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An endogenous timing analysis of international duopoly with transboundary stock pollution

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Abstract

This paper looks into potential determinants of the mode of international competition in a polluting good market by analyzing a so-called timing game between two environmentally concerned governments. From the equilibrium results of our intergovernmental game based on an international duopoly model with transboundary stock pollution, we show how an exact form of international competition depends on the magnitudes of international transportation coefficients of pollutant emissions and decay rates of pollutant stocks in respective countries as well as on other environmental and economic variables.

Keywords: international duopoly, transboundary pollution, stock pollution, gains from trade, endogenous timing.

JEL Classification: F10, F12, Q20.

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1 Introduction

The large literature on the gains-from-trade proposition initiated by Samuelson (1939) generally asserts that free trade is gainful to all the participating nations.[1] Moreover, the so-called 'new trade theory' which incorporates imperfect competition and/or increasing returns has found a new source for gains from trade. Among others, Markusen (1981) makes it clear that the opening of trade promotes competition, from which trading countries can gain. However, environmental considerations are usually missing in the existing literature on gains from trade under imperfect competition although such considerations seem to play an important part in recent negotiations over more liberalized trade regimes at both global and regional scales. This is exemplified in the debates over NAFTA where freer commercial interactions in North America was opposed by some partly on the ground of environmental protection. Similar arguments have been frequently made by some environmental groups who persistently resist so-called globalization, as was symbolically manifested in their feverish oppositions towards the World Trade Organization (WTO) Round Talk in Seattle in 1999.

As it would be easily imaginable, possible existence of gains from international trade in a polluting product is dependent on the market structure as well as the preferences of citizens. On top of these, when the pollution issue is transboundary by nature, the welfare impact of international trade could also depend on the physical characteristics of a pollution issue. The welfare implications of environmental variables in transboundary pollution problems have been typically investigated in game-theoretic studies without accounting for international trade. Important examples of such studies are Mäler (1990) and Tahvonen, Kaitala, and Pohjola (1993) for flow pollution, and Kaitala, Pohjola and Tahvonen (1992) and Mäler and de Zeeuw (1998) for stock pollution.

Conversely, though, there might be situations where the form of an international competition in a polluting good market should not be given *a priori* and could be influenced by the characteristics of the environmental problem at hand. This paper aims to explore such a possibility by explicitly modeling the strategic interaction surrounding

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[1]Kemp and Wan (1972) and Grandmont and McFadden (1972) generalize Samuelson's (1939) proposition in a quite general Arrow-Debreu model.

governmental decision makings which, consequently, determine possible modes of international trade in the presence of transboundary pollution associated with the production of a potentially tradable good. In particular, we demonstrate that both Stackelberg and Cournot-Nash types of competition, in addition to the autarkic situation, are possible equilibrium outcomes concerning the tradeable polluting good market, depending upon the magnitudes of certain economic and environmental variables, such as discount rate, marginal damage cost of pollution and decay rates of pollution stocks, as well as how similar the concerned nations are in these aspects.

This paper is structured as follows. Section 2 presents our basic model of an economy with transboundary stock pollution and derives its autarkic outcome. By extending the model to a two-country world, the next section characterizes potential free trade outcomes in two different modes of international competition. Then, Section 4 describes a game between two governments and discusses its equilibrium results. The last section contains brief concluding remarks.

2 Autarky

This section develops our basic model and describes its autarkic outcome. The model is comprised of two countries (Home and Foreign), two goods (Goods 1 and 2), and one primary factor (labor). We assume that both countries share the identical preferences and production technologies, and produce both goods. In the following description of our model, we focus on Home since the Foreign country can be described symmetrically. We denote each Foreign variable by attaching an asterisk (*). Furthermore, Good 2 serves as a numeraire and is produced with a unitary input coefficient so that the wage rate is internationally equalized and fixed at unity. In the autarkic case, Good 1 is monopolistically supplied by a single domestic firm and $c > 0$ units of labor are required to produce one unit of Good 1. Hence, the marginal cost of production is assumed to be constant at c. In addition, we suppose that the production of one unit of Good 1 entails one unit of pollutant emissions.

Now, let us assume that the utility function of a representative consumer in Home

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can be specified by

$$V = \int_0^\infty e^{-rt} \left[AC_1 - \frac{C_1^2}{2} + C_2 - sZ \right] dt, \quad A > c, \quad s > 0, \tag{1}$$

where V is the consumer's utility level, r is its discount rate, C_i, $i = 1, 2$, is the consumption of each good, s is the constant marginal damage cost of its pollutant stock, and Z is the pollution stock level in this country. We assume, however, that the generation of the pollutant emissions associated with the consumption of Good 1 is treated as a negative externality by this consumer and, therefore, out of its control. Hence, the consumer optimizes its utility by ignoring the adverse effects of the stock pollution completely. This is essentially the same as the case where the consumer optimizes its instantaneous utility without considering the environmental damage cost.

