

# Population Aging, Sustainability of Public Debt, and Endogenous Growth

Ken Tabata\*

This paper examines how population aging influences the sustainability of public debt in an overlapping generations model with unfunded social security. We show that population aging increases the government's risk of bankruptcy and negatively affects economic growth when the population growth rate is sufficiently low. We also show that social security reform reduces the government's risk of bankruptcy and positively affects economic growth.

Ken Tabata

JEL : E62, H55, H63, O40

Keywords : population aging, social security, public debt, economic growth

## 1 Introduction

This paper develops an overlapping generations model of growth and public debt according to the model suggested by Bräuninger (2005), and then uses this framework to analyze the economic impact of population aging and social security reform. The situation considered here is that an aging population causes a heavy burden of social security payments, while social security tax alone would not be sufficient to finance such payments fully. Public debt is introduced in order to supplement this

---

\* Address: School of Economics, Kwansei Gakuin University, 1-155 Uegahara Ichiban-cho, Nishinomiya 662-8501, Hyogo, Japan;  
E-mail: tabataken@kwansei.ac.jp

balance. In such a situation, we examine how population aging influences the sustainability of public debt and economic growth. Furthermore, the economic impact of social security reform will also be examined.

The present analysis is motivated by two important issues in Japan, population aging and government budget deficits. Population aging is triggered by a rise in life expectancy and a decline in the fertility rate. According to demographic projections by the United Nations Population Division, the old-age dependency ratio (the share of the total population aged 65 and over) in Japan is expected to grow from 19.7% in 2005 to 37.7% in 2050, while the working population share (the share of the total population aged 15–64) is expected to decline from 66.4% in 2000 to 51.1% in 2050.<sup>1)</sup> In addition, the public debt in Japan exploded after the collapse of the “bubble economy”. According to OECD Economic Outlook 2006, the debt/GDP ratio in Japan increased from 74.7% in 1993 to 177.6% in 2007. Because the Japanese public pension scheme is based on a pay-as-you-go pension scheme, population aging leads to a greater burden of social security payments on younger generations. In addition, as stressed by Ono (2003), because the social security payments are partly funded by the state budget, government deficits and social security finance are closely related to each other. Therefore, this paper constructs a growth model that explicitly incorporates unfunded social security and public debt, and analyzes how population aging and social security reform influence the sustainability of public debt. Note that population aging is a common feature of developed countries. Many developed countries also employ pay-as-you-go pension schemes. Moreover, most developed countries have experienced continual budget deficits since the mid 1970s. Therefore, the implications of this paper

---

1) Source: The World Population Prospects: The 2006 Revision Population Database.

are also applicable to many other developed countries.

Existing studies such as Pecchenino and Polland (1997), Pecchenino and Utendorf (1999), and Futagami and Nakajima (2001) analyzed unfunded social security and population aging in economic growth models. However, these studies did not consider the issue of public debt explicitly. Previous studies such as Chalk (2000), Bräuninger (2005) and Yakita (2008) examined the sustainability of public debt using an overlapping generations model. For example, Bräuninger (2005) employed a Romer (1986) type endogenous growth model and showed that there is a critical deficit–GDP ratio. If the deficit–GDP ratio is below the critical level, there exists a stable steady-growth path. However, if the deficit–GDP ratio is above the critical level, the bond-financed deficit is not sustainable. However, these studies do not consider the issue of social security explicitly. To my knowledge, Gertler (1997) and Ono (2003) are two exceptional analytical studies that explicitly considered both the issues of social security and public debt. Gertler (1997), who modified the Blanchard (1985) and Weil (1989) framework, analyzed social security as financed by public debt. Ono (2003) analyzed a social security policy with public debt using a Diamond (1965) type overlapping generations model. However, these studies employed neoclassical growth models. Thus they cannot examine the growth effects of population aging and social security reform. Moreover, they did not focus their attention on the sustainability of public debt. Therefore, this paper extends a Bräuninger (2005) type endogenous growth model by introducing unfunded social security and examines how population aging and social security reform influence economic growth and the welfare of agents.

The main findings of this paper are as follows. First, population aging caused by the decline in the population growth rate increases the government’s risk of bankruptcy, while a reduction in the old-age

pension benefit contributes to the sustainability of public debt. Second, population aging caused by the decline in the population growth rate negatively affects economic growth when the population growth rate is low, but it positively affects economic growth when the population growth rate is high. In addition, a reduction in the old-age pension benefit positively affects economic growth.

This paper is organized as follows. In Section 2, we present the basic model. Section 3 shows how population aging or a reduction in the old-age pension benefit influences the sustainability of public debt. In Section 4, we examine how population aging or a reduction in the old-age pension benefit influences the long-run growth rate of the economy. Finally, Section 5 provides concluding remarks.

## 2 The model

### 2.1 Basic setup

We consider a one-sector, endogenous growth model with unfunded social security. In each period  $t$ , a new generation, called generation  $t$ , is born and is composed of a continuum of  $N_t > 0$  units of identical agents. The population growth rate is assumed to be constant at  $n \in (-1, \infty)$  and satisfies  $N_t = (1 + n)N_{t-1}$ . Agents in this economy live for two periods, youth and old age. In each period  $t$ , there exist only two generations, the active working young and the retired old. Therefore, the share of old dependents relative to the young working population in period  $t$  is given by  $\frac{N_{t-1}}{N_t} = \frac{1}{1+n}$ . Thus, population aging would be triggered if there were a decrease in the population growth rate  $n$ .

Agents derive utility from their own consumption in both youth and old age. Thus, the lifetime utility of the agent in generation  $t$  is expressed as:

$$u_t = \ln c_{1,t} + \beta \ln c_{2,t+1}, \quad (1)$$

where  $c_{1,t}$  and  $c_{2,t+1}$  denote the consumption when young and old, and  $\beta$  is a time discount rate.

