A Second Excess Entry Theorem

Kenji Fujiwara

Constructing a two-country oligopolistic model, this paper explores another possibility of excess entry in open economies. The model comprises two stages in which the government of each country chooses the number of firms in the first stage and the oligopolistic firms play a Cournot-Nash game in the second stage. We show that the number of firms determined in the subgame perfect equilibrium of this model is larger than the socially optimal one, but smaller than that in the free entry equilibrium. The implication of our result for coordinating competition policies in the WTO forum is discussed.

Kenji Fujiwara*

JEL：F10, F12, F13, L13

Key words：excess entry, international oligopoly, scale economies, strategic competition policy

1 Introduction

There is a growing interest in 'trade and the competition policy' in the forum of the World Trade Organization (WTO). The WTO meeting in Singapore in 1996 decided to set up three working groups one of which deals with the competition policy issue and is discussing how member countries harmonize their policy. Despite these backgrounds, there are few studies discussing competition policies in a globalized world.

The purpose of this paper is to propose a theoretical framework to address the above question. To this end, we extend a theory of excess entry in an oligopoly to a two-country model and compare three equilibria. The

* School of Economics, Kwansei Gakuin University. Uegahara 1-1-155, Nishinomiya, 662-8501, Japan. Tel: +81-798-54-7066. Fax: +81-798-51-9044. E-mail: kenjifujiwara@nyc.odn.ne.jp.

first is a free entry equilibrium in which the number of oligopolists is determined in such a way to satisfy the zero profit condition. The second is a subgame perfect Nash equilibrium of the following two-stage game; each country's government chooses its number of firms in the first stage and firms play a Cournot-Nash game in the second stage. And, the third is a social optimum in which the number of firms is calculated to maximize the world welfare. We will show that the number of firms in the second equilibrium exceeds the counterpart in the third equilibrium while being short of the one in the first equilibrium. That is, the noncooporative behavior of self-interested governments reduces the tendency toward excess entry, but can not achieve the world optimum. This result might be a rationale for the multilateral coordination of competition policies by the WTO members.

It is helpful to mention the plausibility of the above subgame perfect Nash equilibrium. In a context of international pollution control, Dockner and Long (1993) state that '$\cdots$ a Nash equilibrium need not be interpreted as an equilibrium that arises in the absence of negotiation. In the context of international pollution control, it may be more appropriate to think of a Nash equilibrium as an outcome of negotiations on agreements that are self-enforcing.' The same remark applies to our argument, namely, the above subgame perfect Nash equilibrium can be viewed as an outcome that comes from the self-enforcing negotiations.

The paper is organized as follows. Section 2 presents a model and derives the free entry equilibrium. Section 3 solves a two-stage game consisting of a policy game and Cournot-Nash competition and then compares the resulting subgame perfect Nash equilibrium with the free entry equilibrium. Computing a world optimum, Section 4 compares it with the subgame perfect equilibrium obtained in the preceding section. Section 5 gives a final remark.

2 A Free Entry Equilibrium

A two-country (Home and Foreign), two-good (Goods 1 and 2), one-factor (labor) model of oligopoly and increasing returns is constructed. The world consists of Home and Foreign both of which produce an oligopolized good (Good 1) and a competitive good (Good 2), which serves as a numeraire. All the Foreign variables are distinguished by attaching an asterisk (*). Both goods are tradable while labor is not. Without loss of generality, one unit of labor produces one unit of Good 2, from which the wage rate is internationally unity.

On the other hand, production of good 1 is subject to increasing returns. There are $n \geq 2$ identical firms in Home and $n^* \geq 2$ in Foreign. All firms have the same technology and the production function of a representative firm, say, firm i, is specified by

$$x_i = l_i^{\alpha}, \quad \alpha > 1.$$

It follows from this specification that not only the average cost but the marginal cost is decreasing in outputs.

