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What Does the Solow Model Tell Us about Economic Growth?

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Abstract

This paper presents, within a framework of the Solow model, evidence that there are significant differences in convergence patterns across subsamples. It shows that although OECD countries and the countries converging to their steady states from above follow a pattern of conditional convergence, those converging to their steady states from below do not. This result is best explained by the idea that technology diffusion has a large effect mainly on the countries converging to their steady states from below.

KEYWORDS: convergence, Solow model, technology diffusion

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There has been controversy over the growth regressions deployed in neoclassical growth models. This is generally referred to as the 'convergence controversy'. The neoclassical growth models suggest that an economy converges to its own steady state. This implies that if we control for the exogenously determined variables such as the population growth rate and the investment rates of physical and human capital, and assume that all of the economies face the same exogenously determined constant growth rate of technology, we should observe that the country with the lower initial output per capita tends to grow faster: conditional convergence. Mankiw, Romer, and Weil (1992) (henceforth, MRW) undertake a cross-country regression analysis and find evidence for conditional convergence by augmenting the Solow model with human capital. Barro and Sala-i-Martin (1992a, 1992b) carry out convergence tests by using regional data on Japan and the US, and observe convergence within these two countries. MRW (1992), and Barro and Sala-i-Martin (1992a, 1992b) both conclude that the estimated speed of convergence, β, is about 0.02 per year around the steady state. This implies a relatively slow speed of convergence: an economy moves halfway to its steady state in about 35 years.[1]

Despite these findings, many economists criticize the neoclassical growth models and their empirical tests. Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) are not satisfied with the assumption of exogenous technological change and establish models that endogenize technological progress. Most important, many researchers argue that the idea of treating technology as a nonrival and non-excludable good in the neoclassical growth models is not appropriate.[2] They argue that it is indefensible to assume a common growth rate of technology and a common initial level of technology in the cross-country regressions. The levels and growth rates of technology somehow should differ across countries.

This paper attempts to show what the conventional analysis of the Solow model tends to miss out, and reconsiders the validity of the model by applying a new cross-country regression method. We allow initial levels and growth rates of technology to vary across countries. To perform the convergence tests, we use the method which not only directly controls for saving and population growth rates but also indirectly controls for initial levels of technology by using capital output ratios (It is, thus, different from panel data studies of growth convergence).

[1] Although the studies by Barro and Sala-i-Martin (1992a, 1992b) do not control for steady state determinants, convergence is observed because the steady states are assumed to be similar across regions.

[2] Romer (1994) argues that important discoveries are usually excludable at least for some periods.

The empirical results show significant differences in convergence patterns across subsamples. When the sample is divided into three subsamples: OECD countries ('OECD' sample), countries converging to their steady states from above ('Above' sample) and countries converging to their steady states from below ('Below' sample), conditional convergence is observed only in the 'OECD' and 'Above' samples. This result is best explained by the idea that technology diffusion has a large effect mainly on the countries converging to their steady states from below. The paper also shows that the estimated coefficient for the speed of convergence could be larger than the conventional estimated value without contradicting the Solow model's prediction.

Section 1 re-examines the motion of an economy within the framework of the Solow model by using capital per labor rather than capital per effective labor. This approach allows us to analyze the effect of the initial level of technology on the motion of the economy. Section 2 describes the shortcomings of MRW's cross-country regressions. Sections 3 and 4 show the empirical results. Section 5 discusses the implication of the results. Section 6 concludes.

1 Alternative Analysis of the Solow Model

A conventional approach to analyzing the motion of an economy in the Solow model is to use capital per effective labor. However, capital per labor is used throughout this paper. This approach allows us to pay greater attention to the level of technology in analyzing the model.

1.1 The Model

We consider a Cobb-Douglas production function case in the Solow (1956) model. It takes the form of labor-augmenting technological progress. The function at time t is, therefore, given by:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha} \quad 0 < \alpha < 1, \tag{1}$$

where Y, K, A and L denote output, capital, the level of technology and labor, respectively. The Solow model assumes that the growth rates of population and technology are exogenously determined. Thus, the level of technology and amount of labor at time t are given by:

$$L(t) = L(0)e^{n\,t} \tag{2}$$

$$A(t) = A(0)e^{g\,t}, \tag{3}$$

where $L(0)$ and $A(0)$ are the initial amount of labor and the initial level of technology, respectively. L and A grow at the exogenously determined rates n and g.

Assuming that the rates of saving and depreciation are exogenous and constant, the evolution of capital can be described as:

$$\dot{K}(t) = sY(t) - \delta K(t), \tag{4}$$

where a dot over a variable, such as $\dot{K}(t)$, denotes differentiation with respect to time, and s and δ are the rates of saving and depreciation, respectively. By defining $k(t)$ as capital per unit of labor, the evolution of $k(t)$ is given by:

$$\dot{k}(t) = sA(0)^{1-\alpha} e^{(1-\alpha)\, g\, t} k(t)^{\alpha} - (n+\delta)k(t). \tag{5}$$

Notice that equation (5) describes the evolution of $K(t)/\ L(t)$ but not the evolution of $K(t)/(A(t)L(t))$. This approach makes it possible to capture the impact of differences in the initial levels of technology $A(0)$ on the levels and growth rates of income per unit of labor.

1.2 The Graphical Analysis of the Dynamics

In this sub-section we analyze the dynamics of the model by using a phase diagram. We draw the diagram in the $(t,\ \ln k)$ space. This seems odd at first since one of the variables is time t. This approach, however, works fine. The only difference from the conventional phase diagram analysis is that the direction of motion of t (one of the variables in the diagram) is exogenously given, that is, t rises regardless of the level of $\ln k$.

To analyze the direction of motion of $\ln k$, we first consider the case when $dk(t)/dt = 0$. By setting $dk(t)/dt = 0$, equation (5) gives:

$$\ln k(t) = \frac{1}{1-\alpha} \ln s - \frac{1}{1-\alpha} \ln(n+\delta) + \ln A(0) + g\, t. \tag{6}$$

Equation (6) describes all combinations of t and $\ln k(t)$ which give zero growth rate of $k(t)$. Since α , s, n, δ and g are constant, equation (6) is a linear line in the $(t, \ln k)$ space. The slope of the line is g. We call this line the '*stationary* $\ln k$ *line*' (Figure 1 shows the line). It shows the collection of points in the $(t, \ln k)$ space such that $dk(t)/dt = 0$ holds. In other words, the stationarity is only local. Note that, the *stationary* $\ln k$ *line* does not at all characterize a steady state. It just provides us the $dk(t)/dt = 0$ locus in the $(t,\ \ln k)$ space

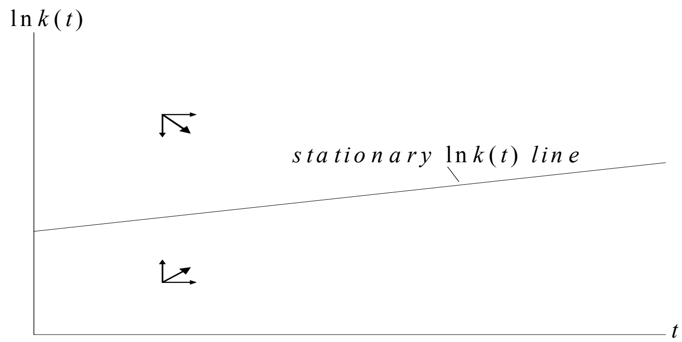

Figure 1: The dynamics of $\ln k(t)$ over time

and it divides the space into two regions. The direction of motion of $\ln k$ depends on whether $\ln k$ is below or above the *stationary* $\ln k$ *line.*

We can find out the direction of motion of $\ln k$ by considering two cases: $dk(t)/dt > 0$ and $dk(t)/dt < 0$. When $dk(t)/dt > 0$, we get the following expression from equation (5).

$$\ln k(t) < \frac{1}{1-\alpha}\ln s - \frac{1}{1-\alpha}\ln(n+\delta) + \ln A(0) + g\,t. \tag{7}$$

Equation (7) implies that the direction of motion of $\ln k$ at a given point of time t is upward (i.e., $dk(t)/dt > 0$) when $\ln k$ is below the *stationary* $\ln k$ *line*. Similarly, when $dk(t)/dt < 0$, we get the following expression from equation (5).

$$\ln k(t) > \frac{1}{1-\alpha}\ln s - \frac{1}{1-\alpha}\ln(n+\delta) + \ln A(0) + g\,t. \tag{8}$$

Equation (8) implies the direction of motion of $\ln k$ at a given point of time t is downward (i.e., $dk(t)/dt < 0$) when $\ln k$ is above the *stationary* $\ln k$ *line*. Thus, since t rises regardless of the level of $\ln k$, the dynamics of $\ln k$ over time can be shown as in Figure 1. The figure shows that $\ln k$ falls over time when $\ln k$ is above the *stationary* $\ln k$ *line* and rises over time when $\ln k$ is below the *stationary* $\ln k$ *line*.

