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Abstract

We consider a model with two countries in which each government redistributes income between two types of individuals (the rich and the poor). This model shows that an increase in the mobility of individuals induces intensive tax competition across countries and lowers the level of redistribution undertaken by each country. However, this lower level of redistribution enhances individuals’ efforts to raise his own labor income and alleviates the consequences of the Samaritan’s dilemma. Welfare evaluation of economic integration should be based on the balance of these two competing effects.

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1 Introduction

One of the main alleged pitfalls of economic integration (e.g., the construction of the European Union) is that it would impede redistributive policies at the national level and threaten the future of the welfare state. The crux of this issue is the potential loss in tax revenues from tax competition. Economic integration, by fostering the mobility of individuals, would exert a negative influence on the ability of each member state to generate an adequate level of tax revenues. As labor becomes more mobile, member states inclined towards substantial redistribution find his capacity to sustain these programs jeopardized by emigration of the rich and immigration of the poor. Consequently, the level of redistribution is likely to reach an inadequate level. Over the last two decades, an increasing amount of research has been devoted to this issue and the literature continues to grow at an impressive speed (e.g., Leito-Monteiro (1997), Hindricks (2001) and Cremer and Pestieau (2004)).

On the other hand, the phenomenon called the Samaritan’s dilemma has been recognized as the inherent social problem in the welfare state, which provides benevolent public assistance to the poor. The benevolent public welfare program tries to assist less fortunate individuals, with more help extended the lower the income of the individual in need. If the recipient realizes that the amount of assistance varies inversely with the amount of income earned, he has an incentive to reduce work and income. In this situation, financial aid exacerbates the condition that brings forth assistance. Economists refer to this situation as the Samaritan’s dilemma. What makes the situation a dilemma from the economist’s point of view is that recipient’s labor supply is inefficient. Inefficiency increases because the anticipated transfer distorts the recipient’s choice of effort to raise his own labor income. This issue also has been the subject of extensive theoretical and empirical analyses (e.g., Bruce and Waldman (1991) and Coate (1995)).

These two issues provide us with insightful points of view to consider the effect of economic integration on the economy of the welfare state. To our knowledge, however, the relationship between these two issues has never been formally examined. Therefore, this paper attempts to fill this gap. This paper is based on the redistributive tax competition model developed by Hindricks (2001), and considers two countries in which each government redistributes income between two types of individuals (the
rich and the poor). The income of the poor is determined by his own effort, whereas the income of the rich is assumed to be fixed. All individuals are assumed to be heterogeneous with respect to his psychological attachment to a country and they are free to choose between two countries (domestic or foreign). In these situations, governments of both countries determine his own redistributive policies, and individuals decide his locations by considering not only his preferences for locations but also for government welfare policies.

The present model departs from Hindricks (2001) by introducing the recipient’s choice of effort to raise his labor income, which enables us to explicitly examine the issue of the Samaritan’s dilemma. Redistributive tax competition among countries motivated by an attempt to attract the rich (and to deter the poor) is wasteful in nature and results in economic distortion. For this reason, normative public economists have called for tax harmonization or super-national governmental intervention to correct this inefficiency (e.g., Cremer (1996)). However, in this paper, we show that the intensive tax competition induced by economic integration may solve economic distortion by alleviating the consequences of the Samaritan’s dilemma. Economic integration limits the ability of the government to redistribute income from the rich to the poor. Anticipation of the decline in the government’s ex post transfer stimulates the recipient’s ex ante incentives to raise his own labor income. In this sense, economic integration seems to play a role of commitment device to preserve each recipient’s ex ante incentives because it prevents ex post government transfer. However, tax competition is wasteful in nature. Thus, the more intensive the tax competition is, the more distorted the ex post allocation of resources becomes. Therefore, any welfare evaluation of economic integration must carefully weigh these costs and benefits.

The result of this paper is also closely related to literature of the time-inconsistency taxation problem. In a closed economy, human capital investment faces a hold-up problem of excessive taxation. Once investment is made, human capital is fixed factor. Thus, government has an incentive to levy high tax rates on human capital, which leads to lower investment in human capital. Recent studies, such as Thum and Uebelmesser (2003) and Andersson and Konrad (2003), stress the role of labor mobility as a key to solving this hold-up problem. In particular, Thum and Uebelmesser (2003) show that factor mobility in the form of mobile labor reduces the incentive of the government to
excessively tax the ex post returns on labor.\footnote{Kehoe (1989) also provides similar arguments with respect to taxation on physical capital.} The present paper differs from Thum and Uebelmesser (2003) in the following ways. First, this paper focuses on efforts made by the beneficiaries of redistributive policy, whereas Thum and Uebelmesser (2003) focus on efforts made by the contributors of redistributive policy. Second, this paper considers the case where both the rich and the poor are mobile, whereas Thum and Uebelmesser (2003) consider the case where only the rich are mobile. Third, this paper focuses on the role of economic integration as a device for enhancing labor mobility, whereas Thum and Uebelmesser (2003) focus on the role of mobility-enhancing education (e.g., foreign languages). Therefore, this paper sheds light on an issue which has not yet been explicitly examined and thus can complement theoretical contributions of the existing literature.

This paper is organized as follows. Section 2 presents the basic model. Section 3 characterizes the properties of migration equilibrium, outcomes of the tax setting game, and the individual’s optimal choice of effort. Section 4 examines a condition under which economic integration is welfare improving. Finally, Section 5 provides concluding remarks.

2 The model

The world consists of two countries \((j = D, F)\), called the domestic \((j = D)\) and the foreign \((j = F)\). The two countries are identical in a sense which we shall describe shortly. In the world, there is a large set \(N\) of individuals with two different types of abilities \((i = H, L)\) called type \(H\) \((i = H)\) and type \(L\) \((i = L)\). The number of type \(H\) individuals is \(N_H\), and that of type \(L\) is \(N_L\), which satisfies \(N = N_H + N_L\). For clarity, we normalize the population size of each type of individual to be unity. Thus, we consider the case in which the total population size of the world \(N\) is 2.

We assume that the income level of a type \(L\) individual is determined by type \(L\) individual’s effort \(e\). We denote the income level of type \(L\) individual as \(y_L(e)\), and assume \(y_L'(e) > 0\) and \(y_L''(e) < 0\). These specifications imply that the type \(L\) individual’s effort \(e\) increases the level of \(y_L\), but the marginal effect decreases as the level of \(e\) becomes large. We can interpret \(e\) in a broader sense. For example, it can be
interpreted as a type $L$ individual’s choice of working time or time cost of education.

