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Hiroaki Ino

School of Economics, Kwansei Gakuin University

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SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho

Nishinomiya 662-8501, Japan

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Hiroaki Ino*
School of Economics, Kwansei Gakuin University

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Abstract

Which is better off for the patentee to license its technology by fixed fee or unit royalties? Kamien and Tauman [8] showed that the fixed fee scheme brings greater private value of the patent in the linear model. We extend their analysis into a general model. Then, the simple fact that the model allows a increasing marginal cost supports the unit royalties scheme. More concretely, the unit royalties scheme is superior to the fixed fee scheme when the number of firms is large.

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*Address: 1-155 Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan. E-mail: hiroakiino@04.alumni.u-tokyo.ac.jp.

1 Introduction

When a firm successfully innovates production process and obtains its patent, which is better off for the patentee to license its technology by fixed fee or unit royalties? Kamien and Tauman [8] showed that the fixed fee scheme brings greater private value of the patent in the linear model. Since this finding contradicts the reality that the unit royalty scheme is adopted in many cases of patent licensing, many studies have been tackling to identify economic aspects that could overturn said finding. Gallini and Wiright [4] and Beggs [2] consider the asymmetric information between licensor and licensee. Wang [17] clarifies the outsider problem of licensor. Sen [12] finds out the integer problem of the number of firms.

In this paper, we extend Kamien and Tauman's analysis into a general model by allowing general cost and demand structure. As for an analysis which intend to generalize a license model, Kamien et al. [6] consider general demands. However, the cost functions are still linear in their model. In our extension, we find the simple fact that the model allows a increasing marginal cost supports the unit royalties scheme. More concretely, the unit royalties scheme is superior to the fixed fee scheme when the number of firms is sufficiently large (when the market is sufficiently competitive).[1]

This finding is explained as follows from the perspective of economics based on the model structure. Under the royalty system, a sales strategy that may be summarized as "small volume sales to a large number of producers" tends to be adopted in order to avoid the problem of capacity limitation (the possibility of decrease in marginal productivity) by diffusing the patented technology among a large number of producers. This allows effective production by those producers, increasing the value of the new technology in the market as a whole. The more competitive the market is, the larger the number of producers to whom the new technology could be licensed, leading to wider use of the technology. Consequently, the license revenues would grow. On the other hand, under the fixed fee system, an opposite sales strategy tends to be adopted. The patentee should restrict the number of licensees to absorb the profit in lump-sum way. In other words, new technology would be disclosed only to a certain degree,

[1] As for the limit results in the field, see Proposition 8 in Kamien and Tauman [8], Kamien and Tauman [7], Taumana and Watanabe [14] and Kishimoto, et al. [9] among others. All of them consider liner costs.

leading to rather monopolistic use of the technology. This would lead to mass production by each licensee, who tends to face the problem of capacity limitation as a result. An increase in marginal cost would reduce the effectiveness of the use of new technology, decreasing the value of the technology in the market as a whole. This structure explains why in contrast to a case where the marginal cost remains constant, in a competitive market with a large number of producers participating therein, the license revenues under the royalty system exceed those under the fixed fee system.

The rest of paper is organized as follows. In Section 2, before considering the license model, we investigate the fundamental futures of the market structure where two (old and new) technologies are used. Then, Section 3 analyzes the unit-royalty scheme and shows where the patentee's license revenue converges as the number of firms goes to infinity. Section 4 analyzes the fixed-fee scheme and by comparing it to the unit-royalty scheme, shows the presented result. Section 5 provides the two related further discussions, the endogenous number of firms and the two-part tariff. Section 6 concludes the paper.

2 Basic model, Monotone comparative statics, and Limit

We will formalize the three-stage game, following Kamien and Tauman [8], where one patentee offers a license contract of its new technology in the first stage, producers decide whether or not to buy the license simultaneously in the second stage, and the licensed producers and the unlicensed producers compete in Cournot fashion[2] in the third stage. In this paper, we consider the situation where the patentee does not produce itself (outsider) and the producers can use old technology even if they are unlicensed. First, in this section, we setup the general model to investigate the fundamental structure of the third stage.

Basic setting. The basic setting of market competition analyzed in the third stage is as follows. The set of producers is $\mathbb{N} = \{1, 2, \ldots, n\}$, where $n \in \mathbb{Z}_{++}$. The strategy of of producer $i \in \mathbb{N}$ is represented by $q_i \in [0, \bar{Q}]$, where $\bar{Q} \in \mathbb{R}_{++}$. The payoff function of producer i is $P(Q)q_i - C_{d_i}(q_i) - F_{d_i}$. The first-order differentiable function $P(Q) : \mathbb{R}_+ \mapsto \mathbb{R}_+$, where $Q =$

[2] As for the analysis of Bertrand competition, see Muto [10].

$\sum_{k=1}^{n} q_k$, is inverse demand which satisfies $P'(Q) < 0$ when $Q < \bar{Q}$ and $P(Q) = 0$ when $Q \geq \bar{Q}$. The first-order differentiable function $C_{d_i}(q_i) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is variable cost which satisfies $C'_{d_i}(q_i) > 0$ and F_{d_i} is fixed cost, where $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ is the parameter which represents firms' cost differences. The revenue $P(Q)q_i$ has decreasing difference in (q_i, q_j),[3] where $j \in \mathbb{N}$ and $j \neq i$.

Cournot competition. Consider the situation wherein the producers are engaged in Cournot competition. Then, the system of n first-order conditions is

$$P(Q) + P'(Q)q_i \leq C'_{d_i}(q_i) \qquad i = 1, \ldots n \tag{1}$$

with equality if $q_i > 0$. Denote the solution of this system (1) by $q_i^*(\mathbf{d})$, which corresponds to the Cournot equilibrium output of producer i. Let producer $i's$ Cournot equilibrium profit excluding fixed cost be $\pi_i^*(\mathbf{d}) = P(Q^*(\mathbf{d}))q_i^*(\mathbf{d}) - C_{d_i}(q_i^*(\mathbf{d}))$, where $Q^*(\mathbf{d}) = \sum_{k=1}^{n} q_k^*(\mathbf{d})$.

Then, by our setting, we can easily implement the monotone comparative statics[4] with respect to producer i's technology parameter d_i.

Lemma 1 *Suppose that $C_{d_i}(q_i)$ has increasing difference in (q_i, d_i). Then, (i) $q_i^*(\mathbf{d})$ is decreasing in d_i, (ii) $q_j^*(\mathbf{d})$ is increasing in d_i, (iii) $Q^*(\mathbf{d})$ is decreasing in d_i, (iv) $\pi_i^*(\mathbf{d})$ is decreasing in d_i, and (v) $\pi_j^*(\mathbf{d})$ is increasing in d_i, where $j \neq i$.*

Since $C_{d_i}(q_i)$ is first-order differentiable, the concept "increasing difference" means that the derivative $C'_{d_i}(q_i)$ is increasing in d_i for all q_i. Thus, Lemma 1 implies the well-known results in Cournot equilibrium that when one producer's marginal cost increases, its own output and profit decrease, the total output decreases, and its rival's output and profit increase.

