-agent literature and also in accordance with observed practice.”

In addition, if we take into account additional managerial aspects, then team-based incentive schemes can be better than stochastic compensation in a single-agent case. For example, suppose the principal faces a credit constraint. In a single-agent case, if the agent fails in the project, then the principal may not be able to pay a high wage to the agent. On the other hand, if the principal adopts a team-based incentive scheme, then she needs to pay high wages to low-performance agents only when most agents succeed in their projects. Because of the profits from other agents' high performance, the principal can pay high wages to low-performance agents and hence she can adopt team incentives even under the credit constraint.

4.2 The Two-Agent Moral-Hazard Model

In Section 4.1, we discussed how team incentives can be effective to implement stochastic compensation. To see the intuitions and properties of such team incentives more precisely, we examine a model in which the principal hires only two loss-averse agents instead of using the outside lottery. The characteristic of a wage scheme is determined by how each agent's wage is related to his colleague's performance. For agent i , a wage scheme $\mathbf{w}^i = (w_{HH}^i, w_{HL}^i, w_{LH}^i, w_{LL}^i)$ exhibits joint performance evaluation (JPE) if $(w_{HH}^i, w_{LH}^i) > (w_{HL}^i, w_{LL}^i)$: given an agent's performance, his wage increases in his colleague's performance.¹⁴ A wage scheme exhibits relative performance evaluation (RPE) if $(w_{HH}^i, w_{LH}^i) < (w_{HL}^i, w_{LL}^i)$: given an agent's performance, his wage decreases in his colleague's performance. Finally, if $(w_{HH}^i, w_{LH}^i) = (w_{HL}^i, w_{LL}^i)$, a wage scheme exhibits independent performance evaluation (IPE): an agent's wage does not depend on his colleague's performance.

In the following analysis, we assume away negative-bonus wage schemes as in Assumption 1.

Assumption 2. $w_{HH} \geq w_{LH}$ and $w_{HL} \geq w_{LL}$.

Assumption 2 holds for virtually any wage schemes in practice. Moreover, Assumption 2 is justified if each agent can secretly dispose his output, as discussed in Section 3.¹⁵

¹⁴The inequality means weak inequality for each component and strict inequality for at least one component.

¹⁵We can also characterize the optimal wage scheme under CPE without imposing Assumption 2. The optimal contract does not change when $\eta(\lambda - 1) \leq 1$, but unlike in Herweg, Müller and Weinschenk (2010) our optimal contract is well-defined even when $\eta(\lambda - 1) > 1$. If Assumption 2 is not imposed and the degree of loss aversion is sufficiently large, then the negative bonuses can be adopted if q_0 is small and if q_1 is either very large or very small. The full characterization is available upon request.

By Assumption 2, the smallest possible wage is either w_{LH} or w_{LL} . First, if the smallest wage is strictly positive, then the principal can reduce the payment without changing the CPE-IC constraint by decreasing the same amount of money from each wage. Thus, the smallest wage must be zero in the optimal contract. Second, due to loss aversion the agent is less willing to work hard if he faces wage uncertainty when he succeeds in his own project. Hence if $w_{HH} \neq w_{HL}$, then the principal can encourage him to work hard by reducing the wage variation when he succeeds. Therefore, we have the following properties of the optimal wage scheme.

Lemma 2. The optimal wage schemes under CPE satisfy (i) $\min\{w_{LH}, w_{LL}\} = 0$ and (ii) $w_{HH} = w_{HL}$.

Lemma 2 implies that team incentives in our model would have different forms from those in the existing literature like Itoh (1991), Che and Yoo (2001), and Kvaløy and Olsen (2006), which show the optimality of team incentives. These studies find that the agent's wage is zero regardless of his colleague's outcome when he fails ($w_{LH} = w_{LL} = 0$), whereas his wage may depend on his colleague's outcome when he succeeds. In contrast, Lemma 2 implies that under loss aversion team incentives become only relevant when the agent fails in his own project: either w_{LH} or w_{LL} can be positive whereas $w_{HH} = w_{HL}$. In what follows, we denote by $w_{HH} = w_{HL} \equiv w$.

By Lemma 2 and Assumption 2, we have two possible types of wage schemes: (i) $w \geq w_{LH} \geq w_{LL} = 0$ and (ii) $w \geq w_{LL} \geq w_{LH} = 0$. We examine the optimal wage scheme in each case. Then, we compare these two cases and derive the optimal wage scheme.

First, we examine case (i): $w \geq w_{LH} \geq w_{LL} = 0$. The principal's problem is to minimize her expected payment given that each agent works:

$$\min_{w, w_{LH}} q_1 w + q_1(1 - q_1)w_{LH}$$

subject to

$$\begin{aligned} & q_1 w + q_1(1 - q_1)w_{LH} - d - \eta(\lambda - 1) [q_1(1 - q_1)^2 w + q_1(1 - q_1)^3 w_{LH} + q_1^2(1 - q_1)(w - w_{LH})] \\ \geq & q_0 w + q_1(1 - q_0)w_{LH} - \eta(\lambda - 1) [q_0(1 - q_0)(1 - q_1)w + q_1(1 - q_0)^2(1 - q_1)w_{LH} \\ & + q_0 q_1(1 - q_0)(w - w_{LH})], \end{aligned} \tag{CPEJ}$$

$$w \geq 0 \quad \text{and} \quad w_{LH} \in [0, w], \tag{LLJ}$$

where (CPEJ) is the CPE-IC constraint and (LLJ) is the limited liability constraint in this case.

Note that (CPEJ) can be rewritten as:

$$[1 - (1 - q_1 - q_0)\eta(\lambda - 1)] w + \left[\underbrace{-q_1}_{\text{(SI)}} + \underbrace{q_1(1 - q_1 - q_0)\eta(\lambda - 1)}_{\text{(LR1)}} + \underbrace{q_1(1 - q_1)(2 - q_1 - q_0)\eta(\lambda - 1)}_{\text{(LR2)}} \right] w_{LH} \geq \frac{d}{\Delta_q}.$$

There are three effects for increasing w_{LH} . We call the effect of (SI) the *standard-incentive effect*, and the effects of (LR1) and (LR2) the *loss-reducing effect*. (SI) comes from the intrinsic utility: increasing w_{LH} reduces the incentive to work hard. (LR1) comes from the gain-loss utility comparing w_{LH} with w . This means increasing w_{LH} encourages the agent to work hard because it reduces wage uncertainty when working, provided $q_1 + q_0 < 1$. (LR2) comes from the gain-loss utility comparing w_{LH} with w_{LL} . This implies that increasing w_{LH} encourages the agent to work hard because it adds wage uncertainty when shirking. Notice that (SI) does not depend on η , λ , while the effects of (LR1) and (LR2) increase as η or λ increases. Hence, in contrast to a classical concave utility, if the agent is sufficiently loss averse he is more likely to work hard when his wage depends on the other's performance ($w_{LH} > w_{LL} = 0$) than when his wage does not ($w_{LH} = w_{LL} = 0$). The loss-reducing effect becomes more crucial than the standard-incentive effect if $\Omega_J \equiv [1 - q_1 - q_0 + q_1(1 - q_1)(2 - q_1 - q_0)]\eta(\lambda - 1) \geq 1$; hence the principal prefers $w_{LH} > 0$ rather than $w_{LH} = 0$ if and only if $\Omega_J \geq 1$.