This paper mainly concerns how a redistributive policy influences the beneficiary’s choice of effort (i.e., the Samaritan’s dilemma). To pursue this purpose, for simplicity, we only explicitly consider the type $L$ individual’s choice of effort. We assume that the income level of the type $H$ individual is exogenously given by $y_H$, and satisfies the condition that $\lim_{e \to \infty} y_L(e) \leq y_H$. This assumption implies that the income level of the type $L$ individual $y_L$ cannot exceed that of the type $H$ individual $y_H$. Therefore, in this paper, the type $L$ individual is assumed to be poor and a beneficiary of redistributive policy, whereas the type $H$ individual is assumed to be rich and a contributor of redistributive policy. In summary, the income level of a type $i$ individual, $y_i$, is expressed as follows.

$$y_i = \begin{cases} y_H & \text{if } i = H, \\ y_L(e) & \text{if } i = L. \end{cases}$$

This pair of income is assumed to be identical across countries (discussed later).

We denote the disutility level of a type $L$ individual’s effort as $\phi(e)$, assuming $\phi'(e) > 0$ and $\phi''(e) > 0$. These specifications imply that the marginal cost of effort increases as the level of $e$ becomes large. Additionally we assume that a type $H$ individual can earn income $y_H$ without suffering any disutility of effort. Thus, the disutility level of a type $i$ individual’s effort, $\phi_i$, is expressed as:

$$\phi_i = \begin{cases} 0 & \text{if } i = H, \\ \phi(e) & \text{if } i = L, \end{cases}$$

where the disutility level of a type $i$ individual is expressed in the numeraire.

In each country $j$, the government is assumed to have perfect information on individual incomes and so the tax system may allow for differentiated treatment of each type of resident. $\tau^j_i$ is the lump sum tax (negative in the case of transfer) imposed on type $i$ individuals living in country $j$. Denoted by $x^j_i \in [0, 1]$, the population size of type $i$ individuals living in country $j$, the budget constraint faced by the government in
country $j$ can be written as:

\[
    x_i^j \tau^j_i + x_i^j \tau^j_F = 0, \quad j = D, F, \tag{3}
\]

assuming purely redistributive taxes.

In this paper, the two economies are linked because both types of individuals (i.e., type $H$ and type $L$) can migrate from one country to the other. Therefore, at the migration equilibrium, all individuals must live in either of the two countries, implying that $x_i^D + x_i^F = 1$ for $i = H, L$.\(^2\)

2.1 Migration

Changing locations involves non-pecuniary costs such as time spent on administrative procedure, retraining, job search, search for housing, some psychological costs of separation from family and friends, adjustments to new culture and environment, etc. To formalize these non-pecuniary costs of moving, following Hindricks (2001), we use a spatial competition model à la Hotelling. Non-pecuniary costs of changing locations are modeled by the psychological attachment of individuals to country $j$, and we assume that this psychological attachment differs among individuals.\(^3\) We also assume that individual psychological attachment to country $j$ is unveiled after a $L$ type individual chooses some form of effort $e$. Thus, type $L$ individuals decide his amount of effort $e$ before they know his own psychological attachment to his location. Here we consider the situation in which individuals gradually discover his own preferences for culture, living environment, job, etc. These factors determine the individual psychological attachment to the location.\(^4\)

The individual psychological attachment to the country $j$ is described by a single taste parameter $\theta \in [0, 1]$. $\theta$ is uniformly distributed within each type of individual

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\(^2\)We call the migration equilibrium $x_i^j$, which is determined by $x_i^D + x_i^F = 1$ for $i = H, L$ and (6) in subsection 2.1.

\(^3\)Models with exogenous incomes require migration costs to avoid the complete migration of some types of individuals.

\(^4\)This assumption is restrictive but simplifies the description of a type $L$ individual’s effort choice problem. Additionally, this simplification enables us to clearly deliver the main implication of this paper without trivial and complicated discussion.
(i.e., type $H$ and type $L$). Suppose a type $i$ individual with preference $\theta \in [0, 1]$ locates in the domestic country. His utility function is expressed as:

$$u(y_i - \tau^D_i - \phi_i - a\theta), \quad i = H, L.$$  

(4)

On the other hand, suppose a type $i$ individual with preference $\theta \in [0, 1]$ locates in the foreign country. His utility function is expressed as:

$$u(y_i - \tau^F_i - \phi_i - a(1 - \theta)), \quad i = H, L,$$  

(5)

where individual psychological attachment to country $j$ is also measured in the numeraire.

The parameter $a (\geq 0)$ expresses the intensity of individual psychological attachment to his location. The larger value of $a$ implies the higher individual’s weight on the psychological attachment to his location relative to the consumption of physical goods. Therefore, if the value of $a$ is large, an individual does not change his location elastically with respect to slight differences in redistributive policies across countries because he does not care about the consumption of physical goods. In particular, if $a \to \infty$, individuals are perfectly immobile, redistributive tax competition is not intense and thus each government has the highest degree of monopoly power over its residents. On the other hand, if the value of $a$ is small, an individual will change his location elastically with respect to slight differences in redistributive policies across countries because he cares about the consumption of physical goods. In particular, if $a \to 0$, individuals are perfectly mobile, the redistributive tax competition is intense and thus each government loses its monopoly power over its own residents. Therefore, in our model, parameter $a$ measures the degree of mobility of individuals with respect to policy changes. Larger values of $a$ imply lower mobility of individuals, whereas smaller values of $a$ imply higher mobility of individuals.

Note that economic integration between two countries lowers the non-pecuniary costs of moving, and thus increases the mobility of individuals with respect to policy changes. For example, the relaxation of mobility regulations (e.g., abolition of border controls, common visa policies, etc.) lowers the time cost of administrative procedures,
job search, search for housing, some psychological costs of migration, while enhancing
the mobility of individuals. This paper concerns how this increase in the mobility of
individuals affects the level of redistribution and the individual’s choice of effort and
welfare. In the present paper, we interpret economic integration as a series of policies
which lower the individual non-pecuniary costs of changing locations and increase the
mobility of individuals with respect to policy changes. Thus parameter $a$ represents
the degree of economic integration. A large value of $a$ implies that the mobility of
individuals is low and two economies are not well integrated. On the other hand, a
small value of $a$ implies that the mobility of individuals is high and two economies are
well integrated.