To define each firm's profit, we introduce the demand side. Assuming a representative consumer in each country, Home's utility function is given by

$$U = \gamma \ln C_1 + C_2, \quad \gamma > 0, \tag{1}$$

which, after utility maximization, yields the demand function of each good:

$$C_1 = \frac{\gamma}{p} \tag{2}$$

$$\begin{aligned} C_2 &= (\text{national income}) - \gamma \\ &= L + n\pi - \gamma, \end{aligned} \tag{3}$$

where p denotes the relative price of Good 1 and π the per-firm profit in Home.[1] Substituting (2) and (3) into (1) yields Home's indirect utility

1) Note that L gives the labor income due to the unitary wage rate.

function:

$$U = \gamma \ln\left(\frac{\gamma}{p}\right) + L + n\pi - \gamma$$
$$= \gamma \ln \gamma - \gamma \ln p + L + n\pi - \gamma$$
$$= n\pi - \gamma \ln p + \gamma(\ln \gamma - 1) + L.$$

Since $\gamma(\ln\gamma - 1) + L$ is constant, we can ignore it from welfare components and define Home's welfare as

$$V(p, n\pi) \equiv n\pi - \gamma \ln p. \tag{4}$$

Foreign's counterpart is analogously defined by

$$V(p, n^*\pi^*) \equiv n^*\pi^* - \gamma \ln p. \tag{5}$$

The rest of this section will focus on a canonical case where the number of firms is determined by the zero profit condition as in Mankiw and Whinston (1986). Because both countries share the identical preference, the market-clearing condition of Good 1 under free trade is

$$\frac{\gamma}{p} + \frac{\gamma}{p} = \sum x_j + \sum x_j^*,$$

which yields the world inverse demand function:

$$p = \frac{2\gamma}{\sum x_j + \sum x_j^*}. \tag{6}$$

Noting that the cost function of Home's representative firm becomes $x_i^{1/\alpha}$, the profit of a representative firm in Home and Foreign is defined by

$$\frac{2\gamma}{\sum x_j + \sum x_j^*} x_i - x_i^{\frac{1}{\alpha}} \tag{7}$$

$$\frac{2\gamma}{\sum x_j + \sum x_j^*} x_i^* - {x_i^*}^{\frac{1}{\alpha}}. \tag{8}$$

Confining attention to the interior maximum in a symmetric equilibrium, each firm's first-order condition for profit maximization becomes[2)]

2) Throughout the paper, we assume away the possibility that a country specializes to the competitive good.

$$\frac{2\gamma}{nx+n^*x^*}\left(1-\frac{x}{nx+n^*x^*}\right)-\frac{1}{\alpha}x^{\frac{1}{\alpha}-1}=0 \tag{9}$$

$$\frac{2\gamma}{nx+n^*x^*}\left(1-\frac{x^*}{nx+n^*x^*}\right)-\frac{1}{\alpha}x^{*\frac{1}{\alpha}-1}=0. \tag{10}$$

Solving (9) and (10) for x and x^* yields a symmetric Cournot-Nash equilibrium output:

$$x=x^*=x^E=\left[\frac{2\alpha\gamma(N-1)}{N^2}\right]^{\alpha}, \tag{11}$$

where superscript E refers to the Nash equilibrium and $N \equiv n+n^*$. Substitution of (11) into (7) gives the maximized profit of each firm:

$$\pi^E \equiv \frac{2\gamma}{Nx^E}x^E-\left(x^E\right)^{\frac{1}{\alpha}}=\frac{2\gamma[N(1-\alpha)+\alpha]}{N^2}. \tag{12}$$

Throughout the paper, an interior maximum has been assumed, for which we need to impose:

Assumption. $N \leq \frac{\alpha}{\alpha-1}$.

This section is closed by deriving the number of firms in the free entry equilibrium. Setting (12) to zero, we have

$$N^E=\frac{\alpha}{\alpha-1},$$

or equivalently

$$n^E=\frac{\alpha}{2(\alpha-1)}=\frac{1}{4}+\frac{\alpha+1}{4(\alpha-1)}. \tag{13}$$

3 Strategic Competition Policy

In the preceding section, the world equilibrium under free entry is characterized. Alternatively, this section considers an equilibrium of a two-stage game in which each country's government determines the number of its firms in the first stage and then each firm plays a quantity-setting game, taking the predetermined number of firms as given. This section is devoted to characterizing the subgame perfect Nash equilibrium of this two-stage game.