Two types of costs. Now, consider that there are two groups of producers, $\mathbb{S}$ and $\mathbb{N}\backslash\mathbb{S}$, where $\mathbb{S} \subseteq \mathbb{N}$. Hereafter, we regard the group of licensed producers as $\mathbb{S}$ and the unlicensed producers

[3] This is equivalent to the stability condition $P'(Q) + P''(Q)q_i \leq 0$ if P is second-order differentiable.

[4] In our basic setting, the abovementioned Cournot competition is a submodular game (Note that $C_{d_i}(q_i)$ is lower semicontinuous in q_i since it is first-order differentiable). Then, as seen in the following lemma, the concept of increasing (decreasing) difference is a well-known condition for the monotone results of comparative statics. See among others, Topkis [15] for modularity and monotone comparative statics, and Vives [16] for the application to oligopoly models.

as $\mathbb{N} \setminus \mathbb{S}$. Thus, these groups differ with respect to their cost functions. Let d_i be the indicator function which satisfies

$$d_i = \begin{cases} 0 & \text{if } i \in \mathbb{S} \\ 1 & \text{if } i \notin \mathbb{S}. \end{cases}$$

Hence, if $C_{d_i}(q_i)$ has increasing difference in (q_i, d_i), the producers in the group $\mathbb{S}$ have the lower marginal cost than the rest of producers. Since all the producers in the same group are respectively identical, each producer's Cournot equilibrium output in the same group is symmetric.[5] Thus, letting the number of elements in $\mathbb{S}$ by s and given the total number of firms n, we can denote these symmetric equilibrium output in each group by $q_0^n(s) = q_i^*(\mathbf{d})$ if $d_i = 0$ ($i \in \mathbb{S}$) and $q_1^n(s) = q_i^*(\mathbf{d})$ if $d_i = 1$ ($i \in \mathbb{N} \setminus \mathbb{S}$). Let each firm's equilibrium profit excluding fixed cost in each group be $\pi_{d_i}^n(s) = P(Q^n(s))q_{d_i}^n(s) - C_{d_i}(q_{d_i}^n(s))$, where $Q^n(s) = sq_0^n(s) + (n-s)q_1^n(s)$.

Under these setting, the following results are obtained as the corollary of Lemma 1.

Corollary 1 *Suppose that $C_{d_i}(q_i)$ has increasing difference in (q_i, d_i). Then, (i) $q_0^n(s) \geq q_1^n(s-1)$, (ii) $q_{d_i}^n(s) \leq q_{d_i}^n(s-1)$ for $d_i = 0, 1$, (iii) $Q^n(s) \geq Q^n(s-1)$, (iv) $\pi_0^n(s) \leq \pi_1^n(s-1)$, and (v) $\pi_{d_i}^n(s) \leq \pi_{d_i}^n(s-1)$ for $d_i = 0, 1$.*

Note that from (i) and (ii) ((iv) and (v)) of this corollary, we have $q_0^n(s) \geq q_1^n(s)$ ($\pi_0^n(s) \geq \pi_1^n(s)$). From (i) to (iii) of this corollary, $sq_0^n(s) \geq (s-1)q_0^n(s-1)$ and $(n-s)q_1^n(s) \leq (n-s+1)q_1^n(n-s+1)$.

Limit quantities. Does the output of each producer become infinitesimally small if the number of producers n is large? The limit outputs will differ depending on which group's members diverges to infinity. By comparing the levels of first-unit marginal costs of two groups, let us denote $\bar{d} = \max_{d \in \{0,1\}} \{C_d'(0)\}$ and $\underline{d} = \min_{d \in \{0,1\}} \{C_d'(0)\}$. Indeed, each producer's output in the group with the higher first-unit marginal cost always converges to zero when n diverges to infinity. On the other hand, to shrink each producer's output in the group with the lower first-unit marginal cost, the number of its own group's members must be large.

Lemma 2 *Take any sequence $\{s_n\}_{n=1}^{\infty}$ with $s_n \leq n$. Then, we have (i) $\forall \epsilon > 0$, $\exists \bar{n}$, $\forall n \geq \bar{n}$, $q_{\bar{d}}^n(s_n) < \epsilon$. In addition, if $\lim_{n \to \infty}(1-\underline{d})s_n + \underline{d}(n - s_n) = \infty$, we have (ii) $\forall \epsilon > 0$, $\exists \bar{n}$, $\forall n \geq \bar{n}$, $q_{\underline{d}}^n(s_n) < \epsilon$.*

[5] Rosen [11] proved that this Cournot game has the unique Nash equilibrium in more general setting. Therefore, there are no solutions except for the symmetric equilibrium that we focus on.

Proof (i) First, we prove that $\forall \epsilon > 0$, $\exists \bar{n}$, $\forall n \geq \bar{n}$, $q_0^n(s_n) < \epsilon$ or $q_1^n(s_n) < \epsilon$. Suppose the contrary, $\exists \epsilon > 0, \forall \bar{n}, \exists n \geq \bar{n}$, $q_0^n(s) \geq \epsilon$ and $q_1^n(s) \geq \epsilon$. Then, we can make up a subsequence $\left\{q_0^{m(n)}\left(s_{m(n)}\right), q_1^{m(n)}\left(s_{m(n)}\right)\right\}$ of $\{q_0^n(s_n), q_1^n(s_n)\}$ such that $q_0^{m(n)}(s_m(n)) \geq \epsilon$ and $q_1^{m(n)}(s_m(n)) \geq \epsilon$. Then,

$$
\begin{aligned}
P(m(n)\epsilon) + P'(m(n)\epsilon)\epsilon &\geq P(Q^n(s_n)) + P'(Q^n(s_n))q_{d_i}^n(s_n) && (\because q_0^n(s) \geq \epsilon \text{ and } q_1^n(s) \geq \epsilon) \\
&= C_0'(q_0^n(s_n)) && (\because \text{the first-order condition (2)}) \\
&\geq C_0'(\epsilon). && (\because q_0^n(s) \geq \epsilon)
\end{aligned}
$$

When $m(n) \to \infty$, the left hand side must become negative – contradiction.