In youth, each agent is endowed with one unit of labor, works inelastically, and obtains wage income. An agent in generation  $t$  divides his or her wage income  $w_t$  between his or her own current consumption  $c_{1,t}$ , savings  $s_t$ , and the payment of social security taxes  $\tau_t w_t$ , where  $\tau_t$  is the rate of social security tax levied on wage income. In old age, agents retire and consume their returns on savings  $(1 + r_{t+1})s_t$  and social security benefits  $z_{t+1}$ .

Thus, the lifetime budget constraints of the agent in generation  $t$  are:

$$c_{1,t} + s_t = (1 - \tau_t)w_t, \quad (2)$$

$$c_{2,t+1} = (1 + r_{t+1})s_t + z_{t+1}. \quad (3)$$

By maximizing (1), subject to (2) and (3), we obtain:

$$s_t = \frac{\beta}{1 + \beta} (1 - \tau_t)w_t - \frac{1}{1 + \beta} \frac{z_{t+1}}{1 + r_{t+1}}. \quad (4)$$

This saving equation states that higher social security benefits  $z_{t+1}$  and tax rate  $\tau_t$  imply lower savings.

Each firm has constant returns-to-scale technology, and the aggregate production function is expressed as  $Y_t = F(K_t, E_t N_t)$ , where  $Y_t$ ,  $K_t$ , and  $N_t$  denote the aggregate levels of output, physical capital, and labor input, respectively.  $E_t$  represents labor productivity, which is assumed to be driven by Romer-type spillovers that emanate from accumulated investment per worker. In order to ensure the existence of a long-run growth path, it is assumed that  $E_t$  takes a particular form as follows:  $E_t = \frac{1}{a} \frac{K_t}{N_t} = \frac{k_t}{a}$ , where  $k_t \equiv K_t/N_t$  and  $a$  is a positive technological

parameter. Representing the production function in effective per capita terms,  $\hat{y}_t = f(\hat{k}_t)$ , where  $\hat{y}_t \equiv Y_t/E_t N_t$ ,  $\hat{k}_t \equiv K_t/E_t N_t$ , and noting that  $\hat{y}_t = f(a)$  and  $\hat{k}_t = a$  hold, we obtain the optimal condition for a representative firm as:

$$\rho_t = f'(a) = \rho, \quad (5)$$

$$w_t = \left[ \frac{f(a) - f'(a)a}{a} \right] k_t = \bar{w}k_t, \quad (6)$$

where  $\rho \equiv f'(a)$  and  $\bar{w} \equiv \frac{f(a) - f'(a)a}{a}$ . Because the market for capital is competitive, the following arbitrage condition holds:

$$1 - \delta + \rho_{t+1} = 1 + r_{t+1} = 1 + r, \quad (7)$$

where  $\delta$  is the depreciation rate of capital. Because  $\rho_{t+1}$  is constant at  $\rho$ , the rate of return on savings  $r_{t+1}$  is also constant at  $r$ .

The government in this economy can impose a social security tax,  $\tau_t$ , on wages. Moreover, it can issue public debt with a one-period maturity in order to finance social security payments. The government funds the expenditure of social security and the repayment of debt with tax receipts and with the revenue from the newly issued debt. Thus the government budget constraint in period  $t$  is given by  $D_{t+1} + \tau_t w_t N_t = z_t N_{t-1} + (1 + r_t) D_t$ . With regard to the left-hand side,  $D_{t+1}$  is the revenue from debt issue, while  $\tau_t w_t N_t$  is the revenue from social security tax. With regard to the right-hand side,  $z_t N_{t-1}$  represents the social security payments, while  $D_t$  is the value of public debt maturing during period  $t$ , and  $r_t$  is the rate of return. Dividing both sides by  $N_t$ , we obtain:

$$(1 + n)d_{t+1} + \tau_t w_t = \frac{z_t}{1 + n} + (1 + r_t)d_t, \quad (8)$$

where  $d_t \equiv \frac{D_t}{N_t}$ .

We assume that the government provides unfunded social security using a defined benefit scheme. Under the defined benefit scheme, there are two forms of social security payments. One is a lump sum transfer

where the payments for old agents in generation  $t - 1$  are a fixed constant  $z_t = z$ . The other is a transfer based on a replacement rate for wages, where the payments for old agents in generation  $t - 1$  are calculated using a replacement rate for the next generation's wages  $z_t = \phi w_t$ , where  $\phi \in [0, 1)$  denotes the replacement ratio. In the theoretical analysis, the former (a lump sum) payment scheme is the general assumption. However, in the real world, many developed countries employ the latter payment scheme characterized by a strong link to current wages. In order to maintain the living standards of the elderly, the pension benefits are adjusted taking into account not only price changes (i.e., price indexation) but also wage changes (i.e., wage indexation). For example, in Japan, the pension benefit level of a salaried worker is set to exceed approximately 50% of the wages of the current working population. Therefore, the replacement ratio is about 50%. In our paper, we focus upon the role of wage indexation and assume that the payments for old agents in generation  $t - 1$  are calculated using a replacement rate for the next generation's wages  $z_t = \phi w_t$ . Existing studies such as Meier (2000) and Brooks (2002) also employ an analogous assumption.<sup>2)</sup>

We also assume that the government employs the following fiscal rule concerning newly issued debt:  $D_{t+1} - D_t = \psi Y_t$ , where  $D_{t+1} - D_t$  expresses the budget deficit in period  $t$ . The government borrows a specified fraction  $\psi$  of national income  $Y_t$  and tries to maintain the budget deficit-GDP ratio constant at  $\psi$ .<sup>3)</sup> This type of fiscal policy

---

2) Another reason for the assumption  $z_t = \phi w_t$  is purely technical. This assumption makes the analysis of this paper employing an endogenous growth model with unfunded social security easier.