Since the second stage has already been solved in the previous section, we concentrate on the first stage. To do so, we begin by defining each government's payoff function. Substituting (11) into (6), the Nash equilibrium price becomes

$$p^E \equiv \frac{2\gamma}{Nx^E} = \frac{(2\gamma)^{1-\alpha}}{\alpha^\alpha} N^{2\alpha-1}(N-1)^{-\alpha}. \tag{14}$$

Hence, further substitution of (12) and (14) into (4) yields the objective function of the Home government:

$$\begin{aligned} V\left(p^E, n\pi^E\right) &= n\pi^E - \gamma \ln p^E \\ &= n\frac{2\gamma[N(1-\alpha)+\alpha]}{N^2} - \gamma \ln\left[\frac{(2\gamma)^{1-\alpha}}{\alpha^\alpha} N^{2\alpha-1}(N-1)^{-\alpha}\right] \\ &= n\frac{2\gamma[N(1-\alpha)+\alpha]}{N^2} - \gamma(2\alpha-1)\ln N + \alpha\gamma\ln(N-1) - \gamma\ln\left(\frac{2^{1-\alpha}\gamma}{\alpha^\alpha}\right). \end{aligned}$$

Because the last term in the right-hand side above is constant, we can redefine Home's objective function as

$$W(n, n^*) \equiv n\frac{2\gamma[N(1-\alpha)+\alpha]}{N^2} - \gamma(2\alpha-1)\ln N + \alpha\gamma\ln(N-1). \tag{15}$$

In the same manner, Foreign's counterpart is defined by[3)]

$$W(n^*, n) \equiv n^*\frac{2\gamma[N(1-\alpha)+\alpha]}{N^2} - \gamma(2\alpha-1)\ln N + \alpha\gamma\ln(N-1). \tag{16}$$

The Home government maximizes (15) and the Foreign government (16), taking the other country's number of firms as given. Then, Home's first-order condition for welfare maximization is[4)]

$$\frac{2\gamma[N(1-\alpha)+\alpha]}{N^2} + 2\gamma n\frac{(1-\alpha)N - 2[N(1-\alpha)+\alpha]}{N^3} - \frac{\gamma(2\alpha-1)}{N} + \frac{\alpha\gamma}{N-1} = 0.$$

Eqs. (15) and (16) allow us to find that both countries choose the same number of firms in the equilibrium, namely, $n = n^*$. Accordingly, the optimal number of n is obtained by setting $N = 2n$ in this first-order condition:

3) Note that both countries have the same function $W(\cdot)$.

4) It is possible to show the second-order condition for maximization.

$$\frac{2n(1-\alpha)+\alpha}{2n^2}+\frac{n(\alpha-1)-\alpha}{2n^2}-\frac{2\alpha-1}{2n}+\frac{\alpha}{2n-1}=0,$$

which gives the number of firms in the subgame perfect Nash equilibrium:

$$n^F=\frac{3\alpha-2}{4(\alpha-1)}=\frac{1}{4}+\frac{2\alpha-1}{4(\alpha-1)}, \tag{17}$$

where superscript F stands for the subgame perfect Nash equilibrium.

Now that we find two equilibrium values of the number of firms, n^E and n^F, we readily compare them. The result is stated in:

Proposition 1. *$n^E > n^F$, i.e., the number of firms in the free entry equilibrium is larger than that in the subgame perfect Nash equilibrium of the competition game.*

Proof. The difference between n^F and n^F is

$$\begin{aligned} n^F - n^E &= \frac{2\alpha-1}{4(\alpha-1)}-\frac{\alpha+1}{4(\alpha-1)} \\ &= \frac{\alpha-2}{4(\alpha-1)}<0, \end{aligned}$$

due to the assumption of $\alpha \in (1,2)$. **Q.E.D.**

Proposition 1 claims that the number of firms is reduced by taking into account the strategic interdependence between the countries. The intuition behind this result is as follows. In the policy game considered, each government determines the number of firms such that it maximizes its social utility including consumer welfare (consumer surplus). This care for consumer welfare leaves the number of firms less than that in the free entry equilibrium.