The analysis and the trade-off between the standard-incentive effect and the loss-reducing effect are similar in the case of (ii): $w \geq w_{LL} \geq w_{LH} = 0$. The principal prefers $w_{LL} > 0$ rather than $w_{LL} = 0$ if and only if $\Omega_R \equiv [1 - q_1 - q_0 + q_1^2(2 - q_1 - q_0)]\eta(\lambda - 1) \geq 1$.

Finally, we compare two cases to derive the optimal wage schemes. If both $\Omega_J < 1$ and $\Omega_R < 1$ hold, then IPE is optimal. Otherwise, team incentives become optimal and the comparison of expected payments determines either JPE or RPE is optimal. By comparing the expected payments under JPE with those under RPE, the cut-off point is given that $\Omega_{JR} \equiv [(1 - q_1 - q_0) - q_1(1 - q_1)^2(2 - q_1 - q_0)]\eta(\lambda - 1)$ is equal to one. When $\Omega_J \geq \Omega_R$, JPE is optimal if $\Omega_{JR} \leq 1$; otherwise RPE is optimal. When $\Omega_J < \Omega_R$, JPE is optimal if $\Omega_{JR} \geq 1$; otherwise RPE is optimal. As a result, we have a full characterization of optimal wage schemes:

Proposition 5. In a two-agent model, the optimal wage scheme is:

1. $\mathbf{w}^I = (w^I, w^I, 0, 0)$ where $w^I = \frac{d}{\Delta_q[1 - (1 - q_1 - q_0)\eta(\lambda - 1)]}$ if both $\Omega_J < 1$ and $\Omega_R < 1$ hold.

2. $\mathbf{w}^J = (w^J, w^J, w^J, 0)$ where $w^J \equiv \frac{d}{\Delta_q(1-q_1)\{1-[1-q_1-q_0-q_1(2-q_1-q_0)]\eta(\lambda-1)\}}$ if one of the following conditions holds; (i) $q_1 \leq \frac{1}{2}$ and $\Omega_R < 1 \leq \Omega_J$, (ii) $q_1 \leq \frac{1}{2}$ and $\Omega_{JR} \leq 1 \leq \Omega_R \leq \Omega_J$, or (iii) $q_1 > \frac{1}{2}$ and $1 \leq \Omega_{JR} < \Omega_J < \Omega_R$.
3. $\mathbf{w}^R = (w^R, w^R, 0, w^R)$ where $w^R = \frac{d}{\Delta_q q_1 \{1-[1-q_1-q_0-(1-q_1)(2-q_1-q_0)]\eta(\lambda-1)\}}$ if one of the following conditions holds; (i) $q_1 \leq \frac{1}{2}$ and $1 < \Omega_{JR} < \Omega_R \leq \Omega_J$, (ii) $q_1 > \frac{1}{2}$ and $\Omega_J < 1 \leq \Omega_R$, or (iii) $q_1 > \frac{1}{2}$ and $\Omega_{JR} < 1 \leq \Omega_J < \Omega_R$.

As we described above, the optimal wage schemes depend on the trade-off between the standard-incentive effect and the loss-reducing effect. In the intrinsic utility, the agent is less willing to work under team incentives than under IPE because he receives same wage with positive probability even when he fails. In the gain-loss utility, however, he is more willing to work under team incentives than under IPE because it reduces his wage uncertainty when working and increases his wage uncertainty when shirking. Proposition 5 provides the following insights on team incentives. First, team incentives become optimal only when $q_0 < 0.648$. When q_0 is large, the agent is very likely to succeed in his project even if he shirks. In other words, his expected loss becomes small even if he shirks. Then, compensating for his failure by team incentives is not optimal for the principal because the standard-incentive effect becomes more crucial than the loss-reducing effect. Consequently, team incentives are not optimal when q_0 is large. Second, when the degree of loss aversion is moderate, i.e. $\eta(\lambda - 1) \leq 1$, the optimal wage scheme exhibits either IPE or JPE.¹⁶ As a typical example, Figure 1 represents the optimal contracts when the degree of loss aversion is moderate such that $\eta = 1$ and $\lambda = 2$. As q_0 and q_1 decrease, the agent's wage uncertainty under working compared to under shirking becomes large, and the agents are less likely to work hard under IPE. Then, the principal's incentive to compensate for the agents' failure increases. As a result, JPE becomes optimal if q_1 is small.

The result that JPE can be optimal even when $\eta(\lambda - 1) \leq 1$ and the principal can hire only two agents is worth emphasizing.¹⁷ This result means that even when the agent does not have large loss-aversion sensitivities and the principal can induce their efforts under IPE, adopting JPE may be still better than IPE. This is because we highlight the loss-reducing effects of the CPE-IC constraint on the optimal wage schemes by assuming the limited

¹⁶Some theoretical literature which analyzes reference-dependent preferences imposes $\eta(\lambda - 1) \leq 1$ as an assumption. See, for example, Herweg, Müller and Weinschenk (2010) or Herweg and Mierendorff (2013).

¹⁷The condition $\eta(\lambda - 1) \leq 1$ corresponds with the “no dominance of gain-loss utility” assumption in Herweg, Müller and Weinschenk (2010). In their Proposition 7, they show that if the assumption is satisfied, then the stochastic compensation for the agent's own failure is not optimal.

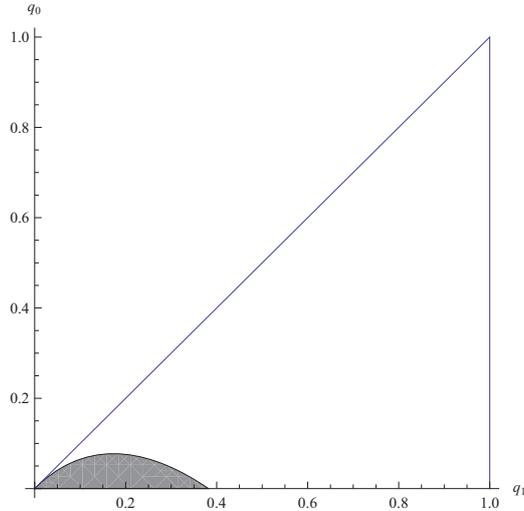


Figure 2: The optimal wage schemes when $\eta = 1$ and $\lambda = 2$. The region of each contract scheme which is optimal under CPE is shown by: IPE=White, JPE=Gray.

liability constraint, and the effects can outweigh the standard-incentive effect even when the degree of loss aversion is moderate.