Letting p denotes the price of Good 1 measured by the price of Good 2, this consumer's utility maximization problem subject to the budget constraint yields the demand function of Good 1:

$$C_1 = A - p.$$

Under autarky, the market-clearing condition is

$$A - p = x,$$

where x is the output of Good 1. Hence, the inverse demand function and the monopoly firm's profit at each time instant are respectively defined by

$$p = A - x, \tag{2}$$
$$\pi \equiv (A - c - x)x. \tag{3}$$

Using (2) and (3), the long-term social welfare of the nation, U, which is the sum of the consumer surplus, the monopolist's profit, and the environmental damage cost of the pollution over the infinite time horizon, can be expressed as

$$U = \int_0^\infty e^{-rt} \left[\frac{(A - p)^2}{2} + \pi - sZ \right] dt. \tag{4}$$

In the subsequent analysis, (4) determines an overall payoff of the government in each situation.

As for the transboundary effects of the pollutant emissions, we assume that one unit of Foreign's (resp. Home's) pollutant emissions x^* (resp. x) increases Home's (resp. Foreign's) current pollutant flow level by the fraction of $\alpha \in [0, 1]$ (resp. α^*) while one unit of domestic emissions increases its own pollutant flow by one unit. In the literature of environmental economics, α and α^* are sometimes referred to as 'transportation coefficients', but we call them 'pollutant import coefficients' here in order to emphasize the directions of the pollutant flow. In the case of global pollution, such as CO_2 that is a culprit of the global warming problem, we have $\alpha = \alpha^* = 1$, while both α and α^* are normally strictly smaller than one and take different values in so-called regional environmental issues, such as the transboundary acid rain problem in Northern Europe. When $\alpha = \alpha^* = 0$, on the other hand, the pollution problem is completely localized. In sum, the pollution levels in the respective countries are described as

$$\dot{Z} = x + \alpha x^* - kZ, \tag{5}$$

$$\dot{Z}^* = x^* + \alpha^* x - k^* Z^*, \tag{6}$$

where $\dot{Z}$ and $\dot{Z}^*$ respectively denote the time derivatives of the pollutant stocks, and k and k^* are so-called decay rates of the pollutant stocks in the respective countries. In this article, we assume that, while the preference-related variables of the consumers, such as discount rates and marginal costs of the pollution, and firms' production costs are exactly symmetric across the two countries, the environmental variables, such as pollutant import coefficients and decay rate of the pollutants can be diverse, and mainly focus on the impacts of the latter variables on the equilibrium outcomes.

Let us now formulate the optimization problem of each country's firm. For simplicity of the exposition, we write the behaviors of the firms in both countries in a completely static fashion although the firms' actual behaviors would not change at all if they maximized their respective profits dynamically since the firms do not care about the stock pollution issue, quite similarly to the consumer who regards the pollution problem as an externality. Again, we focus on the Home firm since its Foreign counterpart acts in exactly the same fashion. Specifically, the Home firm solves the following problem in the autarkic case:

$$\max_x (A - c - x)x,$$

whose solution can be immediately obtained as

$$x^A = \frac{A - c}{2},$$ (7)

where the superscript A represents the autarkic outcome. Also, the autarkic price is derived from the demand function as

$$p^A = \frac{A + c}{2}.$$ (8)

Substituting (7) into (5), we have[2]

$$\dot{Z}^A = (1 + \alpha)\frac{A - c}{2} - kZ^A,$$

where Z^A is the pollutant stock in Home under autarky. This is a simple first-order linear ordinary differential equation, which can be easily solved and yields the path of the pollutant stock over time in Home as

$$Z^A = e^{-kt}Z_0 + (1 - e^{-kt})\frac{(1 + \alpha)(A - c)}{2k},$$ (9)

where Z_0 is the initial pollutant stock level in Home.

Throughout this paper, let us express the total value of social welfare of each country by excluding the influence of the initial pollutant stock level. This is acceptable for our main purpose here, i.e., to analyze an strategic interaction between the governments by comparing their individual relative payoffs under the respective situations, because the marginal damage cost of the pollution is assumed constant. If the damage cost function were nonlinear, the initial pollutant stock level would matter. Hence, by substituting the time-path of the pollutant stock in (9) except the term involving Z_0, as well as (3) and (8) into (4), the payoff of the Home government in the autarkic outcome can be written as

$$U^A = \frac{3}{8}(A - c)^2 \int_0^\infty e^{-rt}dt - \frac{s(1 + \alpha)(A - c)}{2k} \int_0^\infty e^{-rt}\left[1 - e^{-kt}\right]dt$$

$$= \frac{3}{8r}(A - c)^2 - \frac{s(1 + \alpha)}{2r(r + k)}(A - c).$$ (10)

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[2]Note that the firm in Foreign produces the same amount of Good 1 as Home's firm in the autarkic case by our symmetry assumptions on the characteristics of the firms as well as the representative consumers across the countries.

It should be noted that, even under autarky, the welfare of Home is affected by the production level in Foreign through the transboundary pollution in (5). Hence, there exists a negative externality associated with the production of Good 1 across the two countries. The next section extends this model to an international duopoly in two different modes of competition.