3) An alternative fiscal rule is that the government sets a constant tax rate,  $\tau$ , and issues public debt to finance the remaining social security expenditure and the repayment of debt. Provided that this alternative fiscal rule is employed, the analytical property of the model is mostly unchanged.

is employed in the EU and other OECD countries. Existing studies such as Bräuninger (2005) and Yakita (2008) also employ an analogous assumption. Dividing both sides by  $N_t$  leads to:

$$(1+n)d_{t+1} = d_t + \psi y_t, \quad (9)$$

where  $y_t \equiv \frac{Y_t}{N_t}$ , and the relations  $y_t = f(a)k_t = (\bar{w} + \rho)k_t$  hold.

Under these social security and fiscal policies, the tax rate is adjusted endogenously to maintain the social security expenditure and the repayment of public debt. From (5)–(9) and  $z_t = \phi w_t$ , the social security tax rate  $\tau_t$  is given by:

$$\tau_t = \frac{\phi}{1+n} + \frac{r}{\bar{w}}x_t - \psi \left(1 + \frac{\rho}{\bar{w}}\right), \quad (10)$$

where  $x_t \equiv \frac{d_t}{k_t}$ . Equation (10) states that a lower population growth rate  $n$ , a higher replacement ratio  $\phi$ , and a higher debt–capital ratio  $x_t$  imply a higher social security tax rate.

Now let us consider the market equilibrium conditions. Noting that an arbitrary opportunity exists between a real asset and bond, we can set the market clearing condition for capital as:  $K_{t+1} + D_{t+1} = s_t N_t$ , which expresses the equality of the total savings by young agents in generation  $t$ ,  $s_t N_t$ , to the sum of the stocks of aggregate physical capital and aggregate public debt. Dividing both sides by  $N_t$  leads to:

$$(1+n)(k_{t+1} + d_{t+1}) = s_t. \quad (11)$$

The commodity market clears automatically according to Walras's law.

## 2.2 Equilibrium

This subsection characterizes the competitive equilibrium allocation of the economy. By inserting  $y_t = (\bar{w} + \rho)k_t$  into (9), we obtain the dynamic evolution of the debt–labor ratio,  $d_t$ , as:

$$\frac{d_{t+1}}{d_t} = \frac{1}{1+n} \left[ 1 + \psi(\bar{w} + \rho) \frac{1}{x_t} \right] \equiv dg(x_t). \quad (12)$$



Let us denote the right-hand side of (12) as  $dg(x_t)$ . Moreover, by inserting (4)–(7), (10), (12) and  $z_t = \phi w_t$  into (11), we obtain the dynamic evolution of the capital–labor ratio,  $k_t$ , as:

$$\begin{aligned} \frac{k_{t+1}}{k_t} &= \\ &= \frac{1}{1+n+\frac{1}{1+\beta}\frac{\phi\bar{w}}{1+r}} \left\{ \frac{\beta}{1+\beta} \left(1 - \frac{\phi}{1+n}\right) \bar{w} - \left(1 + \frac{\beta}{1+\beta} r\right) x_t - \frac{\psi(\bar{w} + \rho)}{1+\beta} \right\} \\ &\equiv kg(x_t). \end{aligned} \quad (13)$$

Let us denote the right-hand side of (13) as  $kg(x_t)$ . The dynamics of the model are completely described by these two difference equations. Because the evolutions of the debt–labor ratio,  $d_t$ , and the capital–labor ratio,  $k_t$ , depend upon the debt–capital ratio,  $x_t$ , the evolution of  $x_t$  is given by:

$$\frac{x_{t+1}}{x_t} = \frac{\frac{d_{t+1}}{d_t}}{\frac{k_{t+1}}{k_t}} = \frac{dg(x_t)}{kg(x_t)}. \quad (14)$$

Therefore, the steady-state equilibrium is characterized by the conditions that  $x_{t+1} = x_t = x$  or:

$$1 + g = dg(x) = dk(x). \quad (15)$$

From (12) and (13), in the steady-state equilibrium, the state variable  $k_t$  and  $d_t$  grow at the same rate  $g$ . Hereafter, we denote the path on which the state variable  $k_t$  and  $d_t$  grow as the same rate as the balanced growth path, and denote the growth rate  $g$  defined in equation (15) as the balanced growth rate.

### 3 Steady state

In this section, we examine the existence and the stability of the steady-state equilibrium. Then, we also examine how population aging or a reduction in the old-age pension benefit influences the sustainability of public debt.

### 3.1 Existence of the steady-state equilibrium

This subsection examines the existence of the steady-state equilibrium. As discussed in Section 2-2, the steady-state equilibrium is characterized by equation (15). From (12) and (13), equation (15) is rewritten as:

$$LH(x; n, \phi) = RH(x), \quad (16)$$

where

$$\begin{aligned} LH(x; n, \phi) &\equiv \frac{1+n}{1+n+\frac{1}{1+\beta}\frac{\phi\bar{w}}{1+r}} \left\{ \frac{\beta}{1+\beta} \left(1 - \frac{\phi}{1+n}\right) \bar{w} - \left(1 + \frac{\beta}{1+\beta} r\right) x_t - \frac{\psi(\bar{w}+\rho)}{1+\beta} \right\}, \end{aligned}$$