Last but not least, notice that this model does not shed light on the aspects which constitute team production; our model has no common noise, no production externalities, no activities among agents such as help, sabotage or mutual monitoring. In this sense, this model differs from the existing literature on team incentives. However, our results indicate that even if we do not explicitly incorporate such aspects of team production, forming teams and introducing team incentives may be beneficial for managers. It helps to understand why teams and team incentives are ubiquitous even when some workplaces do not seem to have the above aspects of team production.

4.3 JPE as a Team Equilibrium

So far, we have considered the CPE-IC condition in which $(a_i, a_j) = (1, 1)$ is a Nash equilibrium for the agents. When the agents could communicate before choosing their actions, however, it would be possible to act coordinately and jointly deviate from $(a_i, a_j) = (1, 1)$. We now examine whether $(a_i, a_j) = (1, 1)$ yields a higher joint utility in teams rather than other effort pairs. If $(a_i, a_j) = (1, 1)$ yields the highest joint payoffs, the agents have no incentives to jointly deviate from $(a_i, a_j) = (1, 1)$.

Following Che and Yoo (2001), we call an action pair (a_i, a_j) a *team equilibrium* if it

attains the highest joint utility in team under the wage scheme $\mathbf{w}^J = (w^J, w^J, w^J, 0)$ in Proposition 5. The next proposition states that $(a_i, a_j) = (1, 1)$ is a team equilibrium if the condition

$$1 - [1 - q_1 - q_0 - q_0(2 - q_1 - q_0)]\eta(\lambda - 1) > 0 \quad (3)$$

holds.

Proposition 6. Suppose (3) is satisfied. Then, $(a_i, a_j) = (1, 1)$ is a unique team equilibrium.

Proposition 6 implies that even though $(a_i, a_j) = (0, 0)$ can also be a Nash equilibrium, it gives lower total utility to the agents and hence is less plausible than $(a_i, a_j) = (1, 1)$. In the proof, we also show that there is no equilibrium such that one agent works and the other agent shirks.¹⁸ Notice that (3) is always satisfied when $\eta(\lambda - 1) \leq 1$. Thus, $(a_i, a_j) = (1, 1)$ is supported as a unique team equilibrium in Figure 1 for all regions where \mathbf{w}^J is the optimal wage scheme.

5 Conclusion

We investigate a moral hazard model with limited liability in which the agents have expectation-based reference-dependent preferences à la Kőszegi and Rabin (2006, 2007). We highlight a trade-off between the standard-incentive effect and the loss-reducing effect on the agent's incentive compatibility constraint, and show the optimality of stochastic compensation schemes under agent loss aversion.

We also examine a multi-agent model and show that team incentives can be used for the loss-reducing device. Our result may help understand why teams and team incentives are ubiquitous in workplace, and team incentives as compensating for the agent's failure may help explain incentive schemes in practice.

¹⁸With a continuous effort choice, under JPE there might exist a team equilibrium in which one agent works very hard and the other agent does not work at all. When the effort cost function is sufficiently convex, however, such an action pair does not become a team equilibrium. This is because by choosing the same level of efforts, the agents can save the total effort cost as keeping the probability of getting high wages constant.

Appendix

Proof of Proposition 1

Proof. In the text. □

Proof of Proposition 2

Proof. Immediate from (2). □

Proof of Lemma 1

Proof. For notational convenience, let $q_a^H = q_a$ and $q_a^L = (1 - q_a)$ where $a \in \{0, 1\}$. The principal's problem is:

$$\begin{aligned}
& \min_{\{w_{Qn}\}} \sum_{n=1}^N p_n [q_1 w_{Hn} + (1 - q_1) w_{Ln}] \\
& \text{s.t.} \quad \sum_{n=1}^N p_n [q_1 w_{Hn} + (1 - q_1) w_{Ln}] \\
& \quad - \frac{1}{2} \sum_{Q \in \{H, L\}} \sum_{\hat{Q} \in \{H, L\}} \sum_{n=1}^N \sum_{m=1}^N q_1^Q q_1^{\hat{Q}} p_n p_m \eta(\lambda - 1) \cdot |w_{Qn} - w_{\hat{Q}m}| - d \\
& \quad \geq \sum_{n=1}^N p_n [q_0 w_{Hn} + (1 - q_0) w_{Ln}] \\
& \quad - \frac{1}{2} \sum_{Q \in \{H, L\}} \sum_{\hat{Q} \in \{H, L\}} \sum_{n=1}^N \sum_{m=1}^N q_0^Q q_0^{\hat{Q}} p_n p_m \eta(\lambda - 1) \cdot |w_{Qn} - w_{\hat{Q}m}|, \tag{4} \\
& \quad \forall Q \forall n \quad w_{Qn} \geq 0, \quad \forall Q \forall n \forall n' \leq n \quad w_{Qn} \geq w_{Qn'}, \quad \forall n \quad w_{Hn} \geq w_{Ln}.
\end{aligned}$$

(4) can be written as:

$$\begin{aligned}
& (q_1 - q_0) \sum_{n=1}^N p_n (w_{Hn} - w_{Ln}) \\
& - \frac{1}{2} \sum_{Q \in \{H, L\}} \sum_{\hat{Q} \in \{H, L\}} \sum_{n=1}^N \sum_{m=1}^N (q_1^Q q_1^{\hat{Q}} - q_0^Q q_0^{\hat{Q}}) p_n p_m \eta(\lambda - 1) \cdot |w_{Qn} - w_{\hat{Q}m}| \geq d. \tag{5}
\end{aligned}$$

(i) Take the following binary lottery: $N = 2$ and $p_1 = p_2 = 1/2$. Consider the following wage scheme such that the agent receives a positive wage for sure if he succeeds in the

project, and he receives the positive wage $w > 0$ with probability $1/2$ if he fails:

$$w_{Qn} = \begin{cases} 0 & \text{if } Q = L \text{ and } n = 1, \\ w & \text{otherwise.} \end{cases}$$

The wage scheme obviously satisfies the LL constraints. Since the agent can receive $w > 0$ with probability $q_1 + (1 - q_1)/2 = (1 + q_1)/2$ if he works and with probability $q_0 + (1 - q_0)/2 = (1 + q_0)/2$ if he shirks, (5) becomes

$$\begin{aligned} & (q_1 - q_0)\frac{1}{2}w - \left[\frac{1 + q_1}{2} \left(1 - \frac{1 + q_1}{2} \right) - \frac{1 + q_0}{2} \left(1 - \frac{1 + q_0}{2} \right) \right] \eta(\lambda - 1)w \geq d \\ \Leftrightarrow w & \geq \frac{2d}{\Delta q[1 + \eta(\lambda - 1)(q_0 + q_1)]}. \end{aligned}$$

Hence, the principal can induce the agent to exert high effort by setting sufficiently large $w > 0$.

(ii) We prove this by contradiction. Suppose that there exists s and $t > s$ such that $w_{Ht} \neq w_{Hs}$ in the optimal wage scheme \mathbf{w} . Since $w_{Ht} \geq w_{Hs}$, we can set $t = s + 1$ without loss of generality. We also set $w_{Hs+1} = w_{HN}$; otherwise we can take another pair of wages which contains the highest wage.

