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This paper develops a model of mixed oligopoly to establish that (i) a horizontal merger involving a public firm is profitable if the public firm is insufficiently privatized, that (ii) such a merger unambiguously raises the outsiders' profit, but that (iii) it involves welfare losses. Therefore, the presence of a semi-public firm is irrelevant to the effect of a merger on the outsiders' profit and welfare while it plays a nontrivial role for the merger profitability.

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JEL : L13, L32, L33

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1 Introduction

Is a horizontal merger beneficial to the inside and outside firms, and welfare? There is a considerable literature to theoretically address this question, most of which is inspired by a seminal work of Salant et al. (1983). Assuming (i) linear demand and marginal cost, (ii) a homogeneous good (perfect substitute), (iii) Cournot-Nash competition, (iv) no cost synergy, and (v) a static setting, they demonstrate that a merger is unprofitable unless the merger involves more than 80 per cent market share. This result is so striking from not only theoretical but also practical points of
view that many successors have considered its robustness by relaxing the above assumptions.¹)

Along with the previous works above, Barcena-Ruiz and Garzon (2003) and Mendez-Naya (2008) highlight the role of partial privatization. Looking at the real world, a public firm is frequently acquired by private firms, making their study interesting. They show that the profitability of a merger depends on the degree of privatization. This paper is also along the same line as theirs, but our aim differs from that of the existing literature at least in three respects.

First, both Barcena-Ruiz and Garzon (2003) and Mendez-Naya (2008) assume a quadratic cost, i.e., increasing marginal cost.²) In other words, they deviate from Salant et al. (1983) in two respects: the presence of a public firm and increasing marginal cost. Given that Farrell and Shapiro (1990) have already proved that Salant et al.’s (1983) result is invalid under increasing marginal cost, the finding established by the above two papers is quite natural. In order to pay attention to the role of a semi-public firm exclusively, we continue to assume constant marginal cost in analysis. Second, these papers assume that the public firm merges with only one private firm. Instead, we allow for an arbitrary number of insider firms and outsider firms, which is in accordance with Salant et al. (1983).


²) Although it is a common practice to assume increasing marginal cost in the literature on mixed oligopoly, the assumption of increasing marginal cost makes the impact of the presence of a public firm less transparent. Furthermore, Barcena-Ruiz and Garzon (2003) incorporate product differentiation, which makes it even harder to compare their result with that of Salant et al. (1983).
finally, our definition of ‘merger profitability’ is different from that in the above predecessors. While we do not claim that our definition is more plausible, our definition of merger profitability has a certain meaning.

Our main results are summarized as follows. First, if the public firm is insufficiently privatized, the merger can be profitable. Second, if the number of insider (resp. outsider) firms increases (resp. decreases), the profitable merger is more likely. Third, such a merger is always profit-enhancing for the outsider firms for any degree of privatization. Finally, the merger unambiguously reduces welfare.

The rest of this paper is structured as follows. Section 2 sketches a basic model and describes a pre-merger equilibrium. Section 3 derives the post-merger equilibrium. Then, Section 4 examines how privatization affects the profitability of a merger. Section 5 considers the effects of a merger on the outsiders’ profit and welfare. Section 6 concludes.

2 A pre-merger equilibrium

As a preliminary, this section characterizes the equilibrium before a merger. In order to make our argument as parallel to Salant et al. (1983) as possible, we consider an oligopolistic model with linear demand and constant marginal cost. The inverse demand function is given by \( p = a - X \), \( a > 0 \) where \( p \) and \( X \) are the price and industry output, respectively. There are potentially three types of firms in the industry: (i) one public firm (indexed by 0), (ii) \( m \geq 2 \) firms participating in the merger, and (iii) \( n \geq 2 \) outsider firms. All of these firms share an identical marginal cost \( c > 0 \) which is less than \( a \).

While \( m + n \) private firms maximize profits, the public firm maximizes the weighted sum of profit \( \pi_0 \) and welfare \( U \) by following the formulation of Matsumura and Kanda (2005):\(^3\)

\[^3\) See also Matsumura (1998) on assuming this type of objective function of the public firm.\]
\[\theta \pi_0 + (1 - \theta)U\]
\[= \theta \pi_0 + (1 - \theta) \left( \frac{X^2}{2} + \pi_0 + \sum \pi_i + \sum \pi_j \right)\]
\[= \pi_0 + (1 - \theta) \left( \frac{X^2}{2} + \sum \pi_i + \sum \pi_j \right) , \quad \theta \in [0, 1], \] (1)

where \(X^2/2\) is consumer surplus, and \(\pi_i\) and \(\pi_j\) are the profit of the private firms. Each individual firm’s profit is defined by

\[\pi_0 = (a - c - X)x_0\] (2)
\[\pi_i = (a - c - X)x_i\] (3)
\[\pi_j = (a - c - X)x_j.\] (4)