and

$$RH(x) \equiv 1 + \psi(\bar{w} + \rho) \frac{1}{x_t}.$$

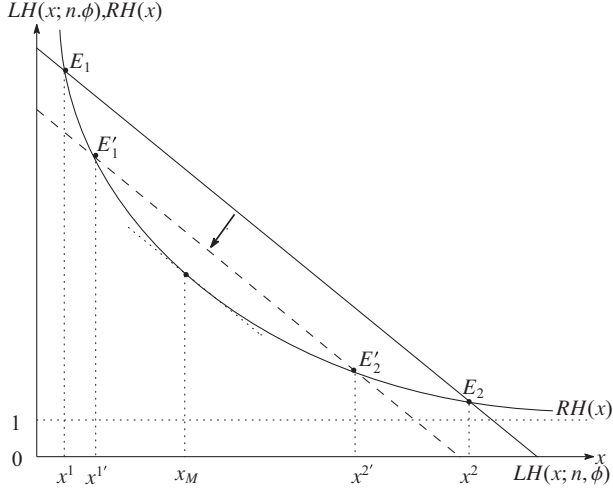
From (16), we can easily confirm that  $\frac{\partial LH}{\partial x} < 0$ ,  $\lim_{x \rightarrow 0} LH(x) < \infty$ ,  $\lim_{x \rightarrow \infty} LH(x) = -\infty$ ,  $\frac{\partial RH}{\partial x} < 0$ ,  $\lim_{x \rightarrow 0} RH(x) = \infty$ ,  $\lim_{x \rightarrow \infty} RH(x) > 0$ . In addition, regarding the left-hand side of (16),  $\frac{\partial LH}{\partial n} > 0$  and  $\frac{\partial LH}{\partial \phi} < 0$  hold. To stress these relationships, we denote the left-hand side of (16) as  $LH(x; n, \phi)$  and the right-hand side of (16) as  $RH(x)$  respectively. Figure 1 illustrates  $LH(x; n, \phi)$  and  $RH(x)$  for given values of  $n$  and  $\phi$ .<sup>4)</sup> From (16), regarding the existence of the steady-state equilibrium, we obtain the following proposition.

**Proposition 1** *Regarding the existence of the steady-state equilibrium, the following statements hold.*

- (1) *There is a critical population growth rate  $\hat{n}$ . If the population growth rate  $n$  is above the critical level  $n > \hat{n}$ , then there are*

---

4) Following Bräuninger (2005) and Yakita (2008), we mainly focus our attention on the regions where the debt-capital ratio  $x_t$  is positive. However, this is only for clarity of explanation. Even if we consider the case where the debt-capital ratio  $x_t$  is negative, the results of this paper do not change at all.

**Figure 1: Existence of equilibria**


two steady states,  $E_1$  and  $E_2$ , where  $E_1$  is characterized by a lower debt-capital ratio  $x^1$ , and  $E_2$  is characterized by a higher debt-capital ratio  $x^2$ . In these steady states,  $0 < x^1 < x^2$ ,  $\frac{dx^1}{dn} < 0$  and  $\frac{dx^2}{dn} > 0$  hold. If the population growth rate  $n$  is below the critical level  $n < \hat{n}$ , then there is no steady state. Only at  $n = \hat{n}$  is there a unique steady-state equilibrium.

- (2) There is a critical replacement ratio  $\hat{\phi}$ . If the replacement ratio  $\phi$  is below the critical level  $\phi < \hat{\phi}$ , then there are two steady states,  $E_1$  and  $E_2$  where  $E_1$  is characterized by a lower debt-capital ratio  $x^1$ , and  $E_2$  is characterized by a higher debt-capital ratio  $x^2$ . In these steady states,  $0 < x^1 < x^2$ ,  $\frac{dx^1}{d\phi} > 0$  and  $\frac{dx^2}{d\phi} < 0$  hold. If the replacement ratio  $\phi$  is above the critical level  $\phi > \hat{\phi}$ , then there is no steady state. Only at  $\phi = \hat{\phi}$  is there is a unique steady-state equilibrium.

The proof of Proposition 1 is given in Appendix A. Proposition 1-1 indicates that there exist two steady states,  $E_1$  and  $E_2$ , when the population growth rate  $n$  is sufficiently high. As described in Figure 1, the decline in the population growth rate  $n$  shifts the function  $LH(x)$  downwards and thus increases the value of  $x^1$ , and lowers the value of  $x^2$ . However, if the population growth rate  $n$  declines further,  $LH(x)$  and  $RH(x)$  may not intersect any more. In this case, there is no steady state. Therefore, there is a critical population growth rate  $\hat{n}$ . If the population growth rate  $n$  is above the critical level  $n > \hat{n}$ , then there are two steady states. However, if the population growth rate  $n$  is below the critical level  $n < \hat{n}$ , then there is no steady state.

Proposition 1-2 also indicates that there exist two steady states,  $E_1$  and  $E_2$ , when the replacement ratio  $\phi$  is sufficiently low. As again described in Figure 1, an increase in the replacement ratio  $\phi$  shifts the function  $LH(x)$  downwards and thus increases the value of  $x^1$  and lowers the value of  $x^2$ . However, if the replacement ratio  $\phi$  is increased further,  $LH(x)$  and  $RH(x)$  may not intersect any more. In this case, there is no steady state. Therefore, there is a critical replacement ratio  $\hat{\phi}$ . If the replacement ratio  $\phi$  is below the critical level  $\phi < \hat{\phi}$ , then there are two steady states. However, if the replacement ratio  $\phi$  is above the critical level  $\phi > \hat{\phi}$ , then there is no steady state.

### 3.2 Stability of the steady-state equilibrium

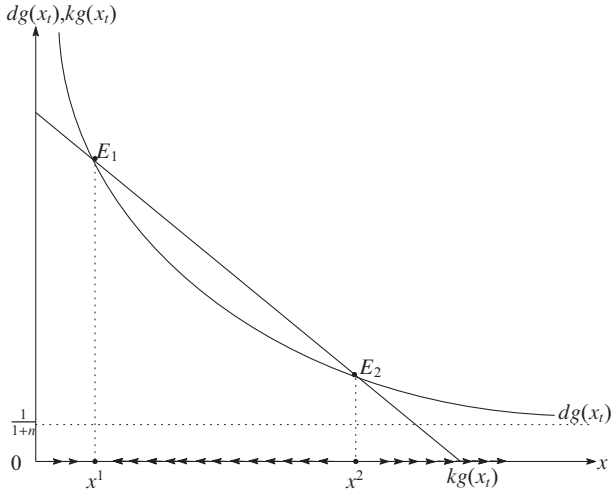
This subsection examines the stability of the steady-state equilibrium. As discussed in Section 2-2, the dynamics of the model are completely described by equations (12), (13) and (14). Figure 2 depicts equations (12) and (13), when there exist two steady states,  $E_1$  and  $E_2$ . Figure 3 depicts equation (12) and (13), when there is no steady state. We denote the right-hand side of equation (12) as  $dg(x_t)$ , and the right-hand

side of equation (13) as  $kg(x_t)$ , respectively. From (12), (13) and (14), regarding the stability of the steady-state equilibrium, we obtain the following proposition.