Because $w_{Hs} \geq w_{Hn}$ and $w_{Hn} \geq w_{Ln}$ for any $n \leq s$, $w_{Hs} \geq w_{Ln}$ holds. This implies that if w_{Ln} satisfying $w_{Hs+1} > w_{Ln} > w_{Hs}$ exists, then $n > s$ must hold. Let $l \geq s + 1$ and $h \geq l$ denote the lowest number and the highest number of n that satisfies $w_{Hs+1} > w_{Ln} > w_{Hs}$, respectively. Define $\sum_{n=l}^h p_n = 0$ if there does not exist n such that $w_{Hs+1} > w_{Ln} > w_{Hs}$.

First, consider a new contract \mathbf{w}' with $\Delta_w > 0$ that replaces w_{Hs} and w_{Hs+1} of \mathbf{w} to $w'_{Hs} = w_{Hs} + p_{s+1}\Delta_w$ and $w'_{Hs+1} = w_{Hs+1} - p_s\Delta_w$, respectively. All elements of \mathbf{w}' satisfy the LL constraints and it has the same ordinal position as the original contract. Then, the difference between the new contract and the original one for the left hand side of (5) is:

$$\begin{aligned} & (q_1^2 - q_0^2)(p_s + p_{s+1})p_s p_{s+1} \eta(\lambda - 1)\Delta_w + 2[q_1(1 - q_1) - q_0(1 - q_0)] \left(\sum_{n=l}^h p_n \right) p_s p_{s+1} \eta(\lambda - 1)\Delta_w \\ & = \left[(q_1 + q_0)(p_s + p_{s+1}) + 2(1 - q_1 - q_0) \left(\sum_{n=l}^h p_n \right) \right] (q_1 - q_0)p_s p_{s+1} \eta(\lambda - 1)\Delta_w. \end{aligned} \quad (6)$$

Notice that (6) is strictly positive if either $p_s + p_{s+1} \geq \sum_{n=l}^h p_n$ or $1 - q_1 - q_0 \geq 0$ holds. In these cases, the principal can relax (5) without violating the LL constraints. Because an expected payment under the new contract is the same as under the original contract, the principal can decrease the expected payment. A contradiction.

Second, suppose that both $p_s + p_{s+1} < \sum_{n=l}^h p_n$ and $(1 - q_1 - q_0) < 0$ hold. Then, we can take $\Delta_w > 0$ such that a new contract that changes the wages from the original contract to $w'_{H_{s+1}} = w_{H_{s+1}} - (1 - q_1)p_h\Delta_w$ and $w'_{L_h} = w_{L_h} + q_1p_{s+1}\Delta_w$, satisfying the LL constraints and has the same ordinal position as the original contract.

Then, the difference between the new contract and the original one for the left hand side of (5) is:

$$\begin{aligned}
& \left\{ \underbrace{(q_1^2 - q_0^2)(1 - q_1)p_{s+1}(1 - p_{s+1})p_h}_{\text{Comparing } w'_{H_{s+1}} \text{ with } \{w'_{H_n}\}_{n=1}^N} + \underbrace{[q_1(1 - q_1) - q_0(1 - q_0)](1 - q_1)p_{s+1}p_h(1 - p_h)}_{\text{Comparing } w'_{H_{s+1}} \text{ with } \{w'_{L_n}\}_{n=1}^N} \right. \\
& - \underbrace{[q_1(1 - q_1) - q_0(1 - q_0)]q_1p_{s+1}(1 - p_{s+1})p_h}_{\text{Comparing } w'_{L_h} \text{ with } \{w'_{H_n}\}_{n=1}^N} - \underbrace{[(1 - q_1)^2 - (1 - q_0)^2]q_1p_{s+1}p_h(1 - p_h)}_{\text{Comparing } w'_{L_h} \text{ with } \{w'_{L_n}\}_{n=1}^N} \\
& \left. + \underbrace{[q_1(1 - q_1) - q_0(1 - q_0)]p_{s+1}p_h[q_1p_{s+1} + (1 - q_1)p_h]}_{\text{Comparing } w'_{H_{s+1}} \text{ with } w'_{L_h}} \right\} \eta(\lambda - 1)\Delta_w \\
& = \left\{ [(q_1 + q_0)(1 - p_{s+1}) + (1 - q_0 - q_1)p_h](1 - q_1) + (1 - q_0)(1 - p_h) \right. \\
& \quad \left. - q_1(1 - q_0 - q_1)(1 - 2p_{s+1}) \right\} (q_1 - q_0)p_{s+1}p_h\eta(\lambda - 1)\Delta_w. \tag{7}
\end{aligned}$$

Notice that $1 - q_1 - q_0 < 0$ implies $(q_1 + q_0)(1 - p_{s+1}) + (1 - q_0 - q_1)p_h > (1 - p_{s+1}) - p_h > 0$, and $p_s + p_{s+1} < \sum_{n=l}^h p_n$ implies $p_{s+1} < \frac{1}{2}$. Hence (7) is strictly positive, and the principal can relax (5) without violating the LL constraints. Because an expected payment under the new contract is the same as under the original contract, the principal can decrease the expected payment—a contradiction.

Let $w_{H_n} = w$ where $n \in \{1, \dots, N\}$. If $w = 0$, then all wages must be zero because $w_{H_n} \geq w_{L_n}$ and the contract does not satisfy (5). Therefore, $w > 0$ in the optimal contract.

(iii) Let $b_1 \equiv w_{L_1}$ and $b_n \equiv w_{L_n} - w_{L_{n-1}}$ for $n \in \{2, \dots, N\}$. Note that $\sum_{n=1}^N p_n w_{L_n} = \sum_{n=1}^N \tilde{p}_n b_n$ where $\tilde{p}_n = \sum_{k=1}^{N-n} p_{N-k}$. Finally, set $b_{N+1} = w - \sum_{n=1}^N b_n$.

Then, the principal's problem can be written as:

$$\begin{aligned}
& \min_{\{b_n\}_{n=1}^{N+1}} q_1(b_{N+1} + \sum_{n=1}^N b_n) + (1 - q_1) \sum_{n=1}^N \tilde{p}_n b_n \\
& \text{s.t. } b_{N+1} + \sum_{n=1}^N b_n - \sum_{n=1}^N \tilde{p}_n b_n \\
& + \left[-(1 - q_1 - q_0) \sum_{n=1}^N p_n (b_{N+1} + \sum_{k=n+1}^N b_k) + (2 - q_1 - q_0) \sum_{n>m} p_n p_m \left(\sum_{k=n}^m b_k \right) \right] \eta(\lambda - 1) = \frac{d}{\Delta_q}, \\
& \forall n \in \{1, \dots, N + 1\} \quad b_n \geq 0.
\end{aligned} \tag{8}$$

This is a linear programming problem. Notice that (8) is a closed set. Because each coefficient of b_n in the principal's objective function is positive and each b_n is bounded from below, there exists $K \in R_{++}$ such that for any n , $b_n > K$ is never optimal. Thus, without loss of generality we can restrict the constraint set to $b_n \leq K$, which attains boundedness of the constraint set. Thus, the problem has a solution.

