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Analytical Stochastic Approach to Budgeting

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Abstract

The work aims to develop the potential contained in the equations of Carlo Masini regarding the complementary themes of planning, programming, perspectives, and business control, both executive and financial, from a strategic point of view.

With the use of a suitable set of symbols, it is intended to further develop the investigation of ex-ante determinations and in particular budgetary determinations through business simulation algorithms and advanced computational engines. The emphasis is on estimated quantities, conjectured quantities, determined systematically together with "economic quantities".

This system of values, integrated with non-monetary quantities and "qualities" feeds into the general information system of the company, which it is intended to analyze with particular reference to budgetary determinations of periodic income, balance sheet capital, and economic capital. We devote a peculiar interest periodic income.

A central point of the research is the clarification of the periodic synthesis of estimated quantities and conjectured quantities through random variables already incorporated into the equations of Italian Scholars, such as Carlo Masini, now in an improved stochastic symbology. This improved formulation makes operational application possible, using advanced business software, to resolve concrete business problems.

The research should be consider as a development of a previous research title "Synthesis, quantitative ex-post and ex-ante determinations and stochastic variables in Economia Aziendale."

1. Risk and uncertainty: probabilistic variables in Periodic Income business decision making

Income is always a factor that orients and optimizes corporate decisions, even though those decisions are not always traced back to it.

This is the case with the special choices relating to product costs, net income or other indicators, which are however all pertinent to the overall logic of income.

To summarize, income can be considered an entity on which all manufacturing companies base the "optimization" of their decisions, regardless of the condition of their legal entity, the operators in charge of their decision-making centres and the market

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structures in which they operate.

"Continuity of income" and "increase in company dimensions" also represent pseudo-objectives that can all be traced back to income, since in the end they are broad and convenient indicators of income in the long term.

The supposition of the certainty of all possible alternatives in terms of negative and positive income components for each of the company's outputs is a useful simplified model for explaining a reality pervaded by incomplete information, great uncertainty, the different dispositions of individual operators and at different moments to operate in conditions of uncertainty.

Nevertheless, recent achievements on Data Science, and IT innovations allow the development of a model with a "preference function" for handling risk, which contrasts disposition towards risk and opportunity of financial results with multiple probabilistic distributions. In reality, the postulate of maximum income includes the risks of negative results and satisfactorily explains the most characteristic implications of economic actions.

In conditions of probabilistic uncertainty, each prescribed alternative is identified with a "distribution" of potential results instead of a single result, thus with a wide range of these "distributions" relating to the various "courses of action". A distribution of possible results is not susceptible to maximization, but the company can choose the alternative, among the range of those possible, with "optimal distribution of results" according to its table of preferences. It does not follow from the above that where uncertainty exists the maximization of income cannot represent the basis for action, but that the postulate in question should be understood as the choice of the alternative with "optimal distribution" of potential results in the context of the process of forecasting and of forming scenarios and the company's "table of preferences". The "maximum", thus reinterpreted as "optimal", which in the non-monetary sense is still a maximum, does not invalidate the consequent generalizations on the topic of business decisions.

It is worth remembering the postulate of continuous reference of the goals of investigation and of the objects considered to the processes of creation of income and business capital, even where the company's determinations involve statistical techniques; statistical methodology should certainly be traced back to the postulate in question, as only in this light can it have precise and detailed significance. In academic developments the tendency is to account for uncertainty in a "quantitative table of probabilities" of the occurrence of the hypothesized fact or quality. So "probability" can represent one of the defining factors of forecasting. The use of forecasting and the uncertainty inherent in it confer on it the character of hypothesis in the context of the reasoning of which it forms part. Probability theory and decision theory use the "results matrix" as a productive tool for distinguishing between certainty, risk and uncertainty and deal first and foremost with rational behaviour in a context where the operator's knowledge of the future is limited. To attain the goal or goals towards which one is moving, a strategy must be chosen from the range of alternatives that will cause the desired result, a choice which is made according to different rules in function of the type and attributes of the independent variables. It should be stressed that it is from "total expected income", which is to some extent predictable, that the opportunity of a choice derives, incorporating into this concept both the immediate and direct result of a decision and its long-term implications. The decision is not isolated, but part of a system of interconnected decisions, each of which will modify those subsequent to it. What we wish to highlight is the difficulty of evaluating these direct and indirect implications, which, at least conceptually, extend indefinitely into the distant future.