**Proposition 2** . *Suppose there are two steady states,  $E_1$  and  $E_2$ , and the steady state  $E_1$  characterized by a lower debt-capital ratio,  $x^1$ , is locally stable, whereas the steady state  $E_2$  characterized by a higher debt-capital ratio,  $x^2$ , is unstable.*

The proof of Proposition 2 is given in Appendix B. The intuition of proposition 2 is explained as follows. From Figure 2, the relation  $kg(x_t) > dg(x_t)$  holds when  $x^1 < x_t < x^2$ . In this region, because the capital-labor ratio  $k_t$  grows faster than the debt-labor ratio  $d_t$ , the debt-capital ratio  $x_t$  decreases monotonically (i.e.,  $\frac{x_{t+1}}{x_t} < 1$ ). However, when  $x_t < x^1$  or  $x_t > x^2$ , the relation  $kg(x_t) < dg(x_t)$  holds. In these regions, the debt-labor ratio  $d_t$  grows faster than the capital-labor ratio

**Figure 2: Two steady-states**



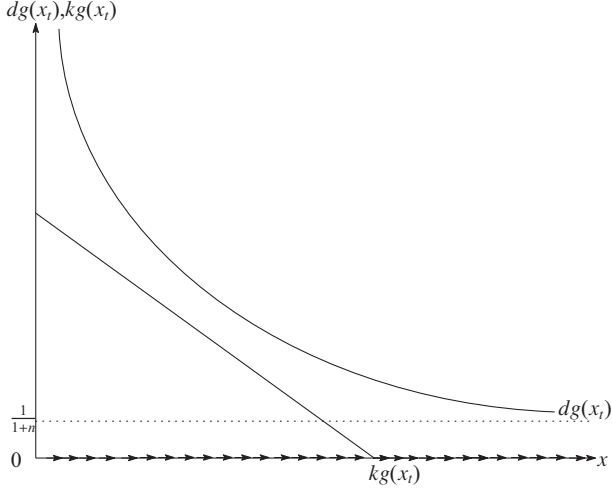
$k_t$ , and the debt–capital ratio  $x_t$  increases monotonically (i.e.,  $\frac{x_{t+1}}{x_t} > 1$ ). Therefore, if the initial debt–capital ratio  $x_0$  is below the critical level  $x^2$ , the debt–capital ratio gradually converges to the steady state  $x^1$ . Thus steady state  $E_1$  characterized by the lower debt–capital ratio  $x^1$  is locally stable. However, if the initial debt–capital ratio  $x_0$  is above the critical level  $x^2$ , the debt–capital ratio  $x_t$  increases monotonically, and the level of the capital–labor ratio  $k_t$  decreases from (13) and will go to zero within a finite number of periods. Once the level of the capital–labor ratio  $k_t$  becomes zero, there is no commodity in the economy, so the value of government debt vanishes. Thus such a situation can be regarded as government bankruptcy.

Figure 3 illustrates the case where there exists no steady state. In this case, as shown in Figure 3, the relation  $dg(x_t) > kg(x_t)$  holds for all  $x_t > 0$ . Thus, regardless of the initial debt–capital ratio  $x_0$ , the debt–capital ratio  $x_t$  increases monotonically (i.e.,  $\frac{x_{t+1}}{x_t} > 1$ ), and the level of the capital–labor ratio  $k_t$  decreases from (13) and goes to zero within a finite number of periods. Hence, the economy always rides on the path to bankruptcy.

Summarizing the results obtained in Sections 3-1 and 3-2, we obtain the following results.

**Result 1** *Regarding the sustainability of the bond-financed deficit, the following statements hold.*

- (1) *The bond-financed deficit is sustainable when the population growth rate  $n$  is sufficiently high to satisfy  $n > \hat{n}$  and the initial debt–capital ratio  $x_0$  is sufficiently low to satisfy  $x_0 < x^2$ .*
- (2) *The bond-financed deficit is sustainable when the replacement ratio  $\phi$  is sufficiently low to satisfy  $\phi < \hat{\phi}$  and the initial debt–capital ratio  $x_0$  is sufficiently low to satisfy  $x_0 < x^2$ .*

**Figure 3: No steady-state**


These results are summarized as follows. The bond-financed deficit is not sustainable when there is no steady state. From Proposition 1, the existence of the steady-state equilibrium is assured when the population growth rate  $n$  is sufficiently high to satisfy  $n > \hat{n}$  or the replacement ratio  $\phi$  is sufficiently low to satisfy  $\phi < \hat{\phi}$ . However, these are not sufficient conditions for the sustainability of public debt. As shown in Proposition 2, the initial debt–capital ratio  $x_0$  plays a crucial role. In order to prevent government bankruptcy, the initial debt–capital ratio  $x_0$  must be sufficiently low to satisfy  $x_0 < x^2$ . Therefore, a sufficiently high population growth rate  $n$ , a sufficiently low replacement ratio  $\phi$  and a sufficiently low initial debt–capital ratio  $x_0$  are sufficient conditions for the sustainability of public debt.

### 3.3 Population aging and social security reform

In this subsection, we examine briefly how population aging caused by the decline in the population growth rate  $n$  influences the sustainability

of public debt. We also examine how a reduction in the old-age pension benefit caused by the decline in the replacement ratio  $\phi$  influences the sustainability of public debt.