Next consider the case $q_{\underline{d}}^n(s_n) < \epsilon$. Take $\epsilon' \in (0, \epsilon]$ such that $\forall q \leq \epsilon'$, $C_{\underline{d}}'(q) < C_{\bar{d}}'(q)$. Then, we must have $q_{\bar{d}}^n(s_n) < \epsilon'$ since the supposition that $q_{\bar{d}}^n(s_n) \geq \epsilon'$ leads a contradiction:

$$
\begin{aligned}
P'(Q^n(s_n))\epsilon' &\leq C_{\underline{d}}'(\epsilon') - P(Q^n(s_n)) && (\because \text{the first-order condition (2)}) \\
&< C_{\bar{d}}'(\epsilon') - P(Q^n(s_n)) && (\because \text{the assumption of } \epsilon) \\
&\leq C_{\bar{d}}'(q_{\bar{d}}^n(s_n)) - P(Q^n(s_n)) && (\because q_{\bar{d}}^n(s_n) \geq \epsilon) \\
&= P'(Q^n(s_n))q_{\bar{d}}^n(s_n) && (\because \text{the first-order condition (2)}) \\
&\Rightarrow \epsilon' > q_{\bar{d}}^n(s_n) && (\because P'(Q^n(s_n)) < 0).
\end{aligned}
$$

(ii) Consider the case where $\underline{d} = 0$. (The proof is similar in the case where $\underline{d} = 1$.) Suppose the contrary, $\exists \epsilon > 0, \forall \bar{n}, \exists n \geq \bar{n}$, $q_0^n(s) \geq \epsilon$. Then, we can make up a subsequence $\left\{q_0^{m(n)}\left(s_{m(n)}\right)\right\}$ of $\{q_0^n(s_n)\}$ such that $q_0^{m(n)}\left(s_{m(n)}\right) \geq \epsilon$. Thus,

$$
\begin{aligned}
& P(s_{m(n)}\epsilon + (n - s_{m(n)})q_1^{m(n)}\left(s_{m(n)}\right)) + P'(s_{m(n)}\epsilon + (n - s_{m(n)})q_1^{m(n)}\left(s_{m(n)}\right))\epsilon \\
& \geq P(Q^{m(n)}\left(s_{m(n)}\right)) + P'(Q^{m(n)}\left(s_{m(n)}\right))q_0^{m(n)}\left(s_{m(n)}\right) \\
& = C_0'(q_0^{m(n)}\left(s_{m(n)}\right)) = C_0'(\epsilon)
\end{aligned}
$$

When $m(n) \to \infty$, we must have $s_{m(n)} \to \infty$ by the assumption that $\lim_{n\to\infty} s_n = \infty$. Therefore, the left hand side must become negative – contradiction. **Q.E.D.**

Innovation. Let the second-order differentiable function $c_{d_i}(q_i) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ be production cost which satisfies $c_{d_i}(0) = 0$[6] and $c'_{d_i}, c''_{d_i} > 0$, and has strictly increasing difference in (q_i, d_i). We represent the innovative (new) technology by $c_0(q_i)$ and the laggard (old) technology $c_1(q_i)$. Assume that $c'_1(0) < P(0)$. Let $\alpha = c'_1(0) - c'_0(0)$, i.e., α is the innovative technology's cost advantage to produce the first unit.

Define the nonnegative-integer number $\hat{s}$ as

$$\hat{s} = \min\{s \in \mathbb{Z}_+ | q_1^n(s) = 0\}.$$

For this number $\hat{s}$, we have the following lemma.

Lemma 3 *Suppose $C_{d_i}(q_i) = c_{d_i}(q_i)$. Then, when n is a sufficiently large number, there uniquely exists $\hat{s} \in (0, n)$ and $q_1^n(s) = 0$ if and only if $s \geq \hat{s}$.*

Proof By Corollary 1 (v), $\pi_1^n(s)$ is decreasing in s. For $s = 0$ (all the producers use the laggard technology), $\pi_1^n(0) > 0$ since $c'_1(0) < P(0)$. For $s = n - 1$ (only one firm use the laggard technology), since $q_0^n(n-1) > 0$ by $c'_0(0) < c'_1(0) < P(0)$, the first-order condition of a producer must be satisfied with equality:

$$P(Q^n(n-1)) + P'(Q^n(n-1))q_0^n(n) = c'_0(q_0^n(n)).$$

By Lemma 2 (ii), we must have $\lim_{n\to\infty} q_0^n(n-1) = 0$. Thus, by diverging n to the infinity in the first-order condition, $\lim_{n\to\infty} P(Q^n(n-1)) = c'_0(0)$. Thus, for sufficiently large n, we must have $P(Q^n(n-1)) < c'_1(0)$. This immediately implies that $q_1^n(n-1) = 0$. As a whole, $\hat{s}$ uniquely exists in $(0, n)$ and $q_1^n(s) = 0$ if and only if $s \geq \hat{s}$. **Q.E.D.**

From this Lemma, we find that $\hat{s}$ is the critical mass of the number of firms using the innovative technology that drive the laggard technology out of the markets. Thus, we call the innovation is *$\hat{s}$-firms drastic*. Hereafter, we assume $\hat{s} > 1$, that is, the innovation is not one-firm drastic.[7]

Define Q^M by $P'(Q^M)Q^M + P(Q^M) = c'_0(0)$ and Q^{QM} by $P(Q^{QM}) = c'_1(0)$. We call the innovation is *efficiently drastic* if $Q^M > Q^{QM}$. Since the efficiently drastic innovation implies

[6] In other words, we consider the situation where the entry costs of producers have sunk. However, we can easily offer a simple modification to consider the U-shaped average cost and endogenize the number of firms. See the further discussion.

[7] The definition of drastic innovation follows Arrow [1]. Sen [12] extends this concept to $\hat{s}$-firms drastic innovation.

that $P(Q^M) < c_1'(0)$, the firm with a constant marginal cost of $c_0'(0)$ can deter the firms with the laggard technology c_1 even under its monopoly price. Note that the innovation can be efficiently drastic even if it is not one-firm drastic since $c_0'' > 0$.

3 Unit royalty scheme

In this section, we investigate the unit royalty scheme wherein the patentee offers $r \in \mathbb{R}_+$, which represents the unit royalty, in the first stage and then, the n producers decide whether or not to buy the license simultaneously in the second stage. Royalties for a patent are collected according to the volume of output produced by use of the patent. Thus, in the third stage, the cost functions of firm i can be described as

$$C_{d_i}(q_i) = c_{d_i}(q_i) + (1 - d_i)rq_i, \qquad\qquad F_{d_i} = 0.$$

Notice that in the unit-royalty scheme, C_{d_i} does not always have increasing difference in (q_i, d_i) depending on the unit-royalty level r. The payoff of patentee is $r\tilde{Q}$, where $\tilde{Q}$ represents the aggregate output of the licensed producers. We suppose that there uniquely exist a subgame perfect equilibrium in this game.

3.1 The third stage

Under the unit royalty scheme, the first-order condition of firm i (1) turns out to be

$$P(Q^n(s)) + P'(Q^n(s))q^n_{d_i}(s) \leq c'_{d_i}(q^n_{d_i}(s)) + (1 - d_i)r \tag{2}$$

with equality if $q^n_{d_i}(s) > 0$. From this, we find that each firm's equilibrium output depends on r. Hence, in this section, we rewrite $q^n_{d_i}(s)$ by a expression including r, $q^n_{d_i}(s, r)$.

