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Who Benefits from Misleading Advertising?

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Abstract

We develop a Hotelling model of horizontally and vertically differentiated brands with misleading advertising competition. We investigate the question of who benefits or loses from the misinformation created by advertising competition and related regulatory policies. We show that the quality gaps between two brands are crucial for determining the effect of misinformation on the firms' profits, aggregate or individual consumer surplus, and national welfare. Although the misinformation tricks consumers into buying products that they would not have purchased otherwise, it may improve welfare even if the advertising does not expand the overall demand for the brands. We also show that, although endogenous advertising competition may lead to a prisoner's dilemma for firms, it makes some consumers better off. We also consider the effects of several regulatory policies, such as advertising taxes, ad valorem and unit taxes on production, comprehensive and partial prohibitions of misleading advertising, government provisions of quality certification or counter-information, and the education of consumers.

Keywords Misinformation; Advertising Competition; Regulation; Product Differentiation;

JEL Code L13; M37; I18;

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1 Introduction

Should governments regulate misleading advertising? In the classical literature on the economics of advertising, an advertisement that misleads and fools consumers is known as *persuasive* advertising, which is usually considered to be potentially anti-competitive.[1] Many developed countries have government agencies (such as the U.S. Food and Drug Administration (FDA) or the Federal Trade Commission (FTC)) that regulate misleading or false advertising and encourage the provision of accurate information to allow consumers to make informed choices. However, Glaeser and Ujhelyi (2010) suggest that a certain amount of misinformation may improve social welfare if a product market is imperfectly competitive. The result comes from the fact that misinformation mitigates the problem of under-consumption of products that results from imperfect competition although it leads consumers to buy products that they would not have purchased otherwise and lowers consumer surplus. In other words, misinformation may improve social welfare by harming consumers on the condition that it induces an overall increase in market demand. Therefore, government regulation and intervention are beneficial to consumers but may be harmful to firms and overall social welfare.

This article investigates misleading advertising competition among brands in a Hotelling model where advertising induces a shift between two brands without expanding the total demand. We consider a situation where two horizontally and vertically differentiated brands compete in price and advertising to take market share from each other. The price and advertising decisions are made strategically by each brand, and the advertising contains misinformation about brand quality, which does not enhance the utility of using the brands but only provides a veneer of quality.

Within the above framework, we address the question of who benefits or loses from misleading advertising competition between two brands and from related regulations. We first show that a certain amount of misinformation may improve social welfare by removing inefficiency due to a misallocation of the products even if the misinformation does not induce an expansion of total demand for the brands. Interestingly, the misinformation actually makes some consumers

[1] For a comprehensive survey of the economic analyses of advertising, see Bagwell (2007).

better off because of the effect that misinformation has on price changes. We also show that the quality gaps between two brands play a crucial role in determining the effect of misinformation on firms' profits, aggregate and individual consumer surpluses, and welfare. Second, the amount of misinformation endogenously given by misleading advertising competition is shown to be excessive from a welfare perspective. If there are no quality gaps between the brands, the advertising competition harms both firms but does not affect the consumers. In this case, no one benefits from the advertising competition. This situation is equivalent to a prisoner's dilemma: each firm individually prefers to send misinformation in order to take market share from its rival, but the industry is collectively worse off if both firms do so. However, if the quality gaps are large, advertising competition increases not only a high-quality firm's profits but also the utility of the consumers who prefer the low-quality brand because of the price-change effects of misinformation.

We also investigate the effects of several regulatory policies, such as advertising taxes, unit and ad valorem taxes on production, comprehensive and partial regulations for misleading advertising, and government provisions of information about brand qualities. Both advertising and ad valorem taxes alleviate the advertising competition between the two brands and improve social welfare. However, the impact of advertising competition on consumers and firms under the two forms of taxation differ. If quality gaps are small, imposing a small advertising (ad valorem) tax on firms increases (decreases) both firms' profits. Conversely, if quality gaps are large, an advertising (ad valorem) tax decreases (increases) the profits of high-quality (low-quality) firms. Consumers are more likely to be better off under the advertising tax than under the ad valorem tax. Interestingly, imposing an advertising tax increases both firms' profits and consumer welfare.

Next, we examine the government's incentives to employ a partial or selective regulation on misleading advertising. If misinformation has different effects on consumers and firms, a policymaker who places different weights on industry profits and consumer welfare may employ different types of arbitrary regulation against each firm's misinformation. We show that a policymaker who weights the consumers' (industry's) benefits more heavily may have an incentive to only prohibit misinformation concerning the high-quality (low-quality) brand. This result occurs because the prohibition of misinformation concerning the high-quality brand lowers (raises) the price of the more (less) heavily consumed brand. We also investigate the government's incentives to provide information about product quality in response to the firms' misinformation. We show that the government's decision to negate or confirm the firms' misinformation depends on the government's bias: a consumerist policymaker may confirm misinformation about the low-quality brand and negate misinformation about the high-quality brand, whereas a neutralist policymaker may do the opposite. These results can be explained by examining the two effects of misinformation: the mischoice and price-change effects.

Finally, we extend the basic model by considering heterogeneous consumers (i.e., naïve and smart consumers) and investigate how misleading advertising competition affects equilibrium outcomes. Advertising competition necessarily reduces the utility of naïve consumers, but it may improve, on average, the utility of smart consumers. We also show that if the quality gaps between the two brands are small, a small decrease in the proportion of naïve consumers will increase both firms' profits, aggregate consumer surplus, and welfare because this decrease weakens the incentives to engage in unprofitable advertising competition. Therefore, in this case, a government policy of educating consumers works well.

The welfare effects of advertising and optimal regulatory policies have been extensively studied by Nelson (1974), Dixit and Norman (1978), Becker and Murphy (1993), Glaeser and Ujhelyi (2010), Hattori and Higashida (2012), and many others. As mentioned above, Glaeser and Ujhelyi (2010) shows that misinformation provided by persuasive or misleading advertising may improve social welfare if a product market is imperfectly competitive. Although our study shares some common features with Glaeser and Ujhelyi (2010), our study differs in several respects. First, we consider a Hotelling model with a horizontally and vertically differentiated product market, whereas their study considers a Cournot model with a homogenous goods market.[2] Second, in their study, advertising induces an overall increase in market demand and thus increases the total consumption of the products, whereas in our study, advertising induces a demand shift between the brands in stead of an expansion of total demand.[3] These two differences are important because

[2]For the welfare effects of misleading advertising and the minimum quality standards in a model of vertical product differentiation, see Hattori and Higashida (2011).

[3]In other words, advertising is a public good among firms in Glaeser and Ujhelyi (2010), whereas advertising is

our result, which shows that misinformation may improve welfare, depends on the quality gaps between the brands. In addition, our result holds even if the misinformation does not increase the total consumption of the brands. Third, our study investigates the strategic interactions between the firms and the government in the transmission of (mis)information, whereas their study treats the government advertising as exogenous. Finally and most importantly, our study considers consumers to be heterogeneous, not only with respect to their tastes (i.e., the location on the Hotelling line) but also in their knowledge of the quality of products (i.e., either smart or naïve consumers). Hence, we can determine what types of consumers benefit or suffer from misinformation and related regulations.

Using a Hotelling model of horizontal product differentiation, Bloch and Manceau (1999) investigate the effect of persuasive advertising on the profitability of firms. They show that in the duopoly case, advertising may induce a decrease in the price of the advertised product if the advertising changes the distribution of consumer tastes between products. However, because their study focuses on the profitability of exogenously given advertising for firms, they do not consider the welfare effects of advertising or endogenous advertising competition. Furthermore, their study does not account for vertical product differentiation or quality differentials between the brands. In contrast, our study considers the welfare effects of persuasive (misleading) advertising and endogenous advertising competition in a Hotelling model of horizontal and vertical product differentiation.[4]

To focus on the welfare and distributional effects of regulations, we do not consider the quality-guarantee effects of advertising, such as the effects of advertising on brands' reputations. In addition, we exclude the signaling-efficiency effect of advertising from our analysis.[5] Following Glaeser and Ujhelyi (2010), we assume that all consumers are naïve in the sense that when purchasing products, they always believe the misinformation provided by firms. However, the

a private good for each firm in our study.

[4]See Chakrabarti and Haller (2011) for a study of *comparative* advertising competition among oligopolists. See Hattori and Higashida (2012) for a study of *generic* advertising in a duopoly model with horizontal product differentiation and advertising spillovers.

[5]For the quality-guarantee and the signaling-efficiency effects of advertising, see Bagwell (2007).

assumption is relaxed in Section 4 when we consider the existence of smart consumers who are never misled by the misinformation and who can identify the true qualities of products when making purchase.

The article is organized as follows. In Section 2, we present the basic theoretical model and investigate the effect of exogenous changes in misinformation. We then derive the equilibrium of the endogenous advertising competition and the second-best optimum. Section 3 examines several regulatory policies, such as advertising and ad valorem taxes, comprehensive and partial regulations, and government advertising. Government biases toward consumers' benefits are also considered in Section 3. Section 4 extends the basic model by incorporating heterogeneous consumers and investigates a policy for educating consumers. Section 5 concludes.

2 Model

Suppose that a continuum of consumers is distributed uniformly on a "Hotelling" line segment $[0, 1]$ with mass 1. The location of an arbitrary consumer $x \in [0, 1]$ is associated with his/her preferences. There are two firms indexed by $i = 1, 2$ in this market. The firms are located at either end of the unit interval, reflecting horizontal product differentiation. Each firm i sells a Brand i at a uniform price p_i.[6] The "perceived" utility of each consumer indexed by x is defined by

$$U(x) = \begin{cases} (v_1 + s_1) - p_1 - tx & \text{if buys Brand 1} \\ (v_2 + s_2) - p_2 - t(1 - x) & \text{if buys Brand 2} \end{cases} \tag{1}$$

where v_i is the true quality of Brand i (or the true benefits that consumers derive from consuming Brand i), s_i is the misinformation about Brand i's quality, and tx and $t(1 - x)$ are the costs of buying a brand different from the consumer ' s ideal choice.[7] Each consumer buys only one unit

[6] Our model setting is similar to that of von der Fehr and Stevik (1998), who suggest a Hotelling framework for persuasive advertising. However, their study focuses on the equilibrium level of advertising, and they investigate neither the welfare implications of persuasive advertising nor the effect of regulatory policies. For a study of informative advertising that uses a Hotelling model, see Tirole (1988).

[7] Because we are not interested in the firms' choice of product differentiation (location), we assume that the transportation costs are linear.

of the brand. When making a purchase, consumers cannot identify the true quality of the brand (v_i). Rather, they only know the perceived quality of those ($(v_i + s_i)$).[8] Thus, the utility expressed by (1) is called "perceived" utility. However, the true or ex-post utility is given by

$$\tilde{U}(x) = \begin{cases} v_1 - p_1 - tx & \text{if buys Brand 1} \\ v_2 - p_2 - t(1-x) & \text{if buys Brand 2.} \end{cases} \tag{2}$$

We assume that v_i (or $v_i + s_i$) is sufficiently large for both Brands $i = 1, 2$ such that all consumers buy either of the brands in equilibrium (i.e., the market is fully covered). Let $\hat{x}$ denote the marginal consumer who is indifferent between purchasing Brands 1 and 2. Then we have

$$\hat{x} = \frac{(v_1 + s_1) - (v_2 + s_2) - (p_1 - p_2)}{2t} + \frac{1}{2}.$$

The profit functions of firm i are given by

$$\pi_i = (p_i - c_i)\, y_i - C(s_i) \tag{3}$$

where $y_1 = \hat{x}$ and $y_2 = (1 - \hat{x})$ are the outputs of firms 1 and 2, respectively; c_i is the constant marginal cost of production; and $C(s_i)$ is the advertising cost for firm i, where we assume $C' > 0$ and $C'' \geq 0$. For the sake of simplicity, the advertising costs are specified by $C(s_i) = k\, s_i^2$, where $k > 0$ represents the cost parameter of misleading advertising. We assume that each firm faces the same advertising cost structure. For example, both firms face the same price of advertising set by the ad agencies.