A Type $i$ individual is free to choose his location, and he chooses the country where
he can attain the highest utility given his knowledge of taxes in both countries (i.e., $\tau_P^i$ and
$\tau_i^f$). Following Hindricks (2001), we assume that the type $L$ individual behaves
like a Stackelberg leader, whereas, the type $H$ individual behaves as a Stackelberg fol-
lower. Thus the type $L$ individual decides his location by anticipating the migration
decisions of type $H$ individuals.\footnote{This assumption might be restrictive, but simplifies analysis of the migration equilibrium where two different types of individuals migrate.} Because each individual will differ in his psychological attachment to country $j$, the migration equilibrium must be characterized by the marginal type $i$ individual, identified by $\theta = x_i$, being indifferent between locating in
either region:

$$y_i - \tau_P^i - ax_i = y_i - \tau_i^f - a(1 - x_i), \quad i = H, L.$$  

(6)

Note that the individual disutility level of effort $\phi_i$ is not taken into account in migration
decisions because it is sunk when an individual decides on his location. From (4) to
(6), a type $i$ individual with $\theta$ less than $x_i$ locates in the domestic country, whereas a
type $i$ individual with $\theta$ more than $x_i$ locates in the foreign country. Therefore, in the
migration equilibrium, the population size of the type $i$ individual living in the country
$j$, $x_i^j$, is expressed as:

$$x_i^j = \begin{cases} 
  x_i & \text{if } j = D, \\
  1 - x_i & \text{if } j = F. 
\end{cases}$$

(7)
2.2 Government

Both types of individuals are assumed to make migration decisions with knowledge of the taxes in both countries. Thus each government chooses its level of taxes that maximize its objective functions, anticipating the migration equilibrium resulting from individual utility maximization. On deciding its poll taxes, each government will be assumed to behave as a Nash player taking, as given, the tax rate of the other country.

The objective function of the government in country \( j \) is assumed to be utilitarian and to depend only on its residents’ level of consumption, which is given by:

\[
V^j = x^j_H v(y_H - \tau^j_H) + x^j_L v(y_L - \tau^j_L), \quad j = D, F,
\]

where \( y_i - \tau^j_i \) is the consumption level of type \( i \) individuals living in county \( j \) and \( v() \) is the government’s evaluation function for the level of \( y_i - \tau^j_i \). The government in country \( j \) assigns a welfare weight \( x^j_i \) for the evaluation of \( y_i - \tau^j_i \) according to its population size \( x^j_i \), and decides the level of taxes, \( \tau^j_H \) and \( \tau^j_L \), given the tax rate in the other country, so as to maximize (8), subject to the revenue constraints in (3) and mobility constraints in (6). Note that the disutility level of a type \( L \) individual’s effort \( \phi(e) \) is not taken into account in the government’s objectives because it is sunk when the level of redistributive tax is determined.

Each government seems to have incentives to redistribute income from type \( H \) individuals to type \( L \) individuals. Anticipating these transfers, a type \( L \) individual may have an incentive to undermine his amount of effort and to rely on the transfer given to him by the government. In this paper, we assume that each government cannot commit to the most desirable redistribution policy ex ante. Thus the government will redistribute more ex post than they would like to commit to ex ante. This is a manifestation of what James Buchanan (1975) termed as the Samaritan’s dilemma.\(^6\)

\(^6\)This kind of discussion is also closely related to the problem of soft budgets. See, e.g., Kornai et al. (2003) as an overview of this problem.
2.3 Type L individual’s effort

A type L individual decides his amounts of effort $e$ before he knows his own psychological attachment $\theta$ to his location. The taste parameter $\theta$ is uniformly distributed within each type of individual. Thus, the expected utility of a type L individual, $W_L$, is expressed as:

$$ W_L = \int_0^{x_L} u(y_L(e) - \tau^D_L - \phi(e) - a\theta)d\theta + \int_{x_L}^1 u(y_L(e) - \tau^F_L - \phi(e) - a(1 - \theta))d\theta. \quad (9) $$

Suppose taste $\theta$, such that it satisfies $\theta \leq x_L$, is given. A type L individual prefers to be located in the domestic country. Thus he attains a level of utility $u(y_L(e) - \tau^D_L - \phi(e) - a\theta))$. On the other hand, suppose taste $\theta$, such that it satisfies $\theta \geq x_L$, is given. A type L individual then prefers to be located in the foreign country. Thus he attains a level of utility $u(y_L(e) - \tau^F_L - \phi(e) - a(1 - \theta))$. Anticipating the outcome of the tax setting game played by both governments (i.e., $\tau^D_L$, $\tau^F_L$), each type L individual decides his amount of effort $e$ so as to maximize (9).

Due to the uncertainty of an individual’s psychological attachment to his location $\theta$, all type L individuals are equally likely to be located in the domestic or in the foreign country. All type L individuals face the same optimal choice problem of effort and undertake the same amount of effort. Consequently, the income level of a type L individual who will be located in the domestic country is equal to that of a type L individual located in the foreign country. This result simplifies the description of the tax setting game played by each government.

Before turning to the descriptions of equilibrium in this economy, we summarize the sequences of decision-making as follows.

1. A type L individual decides his amount of effort $e$, and thus the income level of a type L individual $y_L(e)$ and that of a type H individual $y_H$ are determined, respectively.

2. The individual’s psychological attachment to country $j$ ($\theta$) is unveiled.

3. Each government competes with one another in setting its redistributive policy.

4. Each type of individuals (i.e., type H and type L) decides whether to locate
in the domestic or in the foreign country given his psychological attachment to

country \( j \) and the tax rate in both countries.\(^7\)

Throughout the present paper, we suppose that a type \( L \) individual’s effort \( e \) is
determined before inter-governmental redistributive tax competition. Thus a type \( L \)
individual decides his amount of effort anticipating its impact on the outcome of the
tax setting game.

3 Determination of equilibrium

In this section, we characterize the properties of equilibrium. Because of the spec-
ification of a type \( L \) individual’s effort choice problem, we can concentrate on the
symmetric outcome at each stage starting with the final stage and working backwards.

3.1 The migration equilibrium

In this subsection, we examine the properties of the migration equilibrium. The mi-
gration equilibrium is characterized by the population size of type \( i \) individuals living
in the domestic country \( x^D_i = x_i \). Thus, we examine how the increase in taxes imposed
on a type \( H \) individual living in the domestic country \( \tau^D_H \) affects the population size of
type \( i \) individuals living in the domestic country, \( x_i \), given the tax in the foreign country
\( \tau^F_H \).