To see the dynamics of $\ln k(t)$ in more detail, we simulate the model by using the following equation: (solving the first-order differential equation (5)

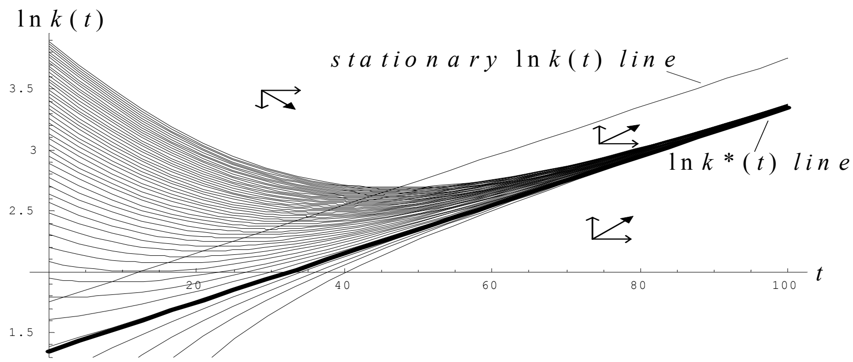

Figure 2: The *stationary* $\ln k(t)$ *line* and the steady state

and taking logs yield the following equation)

$$\ln k(t) = \frac{1}{1-\alpha}\ln[k(0)^{1-\alpha}e^{-(n+\delta)\,(1-\alpha)t}(g+n+\delta)+s\,A(0)^{1-\alpha}(e^{g\,(1-\alpha)t}-e^{-(n+\delta)\,(1-\alpha)t})] -\frac{1}{1-\alpha}\ln(n+g+\delta). \tag{9}$$

To carry out simulations, we apply commonly used values for the parameters: $\alpha = 0.33$, $g = 0.02$, $n = 0.015$, $\delta = 0.05$ and $s = 0.21$. Furthermore, at this stage, we assume that the initial level of technology, $A(0)$, is fixed at 1. For convenience, the initial levels of capital per unit of labor, $k(0)$, are set between 0.01 and 50 (since the purpose of this analysis is to graphically capture the dynamics of $k(t)$, the choices of $k(0)$ and $A(0)$ do not cause any serious problems in the analysis).

Figure 2 shows the movement of $\ln k(t)$ over time for various levels of $\ln k(0)$. Each curve represents the movement of $\ln k(t)$ over time for the corresponding $\ln k(0)$. The higher position of the curve implies the higher level of $k(0)$. Figure 2 confirms the point made before. $\ln k(t)$ falls (rises) over time when $\ln k(t)$ is above (below) the *stationary* $\ln k$ *line*. On the *stationary* $\ln k$ *line*, the growth rate of $k(t)$ is zero (i.e., the *stationary* $\ln k$ *line* intersects with $\ln k(t)$ curves at the bottom of $\ln k(t)$ curves). Figure 2 also shows that $\ln k(t)$ converges to a line which is parallel to the *stationary* $\ln k$ *line*. We call this line the '$\ln k^*$ *line'*. It describes the steady state path of $\ln k(t)$. Since the *stationary* $\ln k$ *line* is given by equation (6), the growth rate of $k(t)$ on the $\ln k^*$ *line* is g. Therefore, $k(t)$ converges to the steady state where the growth rate of $k(t)$ is g. The level of $k(t)$ in the steady state is denoted as $k^*(t)$. The important fact here is that the position of the $\ln k^*$ *line* depends on that of the *stationary* $\ln k$ *line* since the *stationary* $\ln k$ *line* governs the dynamics

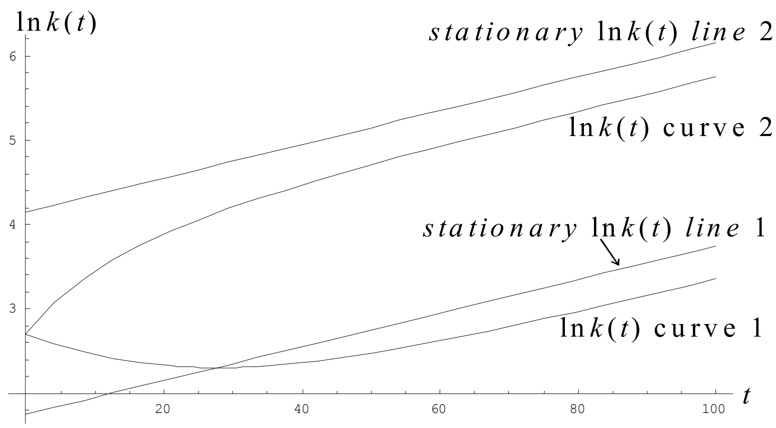

Figure 3: The impact of the difference in $A(0)$

of $\ln k(t)$. The higher position of the *stationary* $\ln k$ *line* leads to the higher position of the $\ln k^*$ *line*.

We next analyze the impact of differences in initial levels of technology on the dynamics of $k(t)$. $\ln k(t)$ curves are traced with two different levels of $A(0)$, ($A(0) = 1$ and 11). The same values for s, α, n, g and δ are used as in the previous analysis, but $k(0)$ is fixed at 15 this time.

Figure 3 shows the analysis. $\ln k(t)$ curves 1 and 2 correspond to the levels of $A(0)$ of 1 and 11, respectively. *Stationary* $\ln k$ *line* 1 and 2 correspond to the levels of $A(0)$ of 1 and 11, respectively. As in Figure 2, each $\ln k(t)$ curve converges to its own $\ln k^*$ *line*. The higher level of $A(0)$ implies the higher position of the *stationary* $\ln k$ *line*. Since the position of the $\ln k^*$ *line* depends on that of the *stationary* $\ln k$ *line* (which is described by equation (6)), $k^*(t)$ depends on α, n, δ, s, g and $A(0)$. Thus, assuming α, δ and g are the same across countries, not only s and n but also $A(0)$ affect the steady state level of $k(t)$ and the non-steady state growth rate of $k(t)$.

Finally, we characterize three different kinds of paths towards the steady state. In Figure 4, $\ln k(t)$ curve 1 shows the dynamics of $\ln k(t)$ when $\ln k(0)$ is less than $\ln k^*(0)$, and $\ln k(t)$ curve 2 shows the dynamics of $\ln k(t)$ when $\ln k(0)$ is greater than $\ln k^*(0)$. The two curves converge to the same $\ln k^*$ *line* since we assume the same levels of $A(0)$, s, n and δ. Points C and D show the starting points for each curve. At point B, $\frac{d\ \ln k(t)}{d\,t} = 0$. At around point A, $\ln k(t) \simeq \ln k^*(t)$. Notice that there are three kinds of paths. Between point C and point B (Path 1), $\ln k(t)$ decreases and the growth rate of $k(t)$ increases. Between point B and point A (Path 2), $\ln k(t)$ and the growth rate of $k(t)$

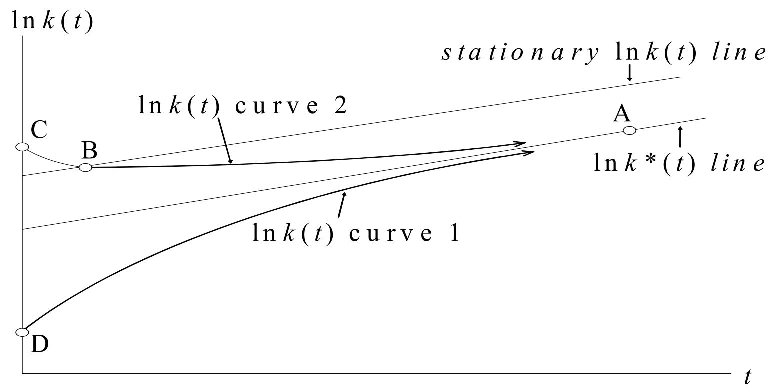

Figure 4: Three types of $\ln k(t)$ paths

both increase. Between point D and point A (Path 3), $\ln k(t)$ increase and the growth rate of $k(t)$ decreases. The economies on Path 3 converge to their steady states from below and the economies on Path 1 and Path 2 converge to their steady states from above.

Cho and Graham (1996) test the Solow growth model and find that many countries (especially, poor ones) converge to their steady states from above. Thus, it might be useful to look at the more details of characteristics of Path 1 and Path 2. The economies on Path 1 run down their capital-labor ratios (and also their capital-*effective* labor ratios) over time. Contrarily, the capital-labor ratios of the economies on Path 2 increase at a lower rate than g to reach their steady states (but their capital-*effective* labor ratios decrease over time). The important point is that it is possible for countries to converge to their steady states from above by increasing their capital-labor ratios.