3   International duopoly

When the two domestic markets of Good 1 described above is fully integrated internationally, the new market-clearing condition becomes

$$C_1 + C_1^* = 2(A - p) = x + x^*,$$

which is inverted to yield

$$p = A - \frac{x + x^*}{2}. \tag{11}$$

This defines the inverse demand function for Good 1 in the international market of Good 1 and each firm's profit function becomes

$$\pi = \left(A - c - \frac{x + x^*}{2}\right) x.$$

First, we consider the case where the two firms are engaged in a Cournot-type competition in this aggregated market. In essence, these firms determine their respective output supply levels concurrently. Specifically, these two firms respectively attempt to solve the following optimization problems:

$$\max_{x} \ \pi = \left(A - c - \frac{x + x^*}{2}\right) x,$$

$$\max_{x^*} \ \pi^* = \left(A - c - \frac{x + x^*}{2}\right) x^*.$$

We can immediately obtain the first-order conditions:

$$A - c - \frac{x^*}{2} - x = 0,$$

$$A - c - \frac{x}{2} - x^* = 0,$$

which lead to their respective reaction functions:

$$x = A - c - \frac{x^*}{2}, \tag{12}$$

$$x^* = A - c - \frac{x}{2}. \tag{13}$$

Solving these equations simultaneously, we can obtain the Cournot-Nash equilibrium levels of output supply for the respective firms:

$$x^C = x^{*C} = \frac{2(A - c)}{3}. \tag{14}$$

Furthermore, the equilibrium price becomes

$$p^C = \frac{A + 2c}{3}. \tag{15}$$

Comparing (8) and (15), we can easily confirm $p^C < p^A$, which implies that the pro-competitive effect of the opening of international trade surfaces under the Cournot-Nash competition.

Moreover, the payoff of the Home government can be obtained in the exact same way as in the autarkic case above. Consequently, we have the Home government's payoff except the effect of the initial pollutant stock level as

$$U^C = \frac{4}{9r}(A - c)^2 - \frac{2s(1 + \alpha)}{3r(r + k)}(A - c). \tag{16}$$

As another possible mode of international duopoly under free trade, we also consider the case where the two firms are engaged in a Stackelberg type competition. In a Stackelberg duopoly game, the leading firm is somehow able to make a credible commitment with respect to the supply level of Good 1 prior to its follower.

Substituting (13) into the definition of π above, the Home firm's profit function, when it acts as the Stackelberg leader, can be described as

$$\pi = \left(A - c - \frac{x + x^*}{2} \right) x$$
$$= \left(\frac{A - c}{2} - \frac{x}{4} \right) x.$$

Thus, from the profit maximization problem of this Stackelberg leader, we can easily derive the following levels of output supply in a Stackelberg equilibrium:

$$x^L = A - c, \tag{17}$$

$$x^F = \frac{A - c}{2}, \tag{18}$$

where x^L and x^F respectively denote the output levels of the leader and the follower. Furthermore, the equilibrium price, p^S, becomes

$$p^S = A - \frac{x^S + x^{*S}}{2} = \frac{A + 3c}{4}. \tag{19}$$

Comparing (15) and (19), we can observe $p^S < p^C$, i.e., the price of Good 1 is lower under the Stackelberg competition than under the Cournot-Nash competition. Hence, the procompetitive effect of international trade is strengthened further in the Stackelberg outcome.

In a similar way to the autarkic case above, substituting (17) and (18) into (5), we can obtain the path of the pollutant stock over time. Then, by substituting this time-path of the pollutant stock as well as (3) and (19) into (4), we have the payoffs of the countries with the leader firm and the follower firm, respectively, as

$$U^L = \frac{17}{32r}(A - c)^2 - \frac{s(2 + \alpha)}{2r(r + k)}(A - c), \tag{20}$$

$$U^F = \frac{13}{32r}(A - c)^2 - \frac{s(1 + 2\alpha)}{2r(r + k)}(A - c). \tag{21}$$

Once again, these payoff values are described by excluding the effects of the initial pollutant stocks just for the simplicity of exposition.

4   Inter-governmental game

In this section, we introduce a game between Home and Foreign governments over their respective firms' roles in the international market of Good 1. We assume that this game takes place, once and for all, prior to the international duopoly game by the firms described above, and the players are the governments of the two nations, instead of the firms themselves as is commonly supposed in the endogenous-timing literature.[3]

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[3]Syropoulos (1994) analyzed the endogenous timing game of governmental trade interventions with different policy instruments in a similar framework to Hamilton and Slutsky (1990). Our model differs

In this study, we suppose that the government can intervene the market only as regards the timing of the move by its own firm in the international market of Good 1 at each time-instant and it does not possess any other kinds of policy measures. We consider that, although the implementation of an environmental policy might be difficult for informational and/or institutional reasons, national governments could make announcements that effectively commit their own firms to the specific timings of the moves in this particular international duopoly, and jointly determine their respective roles in this market.