The timing of events is as follows. In the first stage, each firm chooses on the amount of misleading advertising needed to deceive and persuade consumers to buy more. In the second stage, each firm decides the price of its own brand. Therefore, this advertising can be considered *strategic advertising* in the sense that firms project their advertising strategies while accounting for how advertising affects future price competition.[9]

[8] Following Nelson's (1970) terms, this article assumes that misleading advertising increases the search qualities of the advertised goods but does not affect their experience qualities.

[9] Notice that a situation where advertising and pricing decisions are made simultaneously by the firms (i.e., non-strategic advertising) does not qualitatively affect the results. The amount of advertising in the case of non-strategic advertising is shown to be greater than that obtained in the case of strategic advertising.

We solve for a subgame-perfect equilibrium by applying backward induction. Solving for the Nash equilibrium prices and quantities in the second stage yields:

$$p_i^*(s_i, s_j) = \frac{(\theta_i - \theta_j) + (s_i - s_j)}{3} + c_i + t, \quad y_i^*(s_i, s_j) = \frac{(\theta_i - \theta_j) + (s_i - s_j)}{6t} + \frac{1}{2}. \tag{4}$$

where $\theta_i \equiv (v_i - c_i)$ represents the quality minus cost parameter of Brand i. We hereafter refer to θ_i simply as the *quality parameter* of Brand i. Without loss of generality, we assume $\theta_1 \geq \theta_2$ throughout the article. Hence, we call Firm 1 (Firm 2) and Brand 1 (Brand 2) the high-quality (low-quality) firm and the high-quality (low-quality) brand, respectively. We find from (4) that $\partial p_i^*/\partial s_i > 0$, $\partial p_i^*/\partial s_j < 0$, $\partial y_i^*/\partial s_i > 0$, and $\partial y_i^*/\partial s_j < 0$: misinformation raises own price and output but lowers those of the rival. Substituting (4) into (3) yields equilibrium profits at the second stage, as $\pi_i^*(s_i, s_j)$. Aggregate consumer surplus at the second-stage equilibrium is defined by

$$CS^*(s_i, s_j) = \int_{x=0}^{1} \tilde{U}(x) = \int_0^{\hat{x}^*} [v_1 - tx - p_1^*]\, dx + \int_{\hat{x}^*}^{1} [v_2 - t(1-x) - p_2^*]\, dx,$$

where $\hat{x}^* = y_1^*$. Notice that CS^* is evaluated using the true quality of the brand v_i instead of $(v_i + s_i)$, which means that misleading advertising affects consumers' utility by inducing changes in their purchasing decisions ($\hat{x}^*$) and affecting the prices of the brands (p_1^* and p_2^*). The welfare at the second-stage equilibrium is defined by $W^*(s_i, s_j) \equiv CS^* + \sum_i \pi_i^*$.

In this section, to ensure that interior solutions are obtained for y_i and s_i, we assume the following.

Assumption 1. For all $\theta_i > 0$, it holds that $0 \leq (\theta_1 - \theta_2) \leq \bar{\theta} \equiv \dfrac{9kt - 1}{3k}$.

□ **The case of no misinformation (Case O).** Here we derive the equilibrium outcomes in a case with no advertising competition and no misinformation (i.e., $s_i = 0$ for all $i = 1, 2$) as a benchmark. We can interpret this case as one in which the government can completely prohibit both firms from producing misleading advertising. The equilibrium outcomes in this case are obtained by substituting $s_i = 0$ into the second-stage equilibrium outcome as follows.

$$\begin{aligned} p_i^O &= \frac{\theta_i - \theta_j}{3} + c_i + t, \quad y_i^O = \hat{x}^O = \frac{\theta_i - \theta_j}{6t} + \frac{1}{2}, \quad \pi_i^O = \frac{(3t + \theta_i - \theta_j)^2}{18t}, \\ CS^O &= \frac{(\theta_1 - \theta_2)^2}{36t} + \frac{\theta_1 + \theta_2}{2} - \frac{5}{4}t, \quad \Pi^O = t + \frac{(\theta_1 - \theta_2)^2}{9t}, \\ W^O &= \frac{5(\theta_1 - \theta_2)^2}{36t} + \frac{\theta_1 + \theta_2}{2} - \frac{1}{4}t, \end{aligned}$$

where the superscript O indicates the equilibrium variable in the case with no misinformation. From Assumption 1, we know that $|\theta_i - \theta_j| < 3t$, which ensures an interior solution of $y_i^O \in [0, 1]$, $\forall i = 1, 2$. For example, if $\theta_1 > \theta_2$, we obtain $p_1^O > p_2^O$, $y_1^O > y_2^O$, and $\pi_1^O > \pi_2^O$. In addition, CS^O, Π^O, and W^O are increasing functions of the quality gaps between the brands.

□ **Exogenous changes in misinformation.** Before investigating the endogenous advertising competition between the firms, we examine the effect of exogenous misinformation on the second-stage equilibrium outcomes. Differentiating π_i^* and π_j^* in s_i, we have

$$\frac{d\pi_i^*}{ds_i} = (p_i^* - c_i)\left\{\frac{\partial y_i}{\partial p_j}\frac{\partial p_j^*}{\partial s_i} + \frac{\partial y_i}{\partial s_i}\right\} - C'(s_i), \quad \frac{d\pi_j^*}{ds_i} = \left(p_j^* - c_j\right)\left\{\frac{\partial y_j}{\partial p_i}\frac{\partial p_i^*}{\partial s_i} + \frac{\partial y_j}{\partial s_i}\right\},$$

where

$$\left\{\frac{\partial y_i}{\partial p_j}\frac{\partial p_j^*}{\partial s_i} + \frac{\partial y_i}{\partial s_i}\right\} = \frac{1}{3t} > 0, \quad \left\{\frac{\partial y_j}{\partial p_i}\frac{\partial p_i^*}{\partial s_i} + \frac{\partial y_j}{\partial s_i}\right\} = -\frac{1}{3t} < 0.$$

Therefore, we find that misinformation about Brand i always increases the revenue of firm i and decreases the profits of firm j.

Using the Leibniz' Rule, we can divide the effect of a change in s_i on consumer surplus CS^* into two components:

$$\begin{aligned} \frac{dCS^*}{ds_1} &= \underbrace{\frac{d\hat{x}^*}{ds_1}\Big[\{v_1 - t\hat{x}^* - p_1^*\} - \{v_2 - t(1 - \hat{x}^*) - p_2^*\}\Big]}_{\text{mischoice effect}} \\ &\quad + \underbrace{\left\{\int_0^{\hat{x}^*}\left(-\frac{\partial p_1^*}{\partial s_1}\right)dx + \int_{\hat{x}^*}^1\left(-\frac{\partial p_2^*}{\partial s_1}\right)dx\right\}}_{\text{price-change effects}} \\ &= -\frac{s_1 - s_2}{6t} + \left\{\underbrace{-\frac{3t + (s_1 - s_2) + (\theta_1 - \theta_2)}{18t}}_{(-)\text{ from raises in } p_1^*} + \underbrace{\frac{3t - (s_1 - s_2) - (\theta_1 - \theta_2)}{18t}}_{(+)\text{ from falls in } p_2^*}\right\}. \end{aligned}$$

FIGURE 1 AROUND HERE

The first term represents the *mischoice effect* of misinformation. This effect is due to the change in purchasing behavior, which is induced by misinformation. Because the marginal consumer is given by equating $\{(v_1 + s_1) - t\hat{x}^* - p_1^*\}$ to $\{(v_2 + s_2) - t(1 - \hat{x}^*) - p_2^*\}$, the mischoice effect becomes non-zero unless $s_1 = s_2$. Notice that this effect decreases as the difference in the amount of misinformation between Brands 1 and 2 decreases. In addition, if $s_j = 0$, any exogenous changes in Brand i's misinformation away from zero (in either a positive or negative direction) will reduce consumer surplus through the mischoice effect. The second term represents the *price-change effect*: a change in utility due to the two brands' price changes, which are induced by misinformation. Notice that the price-change effect of increasing s_1 is positive for the consumers who bought Brand 2 and negative for the consumers who bought Brand 1 when evaluated at $s_1 = s_2 = 0$.

Finally, we obtain the effect of a change in s_i on W^* as follows.

$$\frac{dW^*}{ds_i} = \frac{-(s_i - s_j) + 2(\theta_i - \theta_j)}{18t} - C'(s_i), \tag{5}$$

where the second-term is the direct cost effect of misinformation. The above equation implies that a small increase in misinformation about the high-quality brand may improve welfare when evaluated at $s_i = s_j$.

Figure 1 graphically illustrates the effect of positive or negative misinformation concerning the brands on the equilibrium outcomes in four cases. In all four cases, the marginal production cost for both firms is assumed to be zero. Hence, the quality gaps are represented simply by the differences in v. The upper-left panel (a) of the figure illustrates the case in which there are no quality gaps between the brands (i.e., $\theta_1 = \theta_2$) and there is positive misinformation about Brand 1. If there is no misinformation, consumers can make their purchasing decisions based on the true quality v_1 and v_2, and firms 1 and 2 charge p_1^O and p_2^O, respectively. Obviously, market share is divided equally between the firms (i.e., the marginal consumer is $\hat{x}^O = 1/2$). An increase in positive misinformation about Brand 1 raises the perceived quality of the brand (from v_1 to v_1^+), which raises the price of Brand 1 (from p_1^O to p_1^+), lowers that of Brand 2 (from p_2^O to p_2^+), and moves the marginal consumer to the right (from $\hat{x}^O$ to $\hat{x}^+$). The change in Firm 1's profits is represented by areas $B + C + E$, the change in Firm 2's profits is represented by $-D - E$, and the change in industry profits is represented by $B + C - D$. The mendacious firm gains, and the truthful firm loses. Because the ex-post consumer surplus should be evaluated using the true qualities (v_1 instead of v_1^+), compared with Case O, the consumers located at $[0, \hat{x}^O]$ suffer a loss resulting from the price increase equal to area B, whereas the consumers located at $[\hat{x}^*, 1]$ gain from the price decrease equal to area D. These two effects approximately represent the price-change effect of misinformation defined above. In other words, among those who did not change their purchasing decisions, the consumers who bought from the mendacious firm lose, and those who bought from the truthful firm gain. In addition, the consumers located at $[\hat{x}^O, \hat{x}^*]$ incorrectly changed their purchasing decisions, and the change in their utility is $-A - C$, which approximately represents the mischoice effect. In other words, the consumers who were misled on their purchasing decisions lose. As a result, the consumers are made worse off overall by the misinformation. The total change in aggregate consumer surplus is $-A - C - B + D$. Welfare is reduced by $(B + C + D) + (-A - C - B + D) = A$ plus the cost of providing the misinformation. Through similar inferences, we find that any unilateral increase (either positive or negative) in misinformation will generate a social loss in a case with equal-quality parameters.

However, misinformation does not necessarily reduce welfare. The upper-right panel (b) of Figure 1 illustrates this situation. If there is no misinformation, the equilibrium is characterized by p_1^O, p_2^O, and $\hat{x}^O$. An increase in positive misinformation about the high-quality brand (Brand 1) changes the equilibrium to p_1^+, p_2^+, and $\hat{x}^+$. The industry profits (i.e., the sum of both firms' profits) increase by $B + C - D$. Aggregate consumer surplus is changed by $-B + D$ because of the price-change effect, by $A - C$ because of the mischoice effect, and by $A - B - C + D$ overall.[10] Therefore, welfare is imcreased by $(B + C - D) + (A - B - C + D) = A$. If the welfare gain from misinformation represented by A outweighs the cost of sending the misinformation, the

[10]In this case, an increase in the misinformation about the high-quality brand decreases consumer surplus. In particular, from $CS^*(s_1, 0) - CS^O = -s_1 \{5s_1 + 4(\theta_1 - \theta_2)\} / (36t)$, we find that an exogenous change in the misinformation about the high-quality brand increases consumer surplus as long as $-4(\theta_1 - \theta_2)/5 < s_1 < 0$.

misinformation will improve net social welfare, as shown in (5).