1.3 The Graphical Analysis of the Speed of Convergence

As Figures 2 and 3 show, an economy converges to its own steady state regardless of where k and A start. We next consider the speed of convergence to the steady state. Corresponding to the analysis in Figure 4, Figure 5 shows the dynamics of an economy in the $(d(\ln k^*(t) - \ln k(t))/dt, \ln k^*(t) - \ln k(t))$ space. X and dX/dt denote $\ln k^*(t) - \ln k(t)$ and $d(\ln k^*(t) - \ln k(t))/dt$, respectively.

The speed of (conditional) convergence is defined as how rapidly the distance between $k^*(t)$ and $k(t)$ vanishes over time. The convergence coefficient

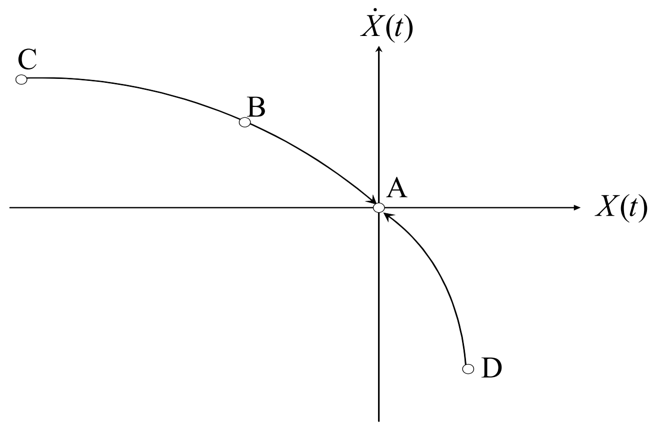

Figure 5: The speed of convergence

is thus given by:

$$\beta(t) = -\frac{\dot{X}(t)}{X(t)}. \tag{10}$$

As we can see in Figure 5, if the economy starts below the steady state, the speed of convergence gets slower over time, but if the economy starts above the steady state, it gets faster over time. Figure 5 also shows that the absolute value of the slope of the curve can be a good approximation of $\beta(t)$ around the steady state (i.e. around point A). Therefore, $\beta(t)$ around the steady state can be given by:

$$\beta(t) \simeq -\frac{d\dot{X}(t)}{d\,X(t)}. \tag{11}$$

After some manipulation (see Appendix 1), equation (11) can be expressed as:

$$\beta(t) \simeq -\frac{d\dot{X}(t)}{d\,X(t)} = (1-\alpha)(n+g+\delta)\left(\frac{y^*(t)}{y(t)}\right)^{\frac{1-\alpha}{\alpha}}. \tag{12}$$

Equation (12) shows that $\beta(t)$ is $(1-\alpha)(n+g+\delta)$ when the economy is at the steady state.[3] Later in this paper, equation (12) will be used to derive a specific equation to test the Solow model and to estimate the speed of convergence.

[3]The convergence rate when the economy is at the steady state is shown also by Barro and Sala-i-Martin (1995, Ch.1).

2 Growth Regression and the Initial Level of Technology

The work of Mankiw, Romer and Weil (1992) shows that the augmented Solow model with human capital can explain a great deal of cross-country income differences without contradicting the model's prediction and obtaining an unrealistic estimate for the elasticity of output with respect to (physical) capital. They also find evidence of conditional convergence. However, there exist some problems in their tests.

Their equations (without a role for human capital) are given by:

$$\ln y_i = a + \frac{\alpha}{1-\alpha}\ln s_i - \frac{\alpha}{1-\alpha}\ln(n_i + g + \delta) + \epsilon_i \tag{13}$$

and

$$\begin{aligned} \ln\left(\frac{y(t)}{y(0)}\right)_i &= (1-e^{-\beta t})a + g\,t + (1-e^{-\beta t})\tfrac{\alpha}{1-\alpha}\ln s_i \\ -(1-e^{-\beta t})\tfrac{\alpha}{1-\alpha}&\ln(n_i+g+\delta) - (1-e^{-\beta t})\ln y(0)_i + (1-e^{-\beta t})\epsilon_i, \end{aligned} \tag{14}$$

where i indexes countries, y_i is income per capita in 1985, $\ln(y(t)/y(0))_i$ is log difference of income per capita 1960-1985, ϵ is a country specific shock, and a is a constant. The growth rate of technology, g, is assumed to be the same across countries. Equation (13) is their estimated equation for the test of the steady state income per labor, and equation (14) is for the test of conditional convergence. The term, β, in equation (14) is the convergence coefficient which is obtained by use of a first-order approximation around the steady state, (i.e. $\beta = (1-\alpha)(n+g+\delta)$). MRW get the implied β from the coefficient on $\ln y(0)$ (thus, they restrict the values of β to be the same across countries to get the generalized value of β). In setting these equations, they assume that $\ln A(0)_i = a + \epsilon_i$. That is, they try to explain the variation in the initial level of technology across countries by using the error term.

MRW assume that the country's initial level of technology is not correlated with the regressors. That is, the error term, ϵ, is assumed to be uncorrelated with independent variables. This is a very weak assumption particularly in equation (14). It is highly unlikely that there is no correlation between ϵ and $\ln y(0)$. In practice, the correlation between ϵ and $\ln y(0)$ seems likely to be quite strong.[4] Thus, the estimated coefficients will be seriously biased (MRW mentioned about this problem in their article).

[4] Hall and Jones (1999) show that differences in the measured productivity levels across countries are quite similar to differences in output per worker.

In the following sections we attempt to estimate the Solow model by taking a different approach from MRW. We assume that technology is not a worldwide good but rather a domestic good. Thus, we allow for different initial levels and growth rates of technology across countries. We also assume that there is no technology diffusion across countries.[5] Thus, technology is completely excludable across countries and differences in technology levels persist over time. Based upon these assumptions, we derive the estimated equation in which the impact of differences in unobservable initial levels of technology is incorporated without relying on the error term.

3 Empirical Test 1

3.1 The Specification

Our aim is to find a way to control for the unobservable initial levels of technology in the regression in order to test the Solow model. We derive our empirical specifications below.

From equations (1), (2), (3) and (4), the equation for the motion of K/AL can be expressed as:

$$\dot{\widehat{k}}(t) = s\widehat{k}(t)^{\alpha} - (n + g + \delta)\widehat{k}(t), \qquad (15)$$

where $\widehat{k}$ is K/AL, capital per effective labor. The standard approach to the Solow model shows that $d\widehat{k}(t)/dt$ is equal to zero at the steady state. Thus equation (15) can be rewritten as:

$$k^{*}(t) = s^{\frac{1}{1-\alpha}}(n + g + \delta)^{\frac{-1}{1-\alpha}}A(t) = s^{\frac{1}{1-\alpha}}(n + g + \delta)^{\frac{-1}{1-\alpha}}A(0)e^{g\,t}, \qquad (16)$$

where $k^{*}(t)$ is the steady state level of capital per labor at time t. From equation (1), the intensive form of the production function is given by:

$$y(t) = A(0)^{1-\alpha}e^{(1-\alpha)\,g\,t}k(t)^{\alpha}. \qquad (17)$$

Thus, the steady state level of output per labor is given by:

$$y^{*}(t) = A(0)^{1-\alpha}e^{(1-\alpha)\,g\,t}k^{*}(t)^{\alpha}. \qquad (18)$$

Substituting equation (16) into equation (18) yields:

$$y^{*}(t) = A(0)s^{\frac{\alpha}{1-\alpha}}(n + g + \delta)^{\frac{-\alpha}{1-\alpha}}e^{g\,t}. \qquad (19)$$

[5] Later in this paper, we consider the possibility of technology diffusion.

The initial level of output per capita is given by (from equation (17)):

$$y(0) = A(0)^{1-\alpha} k(0)^{\alpha}. \tag{20}$$

Solving this expression with respect to $A(0)$ yields:

$$A(0) = \left(\frac{k(0)}{y(0)}\right)^{\frac{-\alpha}{1-\alpha}} y(0). \tag{21}$$

Substituting equation (21) into equation (19) and taking logs give:

$$\ln\left(\frac{y^*(t)}{y(0)}\right)_i - g_i\, t = -\frac{\alpha}{1-\alpha}\ln\left(\frac{k(0)}{y(0)}\right)_i + \frac{\alpha}{1-\alpha}\ln s_i - \frac{\alpha}{1-\alpha}\ln(n_i + g_i + \delta), \tag{22}$$

where i indexes countries and δ is assume to be constant across countries.