For simplicity, the strategy space of our inter-governmental game is restricted to {1, 2, no trade} and we only consider pure strategies. When one government chooses 1 and the other chooses 2, the former nation's firm assumes the role of the Stackelberg leader in the infinitely-repeated international duopolistic competition that follows, while the firm in the latter country becomes the Stackelberg follower. When the two governments choose the same number, the mode of international competition becomes that of the Cournot-Nash type, i.e., the concurrent choices of output supply levels by the two firms at each time-instant. Moreover, we suppose that a firm cannot export its product when its government decides to close the domestic market to import from the other country. Hence, when at least one of the two governments chooses 'no trade', the autarkic situation arises in each country. The payoff matrix of this inter-governmental game is described in Figure 1, with all the payoff values corresponding to the ones described in the previous sections.

(Figure 1 around here)

Observing Figure 1, in combination with the payoffs of the governments under different circumstances obtained above, we can derive the Nash equilibrium outcomes of this inter-governmental game and, therefore, the subgame-perfect Nash equilibrium outcomes of the whole game, possibly including the international duopolistic competition between the two firms afterward unless the outcome of the inter-governmental game is 'autarky'. As we discuss below, the subgame-perfect Nash equilibrium outcomes of this whole game

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from his framework in that an outcome of the governmental interaction determines the timings of moves in a game where they themselves do not participate directly.

can be categorized into several different classes, depending upon the values of the economic and environmental variables. In particular, we focus upon the magnitudes of the two environmental variables, i.e., k, the decay rate of the pollutant stock, and α, the transboundary pollutant import coefficient, as well as upon whether the two countries are symmetric or not in these respects.

4.1 Symmetric case

We begin our analysis of the inter-governmental game above with a simple case where the two countries share the same values of k and α. That is, we suppose that $k = k^*$ and $\alpha = \alpha^*$ in this subsection. Despite the fact that the two countries are completely symmetric in both environmental and economic aspects, we have indeed various possibilities as equilibrium outcomes of this game, depending on the magnitudes of these variables. The first finding is summarized in the following statement.

Proposition 1. *The Cournot-Nash competition in the international market can be a subgame-perfect Nash equilibrium outcome for the values of k and α that satisfy the following inequality*:
$$k + r > \frac{12s(1 + \alpha)}{5(A - c)}, \tag{22}$$
and, at least, one of the following two inequalities:
$$k + r < \frac{48s(2 + \alpha)}{25(A - c)}, \tag{23}$$
$$k + r > \frac{48s(1 - 2\alpha)}{11(A - c)}. \tag{24}$$

Proof. By construction, the autarkic situation always constitutes a subgame-perfect Nash equilibrium of the whole game. This is because, whatever action it may take, a government's payoff is the same autarkic one when the other government chooses 'no trade' in the inter-governmental game. We now attempt to show that the Cournot-Nash competition can also be an equilibrium outcome under the conditions described above. In order for the Cournot-Nash type competition to be a subgame-perfect Nash equilibrium

in addition to the autarkic situation, the government must, at least, prefer the Cournot-Nash outcome to the autarkic one. Taking the ratio of U^A in (10) and U^C in (16), and setting $U^A/U^C < 1$ yields (22). Moreover, we need to exclude the circumstances under which one government wishes its firm to be the Stackelberg leader and the other wishes its firms to be the Stackelberg follower at the same time, in comparison with having their firms compete in the Cournot-Nash fashion. Taking the ratio of U^L in (20) and U^C and setting $U^L/U^C < 1$ leads to (23). If this inequality is met, the government prefers to have its firm become one of the Cournot-Nash competitors to having its firm become the Stackelberg leader. On the other hand, the government would be better off by having its firm become a Cournot-Nash competitor rather than the Stackelberg follower if $U^F/U^C < 1$ holds. Taking the ratio of U^F and U^C and setting $U^F/U^C < 1$ leads to (24).[4] **Q.E.D.**

Proposition 1 implies that, if the actual values of the variables fall in the region C in Figure 2 with α on the horizontal axis and $(k+r)$ on the vertical axis,[5] the Cournot-Nash competition in the international market can bring net gain from trade to each country.

<center>(**Figure 2 around here**)</center>

More interestingly, in addition to the Cournot-Nash type competition, there is a possibility that a Stackelberg-type competition arises as an equilibrium outcome in the international market even with two completely symmetric countries.

Proposition 2. *The Stackelberg-type competition emerges as a subgame-perfect Nash*

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[4]Condition (24) is meaningful only if $\alpha < 1/2$ because $U^F > U^C$ trivially holds for any $\alpha > 1/2$.