Conversely, as the lower-left panel (c) of Figure 1 indicates, positive misinformation about the low-quality brand necessarily generates social loss, represented by A. In this case, consumer surplus may be increased by the misinformation (the area $(-A+B+C-D)$), whereas the industry profits are necessarily reduced.[11]

The lower-right panel (d) of Figure 1 illustrates the effect of negative misinformation concerning the low-quality brand in the case of $\theta_1 > \theta_2$. The example of providing negative misinformation is the health and safety alert for some products that is sometimes excessively provided by government or some third parties. The negative misinformation about Brand 2 lowers the perceived quality of the brand (from v_2 to v_2^-), which lowers p_2 to p_2^-, raises p_1 to p_1^-, and moves the marginal consumer to the right (from $\hat{x}^O$ to $\hat{x}^-$). The change in industry profits is represented by $B + C - D$, and the change in consumer surplus is represented by $A - B - C + D$ (the price change effect is represented by $-B + D$, and the mischoice effect is represented by $A - C$). Therefore, sending negative misinformation about the low-quality brand may improve social welfare, as shown in (5).

□ **Endogenous misinformation through advertising competition (Case S).** Now we consider the endogenous transmission of misinformation by the firms and derive the equilibrium in the first stage. In the first stage, each firm simultaneously chooses an amount of misleading advertising. The first-order condition for profit maximization yields the following reaction function:

$$s_i = R_i(s_j) \equiv \frac{3t + \theta_i - \theta_j}{18kt - 1} - \frac{1}{18kt - 1} s_j. \tag{6}$$

From Assumption 1, we know that $(9kt - 1) \geq 0$, which ensures the second-order conditions for an interior solutions and the stability condition of Nash equilibrium hold. We find that the advertising choices are strategic substitutes (i.e., the reaction functions have negative slope).

[11] In particular, from $CS^*(0, s_2) - CS^O = s_2 \{4(\theta_1 - \theta_2) - 5s_2\} / (36t)$, we find that an exogenous change in the misinformation about the low-quality brand increases consumer surplus as long as $0 < s_2 < 4(\theta_1 - \theta_2)/5$.

The equilibrium amount of misinformation can be derived by solving (6) for $i = 1, 2$:

$$s_i^S = \frac{\theta_i - \theta_j}{2\,(9kt - 1)} + \frac{1}{6k} \geq 0, \tag{7}$$

where the superscript S indicates a variable in the subgame-perfect equilibrium of the "strategic advertising" case. From (7), we find $s_1^S - s_2^S = (\theta_1 - \theta_2)/\,(9kt - 1) \geq 0$: Firm 1 (the high-quality firm) engages in more misleading advertising than Firm 2 (the low-quality firm). We also find that $ds_1^S/dt \leq 0$ and $ds_2^S/dt \geq 0$, which means that the more differentiated the brands are, the less (more) misinformation about the high-quality (low-quality) brand will be sent. This result occurs because an increase in t relaxes the price competition between the firms and raises the prices of both brands. However, when the misinformation about both brands are fixed, an increase in t decreases the market share of the high-quality brand but increases the market share of the low-quality brand. Therefore, an increase in t decreases (increases) the marginal benefits of sending misinformation for the high-quality (low-quality) firm.

Substituting s_i^S into the second-stage equilibrium outcome, we have

$$p_i^S = \frac{3kt\,(\theta_i - \theta_j)}{(9kt - 1)} + c_i + t, \quad y_i^S = \frac{3k\,(\theta_i - \theta_j)}{2\,(9kt - 1)} + \frac{1}{2}, \tag{8}$$

$$\pi_i^S = \frac{(18kt - 1)\,[(9kt - 1) + 3k\,(\theta_i - \theta_j)]^2}{36k\,(9kt - 1)^2}, \tag{9}$$

$$\Pi^S = \frac{k\,(18kt - 1)}{2\,(9kt - 1)^2}\,(\theta_1 - \theta_2)^2 + t - \frac{1}{18k}, \tag{10}$$

$$CS^S = \frac{3k\,(3kt - 2)}{4\,(9kt - 1)^2}\,(\theta_1 - \theta_2)^2 + \frac{\theta_1 + \theta_2}{2} - \frac{5}{4}t, \tag{11}$$

$$W^S = \frac{k(45kt - 8)}{4(9kt - 1)^2}(\theta_1 - \theta_2)^2 + \frac{\theta_1 + \theta_2}{2} - \frac{1}{18k} - \frac{1}{4}t. \tag{12}$$

Assumption 1 ensures that the equilibrium outputs of both firms are positive (i.e., $y_i^S > 0, 1$ holds for both firms in equilibrium). We find from (10) that an increase in the quality gaps between the brands always increases the industry profits. This finding is consistent with the case of no misinformation. However, an increase in the quality gaps (with the average remaining constant) decreases consumer surplus and welfare if k and/or t are small enough that $3kt < 2$ and $45kt < 8$, respectively. This finding contrasts sharply with the case of no misinformation, where the quality gap increases consumer surplus and welfare (see CS^O and W^O in Section 2.1). In the case of advertising competition, the quality gap makes the advertising competition more severe, especially if k and/or t are smaller. Therefore, consumer surplus and welfare decrease because of the increased amount of misinformation.

In the following, we compare the outcomes in case O with those in case S. First, comparing the equilibrium prices and outputs yields

$$p_i^S - p_i^O = \frac{\theta_i - \theta_j}{3\,(9kt-1)}, \quad y_i^S - y_i^O = \frac{\theta_i - \theta_j}{6t\,(9kt-1)}. \tag{13}$$

If $\theta_1 = \theta_2$, the equilibrium prices and outputs are identical in the two cases. If $\theta_1 > \theta_2$, advertising competition lowers (increases) the price of the low-quality (high-quality) brand and shrinks (expands) the market share of the low-quality (high-quality) brand.

Second, we compare the equilibrium profits in the two cases:

$$\pi_1^S - \pi_1^O = -\frac{1}{36k} + \frac{1}{6\,(9kt-1)}(\theta_1 - \theta_2) + \frac{27kt-2}{36t\,(9kt-1)^2}(\theta_1 - \theta_2)^2, \tag{14}$$

$$\pi_2^S - \pi_2^O = -\frac{1}{36k} - \frac{1}{6\,(9kt-1)}(\theta_1 - \theta_2) + \frac{27kt-2}{36t\,(9kt-1)^2}(\theta_1 - \theta_2)^2 \leq 0, \tag{15}$$

$$\Pi^S - \Pi^O = -\frac{1}{18k} + \frac{27kt-2}{18t\,(9kt-1)^2}(\theta_1 - \theta_2)^2. \tag{16}$$

We can easily find that if $\theta_1 = \theta_2$, then advertising competition is harmful to both firms. Although the firms pay advertising costs, they both gain nothing. The situation is equivalent to a prisoner's dilemma: each firm individually prefers to send misinformation to capture market share from its rival, but the industry is collectively worse off if both firms do so.[12] However, if the quality gaps are large, advertising competition may render the high-quality firm (Firm 1) better off, whereas the low-quality firm (Firm 2) necessarily becomes worse off. This result occurs because advertising competition increases Firm 1's revenues (from (13)) and its advertising costs. In

[12] In his seminal work, Pigou (1932, part 2, chapter 9) pointed out the prisoners' dilemma-like situation of advertising competition by noting, "Secondly, it may happen that the expenditures on advertisement made by competing monopolists will simply neutralise one another, and leave the industrial position exactly as it would have been if neither had expended anything. For, clearly, if each of two rivals makes equal efforts to attract the favour of the public away from the other, the total result is the same as it would have been if neither had made any effort at all."

contrast, advertising competition decreases Firm 2's revenues but increases its advertising costs.[13] In addition, Π^S is larger (smaller) than Π^O if the quality gaps between the brands are larger (smaller) because the smaller the gaps between the brands are, the more likely that the advertising competition becomes wasteful for both firms.[14]

Third, comparing CS^S and CS^O, we have

$$CS^S - CS^O = -\frac{(36kt+1)(\theta_1-\theta_2)^2}{36t(9kt-1)^2} \leq 0. \tag{17}$$

If $\theta_1 = \theta_2$, neither individual nor aggregate consumer surplus is affected by advertising competition. In contrast, if $\theta_1 \neq \theta_2$, advertising competition decreases aggregate consumer surplus because it leads consumers to make the wrong brand choice and raises the price of the more consumed brand (Brand 1), although it also lowers the price of the less consumed brand (Brand 2). The greater the difference between θ_1 and θ_2 is, more advertising competition reduces aggregate consumer surplus. However, as a result of advertising competition, the consumers who bought the high-quality brand are worse off, and those who bought the low-quality brand are better off.

Finally, we compare social welfare in the two cases:

$$W^S - W^O = -\frac{1}{18k} + \frac{18kt-5}{36t(9kt-1)^2}(\theta_1-\theta_2)^2 < 0, \tag{18}$$

where the sign condition is from Assumption 1.[15] Therefore, we find that advertising competition always reduces social welfare.

If there are no quality gaps between the brands ($\theta_1 = \theta_2$), advertising competition decreases the profits of both firms, has no effect on consumer surplus, and thus reduces social welfare. In

[13] In detail, $\pi_1^S > \pi_1^O$ holds if

$$(\theta_1-\theta_2) > \frac{(9kt-1)\left[\sqrt{9k^2t^2+kt(27kt-2)}-3kt\right]}{k(27kt-2)}.$$

[14] In detail, $\Pi^S < \Pi^O$ holds when

$$(\theta_1-\theta_2)^2 < \frac{t(9kt-1)^2}{k(27kt-2)} < \bar{\theta}^2.$$

[15] Substituting $\bar{\theta}$ in Assumption 1 into $(\theta_1-\theta_2)$ in (18), we have

$$(W^S-W^O)\Big|_{(\theta_1-\theta_2)<\bar{\theta}} < (W^S-W^O)\Big|_{(\theta_1-\theta_2)=\bar{\theta}} = -\frac{5}{324k^2t} < 0.$$

FIGURE 2 AROUND HERE

this case, *no one benefits from advertising competition.* Government regulations for prohibiting misleading advertising will be Pareto-improving. If $\theta_1 > \theta_2$, the consumers located at $[\hat{x}^S, 1]$ will benefit from advertising competition, those located at $[0, \hat{x}^S]$ and the low-quality firm will suffer, and the high-quality firm may benefit. However, also in this case, advertising competition necessarily reduces welfare due to the socially inefficient duplication of the advertising costs.

Figure 2 illustrates the comparison between the two equilibria. Panels (a) and (b) of the figure illustrate the cases of equal and unequal quality between the brands, respectively. We can see from the left panel that advertising competition between firms of equal quality does not affect equilibrium prices, market shares, or aggregate and individual consumer surpluses. Notice that the firms obtain nothing from advertising competition and simply bear the burden of advertising costs. Thus, the welfare is reduced by exactly the same amount as the costs.

In the right panel (b), both firms send positive misinformation about their own brands, but the high-quality firm (Firm 1) provides a greater amount of misinformation than the low-quality firm (Firm 2). Therefore, the price of the high-quality (low-quality) brand is increased (decreased) by advertising competition. In this case, the welfare gain is represented by the shaded area. However, the welfare gain is necessarily dominated by the two firms' total advertising costs, and the welfare under advertising competition is necessarily smaller than it would be in the case of no misinformation.

□ **The second-best amount of misinformation.** Here we derive the socially optimal (second-best) amount of misinformation given the duopoly market structure. The second-best amount of advertising, denoted as s_1^{SB} and s_2^{SB}, can be derived by solving the following problem: $\max_{s_1, s_2} W^*$. Then we have

$$s_i^{SB} = \frac{\theta_i - \theta_j}{18kt + 1}. \tag{19}$$

We find that the second-best amount of misinformation concerning the high-quality (low-quality)

FIGURE 3 AROUND HERE

brand is positive (negative) if $\theta_1 \neq \theta_2$, but is zero if $\theta_1 = \theta_2$.[16]

Comparing (19) with (7) and using Assumption 1, we have

$$s_1^S - s_1^{SB} = \frac{3\,(\theta_1 - \theta_2)}{2(9kt-1)(18kt+1)} + \frac{1}{6k} > 0,$$
$$s_2^S - s_2^{SB} = -\frac{3\,(\theta_1 - \theta_2)}{2(9kt-1)(18kt+1)} + \frac{1}{6k} > 0,$$

which implies that both brands send an excessive amount of misinformation through advertising competition.