By substituting (3) into (6), we obtain the following two equations.

\[
\begin{align*}
y_H - \tau^D_H - ax_H &= y_H - \tau^F_H - a(1 - x_H), \tag{10} \\
y_L + \frac{x_H}{x_L} \tau^D_H - ax_L &= y_L + \frac{1 - x_H}{1 - x_L} \tau^F_H - a(1 - x_L). \tag{11}
\end{align*}
\]

(10) characterizes the migration equilibrium of type \( H \) individuals. From (10), in the
migration equilibrium, the population size of type \( H \) individuals living in the domestic

\(^7\)More precisely, a type \( L \) individual will behave as a Stackelberg leader, whereas a type \( H \) individual
will behave as a follower. That is, a type \( L \) individual decides his location anticipating the migration
decision of type \( H \) individuals.
country $x_H$ is determined by the tax imposed on type $H$ individuals living in the domestic country $\tau_H^D$, that in the foreign country $\tau_H^F$, and the mobility of individuals $a$. To stress these relationships, we describe $x_H$ as $x_H = x_H(\tau_H^D, \tau_H^F, a)$. By differentiating (10), with respect to $\tau_H^D$, we obtain

$$\frac{\partial x_H}{\partial \tau_H^D} = -\frac{1}{2a}.$$  \hspace{1cm} (12)

Migration equilibrium of type $L$ individuals depends on the migration equilibrium of type $H$ individuals $x_H(\tau_H^D, \tau_H^F, a)$, because a type $L$ individual decides his locations anticipating the migration decisions of type $H$ individuals. To reflect this fact we rewrite (11) as:

$$y_L + \frac{x_H(\tau_H^D, \tau_H^F, a)}{x_L} \tau_H^D - a x_L = y_L + \frac{1 - x_H(\tau_H^D, \tau_H^F, a)}{1 - x_L} \tau_H^F - a(1 - x_L),$$  \hspace{1cm} (13)

which characterizes the migration equilibrium of type $L$ individuals. From (13), the population size of type $L$ individuals living in the domestic country $x_L$ also depends on $\tau_H^D$, $\tau_H^F$ and $a$. Thus we describe $x_L$ as $x_L = x_L(\tau_H^D, \tau_H^F, a)$. By differentiating (13), with respect to $\tau_H^D$, we obtain:

$$\frac{\partial x_L}{\partial \tau_H^D} = \frac{\frac{\partial y_L}{\partial x_L} - \frac{x_H(\tau_H^D, \tau_H^F, a)}{x_L} \frac{\partial \tau_H^D}{\partial \tau_H^D}}{\frac{1 - x_H(\tau_H^D, \tau_H^F, a)}{1 - x_L} \tau_H^F + \frac{\partial y_L}{\partial x_L} \tau_H^D + 2a}.$$  \hspace{1cm} (14)

Appendix A explains the derivation of (14). In this paper, we can concentrate on the symmetric equilibrium due to the specification of a type $L$ individual’s effort choice problem. In the symmetric equilibrium, both governments choose the same tax rate and the population is equally divided between the two countries. Therefore, if $\tau_H^D = \tau_H^F = \tau_H$, $\tau_L^D = \tau_L^F = \tau_L$, then $x_H = x_L = \frac{1}{2}$. Additionally the relation $\tau_L = -\tau_H$ holds from government budget constraints in (3). Then, we have the following proposition:

**Proposition 1** In symmetric equilibrium,

1. the population size of type $H$ residents decreases with the tax imposed on type $H$ residents, and
2. the population size of type L residents decreases (increases) with the tax imposed on type H residents if $\tau_H > \frac{\theta}{2}$ ($\tau_H < \frac{\theta}{2}$).

We leave this proposition because (12) and (14) are replaced by the following relations in symmetric equilibrium:

$$\frac{\partial x_H}{\partial \tau_H} = \frac{-1}{2a} < 0,$$  \hspace{1cm} (15)

$$\frac{\partial x_L}{\partial \tau_H} = \frac{1 - \frac{2\tau_H}{a}}{2a + 2\tau_H} \begin{cases} \leq 0 & \text{if } \tau_H \geq \frac{\theta}{2}, \\ \geq 0 & \text{if } \tau_H \leq \frac{\theta}{2}. \end{cases} \hspace{1cm} (16)$$

(15) implies that the increase in tax imposed on type H residents $\tau_H$ decreases the population size of type H residents $x_H$, simply because it increases the tax burden of type H individuals. On the other hand, (16) implies that the effect of $\tau_H$ on the population size of type L residents $x_L$ depends on the mobility of individuals $a$. Suppose $\tau_H \geq \frac{\theta}{2}$ ($\tau_H \leq \frac{\theta}{2}$), the increase in $\tau_H$ lowers (increases) the level of $x_L$. We can explain this result as follows. The higher (lower) $\tau_H$ implies that the higher (lower) tax burden of type H individuals, and the lower (higher) $a$ implies higher (lower) mobility of type H individuals. Thus, suppose $\tau_H$ is sufficiently high and $a$ is sufficiently low to satisfy $\tau_H \geq \frac{\theta}{2}$, the increase in $\tau_H$ induces so many type H individuals to leave, and thus type L individuals also find it profitable to leave. On the other hand, suppose $\tau_H$ is sufficiently low and $a$ is sufficiently high to satisfy $\tau_H \leq \frac{\theta}{2}$, not so many type H individuals leave due to the increase in $\tau_H$, and thus type L individuals also find it profitable not to leave.

### 3.2 The optimal tax schemes

In this subsection, we characterize the equilibrium outcome of the tax setting game played by each government. Because we focus on the symmetric equilibrium, we only explicitly describe the domestic government’s redistributive policy choice problem.

Taking as given the tax rate in the foreign country $\tau_H^F$, and anticipating the migration equilibrium $x_i(\tau_H^D, \tau_H^F, a)$, the domestic government’s problem at this stage is expressed by:

$$\max_{\tau_H^D} V^D = x_H^D v(y_H - \tau_H^D) + x_L^D v(y_L(e) + \frac{x_H^D}{x_L^D} \tau_H^F),$$  \hspace{1cm} (17)
where \( x^0_i = x_i(\tau^D_H, \tau^L_H, a) \). The first order condition for \( \tau^D_H \) becomes

\[
\frac{v'(y_H - \tau^D_H)}{v'(y_L(e) + \frac{\partial v}{\partial \tau^D_H})} \leq 1 + \epsilon^D_{\tau_H},
\]

where

\[
\epsilon^D_{\tau_H} \equiv \frac{\tau^D_H \partial x^D_H}{x^D_H \partial \tau^D_H} = \frac{\tau^D_H \partial x^D_H}{x^D_H \partial \tau^D_H},
\]

and the strict inequality holds when \( \tau^D_H = 0 \). \( \epsilon^D_{\tau_H} \) is the sum of the elasticities of domestic residents with respect to \( \tau^D_H \) (i.e., \( \epsilon^D_{\tau_H} \equiv \frac{\tau^D_H \partial x^D_H}{x^D_H \partial \tau^D_H} \)). In symmetric equilibrium, the above equation is expressed as:

\[
\frac{v'(y_H - \tau_H)}{v'(y_L(e) + \tau_H)} \leq \frac{1}{1 + \frac{1}{n} \tau_H}.
\]

Appendix B explains the derivations of (19) and its properties. (19) yields the following proposition on the tax imposed on type \( H \) residents.