[5]Since k and r virtually play the same roles in our results, we mainly express them in such a combination.

equilibrium outcome when k and α satisfy the following four conditions simultaneously:

$$r + k > \frac{16s}{5(A-c)}, \tag{25}$$

$$r + k > \frac{16s\alpha}{A-c}, \tag{26}$$

$$r + k > \frac{48s(2+\alpha)}{25(A-c)}, \tag{27}$$

$$r + k < \frac{48s(1-2\alpha)}{11(A-c)}. \tag{28}$$

Furthermore, if the following inequality is satisfied, the first-mover, i.e., the Stackelberg leader, is relatively better off than the second mover, i.e., the Stackelberg follower, and, otherwise, the second mover has an advantage:

$$r + k > \frac{4s(1-\alpha)}{A-c}, \tag{29}$$

Proof. In order for a Stackelberg competition to be a Nash equilibrium outcome of the inter-governmental game, first of all, the Stackelberg outcome has to be superior to the autarkic outcome for both countries. Such conditions are given by $U^A/U^L < 1$ and $U^A/U^F < 1$, which are respectively translated into (25) and (26). Moreover, these two countries must simultaneously be better off under the Stackelberg equilibrium in comparison with the Cournot-Nash equilibrium. Hence, it must be the case that $U^L/U^C > 1$ or $U^F/U^C > 1$, which are respectively transformed into (27) and (28). Moreover, we have the case of the first mover advantage if $U^L/U^F > 1$, which can be expressed as (29). **Q.E.D.**

The region of having a Stackelberg-type competition as a subgame perfect Nash equilibrium outcome is depicted as region S in Figure 2. Even when the Cournot-Nash outcome is dominated by the autarkic outcome for both countries, under the conditions of Proposition 2 the Stackelberg outcome can constitute an equilibrium and free trade becomes mutually beneficial to the respective countries. We need to add, however, that the likelihood of having a Stackelberg-type competition as an equilibrium outcome would be quite slim in this symmetric case, as we can infer from the very limited area of region S in Figure 2

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When neither of the conditions of Proposition 1 and Proposition 2 is met, autarky is the only equilibrium outcome. The region that satisfies the conditions for the realization of such an outcome is depicted as the non-shaded region A in Figure 2. Irrespective of the pollutant import coefficient (α), as long as the decay rate of the pollutant stock (k) is sufficiently low given the discount rate (r), the government would be strictly better off by remaining in the autarkic situation and, as a consequence, the international trade of Good 1 may not materialize between the two countries. In other words, if a country is sufficiently vulnerable to the pollution issue, in terms of a slow decay of the pollutant stock, it rationally opts for autarky for the fear of the long-term effect of the pollution, even though free trade in the good itself could be mutually gainful.

In one extreme case of $\alpha = \alpha^* = 0$, where the pollution problem is completely localized, all of the three types of outcomes, i.e., autarky, Cournot-Nash and Stackelberg competitions in the aggregated market, are possible, depending on the magnitude of the decay rate of the pollutant stock, k. When $k + r < \frac{12s}{5(A-c)}$ holds, autarky is the only equilibrium outcome. When $\frac{96s}{25(A-c)} < k + r < \frac{48s}{11(A-c)}$ holds, on the other hand, the Stackelberg type competition becomes an equilibrium outcome. When either $\frac{12s}{5(A-c)} < k + r < \frac{96s}{25(A-c)}$ or $\frac{48s}{11(A-c)} < k + r$ holds, the Cournot-Nash type competition can materialize. In summary, if the pollutant decay rate is sufficiently high, i.e., $k > \frac{12s}{5(A-c)} - r$, the gain from trade can be captured by each country in this completely localized pollution problem.

In the other extreme case of $\alpha = \alpha^* = 1$, on the other hand, the Stackelberg-type competition never occurs since no country would be content with allowing its firm to become the Stackelberg follower. If $k + r < \frac{24s}{5(A-c)}$ holds, autarky is the only equilibrium outcome and, otherwise, the Cournot-Nash type competition can also be an equilibrium outcome. Hence, in a global pollution problem with two symmetric nations, the gains of trade is less likely to materialize than in the localized pollution case, with $k = \frac{24}{5(A-c)} - r$ is the threshold value of having the gains from trade in terms of the decay rate, instead of $k = \frac{12}{5(A-c)} - r$. As we can easily imagine, the gain from trade is more likely to materialize as the pollution issue is more localized since the line, $U^A = U^C$, is monotone increasing in α in Figure 2.

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4.2 Asymmetric case

When the two countries are asymmetric in terms of having different values of decay rates of their respective pollutant stocks (k) and pollutant import coefficients (α), we need to examine each government's preferred outcomes separately. Accordingly, we have far greater possibilities as potential equilibrium outcomes than in the symmetric case above. Especially, the possibility of a Stackelberg equilibrium significantly expands in the asymmetric case, compared to the symmetric case. In order to simplify the following exposition, as a possible form of Stackelberg-type competition, we focus on the case where Home's firm is the Stackelberg leader and Foreign's firm is the follower. It should be noted that exactly the same argument holds even when the roles in a Stackelberg equilibrium are reversed between the two firms.

Before discussing a Stackelberg equilibrium outcome, we first present the next statement concerning the Cournot-Nash outcome.