The public firm chooses \(x_0\) to maximize (1) whereas private firms respectively maximize (3) and (4) with a Cournot-Nash conjecture. Then, the system of the first-order conditions for the associated maximization problem is

\[
\begin{bmatrix}
\theta + 1 & m & n \\
1 & m + 1 & n \\
1 & m & n + 1
\end{bmatrix}
\begin{bmatrix}
x_0 \\
x_i \\
x_j
\end{bmatrix} =
\begin{bmatrix}
a - c \\
a - c \\
a - c
\end{bmatrix},
\]

which leads to the following outputs and profits in equilibrium:

\[x_0 = \frac{a - c}{\theta(m + n + 1) + 1}, \quad x_i = x_j = \frac{\theta(a - c)}{\theta(m + n + 1) + 1}\] (5)

\[\pi_{0}^{pre} = \theta \left[ \frac{a - c}{\theta(m + n + 1) + 1} \right]^2\] (6)

\[\pi_{i}^{pre} = \pi_{j}^{pre} = \left[ \frac{\theta(a - c)}{\theta(m + n + 1) + 1} \right]^2,\] (7)

where superscript \(pre\) indicates the pre-merger equilibrium.

3 A post-merger equilibrium

This section moves on to the situation in which the public firm and \(m\) firms merge and behave as a cartel. After the merger, each insider firm
chooses output to maximize the modified objective:

\[ \theta \left( \pi_0 + \sum \pi_i \right) + (1 - \theta) \left( \frac{X^2}{2} + \pi_0 + \sum \pi_i + \sum \pi_j \right) \]

\[ = \pi_0 + \sum \pi_i + (1 - \theta) \left( \frac{X^2}{2} + \sum \pi_j \right). \quad (8) \]

That is, the cartel chooses the output of each insider firm to maximize the weighted sum of joint profit and welfare. On the other hand, the problem of the outsider firms is the same as that in the previous section. The system of first-order conditions in this case is abbreviated to

\[ \begin{bmatrix} (\theta + 1)(m + 1) & n \\ m + 1 & n + 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_j \end{bmatrix} = \begin{bmatrix} a - c \\ a - c \end{bmatrix}, \]

where use is made of the symmetry assumption that \( x_0 = x_i \) for insider firms. Solving this system allows us to find the equilibrium outputs and profits:

\[ x_0 = \frac{a - c}{(m + 1)[\theta(n + 1) + 1]}, \quad x_j = \frac{\theta(a - c)}{\theta(n + 1) + 1} \]

\[ \pi_0^{\text{post}} = \frac{\theta}{m + 1} \left[ \frac{a - c}{\theta(n + 1) + 1} \right]^2 \]

\[ \pi_j^{\text{post}} = \left[ \frac{\theta(a - c)}{\theta(n + 1) + 1} \right]^2, \quad (11) \]

where superscript \( \text{post} \) stands for the post-merger equilibrium.

4 Merger profitability and privatization

Having computed the equilibrium profits before and after the merger, we are ready to examine how the merger profitability is affected by privatization. We define the merger profitability as the difference between the aggregate profit of the insider firms after merger and that before merger. Making use of (6), (7) and (10), it is given by
\[(m + 1)\pi_0^{post} - \pi_0^{pre} - m\pi_i^{pre}\]

\[= \theta \left[ \frac{a - c}{\theta(n + 1) + 1} \right]^2 - \theta \left[ \frac{a - c}{\theta(m + n + 1) + 1} \right]^2 - m \left[ \frac{\theta(a - c)}{\theta(m + n + 1) + 1} \right]^2\]

\[= \frac{\theta(a - c)^2 \Gamma}{[\theta(n + 1) + 1]^2[\theta(m + n + 1) + 1]^2}\]  

\[\Gamma \equiv [\theta(m + n + 1) + 1]^2 - (\theta m + 1)[\theta(n + 1) + 1]^2.\]  

The rest of our task in this section is to identify a few properties of \(\Gamma\). In particular, we pay special attention to how \(\theta\) affects the sign of \(\Gamma\). For this purpose, it is convenient to rewrite (13) as follows.

\[\Gamma = \theta m \left[ -(n + 1)^2 \theta^2 + m\theta + 1 \right],\]  

which immediately yields:

**Proposition 1.** A merger is profitable if and only if

\[\theta < \bar{\theta} \equiv \frac{m + \sqrt{m^2 + 4(n + 1)^2}}{2(n + 1)^2}.\]  

An inspection of \(\bar{\theta}\) in (15) leads to:

**Lemma 1.** \(\bar{\theta}\) is increasing in \(m\) and decreasing in \(n\).

Lemma 1 claims that an increase in the number of insider firms makes the merger more profitable and vice versa if the number of outsider firms increases.