We assume

$$g_i = \overline{g} + \widetilde{g}_i,$$

where $\overline{g}$ represents the rate of technology growth which is constant across countries and $\widetilde{g}_i$ reflects deviations in technology growth from $\overline{g}$. By use of a first order Taylor-Series expansion of $\ln(n_i + g_i + \delta)$ around $g_i = \overline{g}$, we get:

$$\ln(n_i + g_i + \delta) \simeq \ln(n_i + \overline{g} + \delta) + \frac{\widetilde{g}_i}{(n_i + \overline{g} + \delta)}.$$

Substitution of this into equation (22) gives the following equation:

$$\ln\left(\frac{y^*(t)}{y(0)}\right)_i - \overline{g}\, t = -\frac{\alpha}{1-\alpha}\ln\left(\frac{k(0)}{y(0)}\right)_i + \frac{\alpha}{1-\alpha}\ln s_i - \frac{\alpha}{1-\alpha}\ln(n_i + \overline{g} + \delta) + \epsilon_i, \tag{23}$$

where

$$\epsilon_i = \left(t - \frac{\alpha}{1-\alpha}\frac{1}{(n_i + \overline{g} + \delta)}\right)\widetilde{g}_i. \tag{24}$$

The restricted version of the equation is given by:

$$\ln\left(\frac{y^*(t)}{y(0)}\right)_i - \overline{g}\, t = -\frac{\alpha}{1-\alpha}\left(\ln\left(\frac{k(0)}{y(0)}\right)_i - \ln s_i + \ln(n_i + \overline{g} + \delta)\right) + \epsilon_i. \tag{25}$$

Equations (23) and (25) are the specifications used in this section. We treat ϵ_i as the error term and it is given by equation (24). We assume that $\widetilde{g}_i$

is random. It is also assumed that the countries are at their steady states at year t (or at least close to their steady state at year t so that most of variation in $\ln(y^*(t)/y(t))$ is due to country-specific demand shocks).

In our estimation, our main goal is not to uncover the true value of the capital share, α. Instead, we think we know that α is about 1/3. Hence, the estimation can help us to learn something about the nature of the error term. If $\widetilde{g}_i$ is independent with $\ln\left(k(0)/y(0)\right)_i$, s_i and n_i, and the countries are close enough to their steady states at year t, we should be able to get the OLS estimator of α close to 1/3.

Another important point is the inclusion of the term $\ln\left(k(0)/y(0)\right)$ in equation (23). Like conventional growth regressions, s and n enter in the regressions so that differences in the rates of saving and population growth are controlled for. The term $\ln\left(k(0)/y(0)\right)$ is unusual one.[6] By rewriting equation (21), one can obtain:

$$\frac{k(0)}{y(0)} = \left(\frac{A(0)}{y(0)}\right)^{-\frac{1-\alpha}{\alpha}}.$$

Thus, the inclusion of $\ln\left(k(0)/y(0)\right)$ in equation (23) implicitly captures the negative correlation between the initial level of per capita output and the subsequent growth rate of per capita output if differences in initial levels of technology are controlled for. In short, the specification directly controls for the rates of saving and population growth and indirectly controls for initial levels of technology.[7]

Our methodology is different from a panel data approach which treats the initial level of technology $A(0)_i$ as a fixed effect. The panel data studies of growth convergence include Islam (1995), Caselli, Esquivel and Lefort (1998), and Bond, Hoeffer and Temple (2001), among many others. The majority of the panel data studies allow only for different initial levels of technology but do not allow for different growth rates of technology.[8] However, our method

[6]Benhabib and Gali (1995) also use the capital-output ratio to control for the unobservable initial levels of technology. In order to control for other determinants of the steady state, they treat them as fixed effects and difference them away by splitting the sample period. However, as Durlauf (1995) argues, this method is likely to reduce the power of the test relative to conventional tests, e.g., MRW (1992), because the null hypothesis is no conditional convergence and the conventional tests reject the null hypothesis by including a set of other control variables.

[7]We do not consider the role of human capital. Its omission is problematic and future research should work to incorporate human capital into the framework.

[8]The studies by Lee, Pesaran and Smith (1997, 1998) are exceptions. They allow for differences in the rates of technology growth. However, they do not report the implied values of α which can provide important information for testing the Solow model. Islam (2003) points out that the implied values of α are likely to be very low when worked out in

allows for differences in growth rates of technology as well as differences in initial levels of technology. Another advantage of our approach over the panel data approach is that it does not have a problem that would arise if one uses the panel data approach to control for initial levels of technology. As argued by many authors, e.g., Temple (1999), Wacziarg (2002), and Durlauf, Johnson and Temple (2004), the fixed effects estimators can worsen the effect of measurement errors if the right hand side variables are fairly persistent over time and measured with white noise errors. This is because the fixed effect estimators could throw away the important between-country variation in the data and be mostly left with noise.

3.2 Data

Data used in this paper are from the Summers and Heston data set version 5.6 (described in Summers and Heston, 1991), the Barro and Lee data set (used in Barro and Lee, 1994), and the King and Levine data set (used in King and Levine, 1994). The average growth rate of the working-age population is used for the population growth rate n where working age is defined as 15 to 64, and the data are constructed by using the Barro and Lee data set. The data on the saving rate s (the average share of real investment in real GDP over the period of 1960-1985) and GDP per equivalent adult y (in 1960 and 1985) are from the Summers and Heston data set. The data on the capital-output ratio k/y (in 1960 and 1985) are from the King and Levine data set.

The sample covers MRW's 'Non-oil' countries in which the dominant industries are not oil production. It consists of 95 'Non-oil' countries for which all necessary data are available. The data set covers the period between 1960 and 1985.

3.3 Results

We divide the sample into three subsample groups. Using equation (22) and substituting 0 for t yield:

$$\left(\frac{1-\alpha}{\alpha}\right)\left(\ln\frac{y^*(0)}{y(0)}\right)_i = -\left(\ln\left(\frac{k(0)}{y(0)}\right)_i - \ln s_i + \ln(n_i + g_i + \delta)\right). \qquad (26)$$

Equation (26) shows whether the country is initially below or above its own steady state. Since g_i is not observable, we use $\overline{g}$ $(= 0.02)$ instead of g_i to

their framework and argues that this may imply some problem in their work.

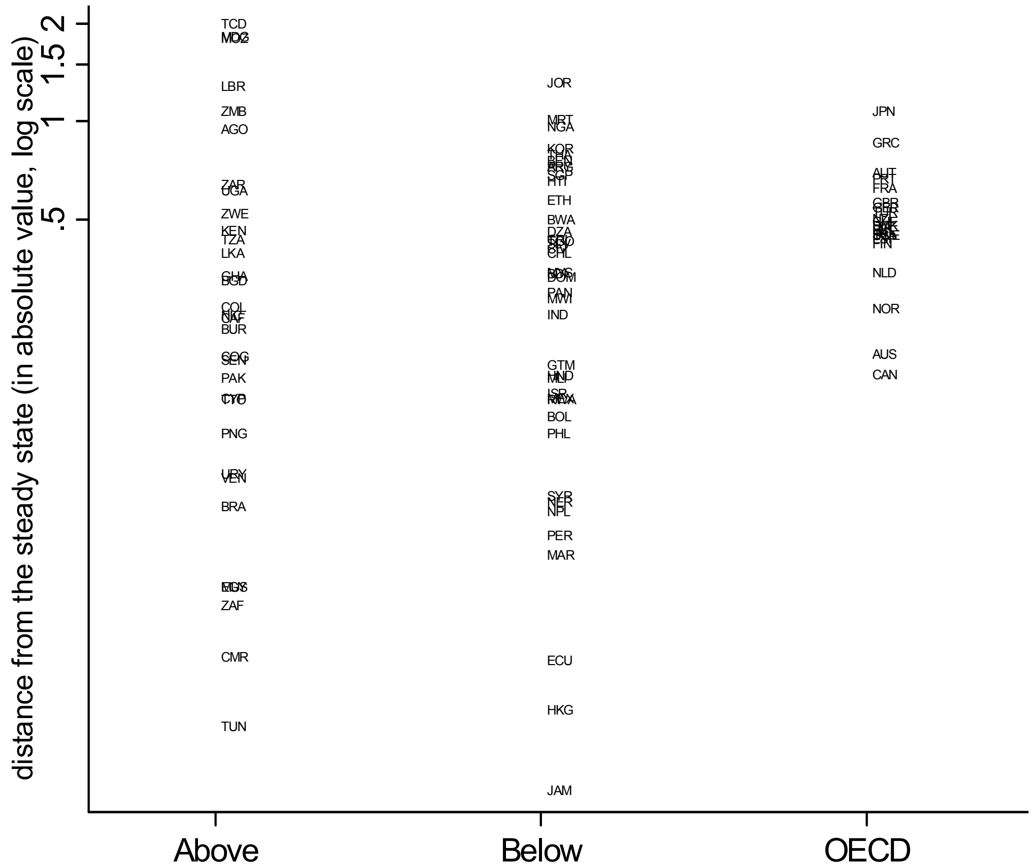

Figure 6: The distance from the steady state

calculate $\left(\frac{1-\alpha}{\alpha}\right)\ln(y^*(0)/y(0))_i$.[9] Treating 1960 as the initial year and assuming that deviations in technology growth from $\overline{g}$ are random and $0 < \alpha < 1$, the countries with the negative (positive) values of $\ln\left(k(0)/y(0)\right)_i - \ln s_i + \ln(n_i + \overline{g} + \delta)$ are likely to be below (above) their steady states in 1960. It turns out that all OECD counties and 41 countries have the negative values and the remaining 32 countries have the positive values. We, therefore, divide the sample into three subsamples; 'OECD' (22 countries), 'Below' (41 countries), and 'Above' (32 countries). Figure 6 shows a plot of the distance from the steady state for all of the countries in the sample (Appendix 2 gives country names, country codes and subsample groups).[10]

[9] We assume that $\overline{g}$ is 0.02 as MRW (1992).