Proposition 3. *When k, α, k^*, and α^* satisfy the following two conditions*:

$$k + r > \frac{12s(1 + \alpha)}{5(A - c)}, \tag{30}$$

$$k^* + r > \frac{12(1 + \alpha^*)}{5(A - c)}, \tag{31}$$

and, in addition, one of the following two conditions, the Cournot-Nash competition becomes an equilibrium outcome and both countries can potentially gain from trade:[6]

$$k + r < \frac{48s(2 + \alpha)}{25(A - c)}, \tag{32}$$

$$k^* + r > \frac{48s(1 - 2\alpha)}{11(A - c)}. \tag{33}$$

Proof. When (30) and (31) are satisfied for the respective countries, both of them can gain by moving from autarky to the Cournot-Nash type competition under free trade. However, a Stackelberg outcome may be even more beneficial than the Cournot-Nash

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[6]Recall that (33) is meaningful only if $\alpha^* < 1/2$ since any value of $\alpha^* > 1/2$ leads to $U^{*C} > U^{*F}$ trivially.

outcome to both nations. Restricting our attention to a Stackelberg equilibrium where Home's firm is the leader and Foreign's firm the follower, we can safely exclude such a possibility if either $U^L/U^C < 1$ or $U^{*F}/U^{*C} < 1$ is satisfied. Each of these conditions can be expressed as (32) and (33). **Q.E.D.**

Again, as in the symmetric case above, the autarkic situation always constitutes a sub-game perfect Nash equilibrium outcome of the whole game. However, this equilibrium is weakly dominated by the other equilibrium outcome of the Cournot-Nash type competition when the conditions of Proposition 3 above are met.

In addition to the Cournot-Nash type competition, a Stackelberg type competition which also weakly dominates the autarkic situation can arise.

Proposition 4. *When k, α, k^*, and α^* satisfy the following four conditions simultaneously, the Stackelberg-type competition with Home's firm being the leader and Foreign's firm being the follower emerges as a subgame-perfect Nash equilibrium outcome.*

$$k + r > \frac{16s}{5(A - c)}, \tag{34}$$

$$k + r < \frac{48s(2 + \alpha)}{25(A - c)}, \tag{35}$$

$$k^* + r > \frac{16s\alpha^*}{(A - c)}, \tag{36}$$

$$k^* + r < \frac{48s(1 - 2\alpha^*)}{11(A - c)}. \tag{37}$$

Proof. In order for this Stackelberg outcome to be a Nash equilibrium of the inter-governmental game, first of all, the Stackelberg outcome has to be superior to the autarkic outcome for both countries. Such conditions are given by $U^A/U^L < 1$ and $U^{*A}/U^{*F} < 1$, which are respectively translated into (34) and (36). Moreover, the respective countries must simultaneously be better off under the Stackelberg equilibrium in comparison with the Cournot-Nash equilibrium. Hence, it must be the case that $U^L/U^C > 1$ or $U^{*F}/U^{*C} > 1$, which are respectively transformed into (35) and (37). **Q.E.D.**

The region of having this type of Stackelberg competition as a subgame-perfect Nash equilibrium outcome is depicted as region S and S^* in Figure 3. It should also be noted that a symmetric argument can be made for the case where Foreign's firm is the Stackelberg leader and Home's firm is its follower.

(Figure 3 around here)

When neither of the conditions of Proposition 3 and Proposition 4 is met, autarky is the only equilibrium outcome. Similarly to the symmetric case above, when its decay rate of its pollutant stock is sufficiently low given the pollutant import coefficient, the government tends to be strictly better off by remaining in the autarkic situation and, as a consequence, the international trade of Good 1 may not materialize between the two countries. Even when the Cournot-Nash outcome is dominated by the autarkic outcome for both countries, however, the Stackelberg outcome can also be an equilibrium and free trade is beneficial to the two countries in this asymmetric case as long as the conditions of Proposition 4 is met. Hence, we can state, at least, in the context of our analytical model, that gain from trade is more likely to be captured by each country with stronger dissimilarity in the environmental characteristics across the country.

In view of Proposition 4, we can obtain further insights into the nature of a Stackelberg outcome. Firstly, it implies that, in order for a Stackelberg outcome to be an equilibrium, there needs to be a country with a fairly small import coefficient of the transboundary pollution. This country must also have a sufficiently low value of the decay rate of the pollutant stock. If these two conditions are concurrently met, this environmentally-vulnerable country would be willing to have its firm become the Stackelberg follower since its consumer benefits from a lower price due to the expanded supply of Good 1 but does not have to suffer too greatly from the transboundary pollution due to the associated expansion of the polluting good production in the other nation as long as the value of the pollutant import coefficient is sufficiently small.

Secondly, when there exists a Stackelberg equilibrium, it is generally the case that the firm in a country with a higher decay rate of the pollutant stock becomes the Stackelberg leader. The country whose firm is the Stackelberg leader is going to experience a significant increase in its domestic production of the polluting good. A country with a higher

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value of k is more resistant to the increased pollutant flow associated with this expanded domestic production and likely to assume the role of the Stackelberg leader.