Figure 3 illustrates the second-best amount of misinformation in the case of $\theta_1 > \theta_2$. The combination of positive misinformation for the high-quality brand and negative misinformation for the low-quality brand is desirable from the welfare point of view because the higher-quality brand would be promoted to consumers. Intrinsically, the marginal consumer between the two brands should be located at point E (i.e., $\hat{x}^E = \{t + (\theta_1 - \theta_2)\}/(2t)$) where the $v_1 - tx$ and $v_2 - t(1-x)$ curves intersect, to obtain the maximum welfare gain. However, without any misinformation, the marginal consumer would be $\hat{x}^O$ because of the higher price of the high-quality brand. Providing the amount of optimal misinformation reduces the distortion by moving the marginal consumer toward point E at the minimum costs of providing misinformation.[17] We should also note that providing optimal amount of misinformation necessarily decreases aggregate consumer surplus because the price-change effect of the misinformation is necessarily negative.

Our results show that an appropriate amount of misinformation would improve social welfare. These findings are closely related to the results obtained by Glaeser and Ujhelyi (2010), who shows that if a product market is imperfectly competitive, a certain amount of misinformation may increase consumption and improve social welfare. In contrast, our results show that a certain

[16]The second-order conditions are

$$\frac{\partial^2 W^*}{\partial s_i^2} = -2k - \frac{1}{18k} < 0, \quad \left(\frac{\partial^2 W^*}{\partial s_1^2} \cdot \frac{\partial^2 W^*}{\partial s_2^2}\right) - \left(\frac{\partial^2 W^*}{\partial s_i \partial s_j}\right)^2 = \frac{2k\,(18kt+1)}{9t} > 0.$$

[17]Because of our assumption of symmetric and increasing cost structure for sending misinformation, the amount of optimal misinformation is the same in absolute value fro the high- and low-quality brands.

amount of misinformation may improve social welfare even if it only shifts demand and does not increase the total amount of consumption: the misinformation improves social welfare by alleviating misallocation between high-quality and low-quality brands.[18] In addition, the effect of misinformation on consumers is uniform among the consumers in Galeser and Ujhelyi (2010), whereas in our study, the effect differs across consumers: some consumers gain from misinformation, whereas others do not.

3 Regulatory Policies

In this section, we investigate several regulatory policies: an advertising tax, ad an valorem tax, comprehensive and partial regulations on misleading advertising, and government provisions of quality certification or counter-information.

□ **Advertising taxes.** Here we investigate the effect of introducing a small advertising tax on firms' profits, consumer surplus, and welfare.[19] Suppose that the government taxes the advertising expenditures of the firms as follows: $\pi_i = (p_i - c_i)y_i - (1+\tau_a)C(s_i)$, where τ_a is the tax on each firm's advertising expenditures. Welfare is defined as $W = CS + \Pi + \tau_a \sum C(s_i)$, where the third term is tax revenue.

The equilibrium amount of misinformation under advertising competition is derived by

$$s_i^{\tau_a} = \frac{3k\left\{3t + (\theta_i - \theta_j)\right\}(1+\tau_a) - 1}{6k(1+\tau_a)\left\{9kt(1+\tau_a) - 1\right\}},$$

where superscript τ_a indicates a variable in the subgame-perfect equilibrium of advertising com-

[18]In other words, misinformation may improve welfare by alleviating the inefficiencies from under-consumption of the high-quality brand and over-consumption of the low-quality brand in our study, whereas misinformation may improve welfare by alleviating the inefficiency from under-consumption of the total consumption in Glaeser and Ujhelyi (2010).

[19]In many European countries, the government imposes a tax on various kinds of advertising. For example, the Swedish government levies a tax on advertising in daily newspapers and other printed media. The Greek government levies on a tax on television advertising. In addition, advertising is taxed in some form in Austria, Belgium, France, the Netherlands, and Spain.

petition with an advertising tax. Then we have

$$\left.\frac{ds_1^{\tau_a}}{d\tau_a}\right|_{\tau_a=0} = -\frac{1}{6k} - \frac{9kt}{2(9kt-1)^2}(\theta_1 - \theta_2) < 0, \tag{20}$$

$$\left.\frac{ds_2^{\tau_a}}{d\tau_a}\right|_{\tau_a=0} = -\frac{1}{6k} + \frac{9kt}{2(9kt-1)^2}(\theta_1 - \theta_2) \gtreqless 0 \;\Leftrightarrow\; (\theta_1 - \theta_2) \gtreqless \left(\frac{9kt-1}{9kt}\right)\bar{\theta}, \tag{21}$$

which implies that a small increase in the advertising tax reduces (raises) the amount of misinformation about the low-quality brand if the quality gaps are small (large), whereas the tax always reduces the amount of misinformation about the high-quality brand. The advertising expenditures of the high-quality firm are larger than those of the low-quality firm. Thus, the advertising tax has a more negative impact on the high-quality firm's incentives to advertise than on the low-quality firm's incentives to advertise.

Hence, we have the following comparative static results.

$$\begin{aligned}
\left.\frac{d\pi_1^{\tau_a}}{d\tau_a}\right|_{\tau_a=0} &= \frac{1}{36k} - \frac{3kt}{2(9kt-1)^2}(\theta_1-\theta_2) - \frac{k(27kt-1)}{4(9kt-1)^3}(\theta_1-\theta_2)^2 \gtreqless 0 \\
&\Leftrightarrow\; (\theta_1-\theta_2) \lesseqgtr \left(\frac{9kt-1}{27kt-1}\right)\bar{\theta} \\
\left.\frac{d\pi_2^{\tau_a}}{d\tau_a}\right|_{\tau_a=0} &= \frac{1}{36k} - \frac{3kt}{2(9kt-1)^2}(\theta_1-\theta_2) - \frac{k(27kt-1)}{4(9kt-1)^3}(\theta_1-\theta_2)^2 > 0, \\
\left.\frac{dCS^{\tau_a}}{d\tau_a}\right|_{\tau_a=0} &= \frac{3k\,(6kt+1)}{2\,(9kt-1)^3}\,(\theta_1-\theta_2)^2 \geq 0, \\
\left.\frac{dW_a^{\tau}}{d\tau_a}\right|_{\tau_a=0} &= \frac{1}{9k} + \frac{3k}{2(9kt-1)^3}(\theta_1-\theta_2)^2 > 0.
\end{aligned}$$

The derivations are presented in the Appendix.

If $\theta_1 = \theta_2$, a small increase in the advertising tax benefits both firms because it reduces unprofitable advertising competition and does not affect the aggregate and individual consumer surpluses. Therefore, in this case, the introduction of an advertising tax is Pareto-improving in a strict sense. However, if the quality gap is substantial, a small increase in the advertising tax benefits the low-quality firm (Firm 2) but may harm the high-quality firm (Firm 1). This result occurs because the advertising tax greatly diminishes the advantages (disadvantages) of advertising competition for the high-quality (low-quality) firm if the quality gaps between the brands are large. The aggregate consumer surplus is necessarily enhanced by a small increase in the advertising tax, but the individual consumer surplus is not. Because the advertising tax raises the price of the low-quality brand, the consumers who prefer the low-quality brand will be made worse off by the tax. However, an advertising tax improves welfare irrespective of the quality gaps.

□ **Ad valorem taxes.** We now consider an ad valorem tax on the outputs. The model setup is the same as the setup in the basic model except for the profits of the firms and the definition of welfare. Firm i's profits are given by $\pi = \{(1-\tau_p)p_i - c\}\, y_i - C(s_i)$, where τ_p is the ad valorem tax rate. In this subsection, we temporarily assume $c_1 = c_2 = c$ for derivational simplicity. Welfare is given by $W = CS + \Pi + \tau_p \sum p_i y_i$, where the third term is tax revenue.

The equilibrium amount of misinformation under advertising competition is derived as

$$s_i^{\tau_p} = \frac{(1-\tau_p)\left\{3k\left(3t + (\theta_i - \theta_j)\right) - (1-\tau_p)\right\}}{6k(9kt - 1 + \tau_p)},$$

where the superscript τ_p indicates a variable in the subgame-perfect equilibrium of the advertising competition with an ad valorem tax. Thus, we have

$$\left.\frac{ds_1^{\tau_p}}{d\tau_p}\right|_{\tau_p=0} = -\frac{1}{6k} - \frac{9kt}{2(9kt-1)^2}(\theta_1 - \theta_2) < 0, \tag{22}$$

$$\left.\frac{ds_2^{\tau_p}}{d\tau_p}\right|_{\tau_p=0} = -\frac{1}{6k} + \frac{9kt}{2(9kt-1)^2}(\theta_1 - \theta_2) \gtreqless 0 \ \Leftrightarrow\ (\theta_1 - \theta_2) \gtreqless \left(\frac{9kt-1}{9kt}\right)\bar{\theta}, \tag{23}$$

which implies that a small increase in the ad valorem tax reduces (raises) the amount of misinformation about the low-quality brand if the quality gaps are small (large). However, a small increase in the ad valorem tax always reduces the amount of misinformation about the high-quality brand. Because the quality gaps $\theta_1 > \theta_2$ means that $p_1^* > p_2^*$ holds in the second stage, Firm 1's incentive to send misinformation is weakened to a greater extent by the ad valorem tax than Firm 2's incentive. Because the amounts of misinformation are strategic substitutes, the decrease in Firm 1's misinformation increases Firm 2's misinformation. Notice that (22) and (23) are identical with (20) and (21): a small increase in the ad valorem tax and that in the advertising tax have the same impact on advertising competition.

Thus, we have the following comparative static results.

$$
\begin{aligned}
\left.\frac{d\pi_1^{\tau_p}}{d\tau_p}\right|_{\tau_p=0} &= -\frac{9kt-1}{18k} - \frac{(9kt-1)^2+81k^2t^2}{6(9kt-1)^2}(\theta_1-\theta_2) - \frac{81k^3t^2}{2(-1+9kt)^3}(\theta_1-\theta_2)^2 < 0,\\
\left.\frac{d\pi_2^{\tau_p}}{d\tau_p}\right|_{\tau_p=0} &= -\frac{9kt-1}{18k} + \frac{(9kt-1)^2+81k^2t^2}{6(9kt-1)^2}(\theta_1-\theta_2) - \frac{81k^3t^2}{2(-1+9kt)^3}(\theta_1-\theta_2)^2 \gtreqless 0\\
&\Leftrightarrow (\theta_1-\theta_2) \gtreqless \left(\frac{9kt-1}{9kt}\right)^2 \bar{\theta}\\
\left.\frac{dCS^{\tau_p}}{d\tau_p}\right|_{\tau_p=0} &= -c + \frac{3k(6kt+1)}{2(9kt-1)^3}(\theta_1-\theta_2)^2 \gtreqless 0,\\
\left.\frac{dW^{\tau_p}}{d\tau_p}\right|_{\tau_p=0} &= \frac{1}{9k} + \frac{3k}{2(9kt-1)^3}(\theta_1-\theta_2)^2 > 0.
\end{aligned}
$$

The derivations are presented in the Appendix.

In the case of no misinformation ($s_1 = s_2 = 0$), imposing a small ad valorem tax on the two brands does not affect the equilibrium market shares or social welfare because the tax revenues simply constitute a transfer from the firms and consumers to the government. However, the presence of advertising competition changes the situation considerably. If $\theta_1 = \theta_2$, a small increase in the ad valorem tax harms both firms because their revenues are decreased although it also reduces an unprofitable advertising competition. The tax may harm consumers because the firms include the tax in their prices, but it necessarily improves welfare. In contrast, if the quality gaps are large, a small increase in the ad valorem tax benefits the low-quality firm despite the tax burden because the tax enhances the competitiveness of the firm with respect to the advertising competition. In addition, also in this case, the ad valorem tax improves welfare. We also note that an advertising tax is more likely to benefit the firms and consumers than an ad valorem tax, but the impacts of the policies on social welfare are the same.

Finally, we briefly mention the case of a unit tax on production. This scenario can be modeled by setting the profits of each firm as $\pi_i = (p_i - c_i - \tau_u)y_i - C(s_i)$, where τ_u is the amount of unit tax. We can easily find from (8) that the unit tax increases the prices of the both brands by the same amount. Therefore, the unit tax has no impact on advertising competition or the amount of misinformation. This finding can be confirmed by the fact that s_i^S, y_i^S, and π_i^S depend only on $(\theta_i - \theta_j)$, which is not affected by the unit tax. Therefore, a small unit tax simply transfers income from consumers to the government and does not affect the advertising competition or social welfare at all.