[10] When the mean value of $|\ln\left(k(0)/y(0)\right)_i - \ln s_i + \ln(n_i + \overline{g} + \delta)|$ is calculated over the sample countries for 1985, the value decreases about by 30 per cent compared to that for

We estimate equations (23) and (25) both with and without an intercept. We assume that $\overline{g}$ is 0.02 and δ is 0.05.[11] Table 1 reports the results of OLS regressions with robust standard errors.

The results support the Solow model for the 'OECD' and 'Above' samples. All of the coefficient estimates in the 'OECD' and 'Above' samples have the signs predicted by the model. Although $\ln(n_i + \overline{g} + \delta)$ does not enter significantly in the regressions with an intercept, all of the coefficient estimates in the regressions without an intercept are statistically significant.[12] The restrictions on the coefficients are not rejected in the 'OECD' and 'Above' samples. The goodness of fit measures are high (e.g., the raw R^2 for the restricted regression is 0.61 for the 'OECD' sample and 0.62 for the 'Above' sample).[13] Most important, in the regressions without an intercept, the implied α is 0.31 for the 'OECD' sample and 0.35 for the 'Above' sample. The data, thus, strongly support the prediction that α is 1/3.[14]

Contrary to the 'OECD' and 'Above' samples, the data for the 'Below' sample fail to support the model. The estimates for the coefficient on $\ln k/y(60)$ and all of the estimates for the coefficients in the restricted regressions are statistically insignificant. The restrictions on the coefficients are rejected. The goodness of fit measures are low (e.g., the raw R^2 for the restricted regression is 0.01). Most important, the implied α of 0.08 in the regression without an intercept is far below the prediction.

1960.

[11] We assume that δ is 0.05 as Barro and Sala-i-Martin (1992a, 1992b). They used the value reported by Jorgenson and Yun (1986, 1990).

[12] If the intercept term is in fact absent as the specification suggests, the slope coefficients may be estimated with far greater precision than with the intercept term left in. Table 1 shows that the intercept terms in the "OECD" and "Above" samples are insignificant not even at the 10 per cent significance level. Thus, one cannot reject the hypothesis that the true intercept is equal to zero, thereby justifying regression through the origin. In the case of the "Below" sample, the estimation shows that the intercept term is significant at the 1 per cent level. As it is discussed later, this can be additional evidence that the data for the "Below" sample does not support the Solow model.

[13] The values of raw R^2 are calculated according to the definition: $Raw\ R^2 = 1 - (1 - \sum_i e_i / \sum_i Y_i)$, where e is the residual from the regression without an intercept and Y is the dependent variable observed.

[14] One of the main reasons that MRW (1992) reject the strict Solow model is that the estimated α for the Solow model without introducing human capital is much too high to be consistent with the conventional value of capital share.

Dependent Variable: (log difference GDP per working-age person 1960-85) - 0.5			
Sample (obs):	OECD(22)	Above(32)	Below(41)
Regression with an intercept			
constant	0.62	−0.68	5.43***
	(1.33)	(3.57)	(2.05)
ln(k/y 60)	−0.61*	−0.45***	−0.25
	(0.30)	(0.13)	(0.22)
ln$s$	1.20**	0.57***	0.65**
	(0.45)	(0.09)	(0.28)
ln(n+$\overline{g}$+δ)	−0.64	−0.86	1.78*
	(0.48)	(1.60)	(0.96)
R^2	0.48	0.54	0.40
Restricted regression:			
constant	−0.13	0.02	−0.07
	(0.17)	(0.08)	(0.11)
ln(k/y 60)-ln$s$+ln(n+$\overline{g}$+δ)	−0.67*	−0.56***	−0.21
	(0.35)	(0.07)	(0.27)
R^2	0.33	0.51	0.02
Test of restriction: p-value	0.10	0.54	0.00
Implied α	0.40	0.36	0.17
Regression without an intercept			
ln(k/y 60)	−0.60**	−0.45***	−0.19
	(0.28)	(0.13)	(0.25)
ln$s$	1.14**	0.57***	0.66**
	(0.40)	(0.08)	(0.31)
ln(n+$\overline{g}$+δ)	−0.85***	−0.56***	−0.58**
	(0.28)	(0.11)	(0.27)
Raw R^2	0.71	0.65	0.31
Restricted regression:			
ln(k/y 60)-ln$s$+ln(n+$\overline{g}$+δ)	−0.44***	−0.54***	−0.09
	(0.10)	(0.05)	(0.18)
Raw R^2	0.61	0.62	0.01
Test of restriction: p-value	0.10	0.51	0.01
Implied α	0.31	0.35	0.08

Notes: s=the average share of real investment in real GDP, $k/y(60)$=the capital-output in 1960, n=the average rate of growth of the working age population, $\overline{g}$=0.02, and δ=0.05. Robust standard errors are in parentheses. * significant at 10% level, ** significant at 5% level and *** significant at 1% level.

Table 1: The test of the Solow model (Test 1)

4 Empirical Test 2

4.1 The Specification

In this section, we perform another empirical test of the Solow model by estimating the speed of convergence towards the steady state. We derive the empirical specifications below.

From equation (12), the convergence coefficient is given by:

$$\beta(t) = (1-\alpha)(n+g+\delta)\left(\frac{y^*(t)}{y(t)}\right)^{\frac{1-\alpha}{\alpha}}. \tag{27}$$

Taking logs of equation (27) gives:

$$\ln\beta(t) = \ln(1-\alpha) + \ln(n+g+\delta) - \frac{1-\alpha}{\alpha}\left(\ln y(t) - \ln y^*(t)\right). \tag{28}$$

From equation (22), we can obtain:

$$\ln y^*(t) = \ln y(0) - \frac{\alpha}{1-\alpha}\ln\frac{k(0)}{y(0)} + \frac{\alpha}{1-\alpha}\ln s - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + g\ t. \tag{29}$$

Substituting this expression into equation (28) and arranging it yield:

$$\ln\left(\frac{y(t)}{y(0)}\right)_i = \frac{\alpha}{1-\alpha}\ln(1-\alpha) - \frac{\alpha}{1-\alpha}\ln\beta_i(t) + g_i t - \frac{\alpha}{1-\alpha}\ln\left(\frac{k(0)}{y(0)}\right)_i + \frac{\alpha}{1-\alpha}\ln s_i. \tag{30}$$

The term, $\beta_i(t)$, in the equation (30) is not directly observable. Hence, we estimate the generalized value of $\beta_i(t)$ across countries.