In one extreme case of $\alpha = \alpha^* = 0$, i.e., when the pollution problem is completely localized, all of the three outcomes are possible, depending on the magnitudes of the decay rates of the pollutant stocks, k and k^*. Especially, when $\frac{96s}{25(A-c)} < k + r$ and $k^* + r < \frac{48s}{11(A-c)}$ simultaneously hold, the Stackelberg type competition with Home's firm being its leader becomes an equilibrium outcome. It should be noted that, unlike the symmetric case above, even when the pollutant decay rate in one country is quite low, the gain from trade can be captured by each country in a localized pollution problem, provided that the other country's pollutant decay rate is sufficiently high in the asymmetric case. In the other extreme case of $\alpha = \alpha^* = 1$, on the other hand, the Stackelberg-type competition never occurs since no country would be content with allowing its firm to become the Stackelberg follower. Nevertheless, the gain from trade can realize to each country even in this global pollution case as long as $\frac{24s}{5(A-c)} < k + r$ and $\frac{24s}{5(A-c)} < k^* + r$ simultaneously hold.

5   Concluding remarks

Our analytical findings might provide some new insights into practical policymaking issues surrounding trade liberalization when a transboundary stock pollution problem is one of each government's interests. Trade liberalization in a good whose production generates transboundary pollutant emissions has two opposing effects: procompetitive effect and pollution-expansion effect. The welfare implications of these effects of international trade could be contingent on certain environmental characteristics of each country, among other things.

In our particular game model, the governments intervene the market with respect to the timings of their firms' entering into the international market and, consequently, determine the roles of their respective firms there. The results of our analysis indicate that decay rates of pollutant stocks and transfer coefficients of transboundary pollution might play significant roles in determining not only the existence of net gain from trade but also the type of competition in the international market of such an product. Al-

though the government tends to forgo the trade opportunity when the stock pollution exerts environmental damages for a prolonged period of time, a smaller pollutant import coefficient and dissimilarity between the two countries on these environmental aspects could nevertheless create an opportunity to bring gains from trade to both nations.

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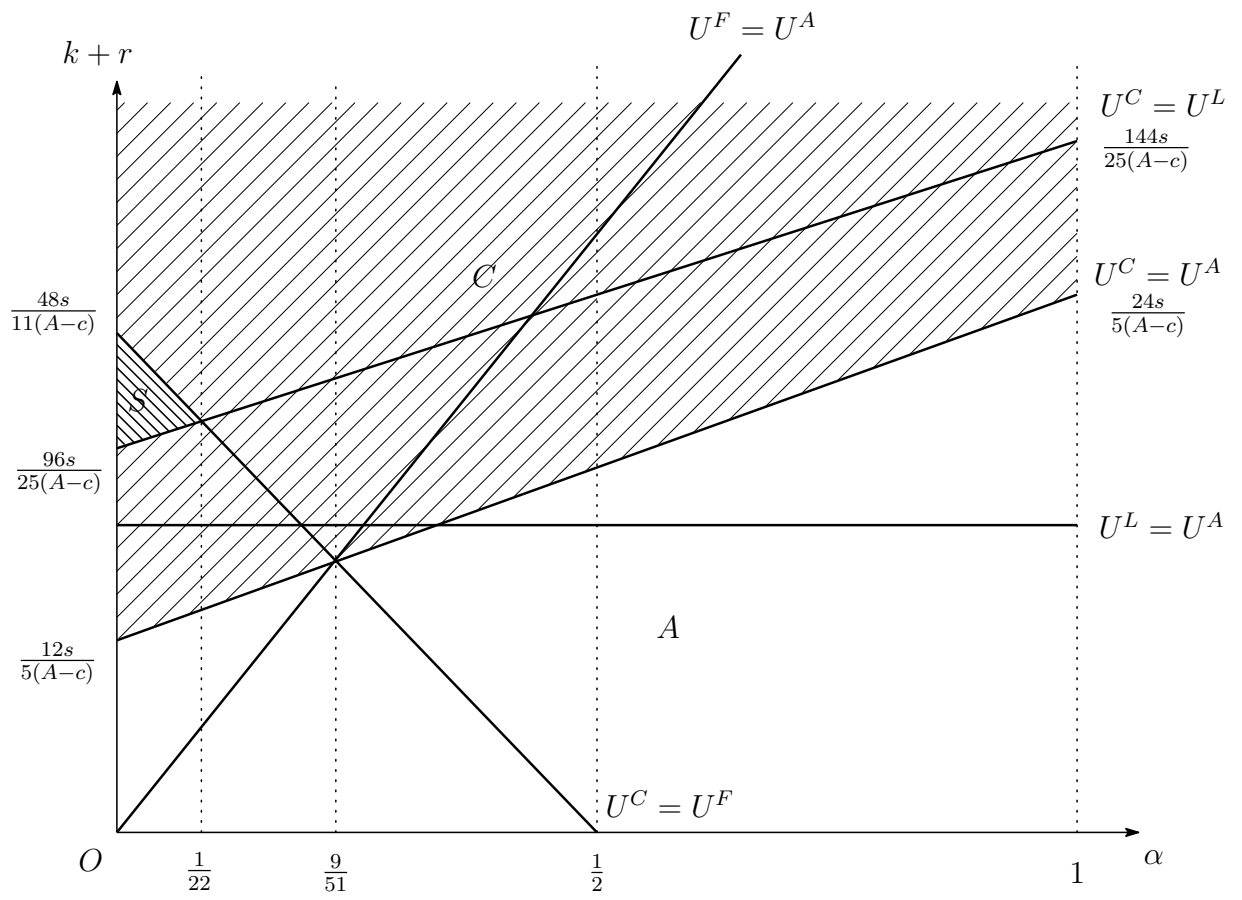| Home\Foreign	1	2	no trade
1	U^C, U^C	U^L, U^F	U^A, U^A
2	U^F, U^L	U^C, U^C	U^A, U^A
no trade	U^A, U^A	U^A, U^A	U^A, U^A