**Remark 1.** In the foregoing arguments, we have allowed for an arbitrary number of inside and outside firms. Let us address what we can infer if the number of insiders is equal to that of outsiders. Then, \(\Gamma\) in (14) simplifies to

\[\Gamma = \theta m \left[ -(n + 1)^2 \theta^2 + m\theta + 1 \right],\]  

\[\text{Remark 1.}\] Note that the number of insiders is \(m + 1\) consisting of \(m\) private firms and one public firm in the present setting.
from which it follows that a merger is profitable if and only if

\[ \theta < \frac{m + \sqrt{m^2 + 4(m + 2)^2}}{2(m + 2)}. \]

Therefore, our model gives a simple counter-example to the losses-from-merger proposition of Salant et al. (1983) since it enables us to state that a merger is profitable in the presence of a semi-public firm even though the number of insiders and outsiders is equal.

5 Welfare effects of a merger

5.1 Effects on the outsiders’ profit

Regarding the effect of a merger on the profit of each outsider firm, we can state:

Lemma 2. A merger increases each outsider firm’s profit unless \( \theta = 0 \).

Proof. Using (7) and (11), we have

\[
\frac{\pi_{j}^{\text{post}}}{\pi_{j}^{\text{pre}}} = \left[ \frac{\theta(m + n + 1) + 1}{\theta(n + 1) + 1} \right]^2,
\]

which is guaranteed to be larger than one unless \( \theta = 0 \). If \( \theta = 0 \), there is no net gain for the outsider firm. **Q.E.D.**

5.2 Effects on welfare

Finally, let us address the welfare effect of a merger. To this end, we need to compute the welfare level in the pre- and post-merger equilibria. Recalling that welfare consists of consumer surplus and the sum of profits in the oligopolistic sector:

\[
U = \frac{X^2}{2} + \pi_0 + m\pi_i + n\pi_j,
\]

substituting (5)-(7) into (16), the pre-merger welfare becomes
Proposition 2. A merger is welfare-reducing unless $\theta = 0$.

Proof. While it is possible to directly compare (17) and (18), we employ another strategy. If we define

$$F(n) \equiv \frac{(\theta n + 1)[\theta(n + 2) + 1]}{[\theta(n + 1) + 1]^2},$$

the pre-merger welfare is expressed by $U^{\text{pre}} = F(n)(a - c)^2/2$ and the post-merger welfare $U^{\text{post}} = F(m + n)(a - c)^2/2$. Consequently, it is fair to say that $U^{\text{post}} < U^{\text{pre}}$ once we can show that $F(n)$ is increasing. To see if this is the case, let us differentiate $F(n)$ to get

$$F'(n) = 2 \left[ \frac{\theta}{[\theta(n + 1) + 1]^3} \right] > 0.$$

Accordingly, we can establish that that $U^{\text{post}} < U^{\text{pre}}$ as is to be proved. Q.E.D.

Therefore, the presence of a semi-public firm suffices to reestablish the losses-from-merger proposition of Salant et al. (1983). In view of that semi-public firms are more or less present in any country, this result can cast a serious skepticism to a merger activity.

Remark 2. In the preceding argument, we have assumed that the merger involves a participation of the public firm. However, this is not always the case and it is possible to make a parallel argument by supposing that the public firm is excluded from the merger. This is a main purpose of a
companion paper, Fujiwara (2010). In that paper, we prove that exactly the same conclusion survives the case in which the public firm is not an insider of the merger.

6 Concluding remarks

We have made clear some implications of privatization for the effect of a merger involving a public firm. We stress that our tack should be regarded as a complement rather than a substitute to the previous studies, e.g., Barcena-Ruiz and Garzon (2003) and Mendez-Naya (2008). Our dissatisfaction comes not from their result but from their deviation from Salant et al. (1983) in assumptions. As already mentioned in Introduction, both of these papers allow for increasing marginal cost as well as a public firm. Hence, there are two possible sources which make their result differ from that of Salant et al. (1983): increasing marginal cost and privatization. In other words, it is difficult to straightforwardly compare their result with that of Salant et al. (1983) in the sense that which factor has a crucial impact on the result. In contrast, we deviate from Salant et al. (1983) only in one respect by maintaining the assumption of constant marginal cost, which has allowed us to find some interesting results on the merger that has never established.

References