2. Market uncertainty and probabilistic Periodic Income determination

It is impossible to explain business decisions and income by means of arguments involving "probability as frequencies". Decisions based on the hypothesis that incomes will probably be positive rather than negative necessarily depend on actions not sustained by observations relating to frequency relations that will tend towards a determined "true" value. In other words, the experience of business operators involves significant classes of data that cannot be measured in a way that will conclude with a "numerically correct average value", as the quantity of observed data increases. Instead, it is as if each class of data were unrepeatable.

Logical adversities deriving from the overly simple hypothesis of maximization of income are also highlighted by other decision-making rules in situations of risk. For example, a choice could be made on the basis of the "variance" of "probabilistic distributions" instead of their "averages". When deciding between a certain strategy linked to a greater expected value, on average, but with a greater distribution (deviations from this average, that is the possibility of a wider range of financial results, that could vary between high income and significant losses) and another strategy that offers a slightly inferior average result but with a lower variance (meaning a best result with lower income than the first potential distribution, but also lower losses), the choice could fall on the latter. The uncertainty inherent in the estimates of business operators can probably not be reduced beyond a certain point through further investigations. Business decisions need to be taken even if the forecasts concerning future events are represented by a variously limited range of alternative hypotheses.

Additional information does not necessarily lead to less uncertainty in income forecasts and sometimes it does not influence the operators' appraisal of the "expected value" and the degree of uncertainty. However this does not mean that the information gathered is irrelevant even if it might contribute to reinforcing both favourable and unfavourable hypotheses to the same degree. One question relevant to the logic of making decisions in conditions of uncertainty is how to evaluate two hypotheses with identical probabilities, but supported by different quantities of information relating to a determined action.

There is no doubt that more information makes the total revision of forecasts, estimates and conjectures less likely in the light of information offered by the implementation that follows the decision. It is worth specifying that a probabilistic evaluation relating to the result of an economic process, for example a planned investment, is a complex evaluation expressing the sum of various simple evaluations. In other words, the composite event to which the general evaluation refers is the result of numerous events that to a first approximation can be observed separately. A business investment, linked to alternative potential scenarios of profit or loss, is associated with the formulation of various probabilities for numerous simple events that the decision-making body is not even able to list fully.

Nonetheless it is possible to some extent to separate out the composite event and thus look into the probabilities attributable to the various prices/costs of the production factors and to the various prices/incomes of the output, to the competence and efficiency of those involved, perhaps to various kinds of characteristics of the production equipment and so on. It goes without saying that fully comprehensive deconstruction is impossible and every time it is taken to a further level of detail the need to revise the previous, less detailed appraisal will become apparent, to ensure consistency within the entire appraisal system. There is plenty of space for applications of probabilistic/subjective and Bayesian principles in business practices. Very often business decisions are taken in conditions of uncertainty and it is hard to fully embrace the principle of maximization of income and of the best value income without recourse to the logic of "subjective probability" and "utility". The problem arises of identifying methods for explaining the process of formulating business decisions in the real world. The fact that the theory of "subjective probability" and "utility" is represented in operational terms does not mean it is possible to experimentally verify the probabilistic evaluations and utility functions of senior management. Experiments with interesting results can be carried out on the utility functions, probabilistic evaluations and strategic attitudes of the subjects. But experimental conditions inevitably have little in common with those of meaningful concrete business decisions.