When $r < \alpha$, since the marginal cost for license payments does not exceed the innovative technology's cost advantage for the first unit, we have $C_0'(0) < C_1'(0)$, that is, $\underline{d} = 0$. Therefore, by Lemma 2, all the producer's output converges to zero when the number of licensed producers is large. Conversely, since $\underline{d} = 1$ when $r > \alpha$, all the producer's output converges to zero when the number of unlicensed producers is large. Furthermore, if $r \notin (\alpha - \delta, \alpha + \delta)$ for some $\delta > 0$, with regard to the level of r, we can find the most slowly converging point in each case. The following lemma states this fact formally.

Lemma 4 *Take $\delta > 0$ arbitrarily. (i) Take any sequence $\{s_n\}_{n=1}^{\infty}$ with $s_n \leq n$ and $\lim_{n\to\infty} s_n = \infty$. Then, $q_0^n(s_n, r)$ and $q_1^n(s_n, r)$ uniformly converge to 0 on $r \in [0, \alpha - \delta]$ when $n \to \infty$. (ii) Take any sequence $\{s_n\}_{n=1}^{\infty}$ with $s_n \leq n$ and $\lim_{n\to\infty}(n - s_n) = \infty$. Then, $q_0^n(s_n, r)$ and $q_1^n(s_n, r)$ uniformly converge to 0 on $r \in [\alpha + \delta, P(0)]$ when $n \to \infty$.*

Proof Pick $\epsilon > 0$ and $\delta > 0$ arbitrarily. (i) First, take any sequence $\{s_n\}_{n=1}^{\infty}$ with $s_n \leq n$ and $\lim_{n\to\infty} s_n = \infty$. We will prove that $\exists n'$, $\forall n \geq n'$, $\forall r \in [0, \alpha - \delta]$, $q_0^n(s_n, r) < \epsilon$ and $q_1^n(s_n, r) < \epsilon$. Since $C_0'(0)$ is increasing in r, $q_1^n(s_n, \alpha - \delta) \geq q_1^n(s_n, r)$ and $q_0^n(s_n, 0) \geq q_0^n(s_n, r)$ for all $r \in [0, \alpha - \delta]$. By Lemma 2(i), $\exists \hat{n}$, $\forall n \geq \hat{n}$, $q_1^n(s_n, \alpha - \delta) < \epsilon$. By Lemma 2(ii), $\exists \check{n}$, $\forall n \geq \check{n}$, $q_0^n(s_n, 0) < \epsilon$. Thus, $\forall n \geq n' = \max\{\hat{n}, \check{n}\}$, $q_0^n(s_n, r) < \epsilon$ and $q_0^n(s_n, r) < \epsilon$ for all $r \in [0, \alpha - \delta]$. (ii) Next, take any sequence $\{s_n\}_{n=1}^{\infty}$ with $s_n \leq n$ and $\lim_{n\to\infty}(n - s_n) = \infty$. Similarly, we can prove that $\exists n''$, $\forall n \geq n''$, $\forall r \in [\alpha + \delta, P(0)]$, $q_0^n(s_n, r) < \epsilon$ and $q_1^n(s_n, r) < \epsilon$. **Q.E.D.**

3.2 The second stage

Let $s^n(r)$ be the equilibrium number of the firms who accept the license with the unit-royalty level r. When n is sufficiently large, as far as $r \notin (\alpha - \delta, \alpha + \delta)$ for some $\delta > 0$, all the producers accept the unit-royalty level $r < \alpha$ and no producers accept the unit-royalty level $r > \alpha$. The following lemma states this formally.

Lemma 5 *$\forall \delta > 0$, $\exists \bar{n}$, $\forall n \geq \bar{n}$,*

$$s^n(r) = \begin{cases} n & \forall r \in [0, \alpha - \delta] \\ 0 & \forall r \in [\alpha + \delta, P(0)]. \end{cases}$$

Proof Consider the case where $r \in [0, \alpha - \delta]$, that is, $C_0'(0) < C_1'(0)$. Take $\epsilon > 0$ such that $\forall q \leq \epsilon$, $C_0'(q) < C_1'(q)$. By Lemma 4, $\exists \hat{n}$, $\forall n \geq \hat{n}$, $q_0^n(n, r) < \epsilon$ for all $r \in [0, \alpha - \delta]$ and $\exists \check{n}$, $\forall n \geq \check{n}$, $q_0^n(n - 1, r) < \epsilon$ and $q_1^n(n - 1, r) < \epsilon$ for all $r \in [0, \alpha - \delta]$. Therefore, $\forall n \geq n' = \max\{\hat{n}, \check{n}\}$, $q_0^n(n, r) < \epsilon$ and $q_0^n(n - 1, r) < \epsilon$ and $q_1^n(n - 1, r) < \epsilon$. Hence, $q \in [0, \epsilon]$ is the relevant range to induce the equilibrium output when $n \geq n'$. Since $C_d'(q)$ has increasing difference in (q, d) when $q \in [0, \epsilon]$, we can applies the Lemma 1(iv) and thus $\pi_0^n(n, r) \geq \pi_1^n(n - 1, r)$ for all $r \in [0, \alpha - \delta]$.

Therefore, when $n \geq n'$, no firms have the incentive to deviate from the situation wherein $s = n$, for all $r \in [0, \alpha - \delta]$. The similar proof is applicable in the case where $\forall r \in [\alpha + \delta, P(0)]$, that is, $C_0'(0) > C_1'(0)$. Thus, we have $\exists n''$, $\forall n \geq n''$, no firms have the incentive to deviate from the situation wherein $s = 0$ for all $r \in [\alpha + \delta, P(0)]$. Let $\bar{n} = \max\{n', n''\}$, we reach the requested result. **Q.E.D.**

We denote firm i's output in the equilibrium path of the subgame after the patentee chooses r by $q_i^n(r)$, the aggregate output by $Q^n(r) = \sum_{i=1}^n q_i^n(r)$, and the patentee's payoff by $\Pi^n(r)$ when the number of firms is n.

3.3 The first stage

Let patentee's equilibrium royalty given the number of firms n be r_R^n and the patentee's equilibrium payoff given the number of firms n be $\Pi_R^n = \Pi^n(r_R^n)$. The following theorem reveals where the equilibrium level of unit royalties goes and how much the patentee can earn by the unit-royalty scheme in the limit.