Let us first examine the effect of population aging. From Proposition 1-1, a lower population growth rate  $n$  is less likely to satisfy the condition  $n > \hat{n}$ , which ensures the existence of the steady-state equilibrium. Moreover, a lower population growth rate  $n$  decreases the critical value of  $x^2$ , as shown in Proposition 1-1, and shrinks the regions of the initial debt-capital ratio  $x_0$  where the economy can ride on the balanced growth path. Therefore, population aging caused by the decline in the population growth rate  $n$  increases the government's risk of bankruptcy by enlarging the burden of social security expenditure and the repayment of public debt.

Next, let us consider the effect of a reduction in the old-age pension benefit. From Proposition 1-2, a lower replacement ratio  $\phi$  is more likely to satisfy the condition  $\phi < \hat{\phi}$ , which ensures the existence of steady-state equilibrium. Moreover, a lower replacement ratio  $\phi$  increases the critical value of  $x^2$  as shown in Proposition 1-2 and expands the region of the initial debt-capital ratio  $x_0$  where the economy can ride on the balanced growth path. Therefore, a reduction in the old-age pension benefit caused by the decline in the replacement ratio  $\phi$  contributes to the sustainability of public debt by mitigating the burden of social security expenditure and the repayment of public debt.

#### 4 Long-run growth

Section 3 shows that population aging increases the government's risk of bankruptcy, while a reduction in the old-age pension benefit contributes to the sustainability of public debt. In this section, we focus on the case where the economy is in a stable steady-state equilibrium



$E_1$ , and we analyze the long-run growth effects of these policies. We examine how population aging caused by a decline in the population growth rate  $n$  influences the balanced growth rate  $g$ . We also examine how a reduction in the old-age pension benefit caused by the decline in the replacement ratio  $\phi$  influences the balanced growth rate  $g$ .

#### 4.1 Population aging

This subsection examines the long-run growth effect of population aging caused by a decline in the population growth rate  $n$ . As shown in Section 2-2, the balanced growth rate  $g$  is described by (13), (15) and (16). However, because the debt-capital ratio,  $x^1$ , is determined by a somewhat complicated form of the implicit function shown in (16), it is difficult to investigate the growth effect of  $n$  analytically. Therefore, in this subsection, we provide some numerical examples. In the numerical simulation, we specify the production function as  $Y_t = AK_t^\alpha (E_t N_t)^{1-\alpha}$ .

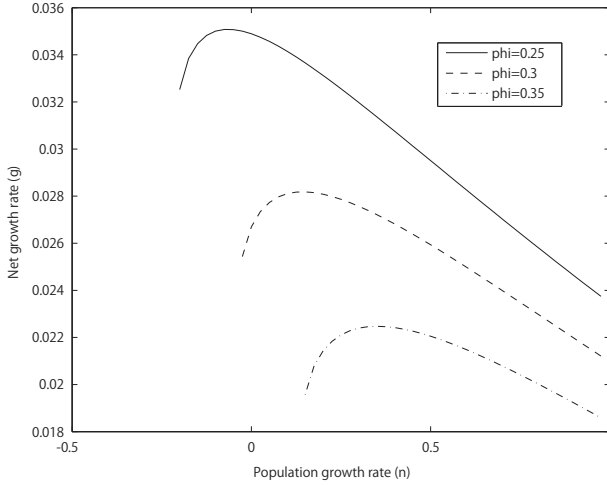
Figure 4 depicts numerical examples of the relationship between the population growth rate  $n$  and the balanced growth rate  $g$  under alternative values of  $\phi$ . The parameters used in the baseline simulations are given in footnote 5.<sup>5)</sup> The figures show that there is a hump-shaped relationship between the population growth rate  $n$  and the balanced growth rate  $g$ .<sup>6)</sup> These results indicate that the growth effect of population aging caused by the decline in the population growth rate  $n$  is positive when the population growth rate  $n$  is high but that it could be negative when the population growth rate is low. These results are intuitively explained as follows.

---

5)  $\alpha = 0.25$ ,  $A = 16$ ,  $a = 1$ ,  $\delta = 1$ ,  $\psi = 0.03$ ,  $\beta = 1$ ,  $n = 0$ ,  $\phi = 0.3$ .

6) Note that the balanced growth rate is defined only when the population growth rate  $n$  is sufficiently high to satisfy  $n > \hat{n}$ . For example, the  $\phi = 0.3$  line in Figure 4 suggests that the critical population growth rate when  $\phi = 0.3$  is  $-0.025$ .

Figure 4: Relationship between  $n$  and  $g$



Equation (13) indicates that the decline in the population growth rate  $n$  has two negative effects and one positive effect on capital accumulation. The first negative effect is the “tax increase effect”. Equation (10) implies that the decline in the population growth rate  $n$  positively affects the tax rate  $\tau$  in the following two ways. First, the decline in  $n$  increases the dependency ratio  $\frac{1}{1+n}$ , which positively affects the tax rate. Second, the decline in  $n$  increases the debt–capital ratio  $x^1$  as shown in Proposition 1-1, which positively affects the tax rate. These two positive effects of the decline in  $n$  on  $\tau$  decrease the net labor earnings and savings of agents and thus negatively affect the capital accumulation. The second negative effect is the “substitution effect from physical capital to public debt”. Proposition 1-2 shows that the decline in  $n$  increases the debt–capital ratio  $x^1$ . This increase in the debt–capital ratio implies a shift in savings from physical capital to public debt. Thus this substitution effect negatively affects the capital accumulation.