As developed by Herweg, Müller and Weinschenk (2010), the solution of the linear programming problem has an extreme point of the constraint, and it is generically unique. The unique solution satisfies that $b_n > 0$ holds for one of $n \in \{1, \dots, N + 1\}$, and $b_m = 0$ holds for any $m \neq n$. By the construction of b_n , we have proven that the optimal wage scheme is binary and generically unique. \square

Proof of Proposition 3

Notice that at the optimal wage scheme, (CPE-IC') must hold with equality because otherwise the principal can decrease w without violating constraints. Thus, the optimal wage is determined by $w(\alpha)$ as in the text, subject to $1 + [1 - (1 - \alpha)(2 - q_1 - q_0)] \eta(\lambda - 1) > 0 \iff \alpha > 1 - \frac{1}{2 - q_1 - q_0} \left[1 + \frac{1}{\eta(\lambda - 1)} \right]$. Note that (CPE-IC') is never satisfied when $\alpha \leq 1 - \frac{1}{2 - q_1 - q_0} \left[1 + \frac{1}{\eta(\lambda - 1)} \right]$.

By substituting $w(\alpha)$ into the expected payment function, the principal's problem becomes:

$$\min_{\alpha \in [0, 1]} W_\alpha \equiv \frac{[q_1 + \alpha(1 - q_1)]d}{\Delta_q(1 - \alpha) \{1 + [1 - (1 - \alpha)(2 - q_1 - q_0)] \eta(\lambda - 1)\}}, \tag{9}$$

subject to $\alpha > 1 - \frac{1}{2 - q_1 - q_0} \left[1 + \frac{1}{\eta(\lambda - 1)} \right]$.

Because (9) is continuously differentiable for all $\alpha \in \left(1 - \frac{1}{2-q_1-q_0} \left[1 + \frac{1}{\eta(\lambda-1)}\right], 1\right)$, the solution satisfies the first-order condition:

$$\frac{dW_\alpha}{d\alpha} = \frac{d}{\Delta_q} \frac{1 + [1 - (2 - q_1 - q_0)(1 + q_1) + 2\alpha q_1(2 - q_1 - q_0) + \alpha^2(1 - q_1)(2 - q_1 - q_0)]\eta(\lambda - 1)}{(1 - \alpha)^2 \{1 + [1 - (1 - \alpha)(2 - q_1 - q_0)]\eta(\lambda - 1)\}^2} \geq 0, \quad (10)$$

which holds with equality if $\alpha^* > 0$.

By solving (10), we get the candidate of the optimal compensation rate α^* as in the statement. Because the numerator of (10) is increasing in α , (10) is also a sufficient condition. \square

Proof of Proposition 4

Proof. First, we check Lemma 1 holds even when the IR constraint is additionally imposed.

Lemma 1 (i) holds, because once we have a contract satisfying the other constraints, then the IR constraint is satisfied by increasing all of the wages at the same amount.

Lemma 1 (ii) holds because the both alternative contracts relax the IR constraint. First, suppose that we take a new contract \mathbf{w}' with $\Delta_w > 0$ which changes w_{H_s} and $w_{H_{s+1}}$ in \mathbf{w} to $w'_{H_s} = w_{H_s} + p_{s+1}\Delta_w$ and $w'_{H_{s+1}} = w_{H_{s+1}} - p_s\Delta_w$, respectively. Then, the difference between the new contract and the original one of the agent's total utility is:

$$q_1^2(p_s + p_{s+1})p_s p_{s+1} \eta(\lambda - 1) \Delta_w > 0.$$

Next, suppose that we take a new contract which changes the wages from the original contract to $w'_{H_{s+1}} = w_{H_{s+1}} - (1 - q_1)p_h\Delta_w$ and $w'_{L_h} = w_{L_h} + q_1 p_{s+1}\Delta_w$. Then, the difference between the new contract and the original one of the agent's total utility is:

$$q_1(1 - q_1)p_{s+1}p_h [q_1 p_{s+1} + (1 - q_1)p_h] \eta(\lambda - 1) \Delta_w > 0.$$

Thus, in each case the IR constraint is relaxed.

To check Lemma 1 (iii), notice that (5) always binds in the optimal contract; otherwise the principal offers a flat-wage contract. The principal's problem in this case is:

$$\begin{aligned}
& \min_{\{b_n\}_{n=1}^{N+1}} q_1(b_{N+1} + \sum_{n=1}^N b_n) + (1 - q_1) \sum_{n=1}^N \tilde{p}_n b_n \\
\text{s.t. } & b_{N+1} + \sum_{n=1}^N b_n - \sum_{n=1}^N \tilde{p}_n b_n \\
& + \left[-(1 - q_1 - q_0) \sum_{n=1}^N p_n (b_{N+1} + \sum_{k=n+1}^N b_k) + (2 - q_1 - q_0) \sum_{n>m} p_n p_m \left(\sum_{k=n}^m b_k \right) \right] \eta(\lambda - 1) = \frac{d}{\Delta_q}, \\
& q_1(b_{N+1} + \sum_{n=1}^N b_n) + (1 - q_1) \sum_{n=1}^N \tilde{p}_n b_n \\
& - \left[q_1(1 - q_1) \sum_{n=1}^N p_n (b_{N+1} + \sum_{k=n+1}^N b_k) + (1 - q_1)^2 \sum_{n>m} p_n p_m \left(\sum_{k=n}^m b_k \right) \right] \eta(\lambda - 1) \geq d, \quad (11) \\
& \forall n \in \{1, \dots, N + 1\} \quad b_n \geq 0.
\end{aligned}$$

If (11) does not bind in the optimal contract, then the proof of Lemma 1 (iii) can be directly applied. Suppose (11) binds in the optimal contract. Then, by solving (11) for b_1 and substituting it into the objective function, the principal's problem becomes a linear programming problem with one equality constraint and non-negative constraints. Thus, the optimal wage scheme is binary and generically unique by the similar logic with the proof of Lemma 1. Set $b_1 = w_L$.

We now characterize the optimal contract. The IR constraint becomes:

$$w_L + [q_1 + \alpha(1 - q_1)] [1 - (1 - \alpha)(1 - q_1)\eta(\lambda - 1)] w(\alpha) \geq d + \bar{u}. \quad (12)$$

If (12) does not bind, the analysis is just same with the proof of Proposition 3 and hence $\alpha = \alpha^*$. We investigate the case of which (12) binds. We first prove $\alpha > 0$ implies $w_L = 0$ by contradiction. Suppose instead $\alpha > 0$ and $w_L > 0$ in the optimal contract. Substituting (12) with equality and $w(\alpha)$ into the objective function and ignoring the LL constraint $w_L \geq 0$, the principal's problem becomes:

$$\min_{\alpha \in [0, 1]} \frac{[q_1 + \alpha(1 - q_1)] (1 - q_1) \eta(\lambda - 1) d}{\Delta_q \{1 + [1 - (1 - \alpha)(2 - q_1 - q_0)] \eta(\lambda - 1)\}} + d + \bar{u}. \quad (13)$$

The first-order derivative of (13) is negative if and only if $\eta(\lambda - 1) > \frac{1 - q_1}{1 - q_0}$. Hence, in the optimal contract the principal chooses either $\alpha = 0$ or $\alpha \rightarrow 1$. The base wage w_L , however, goes to negative infinity as α goes to one—a contradiction. Thus, if $\alpha > 0$ holds in the optimal contract, then the LL constraint binds.