Theorem 1

(i) If the innovation is not efficiently drastic, we have $\lim_{n\to\infty} r_R^n = c_1'(0) - c_0'(0)$ *and*

$$\lim_{n\to\infty} \Pi_R^n = [c_1'(0) - c_0'(0)]Q^{QM}.$$

(ii) If the innovation is efficiently drastic, we have $\lim_{n\to\infty} r_R^n = P(Q^M) - c_0'(0)$ *and*

$$\lim_{n\to\infty} \Pi_R^n = [P(Q^M) - c_0'(0)]Q^M.$$

Proof Pick $\delta > 0$ arbitrarily. By Lemma 5, $\exists \bar{n}$, $\forall n \geq \bar{n}$, $s^n(r) = n$ if $r \in [0, \alpha - \delta]$ and $s^n(r) = 0$ if $r \in [\alpha + \delta, P(0)]$. Henceforth, we focus on the case when $n \geq \bar{n}$. Then, if the patentee chooses $r \in [\alpha + \delta, P(0)]$, its payoff $\Pi^n(r)$ is always zero. On the other hand, if $r \in [0, \alpha - \delta]$, since $q_i^n(r) = q_0^n(n, r) > 0$ for all i by Lemma 5, it must be satisfied that

$$\sup_{r\in[0,\alpha-\delta]} |P(Q^n(r)) - C_0'(0)| = \sup_{r\in[0,\alpha-\delta]} |C_0'(q_0^n(n,r)) - C_0'(0) - P'(nq_0^n(n,r))q_0^n(n,r)|$$

by the first-order condition (2). The right hand side converges to zero when $n \to \infty$ since by Lemma 4 (i), $q_0^n(n,r)$ uniformly converges to 0 on $r \in [0, \alpha - \delta]$ when $n \to \infty$. Therefore, when $n \to \infty$, $P(Q^n(r))$ uniformly converges to $C_0'(0)$ on $r \in [0, \alpha - \delta]$. As a whole, the patentee's payoff $\Pi^n(r)$ uniformly converges to 0 on $[\alpha + \delta, P(0)]$ and to $rP^{-1}(C_0'(0)) = rP^{-1}(c_0'(0) + r)$ on $r \in [0, \alpha - \delta]$.

If the innovation is not efficiently drastic, $c_1'(0) \leq P(Q^M)$. Thus, from the above,

$$\lim_{n\to\infty} \operatorname*{argmax}_{r\in[0,P(0)]\setminus(\alpha-\delta,\alpha+\delta)} \Pi^n(r) = \alpha - \delta, \tag{3}$$

where use was made of the mathematical result of Lemma 8 provided in the appendix. Since we can take arbitrarily small $\delta > 0$, we must have $\lim_{n\to\infty} r_R^n = \alpha$. (Suppose the contrary. Then, there exists $\delta' > 0$, we can make up a subsequence $\left\{r^{m(n)}\right\}$ of $\{r_R^n\}$ such that $|r^{m(n)} - \alpha| \geq \delta'$ for all $m(n)$. Taking $\delta \in (0, \delta')$ leads a contradiction since (3) implies $r_R^n = \operatorname{argmax}_{r\in[0,P(0)]} \Pi^n(r)$ must be in $(\alpha - \delta', \alpha + \delta')$ for sufficiently large n.) The fact that $\lim_{n\to\infty} \Pi_R^n = r_R^n P^{-1}(c_0'(0) + r_R^n) = \alpha P^{-1}(c_1'(0)) = \alpha Q^{QM}$ immediately follows from this conclusion.

If the innovation is efficiently drastic, $c_1'(0) > P(Q^M)$. Thus, from the above, $\forall \delta < c_1'(0) - P(Q^M)$,

$$\begin{aligned}
&\lim_{n\to\infty} \operatorname*{argmax}_{r\in[0,P(0)]\setminus(\alpha-\delta,\alpha+\delta)} \Pi^n(r) = P(Q^M) - c_0'(0), \\
&\lim_{n\to\infty} \max_{r\in[0,P(0)]\setminus(\alpha-\delta,\alpha+\delta)} \Pi^n(r) = [P(Q^M) - c_0'(0)]Q^M,
\end{aligned} \tag{4}$$

by Lemma 8. Since by the definition of Q^M, $[P(Q^M) - c_0'(0)]Q^M$ is greater than $rQ^n(r)$ when $r \neq P(Q^M) - c_0'(0)$, we must have $\lim_{n\to\infty} r_R^n = P(Q^M) - c_0'(0)$. (Suppose that $\{r_R^n\}$ does not converges to $P(Q^M) - c_0'(0)$. Then, there exists $\delta' > 0$, we can make up a subsequence $\left\{r^{m(n)}\right\}$ of $\{r_R^n\}$ such that $|r^{m(n)} - [P(Q^M) - c_0'(0)]| \geq \delta'$ for all $m(n)$. Thus, $\exists \Delta > 0$, for all $m(n)$, $[P(Q^M) - c_0'(0)]Q^M - r^{m(n)}P(Q^{m(n)}(r_R^{m(n)})) > \Delta$. By taking $\delta \in (0, \delta')$, this contradicts to (4).) The fact that $\lim_{n\to\infty} \Pi_R^n = [P(Q^M) - c_0'(0)]Q^M$ immediately follows from this conclusion. **Q.E.D.**

Intuitively, the result of this theorem is very simple if we see the figure 1. In the unit-royalty scheme, *all* the producers are licensed or unlicensed when $n \to \infty$ (by Lemma 5). Then, in the limit, since each producer produces infinitesimally small output (by Lemma 2), the new technology's production approximates that produced with a most efficient constant marginal cost $c_0'(0)$ as a whole, as well as the old technology's production approximates that produced with a constant marginal cost $c_1'(0)$. Therefore, when (i) the innovation is not efficiently drastic, i.e., $c_1'(0) \leq P(Q^M)$, the limit royalty will be the level which just undercuts the old technology's marginal cost $c_1'(0)$ and the patentee can earn the undercutting profit[8] (the area ABCD in the figure). When (ii) the innovation is efficiently drastic, i.e., $c_1'(0) > P(Q^M)$, the limit royalty will be the level which just realizes the monopoly profit (the area AEFG in the figure).

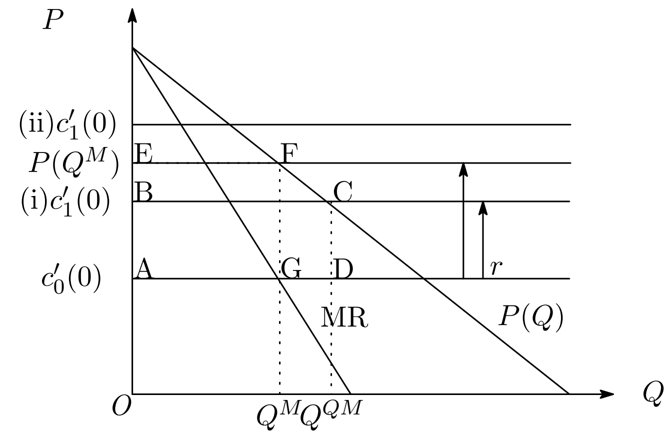

Figure 1: Limit profit in the unit royalty scheme

4 Fixed fee scheme

In this section, we investigate the fixed-fee scheme wherein the patentee offers $f \in \mathbb{R}_+$, which represents the fixed fee, in the first stage and then, the n producers decide whether or not to accept the license simultaneously in the second stage. Under the fixed fee system, a producer who has paid a certain amount of fee is entitled to use a patented technology regardless of production volume. Thus, in the third stage, the cost functions of producer i can be described as

$$C_{d_i}(q_i) = c_{d_i}(q_i), \qquad F_{d_i} = (1 - d_i)f.$$

[8]Ino and kawamori [5] refer this situation as *quasi-monopoly*. Thus, we uses the notation Q^{QM}.