From equation (27), denoting $(y^*(t)/y(t))_i$ as $z_i(t)$, we can rewrite $\beta_i(t)$ as:

$$\beta_i(t) = (1-\alpha)(\overline{n}+\overline{g}+\delta+\widetilde{n}_i+\widetilde{g}_i)\left(\overline{z}(t)+\widetilde{z}_i(t)\right)^{\frac{1-\alpha}{\alpha}},$$

where $\overline{n}$ is the mean population growth rate, $\widetilde{n}_i$ is the deviation from $\overline{n}$, $\overline{z}(t)$ is the mean of $(y^*(t)/y(t))_i$, and $\widetilde{z}_i(t)$ is the deviation from $\overline{z}(t)$. We then define the generalized value of $\beta_i(t)$ as below:

$$\overline{\beta}(t) = (1-\alpha)(\overline{n}+\overline{g}+\delta)\overline{z}(t)^{\frac{1-\alpha}{\alpha}}.$$

Denoting $\widetilde{\beta}_i(t)$ as $\beta_i(t)/\overline{\beta}(t)$, we can get:

$$\ln \widetilde{\beta}_i(t) = \ln \beta_i(t) - \ln \overline{\beta}(t)$$
$$= \ln \left(\frac{\overline{n} + \overline{g} + \delta + \widetilde{n}_i + \widetilde{g}_i}{\overline{n} + \overline{g} + \delta} \right) + \frac{1-\alpha}{\alpha} \ln \left(\frac{\overline{z}(t) + \widetilde{z}_i(t)}{\overline{z}(t)} \right). \quad (31)$$

A first order Taylor-Series expansion of $\ln((\overline{n} + \overline{g} + \delta + \widetilde{n}_i + \widetilde{g}_i)/(\overline{n} + \overline{g} + \delta))$ around $g_i = \overline{g}$ gives:

$$\ln \left(\frac{\overline{n} + \overline{g} + \delta + \widetilde{n}_i + \widetilde{g}_i}{\overline{n} + \overline{g} + \delta} \right) \simeq \ln \left(\frac{\overline{n} + \overline{g} + \delta + \widetilde{n}_i}{\overline{n} + \overline{g} + \delta} \right) + \frac{\widetilde{g}_i}{\overline{n} + \overline{g} + \delta + \widetilde{n}_i}. \quad (32)$$

Similarly, a first order Taylor-Series expansion of $\ln\left((\overline{z}(t) + \widetilde{z}_i(t))/\overline{z}(t)\right)$ around $z_i(t) = \overline{z}(t)$ gives:

$$\ln \left(\frac{\overline{z}(t) + \widetilde{z}_i(t)}{\overline{z}(t)} \right) \simeq \frac{\widetilde{z}_i(t)}{\overline{z}(t)}. \quad (33)$$

By substituting equations (32) and (33) into equation (31), we can get:

$$\ln \beta_i(t) = \ln \overline{\beta}(t) + \ln \left(\frac{\overline{n} + \overline{g} + \delta + \widetilde{n}_i}{\overline{n} + \overline{g} + \delta} \right) + \frac{\widetilde{g}_i}{\overline{n} + \overline{g} + \delta + \widetilde{n}_i} + \frac{\widetilde{z}_i(t)}{\overline{z}(t)}.$$

Substituting this into equation (30) gives:

$$\ln \left(\frac{y(t)}{y(0)} \right)_i - \overline{g}\, t = \frac{\alpha}{1-\alpha} \left(\ln(1-\alpha) - \ln \overline{\beta}(t) \right) - \frac{\alpha}{1-\alpha} \ln \left(\frac{k(0)}{y(0)} \right)_i$$
$$+ \frac{\alpha}{1-\alpha} \ln s_i - \frac{\alpha}{1-\alpha} \ln \left(\frac{n_i + \overline{g} + \delta}{\overline{n} + \overline{g} + \delta} \right) + \varepsilon_i, \quad (34)$$

where

$$\varepsilon_i = \left(t - \frac{\alpha}{1-\alpha} \frac{1}{n_i + \overline{g} + \delta} \right) \widetilde{g}_i - \frac{\alpha}{1-\alpha} \frac{\widetilde{z}_i(t)}{\overline{z}(t)}. \quad (35)$$

The restricted version of equation(34) is given by:

$$\ln \left(\frac{y(t)}{y(0)} \right)_i - \overline{g}\, t = \frac{\alpha}{1-\alpha} \left(\ln(1-\alpha) - \ln \overline{\beta}(t) \right)$$
$$- \frac{\alpha}{1-\alpha} \left(\ln \left(\frac{k(0)}{y(0)} \right)_i - \ln s_i + \ln \left(\frac{n_i + \overline{g} + \delta}{\overline{n} + \overline{g} + \delta} \right) \right) + \varepsilon_i. \quad (36)$$

Sample (obs):	OECD (22)	Above (32)	Below (41)
Mean	0.067	0.048	0.063
Standard deviation	0.006	0.023	0.014
Minimum value	0.054	0.006	0.028
Maximum value	0.080	0.083	0.091

Table 2: The calculated $\overline{\beta}$ values

Equations (34) and (36) are the regression specifications used in this section. We treat ε_i as the error term and it is given by equation (35). As in Section 3, we assume that $\widetilde{g}_i$ is random and countries are close enough to their steady states at year t so that most of the variation in $\widetilde{z}_i(t)$ is due to demand shocks.

The implication of equation (34) is similar to that of equation (23). It implies conditional convergence. By estimating equation (36) with OLS, we can get the estimate for $\alpha/(1-\alpha)$. This, in turn, gives the estimate for $\overline{\beta}(t)$ by looking at the estimate for the constant term, $(\alpha/(1-\alpha))\left(\ln(1-\alpha) - \ln\overline{\beta}(t)\right)$. As mentioned before, our main goal here is to check the validity of the Solow model by looking at the estimated value of α, which should be about 1/3.

4.2 Results

Before estimating equations (34) and (36), we provide a supplement for the regression tests. By substituting 0 for t in equation (28) and using equation (26), we can obtain:

$$\ln\beta(0)_i = \ln(1-\alpha) - \ln\left(\frac{k(0)}{y(0)}\right)_i + \ln s_i. \tag{37}$$

Assuming $\alpha = 1/3$, we can calculate $\ln\beta$ for each country by using equation (37). We then calculate $\overline{\beta}(0)$ by simply taking the mean value across countries. Taking 1985 as the initial year, Table 2 gives the results.

Table 2 reports that the calculated $\overline{\beta}$ value in year 1985 is 0.067 for the 'OECD' sample, 0.048 for the 'Above' sample and 0.063 for the 'Below' sample. Thus, if the model describes the mechanism of economic growth well, the OLS regression estimates on equation (36) should be able to give the estimated $\overline{\beta}$ values that are close to the values of $\overline{\beta}$ reported in Table 2 and also give the estimated α values which are close to 1/3.

The regression results based on specifications (34) and (36) are given in Table 3. The results show that the data for the 'OECD' and 'Above' samples

Dependent Variable: (log difference GDP per working-age person 1960-85) - 0.5			
Sample (obs):	OECD (22)	Above (32)	Below (41)
constant	2.23**	1.32***	1.31**
	(0.80)	(0.30)	(0.56)
$\ln(k/y\ 60)$	-0.61*	-0.45***	-0.25
	(0.30)	(0.13)	(0.22)
$\ln s$	1.20**	0.57***	0.65**
	(0.45)	(0.09)	(0.28)
$\ln\left(\frac{n+\overline{g}+\delta}{\overline{n}+\overline{g}+\delta}\right)$	-0.64	-0.86	1.78*
	(0.48)	(1.60)	(0.96)
R^2	0.48	0.54	0.40
Restricted regression:			
constant	1.55**	1.31***	0.40
	(0.71)	(0.24)	(0.55)
$\ln(k/y\ 60)-\ln s+\ln\left(\frac{n+\overline{g}+\delta}{\overline{n}+\overline{g}+\delta}\right)$	-0.67*	-0.56***	-0.21
	(0.35)	(0.07)	(0.27)
R^2	0.33	0.51	0.02
Test of restriction: p-value	0.10	0.54	0.00
Implied α	0.40	0.36	0.17
Implied $\overline{\beta}$	0.059	0.060	0.118

See notes to Table 1. Data here are identical except for $\overline{n}$ which is the average of n in each subsample.

Table 3: The test of the Solow model (Test 2)

seem to support the Solow model. All of the coefficient estimates for the two samples have the signs predicted by the model and most of them are statistically significant.[15] The restrictions on the coefficients are not rejected in the 'OECD' and 'Above' samples, and the values of R^2 are reasonably high. Most important, the implied values of $\overline{\beta}$ for the 'OECD' and 'Above' samples are reasonably close to the values shown in Table 2 and the implied values of α are very close to the conventional value of capital share.

In the case of the 'Below' sample, the results in Table 3, however, do not support the model. The coefficient on $\ln\left((n_i+\overline{g}+\delta)/(\overline{n}+\overline{g}+\delta)\right)$ is of the wrong sign and significant. The estimate for the coefficient on $\ln k/y(60)$

[15] The robust standard errors of the coefficient on $\ln\left((n_i+\overline{g}+\delta)/(\overline{n}+\overline{g}+\delta)\right)$ are large in the 'OECD' and 'Above' samples. This could be because the constant term, $(\alpha/(1-\alpha))\left(\ln(1-\alpha)-\ln\overline{\beta}(t)\right)$, and $\ln\left((n_i+\overline{g}+\delta)/(\overline{n}+\overline{g}+\delta)\right)$ are both functions of $\overline{n}+\overline{g}+\delta$.

in the unrestricted regression and all of the estimates for the coefficients in the restricted regression are statistically insignificant. The R^2 of 0.02 in the restricted regression is poor and the restrictions are rejected. The implied $\overline{\beta}$ is far from the value shown in Table 2 and the implied α is about half the size of the conventional value of capital share.