By substituting the value of $w(\alpha)$ and $w_L = 0$ into (12), we have:

$$\frac{[q_1 + \alpha(1 - q_1)] [1 - (1 - \alpha)(1 - q_1)\eta(\lambda - 1)]}{(q_1 - q_0)(1 - \alpha) \{1 + [1 - (1 - \alpha)(2 - q_1 - q_0)]\eta(\lambda - 1)\}} \geq \frac{d + \bar{u}}{d} \quad (14)$$

Note that the left hand side of (14) goes to infinity as $\alpha \rightarrow 1$. Hence, for any parameters there exists $\alpha \in [0, 1)$ which satisfies (14) with equality.

In general, there exist multiple α which satisfies (14) with equality. In such a case, the principal chooses α which minimizes her expected payment. Because the numerator of (10) is increasing in α , the principal's expected payment is decreasing in $\alpha < \alpha^*$ and is increasing in $\alpha > \alpha^*$. Let α^l denote the highest $\alpha < \alpha^*$ which satisfies (14) with equality. Similarly, let α^h denote the lowest $\alpha > \alpha^*$ which satisfies (14) with equality. Then, the optimal compensation rate is determined as:

$$\alpha^{IR} \equiv \operatorname{argmin}_{\alpha \in \{\alpha^h, \alpha^l\}} W_\alpha.$$

Note that α^{IR} is well-defined for all parameters. □

Proof of Corollary 1

We first prove $w_L = 0$ by contradiction. Suppose $w_L > 0$ in the optimal contract. Then, (12) binds because otherwise the principal can decrease both $w(\alpha)$ and w_L by a same amount. In this case, (12) holds with $\alpha = \alpha^*$ and $w_L = 0$ if and only if $\eta(\lambda - 1) \leq 1/(1 - q_0)$. The principal prefers such a wage scheme. Since the principal chooses $w_L > 0$ in the optimal contract, $\eta(\lambda - 1) > 1/(1 - q_0)$ must hold. Then, the first-order derivative of (13) is always negative, the principal chooses $\alpha \rightarrow 1$, and the base wage w_L goes to negative infinity—a contradiction.

When $\bar{u} = 0$, (14) becomes:

$$[\alpha(1 - q_0) + q_0] \cdot [1 - (1 - \alpha)(1 - q_0)\eta(\lambda - 1)] \geq 0.$$

Because $\alpha(1 - q_0) + q_0 > 0$, the condition is equivalent to:

$$\alpha \geq 1 - \frac{1}{(1 - q_0)\eta(\lambda - 1)}.$$

Thus, $\alpha^{IR} = 1 - \frac{1}{(1 - q_0)\eta(\lambda - 1)}$ when $\bar{u} = 0$. □

Proof of Lemma 2

Notice that in the two-agent moral-hazard model, the CPE-IC condition can be rewritten as

$$\begin{aligned}
& q_1 w_{HH} + (1 - q_1) w_{HL} - q_1 w_{LH} - (1 - q_1) w_{LL} \\
& - q_1(1 - q_1)(q_1 + q_0)\eta(\lambda - 1)|w_{HH} - w_{HL}| - q_1^2(1 - q_1 - q_0)\eta(\lambda - 1)|w_{HH} - w_{LH}| \\
& - q_1(1 - q_1)(1 - q_1 - q_0)x\eta(\lambda - 1)|w_{HH} - w_{LL}| - q_1(1 - q_1)(1 - q_1 - q_0)\eta(\lambda - 1)|w_{HL} - w_{LH}| \\
& - (1 - q_1)^2(1 - q_1 - q_0)\eta(\lambda - 1)|w_{HL} - w_{LL}| + q_1(1 - q_1)(2 - q_1 - q_0)\eta(\lambda - 1)|w_{LH} - w_{LL}| \\
& \geq \frac{d}{\Delta_q}. \tag{CPE'}
\end{aligned}$$

Proof. (i) We prove this by contradiction. Suppose that $\mathbf{w} = (w_{HH}, w_{HL}, w_{LH}, w_{LL})$ which satisfies $\min\{w_{LH}, w_{LL}\} > 0$ is the optimal wage scheme. By Assumption 2, we can reduce the same amount from each possible wage without violating the LL constraints. Also, reducing the same amount from all payments does not affect (CPE'). Thus, the principal can decrease the expected payment. A contradiction.

(ii) We prove this by contradiction. Suppose $\mathbf{w} = (w_{HH}, w_{HL}, w_{LH}, w_{LL})$ is the optimal wage scheme.

Consider a case in which $w_{HH} > w_{HL}$. Then, we can take $\Delta_w > 0$ such that a new contract $\bar{\mathbf{w}} = (w_{HH} - (1 - q_1)\Delta_w, w_{HL} + q_1\Delta_w, w_{LH}, w_{LL})$ satisfies the LL constraints and has the same ordinal position as the original contract.

First, suppose that $w_{HH} > w_{HL}$. If $w_{HL} \geq w_{LH}$, the difference between the new contract and the original contract for the left hand side of (CPE') is:

$$C(\bar{\mathbf{w}}) - C(\mathbf{w}) = q_1(1 - q_1)(q_1 + q_0)\eta(\lambda - 1)\Delta_w > 0.$$

where we denote the left hand side of (CPE') as $C(\mathbf{w}')$ when a wage scheme is \mathbf{w}' . If $w_{HH} > w_{LH} > w_{HL}$, the difference between the new contract and the original contract for the left hand side of (CPE') is:

$$\begin{aligned}
C(\bar{\mathbf{w}}) - C(\mathbf{w}) &= q_1(1 - q_1)(q_1 + q_0)\eta(\lambda - 1)\Delta_w + 2q_1^2(1 - q_1)(1 - q_1 - q_0)\eta(\lambda - 1)\Delta_w \\
&= q_1(1 - q_1) [(1 - q_1)(q_1 + q_0) + q_1(2 - q_1 - q_0)] \eta(\lambda - 1)\Delta_w \\
&> 0.
\end{aligned}$$

Thus, the principal can relax (CPE') without violating the LL constraints. Because an expected payment under the new contract is the same as under the original contract, the principal can decrease the expected payment. A contradiction.