Notice that in the fixed fee scheme, C_{d_i} always has increasing difference in (q_i, d_i) regardless of the fixed-fee level f. The payoff of patentee is $\tilde{n}f$, where $\tilde{n}$ represents the number of producers who accept the license. We suppose that there uniquely exist a subgame perfect equilibrium in this game.

4.1 The third stage

For given s and f, the first-order condition (1) is not directly affected by f. Thus, we find that each firm's equilibrium output $q^n_{d_i}(s)$ does not depend on f. The profit of firm i is

$$\pi^n_{d_i}(s) = P(Q^n(s))q^n_{d_i}(s) - c_{d_i}(q^n_{d_i}(s)) - (1 - d_i)f.$$

Observe that $\pi^n_{d_i}(s)$ is decreasing in f when $d_i = 0$ while does not depend on f when $d_i = 1$.

The following lemma states that if n is sufficiently large, one producer's incentive to participate in the licensed group is greater, the smaller the number of licensed group's member is.

Lemma 6 *$\exists \bar{n}$, $\forall n \geq \bar{n}$, $\forall s \in \{1, 2, \ldots, n-1\}$,*

$$\pi^n_0(s) - \pi^n_1(s-1) > \pi^n_0(s+1) - \pi^n_1(s).$$

Proof First, we show that for $s < \hat{s}$, $P(Q^n(s))$ converges to $c'_1(0)$ when $n \to \infty$. For $s \in \{0, 1, \ldots, \hat{s}-1\}$, since $q^n_1(s) > 0$ by Lemma 3, the first-order condition of a unlicensed firm must be satisfied with equality:

$$P(Q^n(s)) + P'(Q^n(s))q^n_1(s) = c'_1(q^n_1(s)).$$

By Lemma 2 (i), we must have $\lim_{n\to\infty} q^n_1(s) = 0$. Thus, by diverging n to the infinity in the first-order condition, $\lim_{n\to\infty} P(Q^n(s)) = c'_1(0)$.

Next, we show that $\forall \epsilon > 0$, $\exists \bar{n}$, $\forall n \geq \bar{n}$, $\pi^n_1(s) < \epsilon$ $\forall s \in \{0, 1, \cdots, n\}$ and in particular, $\pi^n_1(s) = 0$ $\forall s \in \{\hat{s}, \hat{s}+1, \cdots, n\}$. Take $\epsilon > 0$ arbitrarily. By Corollary 1 (v), $\forall s$, $\pi^n_1(s) \leq \pi^n_1(s-1)$. Thus, if $\pi^n_1(0) < \epsilon$, we immediately obtain $\forall s$, $\pi^n_1(s) < \epsilon$. When $s < \hat{s}$, from the first paragraph,

$\lim_{n\to\infty} P(Q^n(s)) = c_1'(0)$. Hence,

$$\lim_{n\to\infty} \pi_1^n(0) = \lim_{n\to\infty} P(nq_1^n(0))q_1^n(0) - c_1(\lim_{n\to\infty} q_1^n(0)) \quad (\because \text{ Continuity of } c_0 \text{ and } \lim_{n\to\infty} q_1^n(0) = 0)$$
$$= c_1'(0) \lim_{n\to\infty} q_1^n(0) - c_1(0) = 0 \quad (\because \lim_{n\to\infty} P(Q^n(s)) = c_1'(0) \text{ and } \lim_{n\to\infty} q_1^n(0) = 0)$$

Furthermore, when $s \geq \hat{s}$, $q_1^n(s) = 0$ by Lemma 3 for sufficiently large n. Therefore, Thus, since $\{0, 1, \ldots, n\}$ is finite set, we can take $\bar{n}$ such that $\forall n \geq \bar{n}$, $\pi_1^n(0) < \epsilon$ for all s and $\pi_1^n(s) = 0$ for $s \geq \hat{s}$.

Finally, we show that $\pi_0^n(s) - \pi_0^n(s+1) > \pi_1^n(s-1) - \pi_1^n(s)$ for sufficiently large n. By Corollary 1 (v) and $q_0^n(s) > 0$, $\pi_0^n(s) - \pi_0^n(s+1) > 0$ for all $s \in \{1, \ldots, n-1\}$. Let

$$\epsilon = \min_{s\in\{1,2,\ldots,\hat{s}\}} [\pi_0^n(s) - \pi_0^n(s+1)].$$

Then, from the above, $\exists \bar{n}$, $\forall n \geq \bar{n}$, $\pi_1^n(s-1) - \pi_1^n(s) < \epsilon$ for all s and indeed, $\pi_1^n(s-1) - \pi_1^n(s) = 0$ for $s > \hat{s}$. Therefore, when $s \leq \hat{s}$, $\pi_0^n(s) - \pi_0^n(s+1) \geq \epsilon > \pi_1^n(s-1) - \pi_1^n(s)$ by the definition of ϵ. When $s \leq \hat{s}$, the requested result holds by $\pi_0^n(s) - \pi_0^n(s+1) > 0 = \pi_1^n(s-1) - \pi_1^n(s)$. **Q.E.D.**

4.2 The second stage

Let $s^n(f)$ be the equilibrium number of firms who accept the license contract with the fixed-fee level f. As easily predicted by the result of lemma 6, if n is sufficiently large, we find the monotone relation such that $s^n(f)$ increases as f decreases.

Lemma 7 *$\exists \bar{n}$, $\forall n \geq \bar{n}$,*

$$s^n(f) = \begin{cases} 0 & \text{if } \pi_0^n(1) - \pi_1^n(0) < f \\ 1 & \text{if } \pi_0^n(2) - \pi_1^n(1) < f \leq \pi_0^n(1) - \pi_1^n(0) \\ \vdots & \\ \hat{s}-1 & \text{if } \pi_0^n(\hat{s}) - \pi_1^n(\hat{s}-1) < f \leq \pi_0^n(\hat{s}-1) - \pi_1^n(\hat{s}-2) \\ \hat{s} & \text{if } \pi_0^n(\hat{s}+1) < f \leq \pi_0^n(\hat{s}) - \pi_1^n(\hat{s}-1) \\ \hat{s}+1 & \text{if } \pi_0^n(\hat{s}+2) < f \leq \pi_0^n(\hat{s}+1) \\ \vdots & \\ n-1 & \text{if } \pi_0^n(n) < f \leq \pi_0^n(n-1) \\ n & \text{if } f \leq \pi_0^n(n). \end{cases}$$

Proof (i) By Lemma 6, we can take $\bar{n} > \hat{s}$ such that $\forall n \geq \bar{n}$, $\pi_0^n(s) - \pi_1^n(s-1)$ is decreasing in $s \in \{1, 2, \ldots, n-1\}$. Consider n greater than or equal to such $\bar{n}$.