□ **Partial regulations on misleading advertising.** Here we investigate the effect of a prohibition on misleading advertising as a means of regulating misinformation.

We examined the effect of comprehensive regulations on misleading advertising for both firms in Section 2.3 by comparing Case O to Case S. Now assume that the government uses "partial" or "selective" regulations on misleading advertising. In other words, the government prohibits misleading advertising for only one firm.[20] We consider two extreme regulation policies: a policy that prohibits the high-quality firm from sending any misinformation but that does not restrict the low-quality firm from advertising and vice versa.

We first consider a prohibition on sending misinformation that only applies to the high-quality firm (Firm 1). In this case, the low-quality firm (Firm 2) chooses its s_2 to maximize its profits given $s_1 = 0$. The corresponding equilibrium outcomes are indicated by the superscript PH (i.e., prohibition only applies to the high-quality firm). Comparing the equilibrium PH with that in Cases O and S, we find that

$$\begin{aligned}
&CS^{PH} \geq CS^{O} > CS^{S} \;\Leftrightarrow\; (\theta_1 - \theta_2) \geq \frac{15t}{72kt+1}, \\
&\Pi^{PH} < \Pi^{O}, \quad W^{PH} < W^{O}.
\end{aligned}$$

The derivations are presented in the Appendix. Compared with comprehensive regulations, a partial regulation that only applies the high-quality firm leads to lower industry profits and welfare. However, if the quality gaps are large, a partial regulation leads to greater aggregate consumer surplus than comprehensive regulations, as illustrated by Figure 1-(c). This results occurs because misinformation about the low-quality brand lowers the price of the more heavily consumed high-quality brand. Furthermore, from (17), we know that $CS^{O} > CS^{S}$ holds for all $\theta_1 > \theta_2$. Therefore, if the government's objective is biased towards the consumers' benefits, a policymaker

[20] Advertising regulations often seem to be arbitrary, and the standards in regulations are often criticized for being ambiguous. For example, regulations that restrict where and how advertising may be done are arbitrary under some advertising regulations. Therefore, a government agency can accept advertising of a certain type and arbitrarily reject other advertising of a similar type.

may prefer to only prohibit misleading advertising by the high-quality firm over prohibiting misleading advertising by both firms or taking no action.

We next consider a partial regulation that only applies to the low-quality firm (Firm 2). In this case, the high-quality firm (Firm 1) chooses its s_1 to maximize its profits given $s_2 = 0$. The corresponding equilibrium outcomes are indicated by the superscript PL (i.e., prohibition only applies to the low-quality firm). Comparing the PL equilibrium with that in Cases O and S, we find that

$$CS^{PL} < CS^{O}, \quad W^{PL} < W^{O},$$
$$\Pi^{PL} \geq \max[\Pi^{O}, \Pi^{S}] \;\Leftrightarrow\; (\theta_1 - \theta_2) \geq \frac{3t(9kt-1)}{27kt-1},$$

The derivations are presented in the Appendix. Although a partial regulation that only applies to the low-quality firm leads to lower aggregate consumer surplus and welfare than comprehensive regulations, a partial regulation may lead to greater industry profits than comprehensive regulations. Furthermore, $\Pi^{PL} > \Pi^{S}$ holds for all $\theta_1 > \theta_2$, as shown in the Appendix. Therefore, if the government's objective is biased towards industry benefits, a policymaker may prefer to only prohibit misleading advertising for the low-quality firm instead of prohibiting misleading advertising for both firms or taking no action.

□ **Government advertising and policy bias.** Next, we consider the government's policies on the provision of information, such as correcting or endorsing firms' misleading advertising. Specifically, on the one hand, the government can send information that counters the misleading information provided by firms. For example, the U.S. Food and Drug Administration is responsible for advancing public health by helping to provide the public the accurate, scientifically based information they need to choose medicines and foods to maintain and improve their health. On the other hand, the government can even send misinformation by certifying and supporting misinformation provided by firms. For example, in many countries (such as the US, Canada, EU, and Japan), the standards for organic food productions are formulated and overseen by the government, and the organic food is certified by the government agencies and distributed with labels conveying that information to consumers. As a result, there appears to be widespread perception amongst consumers that such organic foods include higher nutritional quality than conventionally produced foods.[21] However, some studies show that organic food has no nutritional or health benefits over conventionally produced food.[22] In this case, the government labels may be considered to convey false and misleading information to consumers.

As shown in the previous subsection, a government bias toward consumer or industry benefits may drastically change the regulatory framework. We investigate how the government behaves in response to misinformation about brand qualities and how this response relates to government bias.

With government information, consumers' utilities (1) are replaced by

$$U(x) = \begin{cases} (v_1 + \phi_1(s_1, g_1)) - p_1 - tx & \text{if buys Brand 1} \\ (v_2 + \phi_2(s_2, g_2)) - p_2 - t(1-x) & \text{if buys Brand 2,} \end{cases}$$

where $\phi_i(s_i, g_i)$ is the amount of misinformation about Brand i, and g_i $(i = 1, 2)$ represents the information provided by the government aimed at affecting the firms' misinformation.[23] We assume the simplest functional form $\phi_i(s_i, g_i) = s_i + g_i$. A positive (negative) value of g_i signifies a policy that attempts to validate the firms' misinformation (to limit the effectiveness of the firms' misinformation).

The equilibrium in the second stage is characterized as

$$p_i^{**}(s_i, s_j, g_i, g_j) = \frac{(v_i - v_j) + (\phi_i - \phi_j)}{3} + c_i + t, \quad y_i^{**}(s_i, s_j, g_i, g_j) = \frac{(v_i - v_j) + (\phi_i - \phi_j)}{6t} + \frac{1}{2}.$$

By using p_i^{**} and y_i^{**}, we obtain the second-stage equilibrium profits π_i^{**} and consumer surplus CS^{**}. To ensure that interior solutions are obtained for y_i, s_i, and g_i, we assume the following (but only in this section):

[21]The United States Department of Agriculture (USDA) certifies products with the organic label that meet certain requirements. However, after the organic food standards became effective in October of 2002, USDA Secretary Dan Glickman clarified that organic certification expressed a production philosophy and that organic labeling did not imply a superior, safer, or healthier product than food not labeled as organic (Winter and Davis 2006).

[22]See Williams (2001) and Dangour et al. (2009) for the study on a difference in nutrient quality between organically and conventionally produced foods.

[23]Notice that we call g_i "information" even though it may actually be misinformation (for example, $g_i > 0$ operates in a manner similar to the misinformation provided by firms).

Assumption 2. For all $\theta_i > 0$, it holds that $0 \leq (\theta_1 - \theta_2) < \tilde{\theta} \equiv \dfrac{t\,(18kt - 4\lambda + 3)}{6kt + 1}$.

If $\lambda = 0$, the assumption is reduced to $\tilde{\theta} = 3t$, which is less restrictive than the condition in Assumption 1. If $\lambda = 1$, the assumption may be more restrictive than Assumption 1 because the government's costs of providing information are separate from those of private firms. Hence, it is possible to provide greater total amount of (mis)information and to drive the low-quality brand out of the market.

The objective of the government is defined by

$$W^{**} = CS^{**} + \lambda \sum_{i=1}^{2} \pi_i^{**} - \sum_{i=1}^{2} C_g(g_i), \tag{24}$$

where $\lambda \leq 1$ is the policymaker's subjective weight on industry profits. If $\lambda < 1$ ($\lambda = 1$), the regulator places more weight on consumer surplus than on the industry profits (weight on the industry profits and consumer surplus equally).[24] The advertising cost faced by the government is $C_g(g_i)$. We assume that $C_g(g_i) = kg_i^2$, which is the same functional form as the cost function for the private firms.[25]

In the first stage, firms and the government respectively respectively choose their amounts of (mis)information to maximize their own profits and the government's objective, which is represented by (24). The order of moves between the firms and the government is not crucial for our results.[26]

Arranging the first-order conditions yields the following reaction functions for each firm and the government (for $i = 1, 2,\ i \neq j$):

$$s_i = \frac{3t + \theta_i - \theta_j}{18kt - 1} - \frac{1}{18kt - 1}\left(s_j + g_j - g_i\right), \tag{25}$$

$$g_i = \frac{(2\lambda - 1)(\theta_i - \theta_j)}{18kt + (5 - 4\lambda)} - \frac{(5 - 4\lambda)}{36kt + 2(5 - 4\lambda)}(s_i - s_j). \tag{26}$$

[24] We do not consider a case of $\lambda > 1$. However, the case can be analyzed as long as $\lambda < 5/4$.

[25] We assume that the total costs for providing (mis)information on Brand i incurred by firm i and the government are $C(s_i) + C(g_i)$ instead of $C(s_i + g_i)$. The assumption can be justified on the grounds that consumers are much more likely to believe information from multiple sources.

[26] When we consider a case where the firms choose their levels of misinformation first before the government chooses its amount of information, the results do not change qualitatively.

The second-order conditions are satisfied under Assumption 1.[27] Notice that the strategic interaction between the firms and the government is asymmetric: Firm i will increase s_i if the government increases g_i, but the government will decrease g_i if Firm i increases s_i.[28]

Solving (25) and (26) yields the subgame-perfect equilibrium value of s_i, g_i, and ϕ_i:

$$
\begin{aligned}
s_i^G &= \frac{(6kt+1)(\theta_i-\theta_j)}{6kt(18kt-4\lambda+3)} + \frac{1}{6k} \geq 0, && (27)\\
g_i^G &= \frac{\{6kt(2\lambda-1)-1\}(\theta_i-\theta_j)}{6kt(18kt-4\lambda+3)} \gtreqless 0, && (28)\\
\phi_i^G &= \frac{2\lambda(\theta_i-\theta_j)}{18kt-4\lambda+3} + \frac{1}{6k} \gtreqless 0, && (29)\\
p_i^G &= \frac{6kt+1}{18kt-4\lambda+3}(\theta_i-\theta_j) + c_i + t, && (30)\\
y_i^G &= \frac{1}{2} + \frac{6kt+1}{2t(18kt-4\lambda+3)}(\theta_i-\theta_j), && (31)
\end{aligned}
$$

where the superscript G indicates an equilibrium variable in a case with government advertising.[29] Note that if $6kt < (>) \ (2\lambda-1)$, the equilibrium share of Brand 1, which is represented by (31), is greater (smaller) than $\hat{x}^E$ and the equilibrium marginal consumer is located on the right (left) side of Point E in Figure 3.

We find that if quality gaps are absent ($\theta_1 = \theta_2$), the government does not have an incentive to send information ($g_i^G = 0$), which holds irrespective of the value of λ. Conversely, if quality gaps are present ($\theta_1 \neq \theta_2$), the amount and type of information provided by the government depends on the government's type (i.e., the value of λ). Consider first the case of $\lambda = 1$, the neutral government case. In this case, we have $g_1^G > 0$ and $g_2^G < 0$ for $6kt > 1$ (i.e., the equilibrium marginal consumer is located on the left of point $\hat{x}^E$), which implies that the government has an incentive to confirm Firm 1's misinformation and negate Firm 2's misinformation. The result may seem to contradict the previous finding that advertising competition yields socially excessive misinformation (see

[27]In detail, the second-order conditions of the government's maximization are

$$
\frac{\partial^2 W^*}{\partial g_i^2} = -2k - \frac{5-4\lambda}{18t} < 0, \quad \frac{\partial^2 W^*}{\partial g_i \partial g_j} = \frac{5-4\lambda}{18t} > 0,
$$
$$
\left(\frac{\partial^2 W^*}{\partial g_i^2}\cdot\frac{\partial^2 W^*}{\partial g_j^2}\right) - \left(\frac{\partial^2 W^*}{\partial g_i \partial g_j}\right)^2 = 4k^2 + \frac{2k(5-4\lambda)}{9t} > 0.
$$

Therefore, $\lambda < 5/4$ is sufficient for the second-order conditions to hold.

[28]As before, the firms choices regarding the amount of misinformation are strategic substitutes.

[29]The sign condition of $s_i^G \geq 0 \ \forall i = 1, 2$ comes from Assumption 2.