Second, suppose that $w_{HL} < w_{LH} = w_{HH}$. By (i) of this Lemma, $w_{LL} = 0$ holds. The left hand side of (CPE') is:

$$C(\mathbf{w}) = (1 - q_1) \{1 - [(1 - q_1 - q_0) - q_1(2 - q_1 - q_0)] \eta(\lambda - 1)\} w_{HL}.$$

Because we suppose that \mathbf{w} satisfies (CPE'), $1 - [(1 - q_1 - q_0) - q_1(2 - q_1 - q_0)] \eta(\lambda - 1) > 0$ must hold. Then we can take $\Delta_w > 0$ such that a new contract $\tilde{\mathbf{w}} = (w_{HH} - (1 - q_1)\Delta_w, w_{HL} + \Delta_w, w_{LH} - (1 - q_1)\Delta_w, w_{LL})$ satisfies the LL constraints and has the same ordinal position as the original contract. The difference between the new contract and the original one for the left hand side of (CPE') is:

$$C(\tilde{\mathbf{w}}) - C(\mathbf{w}) = (1 - q_1) \{1 - [(1 - q_1 - q_0) - q_1(2 - q_1 - q_0)] \eta(\lambda - 1)\} \Delta_w > 0.$$

Thus, the principal can relax (CPE') without violating the LL constraints. Because an expected payment under the new contract is the same as under the original contract, the principal can decrease the expected payment. A contradiction.

We can prove this in the case where $w_{HH} < w_{HL}$ in the same way except for taking $\bar{\mathbf{w}} = (w_{HH} + (1 - q_1)\Delta_w, w_{HL} - q_1\Delta_w, w_{LH}, w_{LL})$ or $\tilde{\mathbf{w}} = (w_{HH} + (1 - q_1)\Delta_w, w_{HL} - q_1^2\Delta_w, w_{LH}, w_{LL} - q_1^2\Delta_w)$ as a new contract. □

Proof of Proposition 5

First, consider the case of (i) $w \geq w_{LH} \geq w_{LL} = 0$. Suppose that $1 - (1 - q_1 - q_0)\eta(\lambda - 1) > 0$. By substituting w which holds (CPEJ) with equality into the objective function, this problem is reduced to:

$$\min_{w_{LH}} \left[1 - \frac{q_1(1 - q_1)(2 - q_1 - q_0)\eta(\lambda - 1)}{1 - (1 - q_1 - q_0)\eta(\lambda - 1)} \right] w_{LH}$$

subject to

$$w_{LH} \in [0, w].$$

If the coefficient of w_{LH} is positive, w_{LH} should be zero. On the other hand, w_{LH} should be equal to w if the coefficient of w_{LH} is not positive. As a result, the optimal w_{LH} is presented by:

$$w_{LH} = \begin{cases} 0 & \text{if } \Omega_J < 1, \\ w & \text{if } \Omega_J \geq 1, \end{cases}$$

where $\Omega_J \equiv [1 - q_1 - q_0 + q_1(1 - q_1)(2 - q_1 - q_0)] \eta(\lambda - 1)$.

Next, suppose that $1 - (1 - q_1 - q_0)\eta(\lambda - 1) \leq 0 < 1 - [1 - q_1 - q_0 - q_1(2 - q_1 - q_0)]\eta(\lambda - 1)$. Because the coefficient of w_{LH} is positive but that of w is negative, the solution exists and $w_{LH} = w$ holds at the optimum.

Finally, suppose that $1 - [1 - q_1 - q_0 - q_1(2 - q_1 - q_0)]\eta(\lambda - 1) \leq 0$. Then, the solution does not exist.

As a result, if $\Omega_J < 1$, the optimal wage scheme is $\mathbf{w}^I = (w^I, w^I, 0, 0)$ where

$$w^I = \frac{d}{\Delta_q[1 - (1 - q_1 - q_0)\eta(\lambda - 1)]},$$

and the expected wage is:

$$W^I = q_1 \frac{d}{\Delta_q[1 - (1 - q_1 - q_0)\eta(\lambda - 1)]}. \quad (\text{IPE})$$

If $\Omega_J > 1$ and $1 - [1 - q_1 - q_0 - q_1(2 - q_1 - q_0)]\eta(\lambda - 1) > 0$, the solution in this case is $\mathbf{w}^J = (w^J, w^J, w^J, 0)$ where

$$w^J \equiv \frac{d}{\Delta_q(1 - q_1) \{1 - [1 - q_1 - q_0 - q_1(2 - q_1 - q_0)]\eta(\lambda - 1)\}},$$

and the expected wage is:

$$W^J = q_1(2 - q_1) \frac{d}{\Delta_q(1 - q_1) \{1 - [1 - q_1 - q_0 - q_1(2 - q_1 - q_0)]\eta(\lambda - 1)\}}. \quad (\text{JPE})$$

If $1 - [1 - q_1 - q_0 - q_1(2 - q_1 - q_0)]\eta(\lambda - 1) \leq 0$, the solution does not exist in this case. This result can be summarized as follows.

Lemma 3. Suppose that $w \geq w_{LH} \geq w_{LL} = 0$. The solution exists if and only if $1 - [1 - q_1 - q_0 - q_1(2 - q_1 - q_0)]\eta(\lambda - 1) > 0$. If it does exist, the optimal wage scheme is $\mathbf{w}^I = (w^I, w^I, 0, 0)$ if $\Omega_J < 1$, and $\mathbf{w}^J = (w^J, w^J, w^J, 0)$ if $\Omega_J \geq 1$.

Second, we examine the case of (ii) $w \geq w_{LL} \geq w_{LH} = 0$. The principal's problem is as follows:

$$\min_{w, w_{LL}} q_1 w + (1 - q_1)^2 w_{LL}$$

subject to

$$\begin{aligned} & [1 - (1 - q_1 - q_0)\eta(\lambda - 1)] w \\ & + [-(1 - q_1) + (1 - q_1)(1 - q_1 - q_0)\eta(\lambda - 1) + q_1(1 - q_1)(2 - q_1 - q_0)\eta(\lambda - 1)] w_{LL} \\ & \geq \frac{d}{\Delta_q}. \end{aligned} \quad (\text{CPER})$$

$$w \geq 0, \quad \text{and} \quad w_{LL} \in [0, w]. \quad (\text{LLR})$$

where (CPE) is the CPE-IC constraint and (LLR) is the limited liability constraint in this case.

Suppose that $1 - (1 - q_1 - q_0)\eta(\lambda - 1) > 0$. By substituting w which holds (CPE) with equality into the objective function, this problem is reduced to:

$$\min_{w_{LL}} \left[1 - \frac{q_1^2(2 - q_1 - q_0)\eta(\lambda - 1)}{1 - (1 - q_1 - q_0)\eta(\lambda - 1)} \right] w_{LL}$$

subject to

$$w_{LL} \in [0, w].$$

If the coefficient of w_{LL} is positive, w_{LL} should be zero. On the other hand, w_{LL} should be equal to w if the coefficient of w_{LL} is not positive. The optimal w_{LL} is presented by:

$$w_{LL} = \begin{cases} 0 & \text{if } \Omega_R < 1, \\ w & \text{if } \Omega_R \geq 1, \end{cases}$$

where $\Omega_R \equiv [1 - q_1 - q_0 + q_1^2(2 - q_1 - q_0)]\eta(\lambda - 1)$.