Figure 1: Payoff matrix of the timing game

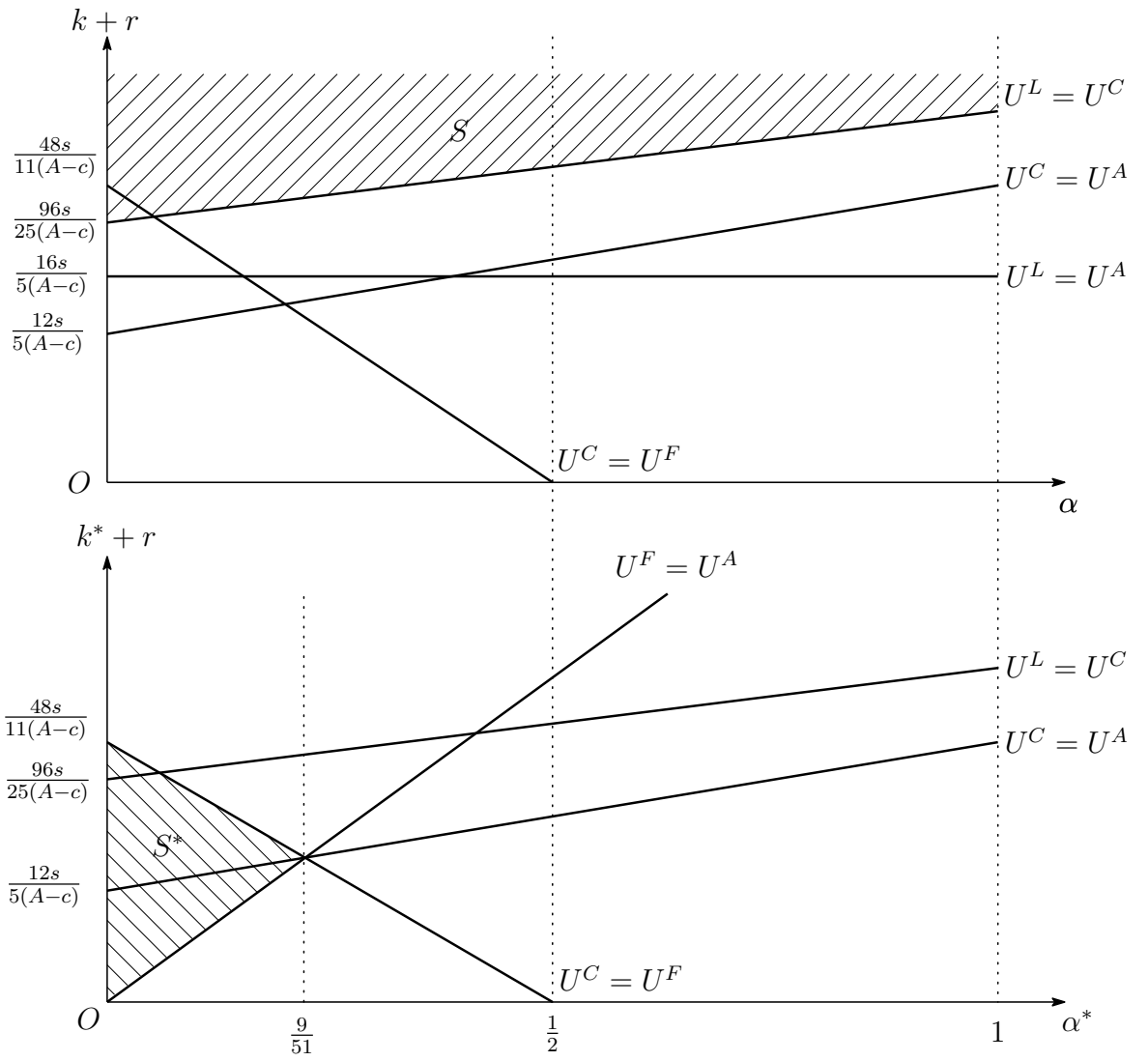Figure 2: Regions for the respective outcomes in the symmetric case

Figure 3: Regions for a Stackelberg outcome in the asymmetric case