FIGURE 4 AROUND HERE

Section 2.4), but the result is reasonable. Because the costs of providing (mis)information for the firms and the government are separable, a certain amount of misinformation can be provided at a lower cost in a case with government advertising than in a case without it. Therefore, from $s_1^{SB} > 0$, we find that if $\lambda = 1$, the government sends positive information (or certifies the misinformation provided by the high-quality firm). However, $g_1^G > 0$, at the same time, makes the high-quality firm send more misinformation because g_1^G raises the market share of the high-quality brand, which increases the marginal benefits of sending misinformation for the high-quality firm.[30] However, if $\lambda = 1$ and $6kt < 1$, then $g_1^G < 0$ and $g_2^G > 0$ holds in equilibrium. This result occurs because if k and/or t are small (i.e., advertising competition is fierce), the amount of Firm 1's misinformation is so large that the equilibrium marginal consumer will be located on the right of point $\hat{x}^E$, which ensures the maximum welfare gain.

Next, consider a case of $\lambda = 0$, the case of an extremely consumerist policymaker. In equilibrium, we have $g_1^G < 0$ and $g_2^G > 0$: the government provides counter-information against the high-quality brand and positive information regarding the low-quality brand. This policy lowers (raises) the price of the high-quality (low-quality) brand and increases the aggregate consumer surplus.

In addition, we find an interesting relationship between the degree of government bias and the information provided by the firms and the government. Figure 4 illustrates the relationship between the amounts of (mis)information provided by the firms and the government in a case where brand quality is unequal ($\theta_1 > \theta_2$). The left panel of the figure illustrates a case with a large k and/or t such that $6kt > (2\lambda - 1)$, whereas the right panel illustrates a case with a small k and/or t such that $6kt < (2\lambda - 1)$. We can confirm the results from the figure. Specifically, from (28), we find that $g_1^G < 0$ and $g_2^G > 0$ hold if

$$\lambda < \bar{\lambda} \equiv \frac{6kt + 1}{12kt},$$

where $\bar{\lambda}$ is necessarily greater than 1/2. We find that if the policymaker's bias is smaller than $\bar{\lambda}$,

[30] This can be confirmed by the strategic relationship between g_1 and s_1, $ds_i/dg_i > 0$, as shown in (25).

then he/she attempts to negate Firm 1's misinformation and confirm Firm 2's misinformation. By doing so, the consumerist policymaker can lower the price of the more purchased high-quality brand and thereby make consumers better off.

4 Naïve and Smart Consumers

Thus far, we have considered a situation where all of the consumers are naïve enough that they always believe the misinformation about the brand qualities. In this section, we extend the model by considering smart consumers who can identify the true quality of the brand without being influenced by misinformation.

Suppose there is a proportion $\delta \in [0, 1]$ of naïve consumers and a proportion $(1 - \delta)$ of smart consumers at each point on the $x \in [0, 1]$ line. The total number of consumers is unity that is the same as before. Because the smart consumers can identify the true quality of the brands when making purchases, their perceived and ex-post utilities are indifferent and are defined by

$$U_{sm}(x) = \begin{cases} v_1 - p_1 - tx & \text{if buys Brand 1} \\ v_2 - p_2 - t(1-x) & \text{if buys Brand 2.} \end{cases}$$

The subscript sm indicates the variables for the smart consumers. The perceived utility of the naïve consumers $U_{na}(x)$ is the same as in (1).

The marginal naïve and smart consumers ($\hat{x}_{na}$ and $\hat{x}_{sm}$), who are indifferent between choosing Brands 1 and 2 are respectively given by

$$\hat{x}_{na} = \frac{(v_1 + s_1) - (v_2 + s_2) - (p_1 - p_2)}{2t} + \frac{1}{2}, \quad \hat{x}_{sm} = \frac{v_1 - v_2 - (p_1 - p_2)}{2t} + \frac{1}{2}. \tag{32}$$

Therefore, the demand for Brands 1 and 2 are $y_1 = \delta\hat{x}_{na} + (1-\delta)\hat{x}_{sm}$ and $y_2 = \delta(1 - \hat{x}_{na}) + (1 - \delta)(1 - \hat{x}_{sm})$, respectively. The second-stage equilibrium prices and outputs can be obtained by

$$p_i^\circ(s_i, s_j, \delta) = \frac{(\theta_i - \theta_j) + \delta(s_i - s_j)}{3} + c_i + t, \quad y_i^\circ(s_i, s_j, \delta) = \frac{(\theta_i - \theta_j) + \delta(s_i - s_j)}{3t} + \frac{1}{2}.$$

Substituting p_i° into (32), we have

$$\hat{x}_{na}^\circ = \frac{(\theta_1 - \theta_2) + (3 - 2\delta)(s_1 - s_2)}{6t} + \frac{1}{2}, \quad \hat{x}_{sm}^\circ = \frac{(\theta_1 - \theta_2) - 2\delta(s_1 - s_2)}{6t} + \frac{1}{2}.$$

FIGURE 5 AROUND HERE

Interestingly, $\hat{x}^{\circ}_{na}$ is increasing in s_1, but $\hat{x}^{\circ}_{sm}$ is decreasing in s_1.

The aggregate consumer surplus for consumer k (k =naïve, smart) is given by

$$CS^{\circ}_k = \int_0^{\hat{x}^{\circ}_k} [v_1 - tx - p^{\circ}_1]dx + \int_{\hat{x}^{\circ}_k}^1 [v_2 - t(1-x) - p^{\circ}_2]dx.$$

The effect of exogenous changes in misinformation on the naïve and smart consumers' consumer surpluses are respectively given by

$$\begin{aligned}
\frac{dCS^{\circ}_{na}}{ds_i} &= \underbrace{\frac{d\hat{x}^{\circ}_{na}}{ds_i}\{(v_1 - t\hat{x}^{\circ}_{na} - p^*_1) - (v_2 - t(1-\hat{x}^{\circ}_{na}) - p^*_2)\}}_{\text{mischoice effect} \neq 0} \\
&\quad \underbrace{+\int_0^{\hat{x}^*_{na}} \left(-\frac{\partial p^*_1}{\partial s_i}\right) dx + \int_{\hat{x}^*_{na}}^1 \left(-\frac{\partial p^*_2}{\partial s_i}\right) dx}_{\text{price}-\text{change effects}} \\
&= -\frac{(3-2\delta)(s_i - s_j)}{6t} - \frac{\delta\{(\theta_i - \theta_j) + (3-2\delta)(s_i - s_j))\}}{9t}, \\
\frac{dCS^{\circ}_{sm}}{ds_i} &= \underbrace{\frac{d\hat{x}^{\circ}_{sm}}{ds_i}\{(v_1 - t\hat{x}^{\circ}_{sm} - p^*_1) - (v_2 - t(1-\hat{x}^{\circ}_{sm}) - p^*_2)\}}_{\text{mischoice effect} = 0} \\
&\quad \underbrace{+\int_0^{\hat{x}^*_{sm}} \left(-\frac{\partial p^*_1}{\partial s_i}\right) dx + \int_{\hat{x}^*_{sm}}^1 \left(-\frac{\partial p^*_2}{\partial s_i}\right) dx}_{\text{price}-\text{change effects}} \\
&= \frac{\delta\{-(\theta_i - \theta_j) + 2\delta(s_i - s_j)\}}{9t}.
\end{aligned}$$

Because the smart consumers are never misled by misinformation, the mischoice effect of misinformation becomes zero for any value of s_i. Therefore, the smart consumers are only affected by misinformation through price-change effects. We also find that when evaluated at $s_1 = s_2$, an increase in misinformation about the high-quality (low-quality) brand always lowers (raises) the aggregate consumer surplus of the smart consumers.

Figure 5 illustrates the effect of an exogenous change in misinformation about the high-quality brand (Brand 1) and the responses by the naïve and smart consumers in cases of equal quality (the left panel) and unequal quality (the right panel). In the left panel of the figure, providing misinformation about the high-quality brand raises Brand 1's price but lowers Brand 2's price. As before, the marginal naïve consumer moves to the right (from $\hat{x}^O$ to $\hat{x}^+_{na}$). However, the marginal smart consumer moves to the left (from $\hat{x}^O$ to $\hat{x}_{sm}^+$) because he/she precisely assesses Brand 1's quality. The change in the consumer surplus of naïve consumers is represented by area $-B-C-D-E+H$ (the mischoice effect is $-B-E$ and the price-change effect is $-C-D+H$), whereas the change in the smart consurmers' surplus is represented by $-A-C+F+G+H$. In this case, the former is negative, but the latter is positive. In total, $\delta(-B-C-D-E+H)+(1-\delta)(-A-C+F+G+H)$ represents the change in the aggregate consumer surplus. Regarding the individual consumer surplus, the smart consumers who are located at $[0, \hat{x}_{sm}^+]$ lose because of the misinformation, whereas those at $[\hat{x}^O, 1]$ gain. In contrast, the naïve consumers who are located at $[0, \hat{x}_{na}^+]$ lose and those who at $[\hat{x}_{na}^+, 1]$ gains from the misinformation. The industry profits are changed by $\delta(C+D+E-H)+(1-\delta)(C-F-G-H)$. Therefore, the welfare change is simply given by $-\delta B-(1-\delta)A$. In the right panel of Figure 5, the misinformation about Brand 1 (v_1 to v_1^+) changes the consumer surplus of the smart consumers by $-A-C+F+G+H$ and that of the naïve consumers by $B-C-D-E+H$. The industry profits are changed by $\delta(C+D+E-H)+(1-\delta)(C-F-G-H)$. Therefore, the welfare change is simply given by $\delta B-(1-\delta)A$.

In the following, we investigate the endogenous misinformation produced by the advertising competition between firms. To ensure that interior solutions are obtained for y_i and s_i, we slightly modify Assumption 1 as follows.

Assumption 3. For all $\theta_i > 0$, it holds that $0 \leq (\theta_1 - \theta_2) \leq \dfrac{9kt-\delta^2}{3k}$.

Deriving the subgame-perfect equilibrium in the same manner as in Section 2.3, we find the following equilibrium values of misinformation, prices, and outputs:

$$s_i^M = \delta\left[\frac{\theta_i-\theta_j}{2\left(9kt-\delta^2\right)}+\frac{1}{6k}\right], \quad p_i^M = \frac{3kt(\theta_i-\theta_j)}{9kt-\delta^2}+c_i+t, \quad y_i^M = \frac{3k(\theta_1-\theta_2)}{2\left(9kt-\delta^2\right)}+\frac{1}{2},$$

where the superscript M refers to an equilibrium variable in this case (i.e., naïve and smart consumers coexist in the economy). Obviously, $s_i^M = s_i^S$ and $p_i^M = p_i^S$ hold if $\delta = 1$, and $s_i^M = 0$ and $p_i^M = p_i^O$ hold if $\delta = 0$. We find that $dp_1^M/d\delta \geq 0$ ($dp_2^M/d\delta \leq 0$) and $dy_1^M/d\delta \geq 0$ ($dy_2^M/d\delta \leq 0$), which suggests that the price and consumption of the high-quality (low-quality) brand increases (decreases) as the proportion of naïve consumers increases. This relationship exists because an increase in the proportion of naïve consumers spurs advertising competition between the firms, which gives the high-quality (low-quality) firm a superior (inferior) position in the subsequent price competition. Because the smart consumers know that the high price of the high-quality brand is partly attributed to the behavior of the naïve consumers, who are deceived by misinformation, the price and market share of the the high-quality brand is more likely to decrease as more consumers become smart shoppers.