**Proposition 2.** In symmetric equilibrium, \( \tau_H(e, a) \) is uniquely determined by (19). Then, \( \tau_H(e, a) \) has the following properties. For all \( e \) and \( a \),

1. \( \frac{\partial \tau_H(e, a)}{\partial e} < 0 \) and
2. \( \frac{\partial \tau_H(e, a)}{\partial a} > 0 \).

Additionally \( \lim_{a \to \infty} \tau_H(e, a) = \frac{y_{LM}(e)}{2} \) and \( \lim_{a \to 0} \tau_H(e, a) = 0 \) hold.

The first part of this proposition means that the increase in a type \( L \) individual’s effort \( e \) lowers the tax rate imposed on type \( H \) residents, decreases the amount of transfers received by type \( L \) residents, and thus lowers the level of redistribution. The increase in a type \( L \) individual’s effort \( e \) decreases the income differences between type \( H \) and type \( L \) individuals, which induces governments to lower its level of redistribution. On the other hand, the second part of this proposition means that the decline in the mobility of individuals (i.e., an increase in \( a \)) increases the tax rate imposed on type \( H \) residents, increases the amount of transfers received by type \( L \) residents, and thus increases the
level of redistribution. Lower mobility of individuals implies less intensive redistributive tax competition across countries. Thus both governments have a higher degree of monopoly power over its residents, and incentives to increase its level of redistribution. In particular, if \( a \to \infty \), then a government will have the highest degree of monopoly power over its residents and undertake full redistribution (i.e., \( \tau_H = \frac{y_H - y_L(e)}{2} \)). On the other hand, if \( a \to 0 \), a government will lose its monopoly power over its residents and the level of redistribution minimizes (i.e., \( \tau_H = 0 \)).

Moreover, suppose the government’s evaluation function \( v() \) is specified as logarithmic function (i.e., \( v(y_i - \tau_i^H) = \ln(y_i - \tau_i^H) \)), we can solve \( \tau_H(e, a) \) as:

\[
\tau_H(e, a) = \frac{1}{2} \{(y_L(e) + a) + [y_L(e) + a]^2 + 2a(y_H - y_L(e))\}^{1/2}.
\] (20)

Appendix C explains the derivation of (20) and its properties. Under this specification, we can see that the relation \( \frac{\partial \tau_H}{\partial a} \frac{\partial y_H}{\partial e} < 0 \) also holds. This result can be summarized as follows.

Corollary 1 In symmetric equilibrium where \( v() \) is specified as a logarithmic function, the marginal decline in the level of redistribution due to the increase in \( e \) becomes larger as the mobility of individuals decreases.

A type \( L \) individual decides his amount of effort \( e \) anticipating these properties of each government’s redistributive policy. We will discuss this point rigorously in the next subsection.

3.3 The optimal effort levels

In this subsection, we characterize the properties of a type \( L \) individual’s choice of effort. The individual’s psychological preference for the country \( j (\theta) \) is unveiled in stage 2. Thus, in stage 1, a type \( L \) individual decides his amount of effort \( e \) so as to maximize his expected utility in (9). From (19), a type \( L \) individual’s effort affects the outcome of the redistributive tax setting game in stage 3. Thus each type \( L \) individual decides his amount of effort anticipating its impact on the outcome of the tax setting
Therefore, a type $L$ individual’s problem at this stage is expressed by:

$$
\max_{e} W_L = \int_{0}^{\infty} u(y_L(e)+\tau_H(e,a)-\phi(e)-a\theta)d\theta + \int_{0}^{1} u(y_L(e)+\tau_H(e,a)-\phi(e)-a(1-\theta))d\theta.
$$

(21)

where the relations $\tau_H^P = \tau_H^F = \tau_H(e,a)$ hold in the symmetric equilibrium.

The first order condition for $e$ becomes

$$
\frac{\partial y_L(e^*)}{\partial e} + \frac{\partial \tau_H(e^*,a)}{\partial e} = \frac{\partial \phi(e^*)}{\partial e}.
$$

(22)

Under some less-restrictive assumptions, the optimal amount of effort $e^*$ is determined uniquely. Additionally, suppose that the government’s evaluation function $v()$ is specified as a logarithmic function (i.e., $v(y_i-\tau_i^j) = \ln(y_i-\tau_i^j)$), the above first order condition for $e$ is represented as:

$$
\frac{1}{2\gamma_L(e^*)}[1 + \frac{\gamma_L(e^*)}{(\gamma_L(e^*))^2 + \alpha^2 + 2\alpha y_H}] = \phi'(e^*).
$$

(23)

Now, we have the following proposition on a type $L$ individual’s optimal level of effort.

**Proposition 3** In symmetric equilibrium where $v()$ is specified as a logarithmic function, a type $L$ individual’s optimal level of effort $e^*$ has the following properties;

1. the existence of redistributive policy decreases $e^*$, and

2. the lower mobility of individuals decreases $e^*$.

Because Proposition 2 shows that $\frac{\partial \tau_H(e^*,a)}{\partial e} < 0$, (22) leads the first part of this proposition. Note that this result represents the phenomenon called the Samaritan’s dilemma. From (23), we can immediately show the second part of this proposition that $\frac{de}{da} < 0$ holds. This result implies that the lower mobility of individuals (i.e., an increase in

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8To clearly convey the main implication of the model, we implicitly assume that a type $L$ individual decides his amount of effort cooperatively (i.e., type $L$ individuals’ collective choice of effort). Even if we consider the case in which each type $L$ individual decides his amount of effort non-cooperatively given the behavior of other type $L$ individuals, the main implication of this paper does not change significantly.
a) makes the problem of the Samaritan’s dilemma more serious. To stress this relationship, we describe \( e^* \) as \( e^>(a) \). As the mobility of individuals becomes low (i.e., as the value of \( a \) becomes large), the marginal decline in the level of redistribution due to the increase in \( e \) becomes large (i.e., \( \frac{\partial \pi}{\partial e} > 0 \)), which induces a type \( L \) individual to undermine his amount of effort more. In particular, if \( a \to \infty \), because each government undertakes full redistribution (i.e., \( \pi_H = \frac{\gamma_H - \gamma_L(e)}{2} \)), the first order condition for \( e \) in (22) is expressed as \( \frac{1}{2}y_L'(e^*) = \phi'(e^*) \) and type \( L \) individuals exert the lowest level of effort. On the other hand, if \( a \to 0 \), because the level of redistribution minimizes (i.e., \( \tau_H = 0 \)), the first order condition for \( e \) in (22) is expressed as \( y_L'(e^*) = \phi'(e^*) \) and type \( L \) individuals exert the highest level of effort.