The one positive effect is the “antidissolving effect”. From (11), the decline in  $n$  increases the ratio of old to young in the population, which mitigates the dissolving of savings into a larger workforce  $\frac{s_t}{1+n}$ . Thus this antidissolving effect positively affects the capital accumulation

From (13) and (16), it is easy to guess that the effect of the decline in  $n$  is likely to be negative when  $n$  is extremely small (i.e.,  $n \rightarrow -1$ ). Because the ratio of old to young in the population is already large, the positive dissolving effect caused by the decline in  $n$  becomes negligible. However, because the social security tax rate is already high because of the high dependency ratio and the high debt–capital ratio, any increase in social security tax caused by the decline in the population growth rate  $n$  provides a considerable negative impact upon capital accumulation. Therefore, when  $n$  is extremely small, the negative tax reduction effect and substitution effect are likely to dominate the positive antidissolving effect. On the other hand, the effect of the decline in  $n$  is likely to be positive when  $n$  is extremely large (i.e.,  $n \rightarrow \infty$ ). Because the ratio of old to young in the population is already small, the positive antidissolving effect caused by the decline in  $n$  becomes more crucial. However, because the social security tax rate is already low because of the low dependency ratio and the low debt–capital ratio, any increase in social security tax caused by the decline in the population growth rate  $n$  provides a fairly small negative impact upon capital accumulation. Therefore, when  $n$  is extremely large, the positive antidissolving effect is likely to dominate the negative tax reduction effect and the substitution effect.

Therefore, these numerical examples suggest that population aging caused by the decline in the population growth rate may not only increase the government’s risk of bankruptcy but also decrease the rate of capital accumulation and the balanced-growth rate when the population growth rate  $n$  is sufficiently low.

## 4.2 Social security reform

This subsection examines the long-run growth effect of a reduction in the old-age pension benefit caused by a decline in the replacement ratio  $\phi$ . The balanced growth rate  $g$  is again described by (13), (15) and (16). From (13), (15) and (16), regarding the effect of  $\phi$  on  $g$ , we obtain the following proposition.

**Proposition 3** . *In the stable steady state  $E_1$ ,  $\frac{dg}{d\phi} < 0$  holds.*

The proof of Proposition 3 is simple. By differentiating (13) with respect to  $\phi$  and noting  $\frac{dx_1}{d\phi} > 0$  from Proposition 1-2, we can easily confirm that the relation  $\frac{dg}{d\phi} < 0$  holds. Proposition 3 shows that the decline in the replacement ratio  $\phi$  increases the balanced growth rate. Equation (13) indicates that the decline in the replacement ratio  $\phi$  has three positive effects on capital accumulation. The first positive effect is the “tax reduction effect”. Equation (10) implies that the decline in  $\phi$  negatively affects the tax rate  $\tau$  in the following two ways. First, the decline in  $\phi$  decreases the level of social security expenditure, which negatively affects the tax rate. Second, the decline in  $\phi$  lowers the debt–capital ratio  $x^1$  as shown in Proposition 1-2, which negatively affects the tax rate. These two negative effects of the decline in  $\phi$  on  $\tau$  increase the net labor earnings and savings of agents and thus positively affect the capital accumulation. The second positive effect is the “substitution effect from public debt to physical capital”. Proposition 1-2 shows that the decline in  $\phi$  lowers the debt–capital ratio  $x^1$ . This reduction in the debt–capital ratio implies a shift in savings from public debt to physical capital. Thus this substitution effect positively affects the capital accumulation. The third positive effect is the “incentive effect”. The decline in  $\phi$  implies the reduction in the old-age pension benefit. Thus, it motivates agents to save more for their old age, which

positively affects the capital accumulation.

Therefore, we can confirm that a reduction in the old-age pension benefit caused by the decline in the replacement ratio  $\phi$  not only contributes to the sustainability of public debt but also increases the rate of capital accumulation and the balanced-growth rate.

## 5 Concluding remarks

This paper examined how population aging influences the sustainability of public debt in an overlapping generations model with unfunded social security. We showed that population aging increases the government's risk of bankruptcy and negatively affects economic growth when the population growth rate is sufficiently low. We also showed that social security reform reduces the government's risk of bankruptcy and positively affects economic growth.

## References

- [1] Blanchard, O.J., (1985). "Debt, deficits, and finite horizons", *Journal of Political Economy* 93, 223–247.
- [2] Bräuninger, M., (2005). "The budget deficit, public debt, and endogenous growth", *Journal of Public Economic Theory* 7, 827–840.
- [3] Brooks, R.J., (2002). "Asset-market effects of the baby boom and social-security reform", *American Economic Review* 92, 402–406.
- [4] Chalk, N.A., (2000). "The sustainability of bond-financed deficits: an overlapping generations approach", *Journal of Monetary Economics* 45, 293–328.
- [5] Diamond, P.A., (1965). "National Debt in a Neoclassical Growth Model", *American Economic Review* 55, 1126–1150.
- [6] Futagami, K. and T. Nakajima, (2001). "Population aging and economic growth", *Journal of Macroeconomics* 23, 31–44.

- [7] Gertler, M., (1997). “Government debt and social security in a life cycle economy”, *NBER Working Paper* No. 6000.
- [8] Meier, V., (2000). “Time Preference and International Migration, and Social Security”, *Journal of Population Economics* 13, 127–146.
- [9] Ono, T., (2003). “Social Security Policy with Public Debt in an Aging Economy”, *Journal of Population Economics* 16, 363–387.
- [10] Pecchenino, R. and P. Pollard, (1997). “The effect of annuities, bequests and aging in an overlapping generations model of endogenous growth”, *Economic Journal* 107, 26–46.
- [11] Pecchenino, R. and K. Utendorf, (1999). “Social security, social welfare and the aging population”, *Journal of Population Economics* 12, 607–623.
- [12] Romer, P.M., (1986). “Increasing returns and long-run growth”, *Journal of Political Economy* 94, 1002–1037.
- [13] Weil, P. (1989). “Overlapping families of infinitely-lived agents”, *Journal of Public Economics* 38, 410–421.
- [14] Yakita, A. (2008). “Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting”, *Journal of Public Economics* 92, 897–914.