5 Implication of the Obtained Results

The results from Sections 3 and 4 show that the Solow model explains the growth mechanism for the 'OECD' and 'Above' samples quite well but not for the 'Below' sample. In this section we examine the origins of these differences and provides a possible explanation.

The empirical tests conducted in the previous sections control for saving rates, population growth rates and initial levels of technology. The rates of technology growth are, however, left untouched and unobservable differences in technology growth rates are reflected in the error terms. Despite no conditioning on the rates of technology growth, convergence is observed in the 'OECD' and 'Above' samples but not in the 'Below' sample. If the Solow model describes the mechanism of economic growth correctly, the significant differences in convergence patterns across subsamples can be attributed to the fact that the growth rates of technology vary randomly in the 'OECD' and 'Above' samples but not in the 'Below' sample. The rates of technological progress may systematically differ across the 'Below' sample countries.

If technological progress is, as argued above, important for explaining the empirical results, we need to explain why the growth rates of technology systematically vary only in the 'Below' sample. Many explanations are possible. One of the most promising might be technology diffusion.[16] In order to incorporate the idea of technology diffusion, we modify the assumptions about technological progress. Firstly, we assume that there is one technologically leading country and the others are followers. As before technology is assumed not to be a worldwide good but rather a domestic good. However, this time, assume that technology diffusion from the leader benefits some countries. Some countries can easily absorb cutting-edge technologies and make full use of them. Adoption of leading technologies can push the country's technology level up over time. A larger gap in the initial technology level between the leader and the follower leads to a greater increase in the follower's level of technology over time due to technology diffusion. This seems reasonable if we think about im-

[16] Barro and Sala-i-Martin (1997), for example, show the model in which technology diffusion generates convergence.

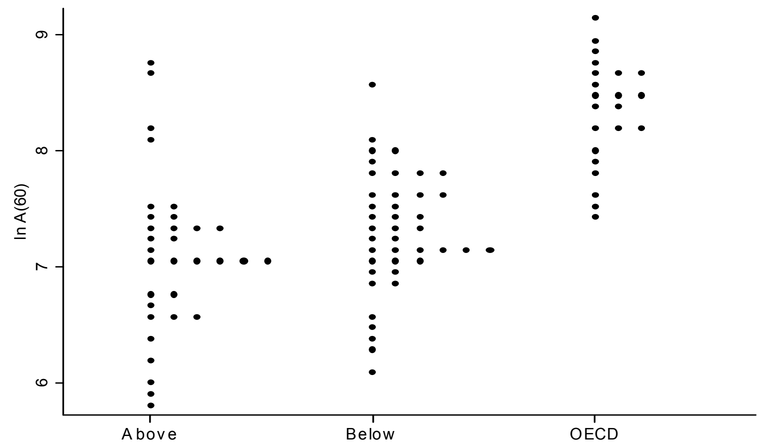

Figure 7: The distribution of $A(0)$

itation of technology. The rates of technology growth, thus, systematically differ across those countries who benefit from technology diffusion. On the other hands, if the follower initially has a similar level of technology as the leader, there is not much to be adopted or imitated by the follower. Thus, the rates of technology growth do not systematically differ across those countries who do not benefit much from technology diffusion. We also assume that it is hard for the follower to adopt or imitate the leading technologies if the follower does not have a baseline technology level since the follower would face a large adoption cost. Here, we interpret 'technology' in a broad sense. The initial level of technology $A(0)$ reflects not just production technology but also institutions, geographies, government policies and also possibly human capital. For example, a bad government policy set a barrier to technology adoption by increasing the amount of investment required for adopting leading technologies. Thus, no or very little technology diffusion occurs in the countries with the very low levels of $A(0)$. Parente and Prescott (1999) emphasize the role of barriers that limit firms' incentives to adopt technology.

If the assumptions above are approximately correct, we should observe that the levels of $A(0)$ of the 'Below' sample countries tend to be between those of the 'OECD' and 'Above' samples. Figure 7 shows the distributions of A in year 1960 for each sample group, where $A(0)$ is calculated by using equation (21). It shows that the levels of $\ln A(0)$ of the 'OECD' sample countries are likely to be much higher than those of the other countries. It may thus be reasonable to say that many of the 'OECD' sample countries have the initial levels of technology just below the leader's $\ln A(0)$. As for other countries, it shows that the levels of $\ln A(0)$ of the 'Above' sample tend to be low and the levels of $\ln A(0)$ of the 'Below' sample tend to be between those of the other two subsamples.[17]

Our regression results also indicate the possibility that the growth rates of technology are not random and are negatively correlated with initial levels of technology in the 'Below' sample due to technology diffusion. Compared with the implied values of α for the 'OECD' and 'Above' samples, the implied value of α for the 'Below' sample is far off below the conventional value of capital share. The low implied value of α for the 'Below' sample can be interpreted as the bias introduced by the correlation between $\ln(k(0)/y(0))_i - \ln s_i + \ln(n_i + \overline{g} + \delta)$ and ϵ_i in equation (25).[18] Denoting $-\alpha/(1-\alpha)$, the coefficient on $\ln(k(0)/y(0))_i - \ln s_i + \ln(n_i + \overline{g} + \delta)$ in equation (25), as x, the degree of the bias is shown by:

$$E(\widehat{x}) - x = \frac{Cov\left[\ln\left(\frac{k(0)}{y(0)}\right)_i - \ln s_i + \ln(n_i + \overline{g} + \delta), \left(t - \frac{\alpha}{1-\alpha}\frac{1}{(n_i+\overline{g}+\delta)}\right)\widetilde{g}_i\right]}{Var\left[\ln\left(\frac{k(0)}{y(0)}\right)_i - \ln s_i + \ln(n_i + \overline{g} + \delta)\right]}.$$

Assuming $\widetilde{g}_i$ is not correlated with $\ln s_i$ and $\ln(n_i + \overline{g} + \delta)$, we can get:

$$E(\widehat{x}) - x = \frac{E\left(\ln\left(\frac{k(0)}{y(0)}\right)_i \widetilde{g}_i\right)\left(t - \frac{\alpha}{1-\alpha}E\left(\frac{1}{n_i+\overline{g}+\delta}\right)\right)}{Var\left[\ln\left(\frac{k(0)}{y(0)}\right)_i - \ln s_i + \ln(n_i + \overline{g} + \delta)\right]}.$$

Since $t - (\alpha/(1-\alpha))E(1/(n_i + \overline{g} + \delta))$ is likely to be positive, the direction of the bias in the estimate of x depends on the sign of $E(\ln(k(0)/y(0))_i\widetilde{g}_i)$.[19] If $\widetilde{g}_i$ is positively correlated with $\ln(k(0)/y(0))_i$, we get an upward bias in the estimate of x (i.e., a downward bias in the estimate of α). Since $A(0)_i$ is negatively correlated with $\ln(k(0)/y(0))_i$, we get a downward bias in the estimate of α if $\widetilde{g}_i$ is negatively correlated with $A(0)_i$.[20] Hence, the very low

[17] The average level of technology in 1960 is higher in the 'Below' sample although the average level of output per working age population in 1960 is higher in the 'Above' sample.

[18] For empirical test 2, the bias is introduced by the correlation between $(\ln(k(0)/y(0)))_i - \ln s_i + \ln((n_i + \overline{g} + \delta)/(\overline{n} + \overline{g} + \delta))$ and ε_i in equation (36).

[19] The maximum value of $1/(n_i + \overline{g} + \delta)$ in our sample is 13.67 and the minimum value is 8.96. We use the 25 years sample period so that t is 25. Thus, assuming $\alpha = 1/3$, the value of $t - (\alpha/(1-\alpha))/(n_i + \overline{g} + \delta)$ is in the range between 18.17 and 20.52 in our sample.

[20] Since $k(0)/y(0) = (A(0)/y(0))^{-(1-\alpha)/\alpha}$ and $\partial(A(0)/y(0))/\partial A(0) = \alpha/y(0) > 0$, $A(0)_i$ is negatively correlated with $\ln(k(0)/y(0)_i$.

estimate of α for the 'Below' sample may suggest the existence of a strong negative relationship between $A(0)_i$ and $\widetilde{g}_i$ in the 'Below' sample countries.[21]

6 Conclusion

This paper analyzes the Solow model both empirically and theoretically by taking a new approach which yields some important implications.

First, it shows the importance of a variation in the initial level of technology in the Solow model. The steady state level of income per capita and the (non-steady state) growth rate of income per capita are sensitive to the initial level of technology. It also shows the nature of convergence from above in a much clearer way than in previous work. Countries converging from above could have positive growth rates of capital per labor for a relatively long period before reaching their steady state paths.