4 Effect of economic integration

In this section, we examine how an increase in the mobility of individuals caused by economic integration affects the level of redistribution, each individual’s choice of effort, and welfare. As described in subsection 2.1, we interpret economic integration as a series of policies which lowers the individual non-pecuniary costs of changing locations and increases the mobility of individuals with respect to policy changes. Thus parameter \( a \) represents the degree of economic integration. A large value of \( a \) implies that the mobility of individuals is low and two economies are not well integrated. On the other hand, a small value of \( a \) implies that the mobility of individuals is high and two economies are well integrated. Thus the effect of economic integration is modeled as a decline in \( a \).

As made explicit in the previous section, an increase in the mobility of individuals (i.e., a decline in \( a \)) induces more intense redistributive tax competition across countries and lowers the level of redistribution that each government undertakes. This lower level of redistribution is regarded as the cost of economic integration. On the other hand, the previous section also shows that this lower level of redistribution enhances a type \( L \) individual’s effort to raise his own labor income and alleviates the consequences of the Samaritan’s dilemma. This higher level of a type \( L \) individual’s effort is regarded as the benefit of economic integration.

The purpose of this section is to compare these costs and benefits and to provide
a condition under which economic integration is welfare improving. To this purpose, we take the following two steps. First, we examine a condition under which economic integration improves the government’s welfare function defined in (8). Second, we examine a condition under which economic integration improves the sum of the expected utilities of both types of individuals, although we do not take the direct welfare effect of the decline in a explicitly into account.

4.1 The government’s welfare

In this subsection, we examine a condition under which economic integration improves the government’s welfare function defined in (8). In symmetric equilibrium (i.e., \( \tau_H^* = \tau_H^L = \tau_H, \ x_H = x_L = \frac{1}{2} \)), the level of the domestic government’s welfare function is expressed as:

\[
V^D = \frac{1}{2}v(y_H - \tau_H(e^*, a)) + \frac{1}{2} v(y_L(e^*) + \tau_H(e^*, a)),
\]

(24)

where \( e^* = e^*(a) \). The foreign government’s welfare function is also expressed in the same equation (i.e., \( V^D = V^F = V \)). By totally differentiating (24) with respect to \( a \), we obtain:

\[
\frac{dV}{da} = \frac{1}{2} v'(y_L(e^*) + \tau_H)[1 - \frac{v'(y_H - \tau_H)}{v(y_L(e^*) + \tau_H)}][\frac{\partial \tau_H}{\partial a} + \frac{\partial \tau_H}{\partial e^*} \frac{\partial e^*}{\partial a} + \frac{\partial y_L}{\partial e^*} \frac{\partial e^*}{\partial a}],
\]

(25)

which expresses the marginal welfare evaluation of further economic integration. From (25), economic integration (i.e., a decline in \( a \)) is welfare improving if and only if:

\[
[1 - \frac{v'(y_H - \tau_H)}{v(y_L(e^*) + \tau_H)}]\left(\frac{\partial \tau_H}{\partial a} + \frac{\partial \tau_H}{\partial e^*} \frac{\partial e^*}{\partial a} + \frac{\partial y_L}{\partial e^*} \frac{\partial e^*}{\partial a}\right) < 0.
\]

The first term \( [1 - \frac{v'(y_H - \tau_H)}{v(y_L(e^*) + \tau_H)}]\left(\frac{\partial \tau_H}{\partial a} + \frac{\partial \tau_H}{\partial e^*} \frac{\partial e^*}{\partial a} + \frac{\partial y_L}{\partial e^*} \frac{\partial e^*}{\partial a}\right) \) represents the marginal cost of the decline in \( a \) induced by the lower level of redistribution. Because \( \frac{\partial \tau_H}{\partial a} > 0 \), \( \frac{\partial \tau_H}{\partial e^*} < 0 \), \( \frac{\partial e^*}{\partial a} < 0 \) and \( \frac{\partial y_L}{\partial e^*} \frac{\partial e^*}{\partial a} < 1 \) from (19), this term is positive. The second term \( \frac{\partial y_L}{\partial e^*} \frac{\partial e^*}{\partial a} \) represents the marginal benefit of the decline in \( a \) induced by the higher level of a type \( L \) individual’s effort. Because \( \frac{\partial y_L}{\partial a} > 0 \) and \( \frac{\partial e^*}{\partial a} < 0 \), this term is negative. The desirability of economic integration depends on the balance of these two terms. An increase in a type
individual’s effort is advantageous, while a decline in the level of redistribution is harmful.

To further examine the conditions under which economic integration improves the government’s welfare, we present numerical simulation results. In the numerical version of the model, for illustrative purpose, the government’s evaluation function is specified as \( v(y_i - \tau_i) = \ln(y_i - \tau_i) \). The disutility function of effort is specified as \( \phi(e) = \Phi e \) implying that the constant marginal cost of effort \( \Phi > 0 \). The income level of a type \( L \) individual is determined by the following parameterization of the function:

\[
y_L(e) = \frac{y_L + \gamma_L e^n}{1 + e^n}, \quad \eta \in (0, 1),
\]

where \( y'_L(e) > 0, y''_L(e) < 0, \lim_{e \to \infty} y_L(e) = \overline{y}_L < y_H \) and \( \lim_{e \to 0} y_L(e) = y_L \). The parameters \( \overline{y}_L \) and \( \eta \) express the efficiency of a type \( L \) individual’s effort. The higher values of \( \overline{y}_L \) and \( \eta \) imply higher marginal effects of \( e \) on \( y_L \). The parameters used in the baseline simulations are given in the footnotes.\(^{10}\)

The solid-line in Figures 1-1 to 1-9 depicts the numerical examples of the relationship between the degree of economic integration \( a \) and the welfare level of governments \( V \) under several alternative values of \( \overline{y}_L \) and \( \eta \). Small values of \( a \) imply that two economies are well integrated. In the figures, the value of ymax expresses the value of \( \overline{y}_L \), and the value of eta expresses the value of \( \eta \). The broken-line depicts the welfare level of the government when the mobility of individuals is lowest (i.e., \( a \to \infty \)), two economies are completely separated, and the government undertakes full redistribution.