Suppose that for $s = 1, \ldots, n-1$, s firms accepts the license contract and

$$\pi_0^n(s+1) - \pi_1^n(s) < f \leq \pi_0^n(s) - \pi_1^n(s-1). \tag{5}$$

Note that if $s > \hat{s}$, the term $\pi_1^n(s)$ is vanished from this condition since it is zero. The deviation profit of a licensed firm is $\pi_1^n(s-1) - [\pi_0^n(s) - f]$ but this is non-positive by the second inequality of (5). The deviation profit of a unlicensed firm is $[\pi_0^n(s+1) - f] - \pi_1^n(s)$ but this is negative by the first inequality of (5). Therefore, $s^n(f) = s$.

Finally, we will consider two extreme cases. Suppose that $\pi_0^n(1) - \pi_1^n(0) < f$ and no firms accept the license contract. The deviation profit of a firm is $[\pi_0^n(1) - f] - \pi_1^n(0)$ but this is negative by the supposition. Thus, in this case, $s^n(f) = 0$. Suppose that $f \leq \pi_0^n(n)$ and all the firms accept the license contract. The deviation profit of a firm is $\pi_1^n(n-1) - [\pi_0^n(n) - f]$ but this is non-positive by the supposition and the fact that $\pi_1^n(n-1) = 0$ since $n \geq \bar{n} > \hat{s}$. Thus, in this case, $s^n(f) = n$. **Q.E.D.**

We denote the patentee's payoff in the equilibrium path of the subgame after the patentee chooses f by $\Pi^n(f) = s^n(f)f$ when the number of firms is n.

4.3 The first stage

Let f_F^n be the equilibrium level of fixed fee for given n. Then, the equilibrium profit of the patentee Π_F^n is defined by

$$\Pi_F^n = \Pi^n(f_F^n) = s^n(f_F^n)f_F^n.$$

The following theorem is our main goal which reveals that it is better off for the patentee to license its technology in the unit-royalty scheme than in the fixed-fee scheme when the number of producers is sufficiently large.

Theorem 2 *Whenever the innovation is efficiently drastic or not,* $\exists \bar{n}$, $\forall n \geq \bar{n}$, $\Pi_F^n < \Pi_R^n$.

Proof Take n' such that $\forall n \geq n'$, $s^n(f)$ is that in Lemma 7. Since the profit of the patentee

$\Pi^n(f) = s^n(f)f$ is maximized when $f = \pi_0^n(s) - \pi_1^n(s-1)$ in the range wherein $s^n(f) = s$,

$$\Pi^n(f) \leq s^n(f)[\pi_0^n(s^n(f)) - \pi_1^n(s^n(f) - 1)] \tag{6}$$

with equality if $f = \pi_0^n(s) - \pi_1^n(s-1)$.

We confirm that $\exists\epsilon > 0$, $\forall n \geq n'$, $\Pi_F^n > \epsilon$. Take $n > n'$ arbitrarily. Then, if f is in $(\pi_0^n(\hat{s}+1), \pi_0^n(\hat{s}) - \pi_1^n(\hat{s}-1)]$, the patentee's payoff is greater than $\hat{s}\pi_0^n(\hat{s}+1)$ by Lemma 7. Since $q_1^n(s^n(f)) = 0$ in this case, $\pi_0^n(\hat{s}+1) = \pi_0^{\hat{s}+1}(\hat{s}+1)$, which equals to the ordinary $\hat{s}+1$-firm Cournot profit regardless of n.

We exclude two polar cases. First, we must have $s^n(f_F^n) > 0$. To the contrary, if $s^n(f_F^n) = 0$, $\Pi_F^n = 0$ contradicts to the above. Next, we must have $\exists s^* > \hat{s}$, $\forall n \geq n'$, $s^n(f_F^n) < s^*$. If not so, we can make up the subsequence $\left(s^{m(n)}\right)$ of $(s^n(f_F^n))$ such that $\lim_{m(n)\to\infty} s^{m(n)} = \infty$. Then,

$$\lim_{m(n)\to\infty} P(Q^{m(n)}(s^{m(n)})) = c_0'(0),$$

where the proof is similar to Lemma 3. Thus, $\lim_{m(n)\to\infty} s^{m(n)}\pi_0^{m(n)}(s^{m(n)}) = 0$. Since $f_F^{m(n)} \leq \pi_0^{m(n)}(s^{m(n)})$ by (6), $\lim_{m(n)\to\infty} \Pi_F^m(n) = 0$. This contradicts to the above.

Hereafter, we focus on the case where $0 < s^n(f) < s^*$. From (6), for all $n \geq n'$,

$$\Pi^n(f) \leq s^n(f)\pi_0^n(s^n(f)) = P(Q^n(s^n(f)))s^n(f)q_0^n(s^n(f)) - s^n(f)c_0(q_0^n(s^n(f))) \tag{7}$$

where the second term of the right hand side can be decomposed as

$$s^n(f)c_0(q_0^n(s^n(f))) = s^n(f)q_0^n(s^n(f))c_0'(0) + s^n(f)\int_0^{q_0^n(s^n(f))}[c_0'(t) - c_0'(0)]dt.$$

From Corollary 1 (ii) and $s^n(f) < s^*$,

$$\int_0^{q_0^n(s^n(f))}[c_0'(t) - c_0'(0)]dt \geq \int_0^{q_0^n(s^*)}[c_0'(t) - c_0'(0)]dt \equiv A > 0,$$

where the last inequality comes from $c_0'' > 0$. By substituting these into (7), for all $n \geq n'$,

$$\begin{aligned} \Pi^n(f) &\leq [P(Q^n(s^n(f))) - c_0'(0)]s^n(f)q_0^n(s^n(f)) - s^n(f)A \\ &\leq [P(Q^n(s^n(f))) - c_0'(0)]Q^n(f) - A \qquad (\because s^n(f) \geq 1). \end{aligned} \tag{8}$$

Observe that since $q_0^n(s^*)$ does not depend on n by $s^* > \hat{s}$ and Lemma 3, so A does not (Thus, the term A never converges to zero if $n \to \infty$). Therefore, by (8), $\exists\delta > 0$, $\exists\bar{n} \geq n'$, for all $n \geq \bar{n}$,

$$\Pi^n(f) < [P(Q^n(s^n(f))) - c_0'(0)]Q^n(f) - \delta \tag{9}$$

Thus, $\Pi^n(f)$ must be less than the profit gained by producing $Q^n(s^n(f))$ with a constant marginal cost $c_0'(0)$. Therefore, $\Pi^n(f)$ never reaches $[P(Q^M) - c_0'(0)]Q^M$.