Next, suppose that $1 - (1 - q_1 - q_0)\eta(\lambda - 1) \leq 0$. Because the coefficient of w_{LL} is positive but that of w is not positive, the solution exists and $w_{LL} = w$ holds at the optimum.

As a result, if $\Omega_R < 1$, the optimal contract in this case is \mathbf{w}^I and the expected wage is W_I . On the other hand, if $\Omega_R \geq 1$, the optimal contract is $\mathbf{w}^R = (w^R, w^R, 0, w^R)$ where

$$w^R = \frac{d}{\Delta_q q_1 \{1 - [1 - q_1 - q_0 - (1 - q_1)(2 - q_1 - q_0)]\eta(\lambda - 1)\}},$$

and the expected wage is:

$$W^R = (q_1^2 - q_1 + 1) \frac{d}{\Delta_q q_1 \{1 - [1 - q_1 - q_0 - (1 - q_1)(2 - q_1 - q_0)]\eta(\lambda - 1)\}}. \quad (\text{RPE})$$

Because $1 - q_1 - q_0 - (1 - q_1)(2 - q_1 - q_0) < 0$, the solution always exists.

Hence, we have the following lemma.

Lemma 4. Suppose that $w \geq w_{LL} \geq w_{LH} = 0$. The solution always exists. The optimal wage scheme is $\mathbf{w}^I = (w^I, w^I, 0, 0)$ if $\Omega_R < 1$, and $\mathbf{w}^R = (w^R, w^R, 0, w^R)$ if $\Omega_R \geq 1$,

where $w^R = \frac{d}{\Delta_q q_1 \{1 - [1 - q_1 - q_0 - (1 - q_1)(2 - q_1 - q_0)]\eta(\lambda - 1)\}}.$

We now derive the optimal wage scheme from Lemma 3 and Lemma 4. We have the following relationship:

$$\Omega_J \gtrless \Omega_R \Leftrightarrow \frac{1}{2} \gtrless q_1.$$

When $q_1 \leq \frac{1}{2}$, we have the following possible cases: (I-1) $\Omega_R \leq \Omega_J < 1$, (I-2) $\Omega_R < 1 \leq \Omega_J$ and (I-3) $1 \leq \Omega_R \leq \Omega_J$.

First, in case (I-1), the optimal wage scheme is \mathbf{w}^I which exhibits IPE. Second, in case (I-2), the optimal wage scheme is \mathbf{w}^J which exhibits JPE. These results are easily derived from Lemma 3 and Lemma 4. Finally, in case (I-3), we should compare between W_J and W_R in order to determine the optimal wage scheme.

$$W_J < W_R \Leftrightarrow (1 - 2q_1) \{1 - [1 - q_1 - q_0 - q_1(1 - q_1)^2(2 - q_1 - q_0)] \eta(\lambda - 1)\} > 0. \quad (15)$$

Because $q_1 \leq \frac{1}{2}$, we have:

$$W_J \leq W_R \Leftrightarrow \Omega_{JR} \equiv [1 - q_1 - q_0 - q_1(1 - q_1)^2(2 - q_1 - q_0)] \eta(\lambda - 1) \leq 1. \quad (16)$$

Thus, when (I-3), the optimal wage scheme is \mathbf{w}^J if (16) is satisfied; otherwise \mathbf{w}^R is the optimal.

Next, when $q_1 > \frac{1}{2}$, we have the following possible cases: (II-1) $\Omega_J < \Omega_R < 1$, (II-2) $\Omega_J < 1 \leq \Omega_R$ and (II-3) $1 \leq \Omega_J < \Omega_R$.

First, by Lemma 3 and 4, the optimal wage scheme is \mathbf{w}^I in case (II-1) while it is \mathbf{w}^R in case (II-2). Next, in the case of (II-3), we should compare between W_J and W_R in order to determine the optimal wage scheme. By (15) and $q_1 > \frac{1}{2}$, we have:

$$W_R < W_J \Leftrightarrow \Omega_{JR} < 1. \quad (17)$$

Thus, when (II-3), the optimal wage scheme is \mathbf{w}^R if (17) is satisfied; otherwise it is \mathbf{w}^J .

Proof of Proposition 6

Proof. Because \mathbf{w}^J is the optimal contract, the CPE-IC constraint holds with equality: $U(1, 1, \mathbf{w}^J|1, 1, \hat{\mathbf{w}}^J) = U(0, 1, \mathbf{w}^J|0, 1, \hat{\mathbf{w}}^J)$. Also, under the payment scheme \mathbf{w}^J , the agent's probability of getting w when he works and his colleague shirks is equal to when he shirks and his colleague works. Hence $U(1, 0, \mathbf{w}^J|1, 0, \hat{\mathbf{w}}^J) = U(0, 1, \mathbf{w}^J|0, 1, \hat{\mathbf{w}}^J) - d < U(1, 1, \mathbf{w}^J|1, 1, \hat{\mathbf{w}}^J)$. Thus, neither $(a_i, a_j) = (1, 0)$ nor $(a_i, a_j) = (0, 1)$ gives the agents the highest joint utility under the payment scheme \mathbf{w}^J .

Next, we show that each agent's total utility when both agents work is higher than when both agents shirk. The agent's total utilities when $(a_i, a_j) = (0, 1)$ and $(a_i, a_j) = (0, 0)$ under \mathbf{w}^J are as follows:

$$\begin{aligned} U(0, 1, \mathbf{w}^J | 0, 1, \hat{\mathbf{w}}^J) &= [1 - (1 - q_1)(1 - q_0)] w^J - (1 - q_1)(1 - q_0) [1 - (1 - q_1)(1 - q_0)] \eta(\lambda - 1) w^J, \\ U(0, 0, \mathbf{w}^J | 0, 0, \hat{\mathbf{w}}^J) &= [1 - (1 - q_0)^2] w^J - (1 - q_0)^2 [1 - (1 - q_0)^2] \eta(\lambda - 1) w^J. \end{aligned}$$

Because (3) implies

$$\begin{aligned} &U(0, 1, \mathbf{w}^J | 0, 1, \hat{\mathbf{w}}^J) - U(0, 0, \mathbf{w}^J | 0, 0, \hat{\mathbf{w}}^J) \\ &= \Delta_q(1 - q_0) [1 + \eta(\lambda - 1) - (1 - q_0)(2 - q_1 - q_0)\eta(\lambda - 1)] w^J \\ &= \Delta_q(1 - q_0) \{1 - [1 - q_1 - q_0 - q_0(2 - q_1 - q_0)] \eta(\lambda - 1)\} w^J \\ &> 0, \end{aligned}$$

we have $U(0, 0, \mathbf{w}^J | 0, 0, \hat{\mathbf{w}}^J) < U(0, 1, \mathbf{w}^J | 0, 1, \hat{\mathbf{w}}^J) = U(1, 1, \mathbf{w}^J | 1, 1, \hat{\mathbf{w}}^J)$.

Therefore, $(a_i, a_j) = (1, 1)$ attains the agents the highest joint utility under the wage scheme \mathbf{w}^J . \square

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