When the values of both \( \overline{y}_L \) and \( \eta \) are sufficiently high (i.e., \( \overline{y}_L = 90, \eta = 0.8 \)), as shown in Figure 1-1, the solid-line always lies above the broken-line. Thus the maximum welfare level of government \( V \) is attained when \( a \) is zero and the two economies are completely integrated. This result implies that the marginal benefit of the decline in \( a \) induced by a type \( L \) individual’s greater effort always dominates the marginal cost

\(^9\)We set the value of \( y_L \) as 0, \( \overline{y}_L \) as 80, \( \eta \) as 0.5 and \( \Phi \) as 0.25, respectively, in the baseline simulations. Additionally, the value of \( a \) is changed from 0 to 3000 in increments of 1.

\(^{10}\)The objective of this numerical analysis is to supplement qualitative results derived from the model. To ensure that our simulation results are not too sensitive to the set of baseline values we have chosen, we conducted extensive sensitivity analyses for several key parameters. Although the authors chose the values of parameters carefully, the quantitative results should be interpreted with caution.
of the decline in $a$ induced by the lower level of redistribution.

On the other hand, as shown in Figure 1-9, when the values of both $\overline{y_l}$ and $\eta$ are sufficiently low (i.e., $\overline{y_l} = 70$, $\eta = 0.2$), the solid-line always lies below or almost equals the broken-line. Thus the maximum welfare level of government $V$ is attained when $a$ is infinite and the two economies are completely separated. This result implies that the marginal cost of the decline in $a$ induced by the lower level of redistribution always dominates the marginal benefit of the decline in $a$ induced by a type $L$ individual’s greater effort.

Remaining figures describe the cases in which both $\overline{y_l}$ and $\eta$ take moderate values. In these cases, the maximum welfare level of the government is attained at the value of $a \in (0, \infty)$. From the figures, we can confirm that the higher values of $\overline{y_l} (\eta)$ are likely to lower the value of $a$ which attains the maximum welfare level of the government. From the government’s point of view, this result implies that two economies should be integrated further if the efficiency level of a type $L$ individual’s effort is greater.

As the efficiency level of a type $L$ individual’s effort becomes greater, the negative welfare effect of a type $L$ individual’s lesser effort becomes more serious. Consequently, the benefit of economic integration, which is induced by a type $L$ individual’s greater effort is likely to dominate the cost of economic integration induced by a lower level of redistribution.

### 4.2 The welfare of individuals

In this subsection, we examine a condition under which economic integration improves the sum of the expected utilities of both types of individuals. In the symmetric equilibrium, the sum of the expected utilities of both types of individuals $W$ is expressed as:

$$W = W_H + W_L,$$  \hspace{1cm} (27)

where

$$W_H = \int_{0}^{1/2} u(y_H - \tau_H(e^*, a) - a\theta)d\theta + \int_{1/2}^{1} u(y_H - \tau_H(e^*, a) - a(1 - \theta))d\theta,$$
\[ W_L = \int_0^{1/2} u(y_L(e^* + \tau_H(e^*, a) - \phi(e^* - a\theta))d\theta + \int_1^{1/2} u(y_L(e^* + \tau_H(e^*, a) - \phi(e^* - a(1-\theta))d\theta. \]

and \( e^* = e^*(a) \). Note that an individual’s psychological attachment to a location and the disutility level of a type \( L \) individual’s effort \( \phi(e^*) \) are explicitly taken into accounts. These aspects differ from the government’s welfare function defined in (24).

Analogous to the previous subsection, we can evaluate the marginal welfare effect of economic integration by differentiating (27) with respect to \( a \). The direct welfare effect of the decline in \( a \) is apparently positive because it decreases the non-pecuniary costs of re-location. However, it is not this direct effect we would like to stress as the effect of economic integration. Therefore, we ignore this direct effect of \( a \) in the following welfare evaluation of economic integration.

By totally differentiating (27) with respect to \( a \), ignoring its direct effect, we obtain:

\[ \frac{dW}{da} = \left( \frac{\Delta U_L - \Delta U_H}{a} \right) \frac{\partial \tau_H}{\partial a} - \frac{\Delta U_H}{a} \frac{\partial \tau_H}{\partial e^*} \frac{\partial e^*}{\partial a}, \tag{28} \]

where

\[ \Delta U_H \equiv 2[u(y_H - \tau_H(e^*, a)) - u(y_H - \tau_H(e^*, a) - \frac{1}{2}(a))] > 0, \]

\[ \Delta U_L \equiv 2[u(y_L(e^*) + \tau_H(e^*, a) - \phi(e^*)) - u(y_L(e^*) + \tau_H(e^*, a) - \phi(e^*) - \frac{1}{2}(a))] > 0. \]

Noting that \( y_H - \tau_H(e^*, a) > y_L(e^*) + \tau_H(e^*, a) \) from (19), we can also confirm that the relation \( \Delta U_L - \Delta U_H > 0 \) holds due to the concavity of the utility function. Appendix D briefly explains the derivation of (28). From (28), economic integration (i.e., a decline in \( a \)) is welfare improving if and only if:

\[ \left( \frac{\Delta U_L - \Delta U_H}{a} \right) \frac{\partial \tau_H}{\partial a} - \frac{\Delta U_H}{a} \frac{\partial \tau_H}{\partial e^*} \frac{\partial e^*}{\partial a} < 0. \]

The first term \( \left( \frac{\Delta U_L - \Delta U_H}{a} \right) \frac{\partial \tau_H}{\partial a} \) represents the marginal cost of the decline in \( a \) induced by the lower level of redistribution. Because \( \left( \frac{\Delta U_L - \Delta U_H}{a} \right) > 0 \) and \( \frac{\partial \tau_H}{\partial a} > 0 \), this term is positive. The second term \( -\frac{\Delta U_H}{a} \frac{\partial \tau_H}{\partial e^*} \frac{\partial e^*}{\partial a} \) represents the marginal benefit of the decline in
$a$ induced by the a type $L$ individual’s greater effort. Because $\frac{\Delta U_H}{a} \frac{\partial \eta}{\partial e} < 0$ and $\frac{\partial c}{\partial a} < 0$, this term is negative. The desirability of economic integration again depends on the balance of these two terms. An increase in a type $L$ individual’s effort is advantageous, while a decline in the level of redistribution is harmful.

To further examine the condition under which economic integration improves the sum of the expected utilities of both types of individuals, we present numerical simulation results. The solid-line in Figures 2-1 to 2-9 depicts numerical examples of the relationship between the degree of economic integration $a$ and the sum of the expected utilities of both types of individuals $W$ under several alternative values of $\overline{y}_L$ and $\eta$. In the figures, the value of $y_{\text{max}}$ expresses the value of $\overline{y}_L$, and the value of eta expresses the value of $\eta$. The broken-line depicts the sum of the expected utilities of both types of individuals when the mobility of individuals is the lowest (i.e., $a \to \infty$), two economies are completely separated, and the government undertakes full redistribution.