The equilibrium profits, the consumer surpluses for the naïve and smart consumers, and the welfare can be derived as follows.

$$\pi_i^M = \frac{\left(18kt-\delta^2\right)\left[3k\left(3t+\theta_i-\theta_j\right)-\delta^2\right]^2}{36k\left(9kt-\delta^2\right)^2}, \tag{33}$$

$$CS_{sm}^M = \frac{(1-\delta)\left(3kt-\delta^2\right)^2}{4t\left(9kt-\delta^2\right)^2}(\theta_1-\theta_2)^2+\frac{(1-\delta)(\theta_1+\theta_2)}{2}-\frac{5t(1-\delta)}{4}, \tag{34}$$

$$CS_{na}^M = \frac{\delta\left(3kt-\delta-\delta^2\right)\left(3kt+\delta-\delta^2\right)}{4t\left(9kt-\delta^2\right)^2}(\theta_1-\theta_2)^2+\frac{\delta\left(\theta_1+\theta_2\right)}{2}-\frac{5\delta t}{4}, \tag{35}$$

$$\begin{aligned} W^M &= \delta CS_{na}^M+(1-\delta)CS_{sm}^M+\sum_{i=1}^{2}\pi_i^M \\ &= \frac{kt\left(45kt-8\delta^2\right)-\delta^3(1-\delta)}{4t\left(9kt-\delta^2\right)^2}(\theta_1-\theta_2)^2+\frac{\theta_1+\theta_2}{2}-\frac{\delta^2}{18k}-\frac{1}{4}t. \end{aligned} \tag{36}$$

First, we compare the consumer surplus of the naïve and smart consumers with that of the consumers in Case O. We obtain

$$CS_{sm}^M - CS^O = -\frac{\delta^2\left(9kt-2\delta^2\right)}{9t\left(9kt-\delta^2\right)^2}(\theta_1-\theta_2) \gtreqless 0 \;\Leftrightarrow\; 9kt \lesseqgtr 2\delta^2,$$

$$CS_{na}^M - CS^O = -\frac{\delta^2\left\{36kt+\left(9-8\delta^2\right)\right\}}{36t\left(9kt-\delta^2\right)^2}(\theta_1-\theta_2) \leq 0.$$

The smart consumers, on average, gain (lose) from advertising competition if the proportion of naïve consumers is large (small) and the advertising costs are small (large), whereas the naïve consumers are always made worse off. However, the aggregate consumer surplus for all consumers $CS^M \equiv \delta CS_{na}^M+(1-\delta)CS_{sm}^M$ is necessarily smaller than CS^O. Therefore, advertising competition reduces the aggregate consumer surplus even if there are some smart consumers in the economy.

Second, we investigate the effect of a decrease in the proportion of naïve consumers on the equilibrium outcomes. Here, the effect of a decrease in δ can be considered as a government policy that educates consumers. Differentiating (33) in δ yields the following:

$$\frac{d\pi_1^M}{d\delta} = -\frac{\delta}{18k} + \frac{3\delta kt}{\left(9kt-\delta^2\right)^2}(\theta_1-\theta_2) + \frac{\delta k\left(27kt-\delta^2\right)}{2\left(9kt-\delta^2\right)^3}(\theta_1-\theta_2)^2, \tag{37}$$

$$\frac{d\pi_2^M}{d\delta} = -\frac{\delta}{18k} - \frac{3\delta kt}{\left(9kt-\delta^2\right)^2}(\theta_1-\theta_2) + \frac{\delta k\left(27kt-\delta^2\right)}{2\left(9kt-\delta^2\right)^3}(\theta_1-\theta_2)^2 \leq 0, \tag{38}$$

The profits of the low-quality firm are a decreasing function of δ because an increase in δ leads to greater advertising competition, which decreases the price and consumption of the low-quality brand. However, the effect of an increase in δ on the profits of the high-quality firm is ambiguous. An increase in δ induces more advertising competition that is unnecessary and costly. However, if the quality gaps are large, this increase also gives a competitive advantage in advertising and price competition to the high-quality firm. Therefore, the greater the quality gaps are, the more likely that an increase in the proportion of naïve consumers benefits the high-quality firm.[31] However, if there are no quality gaps ($\theta_1 = \theta_2$), we have $d\pi_i^M/d\delta = -\delta/(18k) < 0$ for both $i = 1, 2$ from (37) and (38), which indicates that an increase in the proportion of naïve consumers reduces both firms' profits due to the unprofitable advertising competition.

Next, differentiating the aggregate consumer surplus (CS^M) in δ yields

$$\frac{dCS^M}{d\delta} = -\frac{\delta\left\{3kt\left(24kt+\delta(9-8\delta)\right)+\delta^3\right\}}{4t\left(9kt-\delta^2\right)^3}(\theta_1-\theta_2)^2 \leq 0,$$

which implies that an increase in the proportion of naïve consumers reduces aggregate consumer surplus. The naïve consumers suffer disutility from the mischoices that are directly induced by increased misinformation. Despite having never been misguided by misinformation, the smart consumers also indirectly suffer disutility from price changes (i.e., the increase (decrease) in the price of the more (less) heavily consumed Brand 1 (Brand 2)).

Finally, differentiating W^M in δ yields

$$\frac{dW^M}{d\delta} = -\frac{\delta}{9k} - \frac{\delta\left(36k^2t^2 - 27kt\delta + 20kt\delta^2 - \delta^3\right)}{4t\left(9kt-\delta^2\right)^3}\left(\theta_1-\theta_2\right)^2 \leq 0.$$

[31]In detail, π_1^M is an increasing function of δ if

$$(\theta_1-\theta_2) > \frac{\left(9kt-\delta^2\right)^2}{3k\left(27kt-\delta^2\right)}.$$

The sign condition is from Assumption 3.[32] Therefore, an increase in the proportion of naïve consumers always reduces social welfare.

The results can be summarized as follows. A policy of educating consumers (to reduce δ) does not affect the consumers but benefits both firms if there are no quality gaps between the brands. If there are some quality gaps between the brands, the policy improves aggregate consumer surplus, the profits of the low-quality firm, and welfare. However, the policy may reduce the profits of the high-quality firm. In addition, because the effect of the policy on individual consumer surpluses is diverse, the policy may have difficulty obtaining enough political support.[33]

5 Conclusion

In this study, we investigate misleading advertising competition in a Hotelling model where two firms compete for market share. Although misinformation makes some consumers worse off, a certain amount of misinformation may improve social welfare by removing the inefficiency due to the misallocation of products, even if the misinformation does not increase total consumption. We show that the quality gaps between the brands play a crucial role in determining the effect of misinformation on the firms' profits, the aggregate and individual consumer surpluses, and

[32]From Assumption 3, the maximum difference between $(\theta_1 - \theta_2)$ is $(9kt - \delta^2)/(3k)$. Thus, we derive

$$\frac{dW^M}{d\delta} \leq \left.\frac{dW^M}{d\delta}\right|_{(\theta_1-\theta_2)=\frac{9kt-\delta^2}{3k}} = -\frac{\delta^2\left\{3kt\left(9-8\delta\right)+\delta^2\right\}}{36k^2t\left(9kt-\delta^2\right)} \leq 0,$$

which implies that $dW^M/d\delta \leq 0$ always holds.

[33]It is interesting to compare our results with those of Hattori and Higashida (2012), who show that an increase in the proportion of naïve consumers necessarily increases the profits of the firms that provide misinformation about their product qualities and may improve social welfare. The difference between our findings and their results depends on the types of the advertising and whether misinformation can expand market size. In their model, firms engage in "generic" advertising, which sends misinformation about the product category. Hence, there are some positive advertising externalities among the firms. In addition, in their study, the misinformation provided by the firms can expand the total market size (i.e., total consumption), which mitigates the underprovision of goods that results from oligopolistic competition. In contrast, in our study, each firm advertises its own product to capture its rival firm's market share, and the total market size is fixed. Thus, there is no efficiency gain from the increase in misinformation caused by an increase in the proportion of naïve consumers.

welfare.

The main contribution of this article is to determine who benefits and who loses from misinformation due to misleading advertising competition between firms and related regulations. We use simple analytical and graphical analyses. First, we show that the amount of misinformation endogenously produced by misleading advertising competition is excessive from a welfare perspective. If there are no quality gaps between the brands, advertising competition harms both firms but does not affect consumers. However, if there are certain quality gaps between the brands, advertising competition benefits the high-quality firm and the consumers who prefer the low-quality brand.

We also investigate the effects of several regulatory policies. Both advertising and ad valorem taxes reduce the degree of advertising competition between the brands and have the same impact on the degree of advertising competition and social welfare. However, these taxes differ in the distributional impacts on consumers and firms. We also show that bias in the policymaker's objective plays a crucial role in the prohibition of misinformation and the government's provision of information. In particular, a consumerist policymaker may prefer to regulate misinformation about the high-quality brand, but he/she may overlook or even certify misinformation about the low-quality brand.

Finally, we extend the basic model by considering a case with heterogeneous consumers (i.e., naïve and smart consumers) to investigate how misleading advertising competition between brands affects the consumers' utilities and the firms' profits. Advertising competition necessarily reduces the utility of naïve consumers but may improve the utility of smart consumers on average. We also show that if the quality gaps between the brands are small, a small decrease in the proportion of naïve consumers will be Pareto-improving by weakening the incentive to engage in unprofitable advertising competition. Therefore, in this case, a government policy of educating consumers works well.

Appendix

■ In the Appendix section, for notational convenience, we define $\Delta_\theta \equiv (\theta_1 - \theta_2) \geq 0$.

□ **Equilibrium under advertising taxes**

The equilibrium profits of each firm, the aggregate consumer surplus, and welfare can be obtained as follows:

$$\begin{aligned}
\pi_1^{\tau_a} &= \frac{18kt(1+\tau_a)-1}{36k(1+\tau_a)} + \frac{18kt(1+\tau_a)-1}{6\left\{9kt(1+\tau_a)-1\right\}}\Delta_\theta + \frac{k(1+\tau_a)\left\{18kt(1+\tau_a)-1\right\}}{4\left\{9kt(1+\tau_a)-1\right\}^2}\Delta_\theta^2,\\
\pi_2^{\tau_a} &= \frac{18kt(1+\tau_a)-1}{36k(1+\tau_a)} - \frac{18kt(1+\tau_a)-1}{6\left\{9kt(1+\tau_a)-1\right\}}\Delta_\theta + \frac{k(1+\tau_a)\left\{18kt(1+\tau_a)-1\right\}}{4\left\{9kt(1+\tau_a)-1\right\}^2}\Delta_\theta^2,\\
CS^{\tau_a} &= \frac{3k(1+\tau_a)\left\{3kt(1+\tau_a)-2\right\}}{4\left\{9kt(1+\tau_a)-1\right\}^2}\Delta_\theta^2 + \frac{\theta_1+\theta_2}{2} - \frac{5}{4}t,
\end{aligned}$$

and $W^{\tau_a} = CS^{\tau_a} + \sum \pi_i^{\tau_a} + \tau_a k\left\{(s_1^{\tau_a})^2 + (s_2^{\tau_a})^2\right\}$. Thus, differentiating the above in τ_a at $\tau_a = 0$, we obtain the comparative static results presented in the main body of the article.

□ **Equilibrium under ad valorem taxes** The equilibrium profits of each firm, the aggregate consumer surplus, and welfare can be obtained as follows:

$$\begin{aligned}
\pi_1^{\tau_p} &= \frac{(1-\tau_p)\left\{18kt-(1-\tau_p)\right\}}{36k} + \frac{(1-\tau_p)\left\{18kt-(1-\tau_p)\right\}}{6\left\{9kt-(1-\tau_p)\right\}}\Delta_\theta + \frac{k(1-\tau_p)\left\{18kt-(1-\tau_p)\right\}}{4\left\{9kt-(1-\tau_p)\right\}^2}\Delta_\theta^2,\\
\pi_2^{\tau_p} &= \frac{(1-\tau_p)\left\{18kt-(1-\tau_p)\right\}}{36k} - \frac{(1-\tau_p)\left\{18kt-(1-\tau_p)\right\}}{6\left\{9kt-(1-\tau_p)\right\}}\Delta_\theta + \frac{k(1-\tau_p)\left\{18kt-(1-\tau_p)\right\}}{4\left\{9kt-(1-\tau_p)\right\}^2}\Delta_\theta^2,\\
CS^{\tau_p} &= \frac{3k\left\{3kt-2(1-\tau_p)\right\}}{4\left\{9kt-(1-\tau_p)\right\}}\Delta_\theta^2 - \frac{c\tau_p}{1-\tau_p} + \frac{\theta_1+\theta_2}{2} - \frac{5}{4}t,
\end{aligned}$$

and $W^{\tau_p} = CS^{\tau_p} + \sum \pi_i^{\tau_p} + \tau_p \sum p_i^{\tau_p} y_i^{\tau_p}$. Thus, differentiating them in τ_p at $\tau_p = 0$, we obtain the comparative static results presented in the main body.