## Appendix A: Proof of Proposition 1

We denote the value of  $x$  such that satisfies the condition  $\frac{\partial LH}{\partial x} = \frac{\partial RH}{\partial x}$  as  $x_M$ . From (16), we can easily confirm that  $x_M = (\frac{RH_2}{LH_1 LH_3})^{1/2}$  where  $LH_1 \equiv \frac{1+n}{1+n+\frac{1}{1+\beta}\frac{\phi\bar{w}}{1+r}}$ ,  $LH_3 \equiv 1 + \frac{\beta}{1+\beta}r$  and  $RH_2 \equiv \psi(\bar{w} + \rho)$ . Since  $\lim_{x \rightarrow 0} LH(x) < \lim_{x \rightarrow 0} RH(x)$  and  $\lim_{x \rightarrow \infty} LH(x) < \lim_{x \rightarrow \infty} RH(x)$ , the following three cases might occur,  $RH(x_M) < LH(x_M)$ ,  $RH(x_M) = LH(x_M)$ , and  $RH(x_M) > LH(x_M)$ . We denote  $RH(x_M) < LH(x_M)$  as case1,  $RH(x_M) = LH(x_M)$  as case 2, and  $RH(x_M) > LH(x_M)$  as case 3 respectively.

If  $RH(x_M) < LH(x_M)$  (i.e. case 1), as shown in Figure 1, the function  $RH(x)$  and  $LH(x)$  have necessarily two intersections, so there are two steady-states,  $x^1$  and  $x^2$ . If  $RH(x_M) = LH(x_M)$  (i.e. case 2), the function  $RH(x)$  and  $LH(x)$  are just tangent at  $x_M$  and there is a unique steady-state. If  $RH(x_M) > LH(x_M)$  (i.e. case 3),  $RH(x)$  and  $LH(x)$  do not intersect and therefore there is no steady-state.

Now notice that  $\frac{\partial LH}{\partial n} > 0$ . Hence, an increase in population growth rate  $n$  increases  $LH(x)$ . Therefore, if the population growth rate  $n$  is sufficiently high to satisfy  $n > \hat{n}$ , the situation called case 1 occurs and there will be two steady-states,  $x^1$  and  $x^2$ . However, if the population growth rate  $n$  is sufficiently low to satisfy  $n < \hat{n}$ , there will be no steady-state. Then critical value of  $\hat{n}$  is defined as the value of  $n$  which yields the situation called case 2.

In addition, by differentiating (16) with respect to  $n$ , we obtain

$$\frac{dx}{dn} = - \frac{\frac{\partial LH}{\partial n}}{\frac{\partial LH}{\partial x} - \frac{\partial RH}{\partial x}}.$$

Since  $\frac{\partial LH}{\partial n} > 0$  and  $0 > \frac{\partial LH}{\partial x} > \frac{\partial RH}{\partial x}$  at  $x^1$  ( $0 > \frac{\partial RH}{\partial x} > \frac{\partial LH}{\partial x}$  at  $x^2$ ) as shown in Figure 1, we can easily confirm that  $\frac{dx^1}{dn} < 0$  ( $\frac{dx^2}{dn} > 0$ ) holds.

Moreover, notice that  $\frac{\partial LH}{\partial \phi} < 0$ . Hence, an increase in replacement ratio  $\phi$  decreases  $LH(x)$ . Therefore, if the replacement ratio  $\phi$  is

sufficiently low to satisfy  $\phi < \hat{\phi}$ , the situation called case 1 occurs and there will be two steady-states,  $x^1$  and  $x^2$ . However, if the replacement ratio  $\phi$  is sufficiently high to satisfy  $\phi > \hat{\phi}$ , there will be no steady-state. Then critical value of  $\hat{\phi}$  is defined as the value of  $\phi$  which yields the situation called case 2.

In addition, by differentiating (16) with respect to  $\phi$ , we obtain

$$\frac{dx}{d\phi} = -\frac{\frac{\partial LH}{\partial \phi}}{\frac{\partial LH}{\partial x} - \frac{\partial RH}{\partial x}}.$$

Since  $\frac{\partial LH}{\partial \phi} < 0$  and  $0 > \frac{\partial LH}{\partial x} > \frac{\partial RH}{\partial x}$  at  $x^1$  ( $0 > \frac{\partial RH}{\partial x} > \frac{\partial LH}{\partial x}$  at  $x^2$ ), we can easily confirm that  $\frac{dx^1}{d\phi} > 0$  ( $\frac{dx^2}{d\phi} < 0$ ) holds.

## Appendix B: Proof of Proposition 2

By differentiating (14) with respect to  $x_t$ , we obtain

$$\frac{dx_{t+1}}{dx_t} = \frac{\partial dg}{\partial x_t} \frac{x_t}{kg} + \frac{dg}{kg} - \frac{dg}{kg} \frac{x_t}{kg} \frac{\partial kg}{\partial x_t}.$$

Close to the steady-state, we have  $dg = kg$  and  $x_{t+1} = x_t = x$ , so the differential simplifies to

$$\frac{dx_{t+1}}{dx_t} = 1 + \frac{x}{kg} \left( \frac{\partial dg}{\partial x} - \frac{\partial kg}{\partial x} \right).$$

From Figure 2, the relation  $\frac{\partial dg}{\partial x} < 0$ ,  $\frac{\partial kg}{\partial x} < 0$  and  $0 > \frac{\partial kg}{\partial x} > \frac{\partial dg}{\partial x}$  ( $0 > \frac{\partial dg}{\partial x} > \frac{\partial kg}{\partial x}$ ) hold at  $x^1$  ( $x^2$ ). Therefore,  $\frac{dx_{t+1}}{dx_t} < 1$  holds at  $x^1$  and  $\frac{dx_{t+1}}{dx_t} > 1$  holds at  $x^2$ . Hence  $x^1$  is locally stable, whereas  $x^2$  is unstable.