4 A Second Excess Entry Theorem

This section compares the socially optimal number of firms with the number of firms in the subgame perfect Nash equilibrium found in the last section. For this purpose, let us calculate the social optimum. We define the 'social optimum' as the solution that maximizes the joint welfare of Home and Foreign. The world welfare is simply defined by the sum of both countries' welfare:[5)]

$$W(n,n^*)+W(n^*,n)=\frac{2\gamma[N(1-\alpha)+\alpha]}{N}-2\gamma(2\alpha-1)\ln N+2\alpha\gamma\ln(N-1).$$

Since this is a function of N only, we need not compute the first-order condition with respect to n and n^* separately. Maximizing the world welfare with respect to N yields[6)]

$$N^S=\frac{1}{2}+\sqrt{\frac{5\alpha-1}{4(\alpha-1)}},$$

where superscript S represents the social optimum. Noting that each country's number of firms is just a half of N leads to

$$n^S=\frac{N^S}{2}=\frac{1}{4}+\sqrt{\frac{5\alpha-1}{16(\alpha-1)}}. \tag{18}$$

It may be constructive to mention that n^S can also be obtained by considering the Nash bargaining problem between the countries. That is, we can verify that n^S is alternatively obtained by maximizing the Nash product:

$$\left[W\left(n,n^*\right)-W^F\right]\left[W\left(n^*,n\right)-W^F\right],$$

where W^F stands for each country's welfare attained in the subgame perfect Nash equilibrium, which is assumed to serve as a disagreement point. Based on (17) and (18), we can prove:

Proposition 2. $n^F > n^S$, *i.e., the number of firms in the subgame perfect*

5) Due to the quasi-linearity of preferences, summation of both countries' welfare entails no serious problem. Uekawa (1994) also employs the sum of welfare as the world welfare in a context of strategic trade policy.

6) The second-order condition is also satisfied since $W(n,n^*)+W(n^*,n)$ is strictly concave in N.

Nash equilibrium of the competition policy game exceeds the socially optimal one.

Proof. Subtracting n^F from n^S, we establish that

$$n^S - n^F = \sqrt{\frac{5\alpha - 1}{4(\alpha - 1)}} - \frac{2\alpha - 1}{4(\alpha - 1)} < 0,$$

for any $\alpha \in (1, 2)$. **Q.E.D.**

Proposition 2 provides a relevant policy implication for the international harmonization of competition policies. It asserts that the noncooporative choice of the number of firms leads to another possibility of excess entry although the resulting world welfare is higher than under free entry. Thus, each country calls for a worldwide harmonization of competition policies in the international forum, e.g., the WTO. In other words, our result sheds light on the need for the coordination of competition policies in the WTO round talks.

5 Concluding Remarks

This paper has sought another possibility of excess entry in an oligopolistic market by constructing the following two-stage game model. The two countries' government seeks to maximize welfare by controlling its number of firms in the first stage and each oligopolist plays a Cournot-Nash game in the second stage. In this setting, we have shown a unique ranking among the number of firms in the free entry equilibrium $\left(n^E\right)$, subgame perfect Nash equilibrium $\left(n^F\right)$, and socially optimal equilibrium $\left(n^S\right)$ such that $n^E > n^F > n^S$ holds. This result immediately appeals a need for international coordination of competition policies since noncooperative policy-making of self-interested governments can not achieve the world optimum. That is, our conclusion provides a rationale for the cooperation among the

WTO participants on competition policy coordination.

References

[1] Dockner, E. J. and N. V. Long (1993), 'International pollution control: cooperative versus noncooperative strategies,' *Journal of Environmental Economics and Management*, 24, 13-29.

[2] Mankiw, N. G. and M. D. Whinston (1986), "Free entry and social inefficiency", *Rand Journal of Economics*, 17, 48-58.

[3] Uekawa, Y. (1994), "Imperfect competition, intra-industry trade, and trade policy", *Economic Studies Quarterly*, 45, 1-13.