□ **Equilibrium under partial regulation** First, we derive the equilibrium outcomes in case PH, where the government only prohibits the high-quality firm from generating misleading advertising. From (6) we have $s_1^{PH} = 0$ and $s_2^{PH} = (3t - \Delta_\theta)/(18kt - 1)$. Substituting them into Π^* and CS^* yields

$$\begin{aligned}
\Pi^{PH} &= \frac{t(324k^2t^2 - 45kt + 2)}{(18kt-1)^2} - \frac{6kt}{(18kt-1)^2}\Delta_\theta + \frac{k(36kt-1)}{(18kt-1)^2}\Delta_\theta^2\\
CS^{PH} &= \frac{(6k\Delta_\theta - 1)\left\{2(3kt-1)\Delta_\theta + 5t\right\}}{4(18kt-1)^2} + \frac{\theta_1+\theta_2}{2} - \frac{5}{4}t,
\end{aligned}$$

and $W^{PH} = CS^{PH} + \Pi^{PH}$. Thus, we have

$$\begin{aligned}
\Pi^{PH} - \Pi^O &= -\frac{(3t-\Delta_\theta)\left\{3t(9kt-1) + (27kt-1)\Delta_\theta\right\}}{9t(18kt-1)^2} < 0,\\
CS^{PH} - CS^O &= \frac{(3t-\Delta_\theta)\left\{(72kt+1)\Delta_\theta - 15t\right\}}{36t(18kt-1)^2} \gtreqless 0 \;\Leftrightarrow\; \Delta_\theta \gtreqless \frac{15t}{72kt+1},\\
W^{PH} - W^O &= -\frac{(3t-\Delta_\theta)\left\{(3t-\Delta_\theta) + 108kt^2 + 4(9kt-1)\Delta_\theta\right\}}{36t(18kt-1)^2} < 0.
\end{aligned}$$

Because $CS^O \geq CS^S$ necessarily holds from (17), we have $CS^{PH} \geq CS^O > CS^S$ for $\Delta_\theta \geq (15t)/(72kt+1)$, $\Pi^{PH} < \Pi^O$, and $W^{PH} < W^O$.

Then, we derive the equilibrium outcomes in case PL, where the government implements $s_2 = 0$. From (6) we have $s_1^{PL} = (3t + \Delta_\theta)/(18kt - 1)$ and $s_2^{PH} = 0$. Substituting them into Π^* and CS^* yields

$$\begin{aligned} \Pi^{PL} &= \frac{t(324k^2t^2 - 45kt + 2)}{(18kt-1)^2} + \frac{6kt}{(18kt-1)^2}\Delta_\theta + \frac{k(36kt-1)}{(18kt-1)^2}\Delta_\theta^2 \\ CS^{PL} &= \frac{(6k\Delta_\theta + 1)\{2(3kt-1)\Delta_\theta - 5t\}}{4(18kt-1)^2} + \frac{\theta_1 + \theta_2}{2} - \frac{5}{4}t, \end{aligned}$$

and $W^{PL} = CS^{PL} + \Pi^{PL}$. Thus, we have

$$\begin{aligned} \Pi^{PL} - \Pi^O &= \frac{(3t+\Delta_\theta)\{(27kt-1)\Delta_\theta - 3t(9kt-1)\}}{9t(18kt-1)^2} \gtreqless 0 \;\Leftrightarrow\; \Delta_\theta \gtreqless \frac{3t(9kt-1)}{27kt-1} \\ CS^{PL} - CS^O &= -\frac{(3t+\Delta_\theta)\{15t + (72kt+1)\Delta_\theta\}}{36t(18kt-1)^2} < 0, \\ W^{PL} - W^O &= -\frac{(3t+\Delta_\theta)\{\Delta_\theta + 15t + 4(9kt-1)(3t-\Delta_\theta))\}}{36t(18kt-1)^2} < 0. \end{aligned}$$

Furthermore, comparing Π^{PL} and Π^S, we have

$$\Pi^{PL} - \Pi^S = \frac{(9kt - 1 - 3k\Delta_\theta)\left\{(9kt-1) + 18kt(9kt-1)^2 + 3k\Delta_\theta\left[1 + 54(9kt-1)\right]\right\}}{18k(18kt-1)^2(9kt-1)^2} > 0,$$

where the sign comes from Assumption 1. Therefore, we have $\Pi^{PL} \geq \max[\Pi^O, \Pi^S]$ when $\Delta_\theta > \{3t(9kt-1)\}/(27kt-1)$, $CS^{PL} < CS^O$, and $W^{PL} < W^O$.

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FIGURE 1

EXOGENOUS CHANGES IN MISINFORMATION FOR HIGH- OR LOW-QUALITY BRAND

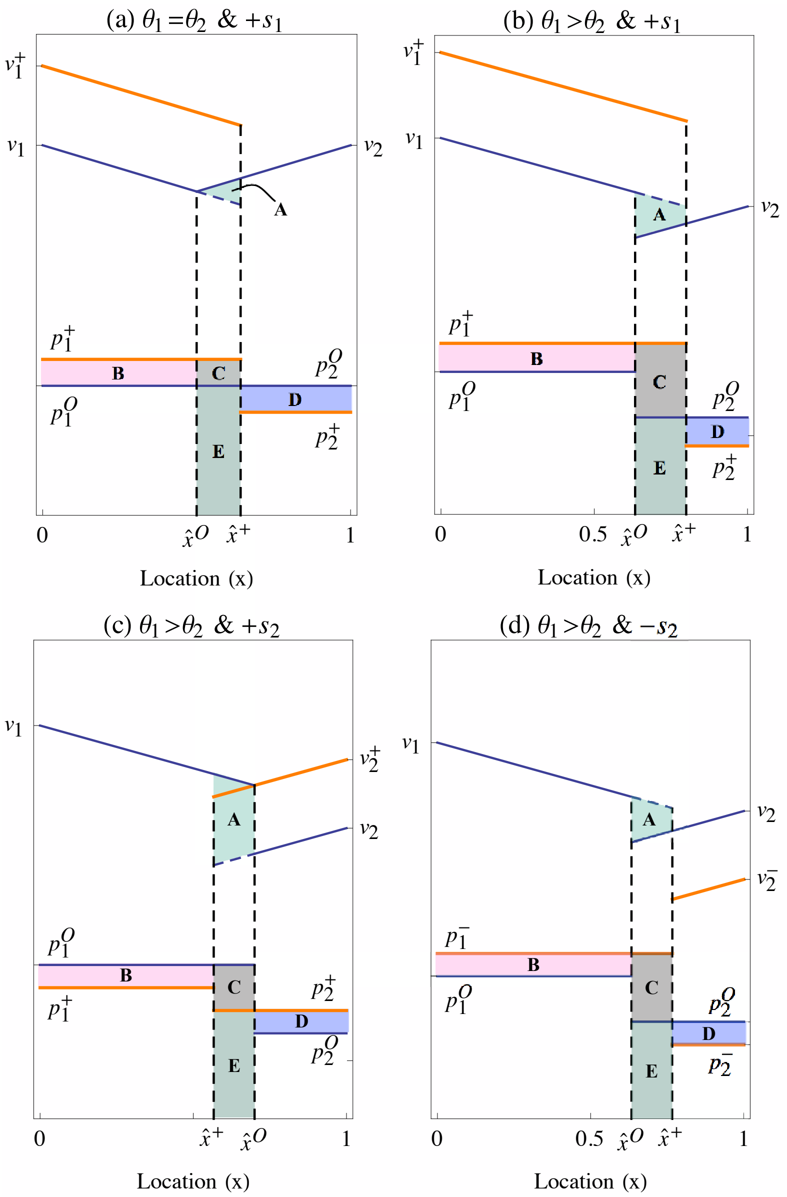

FIGURE 2

WELFARE EFFECT OF ADVERTISING COMPETITION

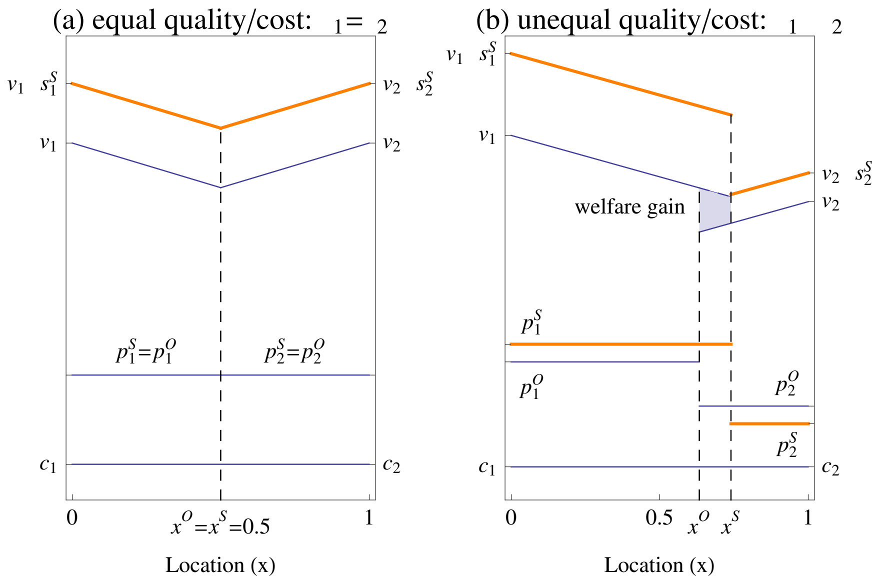

FIGURE 3

THE SECOND-BEST MISINFORMATION

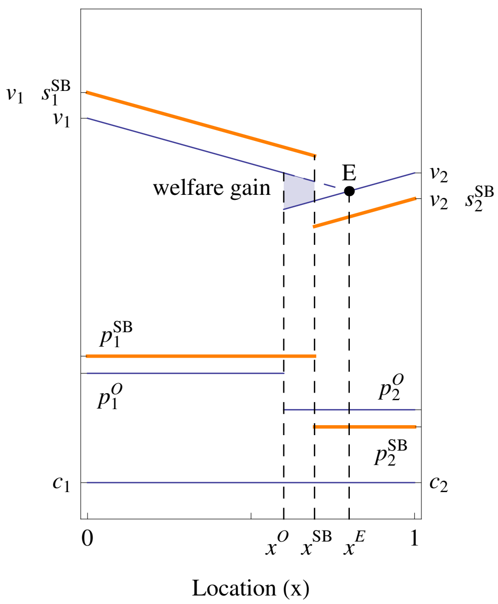

FIGURE 4

PRIVATE AND PUBLIC (MIS)INFORMATION AND GOVERNMENT BIAS

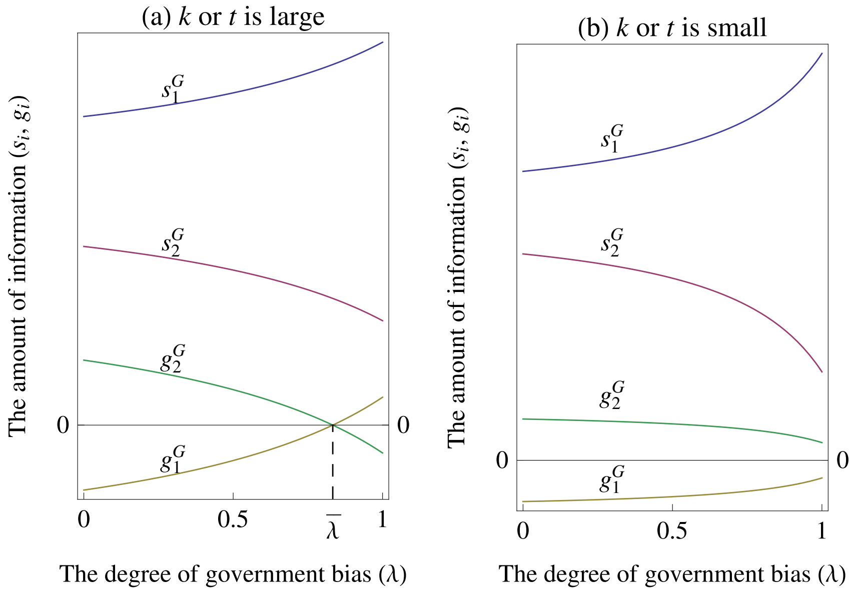

FIGURE 5

THE EFFECT OF MISINFORMATION ON NAIVE AND SMART CONSUMERS