We also construct a new method to test the Solow model by allowing for cross-country differences in initial levels and growth rates of technology. The results show that the Solow model can explain well growth mechanisms only for the 'OECD' and 'Above' samples. Conditional convergence due to diminishing returns to capital is observed in these two subsamples. The implied value of the speed of convergence is about 0.06 and it is much higher than the conventional estimated value of 0.02. To the contrary, the results show no evidence of conditional convergence in the 'Below' sample. It is argued that the significant differences in convergence patterns across subsamples come from the fact that technology diffusion has a large impact mainly on the 'Below' sample countries.

[21] We could also get the downward bias in the estimate of α if the countries are far from their steady states at the end of the sample period. However, this is unlikely to be the reason why the implied value of α for the 'Below' sample is much lower than the implied values of α for the other two samples. When the value of $|((1-\alpha)/\alpha)\ln(y^*(t)/y(t))_i|$ with $\widetilde{g}_i = \overline{g}$ is calculated for all of the countries in the sample by using equation (26), the mean value in the 'Below' sample is lowest and the standard deviation in the 'Below' sample is lower than that in the 'Above' sample.

Appendix 1: The speed of convergence, β

The coefficient of the speed of convergence is given by:

$$\beta(t) \simeq -\frac{d\,\dot{X}(t)}{d\,X(t)} = -\frac{d\,\left(\frac{d\,(\ln k^*(t) - \ln k(t))}{d\,t}\right)}{d\,\left(\ln k^*(t) - \ln k(t)\right)}. \tag{A.1}$$

Using the facts:

$$\ln y(t) = (1-\alpha)\ln A(t) + \alpha \ln k(t)$$

and

$$\ln y^*(t) = (1-\alpha)\ln A(t) + \alpha \ln k^*(t),$$

the distance between $\ln k^*(t)$ and $\ln k(t)$ is given by:

$$\ln k^*(t) - \ln k(t) = \frac{1}{\alpha}\left(\ln y^*(t) - \ln y(t)\right). \tag{A.2}$$

Substituting equation (A.2) into equation (A.1) yields:

$$\begin{aligned} \beta(t) &\simeq -d\,\left(\frac{d\,(\ln y^*(t) - \ln y(t))}{d\,t}\right)/d\,\left(\ln y^*(t) - \ln y(t)\right) \\ &= -d\,\left(g - \frac{\dot{y}(t)}{y(t)}\right)/d\,(\ln \frac{y^*(t)}{y(t)}) = d\,\left(\frac{\dot{y}(t)}{y(t)}\right)/d\,(\ln \frac{y^*(t)}{y(t)}). \end{aligned} \tag{A.3}$$

Since $y(t) = A(t)^{1-\alpha}k(t)^{\alpha}$, the growth rate of output per labor is given by:

$$\frac{\dot{y}(t)}{y(t)} = (1-\alpha)g + \alpha\frac{\dot{k}(t)}{k(t)}. \tag{A.4}$$

By using the equation for the capital accumulation, the growth rate of capital per labor is given by:

$$\frac{\dot{k}(t)}{k(t)} = s\,A(t)^{1-\alpha}k(t)^{\alpha-1} - (n+\delta). \tag{A.5}$$

Substituting equation (A.5) into equation (A.4) yields:

$$\frac{\dot{y}(t)}{y(t)} = (1-\alpha)g + \alpha\left(s\,A(t)^{1-\alpha}k(t)^{\alpha-1} - (n+\delta)\right). \tag{A.6}$$

Since $s\,A(t)^{-\alpha}k(t)^{\alpha} - (n+g+\delta)\,k(t)\,A(t)^{-1} = 0$ at the steady state,

$$k^*(t)^{\alpha-1} = \frac{n+g+\delta}{s}k(t)\,A(t)^{\alpha-1}.$$

Solving this equation for $A(t)$ and substituting it into equation (A.6) yield:

$$\frac{\dot{y}(t)}{y(t)} = (1-\alpha)g + \alpha\left((n+g+\delta)\left(\frac{k(t)}{k^*(t)}\right)^{\alpha-1} - (n+\delta)\right). \qquad \text{(A.7)}$$

Rewriting equation (A.2) gives:

$$\frac{k(t)}{k^*(t)} = \left(\frac{y(t)}{y^*(t)}\right)^{\frac{1}{\alpha}}.$$

Substituting this expression into equation (A.7) yields:

$$\frac{\dot{y}(t)}{y(t)} = (1-\alpha)g + \alpha\left((n+g+\delta)\left(\frac{y^*(t)}{y(t)}\right)^{\frac{1-\alpha}{\alpha}} - (n+\delta)\right). \qquad \text{(A.8)}$$

By substituting equation (A.8) into equation (A.3), the coefficient for the speed of convergence is thus given by:

$$\begin{aligned}\beta(t) \simeq \frac{d}{d\ \ln \frac{y^*(t)}{y(t)}}\left((1-\alpha)g + \alpha\left((n+g+\delta)\left(\frac{y^*(t)}{y(t)}\right)^{\frac{1-\alpha}{\alpha}} - (n+\delta)\right)\right) \\ = \frac{y^*(t)}{y(t)}\left(\frac{1-\alpha}{\alpha}\alpha(n+g+\delta)\left(\frac{y^*(t)}{y(t)}\right)^{\frac{1-2\alpha}{\alpha}}\right) \\ = (1-\alpha)(n+g+\delta)\left(\frac{y(t)}{y^*(t)}\right)^{\frac{\alpha-1}{\alpha}}.\end{aligned}$$

When the economy is at the steady state, $\beta(t)$ is equal to $(1-\alpha)(n+g+\delta)$.

Appendix 2: Subsample groups of countries

Name	Code	Group	Name	Code	Group	Name	Code	Group
Angola	AGO	Above	Algeria	DZA	Below	Paraguay	PRY	Below
Bangladesh	BGD	Above	Argentina	ARG	Below	Peru	PER	Below
Brazil	BRA	Above	Benin	BEN	Below	Philippine	PHL	Below
Cameroon	CMR	Above	Bolivia	BOL	Below	Rwanda	RWA	Below
Centr. African R.	CAF	Above	Botswana	BWA	Below	Singapore	SGP	Below
Chad	TCD	Above	Burundi	BDI	Below	Somalia	SOM	Below
Colombia	COL	Above	Chile	CHL	Below	Syria	SYR	Below
Congo	COG	Above	Costa Rica	CRI	Below	Thailand	THA	Below
Cyprus	CYP	Above	Cote d'Ivoire	CIV	Below	Togo	TGO	Below
Egypt	EGY	Above	Dominican R.	DOM	Below	Australia	AUS	OECD
Ghana	GHA	Above	Ecuador	ECU	Below	Austria	AUT	OECD
Kenya	KEN	Above	El Salvador	SLV	Below	Belgium	BEL	OECD
Liberia	LBR	Above	Ethiopia	ETH	Below	Canada	CAN	OECD
Madagascar	MDG	Above	Guatemala	GTM	Below	Denmark	DNK	OECD
Mauritius	MUS	Above	Haiti	HTI	Below	Finland	FIN	OECD
Mozambique	MOZ	Above	Honduras	HND	Below	France	FRA	OECD
Myanmar	BUR	Above	Hong Kong	HKG	Below	Germany	GER	OECD
Nicaragua	NIC	Above	India	IDN	Below	Greece	GRC	OECD
Pakistan	PAK	Above	Israel	ISR	Below	Ireland	IRL	OECD
Papua New Guinea	PNG	Above	Jamaica	JAM	Below	Italy	ITA	OECD
Senegal	SEN	Above	Jordan	JOR	Below	Japan	JPN	OECD
South Africa	ZAF	Above	Korea	KOR	Below	Netherlands	NLD	OECD
Sri Lanka	LKA	Above	Malawi	MWI	Below	New Zealand	NZL	OECD
Tanzania	TZA	Above	Malaysia	MYS	Below	Norway	NOR	OECD
Trinidad	TTO	Above	Mali	MLI	Below	Portugal	PRT	OECD
Tunisia	TUN	Above	Mauritania	MRT	Below	Spain	ESP	OECD
Uganda	UGA	Above	Mexico	MEX	Below	Sweden	SWE	OECD
Uruguay	URY	Above	Morocco	MAR	Below	Switzerland	CHE	OECD
Venezuela	VEN	Above	Nepal	NPL	Below	Turkey	TUR	OECD
Zaire	ZAR	Above	Niger	NER	Below	U. K.	GBR	OECD
Zambia	ZMB	Above	Nigeria	NGA	Below	U.S.A.	USA	OECD
Zimbabwe	ZWE	Above	Panama	PAN	Below			

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