When $s^n(f) < \hat{s}$, since $\lim_{n\to\infty} P(Q^n(s^n(f))) = c_1'(0)$ by the first paragraph in the proof of Lemma 6,

$$\lim_{n\to\infty} P(Q^n(s^n(f)))Q^n(f) = c_1'(0)Q^{QM}. \tag{10}$$

Therefore, by (9), $\exists\delta > 0$, $\exists\bar{n} \geq n'$, for all $n \geq \bar{n}$,

$$\Pi^n(f) < [c_1'(0) - c_0'(0)]Q^{QM} - \delta. \tag{11}$$

Therefore, $\Pi^n(f)$ never reaches $[c_1'(0) - c_0'(0)]Q^{QM}$.

From these facts and Theorem 1, we have the requested result. **Q.E.D.**

Intuition of this theorem is clear. As seen in Lemma 5 and Theorem 1, all the producers are licensed in the unit-royalty scheme. Since the unit royalty raises the marginal cost, competition among the licensed producers does not induce a low price. Thus, the patent holder can enjoy the difference between the price and the reduced marginal-production cost under "small volume sales to a large number of producers". In contrast, the feature of the optimal license contract in the fixed-fee scheme is "large volume sales to a small number of producers". This is because if all the producers use the innovative technology, competition among them lowers the price since the fixed fee does not raise the marginal cost. Thus, to absorb the profit in lump-sum way, the patent holder should restrict the number of licensed producers. However, since the cost is convex, this feature of the fixed-fee scheme yields the disadvantage with respect to the production efficiency. Note that under the convex costs, the larger the number of licensed producers, the lower the total cost is. Thus, when n is large, this disadvantage is more likely to dominate the advantage of the fixed-fee scheme that is found by Kamien and Tauman [8].

5 Further Discussions

Endogenous number of firms In the body of paper, we avoid discussing the set-up or entry costs of the producers and endogenizing the number of firms. In other words, we assume that the entry costs of producers have sunk. Thus, at a glance, we only consider the increasing average cost and make n diverge exogenously. However, we can easily offer a simple modification to consider the U-shaped average cost and endogenize the number of firms. Suppose that potential producers need to pay non-sunk fixed costs, $F > 0$ to enter the market and they choose to enter or not before the patentee offers license contract (Stage 0).[9] Consider that n is determined endogenously by a zero-profit condition in this stage. Then, the post-entry market (the subgame after n is determined) is exactly the same one analyzed in this paper. Producers will enter the market if F is less than its post-entry profit analyzed in our paper (note that each producer earns positive post-entry profit when n is finite). The post-entry market grows into a competitive one as F goes to zero. Since $n \to \infty$ when $F \to 0$, this limit outcome is the same as that in the body of the paper.

Two-part tariff Thus far, we consider the unit-royalty scheme and the fixed-fee scheme separately. However, it is often discussed that the combination of the unit and the fixed fee (two-part tariff) brings greater private value of the patent than each scheme.[10] Thus, it is worth to discuss what happens in our limit results if we allow the combination of two schemes: the patentee offers $(r, f) \in \mathbb{R}^2_+$ in the first stage and the cost functions of producer i in the third stage can be described as

$$C_{d_i}(q_i) = c_{d_i}(q_i) + (1 - d_i)rq, \qquad\qquad F_{d_i} = (1 - d_i)f.$$

Even under this modification, we can show that in the optimal license contract, r converges to the same values provided in Theorem 1 and f converges to zero as $n \to \infty$. In other words, when the market is sufficiently competitive, the patentee solely uses the unit royalty even if the combination with fixed fee is feasible. This is because the unit royalty alone can attain the maximized profit (monopoly or under-cutting profit) as seen in Theorem 1 and a positive fixed fee disturbs the efficient use of technology for the similar reason to Theorem 2.

[9]De Meza [3] considers another timing of entry. In his license model, the entries of producers occur between Stage 1 and Stage 2.

[10]See Sen and Tauman [13] among others.

6 Concluding Remarks

We have shown that the unit-royalty scheme is superior to the fixed-fee scheme when the market is sufficiently competitive (Theorem 2). Kamien and Tauman [8] showed that as the market goes purely competitive ($n \to \infty$), the patentee's profit using the fixed fee or using the unit royalty goes indifferent under the linear costs (Proposition 8(2) in their paper). Combining these results, we can state that in the convex-cost environment, when the market goes purely competitive, the patentee's profits using the royalty licensing is higher than or equal to that using the fixed fee licensing with equality if and only if cost functions are linear. Therefore, the result of Kamien and Tauman is a boundary solution in the sense that the fixed fee licensing always dominates the unit royalty licensing if and only if marginal costs are constant.

Our result provides an important implication for pro-patent policies. Sometimes, pro-patent policies draw criticism for lack of availability of new technologies. An intuition behind this criticism is that strong protection of patents ensures patentees' monopolistic license revenue and it may lead to exclusive supplies of new technologies. As a result, it seems that the new technologies do not widespread due to pro-patent policies. As for the the fixed-fee scheme, this is true since the the fixed-fee scheme induces the strategy, "large volume sales to a small number of producers." However, when the market is sufficiently competitive, our result indicates that the technology which should be common (convex cost) would widespread even under the perfect protection of the patent. This is because the unit royalty scheme induces the strategy, "small volume sales to a large number of producers". Therefore, in this mean, for the availability of new technology, the important thing is to improve market competition along with pro-patent policies.

Appendix

We provide a mathematical fact suitable for our analysis. Roughly speaking, this fact tells us that if the "limit payoff function" f, which the sequence of payoff function (f^n) uniformly converges to, has the unique maximizer on the compact area, the sequence of maximizers for (f^n) cannot escape from that value as n goes to infinity. Thus, once this "limit payoff function" is identified, we can find the convergent point of the equilibria without characterizing each finite maximization problem. Pointwise convergence is not sufficient to prove the lemma below. Uniform convergence is the key condition.

Lemma 8 (Solution of maximization in the limit)
Let $D \subset \mathbb{R}$ be compact. Suppose that a sequence of functions $(f^n : D \to \mathbb{R})_{n \in \mathbf{N}}$ uniformly converges to a continuous function $f : D \to \mathbb{R}$. If f has the unique maximizer denoted by $x^* \in D$ and f^n has a maximizer denoted by $x^n \in D$ for each n, we have $x^* = \lim_{n \to \infty} x^n$.

Proof See Ino and Kawamori [5] **Q.E.D.**

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