Analogous to the previous subsection, we can find that a higher value of $\overline{y}_L$ ($\eta$) is likely to decrease the value of $a$ which attains the maximum value of the sum of the expected utilities of both types of individuals. As the efficiency level of a type $L$ individual’s effort becomes higher, the negative welfare effect of a type $L$ individual’s lesser effort becomes more serious. Consequently, the benefit of economic integration, which is induced by a type $L$ individual’s greater level of effort is likely to dominate the cost of economic integration induced by the lower level of redistribution. However, note that (27) explicitly takes into account the disutility level of a type $L$ individual’s efforts $\phi(e)$, which differs from the government’s welfare function. The disutility level of effort increases with the increase in $e$ induced by economic integration. Thus, the cost of economic integration is evaluated to be higher relative to the case of the government’s welfare. Consequently, given the values of $\overline{y}_L$ and $\eta$, the value of $a$ which attains the maximum value of the sum of the expected utilities of both types of individuals is higher than the value of $a$ which attains the maximum welfare value of the government. Given the efficiency level of a type $L$ individual’s efforts, this result implies that the government’s marginal evaluation of economic integration is higher relative to the individuals’ evaluation of economic integration because the government does not explicitly take into account the disutility level of type $L$ individual’s efforts.
5 Concluding Remarks

In the present paper, we have studied the circumstances under which an increase in the mobility of individuals caused by economic integration improves the welfare of individuals. We used a model with two countries in which each government redistributes income between two types of individuals. The model in this paper shows that an increase in the mobility of individuals caused by economic integration induces intensive tax competition across countries and lowers the level of redistribution that each country undertakes. However, this lower level of redistribution enhances each individual’s effort in raising his own labor income and alleviates the consequences of the Samaritan’s dilemma. Welfare evaluation of economic integration should be based on the balance of these two competing effects.

Appendix A

(11) is rewritten as

\[ g(x_L, \tau_H^D) = \frac{1 - x_H^r}{1 - x_L} - \frac{x_H^*}{x_L} \tau_H^D + 2ax_L - a = 0. \]

Noting the relation \( \frac{\partial g}{\partial \tau_H^D} = -\frac{1}{2a} \) holds from (10), we have

\[ \frac{\partial g}{\partial x_L} = \frac{1 - x_H^r}{(1 - x_L)^2} \tau_H^r + \frac{x_H^*}{(x_L)^2} \tau_H^D + 2a, \]

\[ \frac{\partial g}{\partial \tau_H^D} = -\left[ \frac{x_H^r}{x_L} - \left( \frac{\tau_H^r}{1 - x_L} + \frac{\tau_H^D}{x_L} \right) \frac{1}{2a} \right]. \]

Thus, applying the implicit function theorem to \( g(x_L, \tau_H^D) \), we obtain (14).
Appendix B

In symmetric equilibrium (i.e., \( \tau^D_H = \tau^D_L = \tau_H, \ x_H = x_L = \frac{1}{2} \)), from (15) and (16), we find \( e^D_\tau = -\frac{\partial \phi^D_H}{\partial \mu_{HH}} = -\mu_{HH}, e^D_\tau = -\frac{\partial \phi^D_H}{\partial \mu_{HL}} = -\mu_{HL} \), and \( e^D_\tau = -\frac{2\mu_{HL}}{a+2\tau_H} = -\frac{2\mu_{HL}}{a+2\tau_H} \). Thus by substituting \( e^D_\tau = -\frac{2\mu_{HL}}{a+2\tau_H} \) into (18), and using the symmetry of the model, we obtain (19).

By totally differentiating (19), we obtain:

\[
\frac{\partial \tau}{\partial e} = \frac{-\frac{2\mu_{HL}}{(1+\frac{a}{2\tau_H})^2} + \frac{\mu_{HL}}{(1+\frac{a}{2\tau_H})^2}}{\frac{1}{(1+\frac{a}{2\tau_H})^2}} < 0,
\]

\[
\frac{\partial \tau}{\partial a} = \frac{-\frac{2\mu_{HL}}{(1+\frac{a}{2\tau_H})^2} + \frac{\mu_{HL}}{(1+\frac{a}{2\tau_H})^2}}{\frac{1}{(1+\frac{a}{2\tau_H})^2}} > 0.
\]

Moreover, when \( a \to \infty \), (19) is rewritten as \( \frac{\mu_{HL}}{(1+\frac{a}{2\tau_H})^2} = 1 \). Thus we can confirm that the relation \( \lim_{a \to \infty} \tau_H(e,a) = \frac{\mu_{HL} - y(e)}{2} \) holds. On the other hand, when \( a \to 0 \), the strict inequality holds from (19). Thus we can confirm that the relation \( \lim_{a \to 0} \tau_H(e,a) = 0 \) holds.

Appendix C

Suppose that the government’s evaluation function \( v() \) is specified as a logarithmic function (i.e., \( v(y_i - \tau^H_i) = \ln(y_i - \tau^H_i) \)). Then, (19) with equality is rewritten as

\[
\tau_H^2 + (y_L(e) + a)\tau_H - \frac{a}{2}(y_H - y_L(e)) = 0.
\]

In our model, the inequality \( \tau_H \leq 0 \) must hold for the existence of redistributive policy. Thus we obtain (20).
Appendix D

By totally differentiating (27) with respect to $a$, noting the first order condition of (22), we obtain:

\[
\frac{dW}{da} = -\left[ \int_0^{1/2} u(y_H - \tau^*_H - a\theta)d\theta + \int_{1/2}^{1} u(y_H - \tau^*_H - a(1 - \theta))d\theta \right] \frac{\partial \tau_H}{\partial a} + \frac{\partial \tau_H}{\partial a} + \frac{\partial \tau_H}{\partial e^*} + \frac{\partial \tau_H}{\partial e^*} \\
+ \Gamma,
\]

where

\[
\Gamma \equiv -\int_0^{1/2} u'(y_H - \tau^*_H - a\theta)d\theta - \int_{1/2}^{1} u'(y_H - \tau^*_H - a(1 - \theta))(1 - \theta)d\theta \\
- \int_0^{1/2} u'(y_L(e^*) + \tau^*_H - \phi(e^*) - a\theta)d\theta - \int_{1/2}^{1} u'(y_L(e^*) + \tau^*_H - \phi(e^*))(1 - \theta)d\theta,
\]

Here the third term $\Gamma$ expresses the direct welfare effect of the decline in $a$. Thus, by ignoring $\Gamma$ and properly rearranging the remaining terms, we obtain (28).

References


Figure 1: Relationship between $a$ and $V$
Figure 2: Relationship between $a$ and $W$