The concept of probability is at the heart of the decision-making process in conditions of uncertainty, a process that in turn emerges as one of the preeminent questions in economics and in EMAR, it must be seen as the central theme of every theory of income. Among the numerous advantages that can ensue from such an emphasis on the major economic themes is a much more critical attitude towards the so-called "objectivist" methods for reaching conclusions in the social and non-social sciences. The rules for taking decisions in conditions of uncertainty strive to offer business operators a ready-made basis for attributing "subjective probability" values to potential "states of nature".

In all likelihood subjectively attributed probabilities will not coincide with those that may be determined objectively, based on the predicted repetition of consistent values found in the past and moreover the former vary from operator to operator. The reasons for underlining such epistemological problems lie in the fact that the arguments of Economia Aziendale should be considered in the light of that constantly evolving interdisciplinary set of knowledge in the aim of formulating meaningful decision-making models. Probability has an important place in economics, more important than in any other field, as it is a realm where uncertainty reigns and the very occurrence of events is a function of the behaviour of operators, determined in turn by variously vague evaluations and probabilistic arguments. It is therefore "probability theory" in the broadest, fullest sense that should be applied.

3. Linear Equations, value systems and probability

The summary of the financial year and of the hth period of the complex system of business events, as far as it pertains to periodic income and balance sheet capital, in the formulation of Prof. Carlo Masini, "Lavoro e Risparmio. Economia d'azienda", UTET, Turin, 1970 (see pp. 104, idem, and "Lavoro e Risparmio. Corso di Economia d'azienda", vol. 2, Publisher Fusi, Pavia, 1968) is as follows:

3.1 Determination of Periodic Income

$$\rho = -\Pi_{t_{h-1}} - {}_eC_h - {}_eP_h^{(L)} - {}_eT_h - {}_eI_h + {}_eR_h - q_{t_h} + \Pi_{t_h} \tag{1}$$

3.2 Distribution of the Periodic Income

$$\rho = g_{t_h} - {}_eP_h^{(L')} - {}_{t_0}P_h^{(S)} \tag{2}$$

3.3 Periodic Income

$$\begin{aligned} &-\Pi_{t_{h-1}} - {}_eC_h - {}_eP_h^{(L)} - {}_eT_h - {}_eI_h + {}_eR_h - q_{t_h} + \Pi_{t_h} + \\ &- g_{t_h} - {}_eP_h^{(L')} - {}_{t_0}P_h^{(S)} \end{aligned} \tag{3}$$

3.4 Periodic Income and Capital

$$\begin{aligned} {}_{t_0}P^{(S)}_{t_h} &+ (-\Pi_{t_{h-1}} - {}_eC_h - {}_eP^{(L)}_h - {}_eT_h - {}_eI_h + {}_eR_h - q_{t_h} + \Pi_{t_h} + \\ &- g_{t_h} - {}_eP^{(L')}_h - {}_{t_0}P^{(S)}_h) = \\ &= (N'_{t_{h-1}} + N_h - N'_{t_h}) \end{aligned} \tag{4}$$

The above linear equations of the periodic income use the following symbolic structure, which should drive also future developments. The itemization describe an introductory probabilistic approach making use of matrix formats:

$\mathscr{I}$ and ι), Periodic Income. Hence, ${}_e\iota_h$ can be used to replace ${}_e\rho_h$.

a_i Production alternatives, more generally, choice alternatives (of either internal Stakeholders or external ones). This notation is used for example in Management Decision Making based on Matrix Algebra Analysis (Linear Algebra), *e.g.* Company Budgeting.

In the symbol a_i, i states the i^{th} alternative, where $i = 1, ..., m$. The alternatives set is defined as $a_i = (a_1, ..., a_m)$.

sn_j States of Nature, where $j = 1, ..., n$. Again, this notation is used in Management Decision Making based on Linear Algebra metrics, *e.g.* Enterprises Risk Analysis.

In the symbol sn_j, j stay for the j^{th} state of nature, where $j = 1, ..., n$. Hence, all states of nature are defined as $sn_j = (sn_1, ..., sn_n)$.

h Administrative period, symbol embedded in ME. The set of period is defined as $h = (1, ..., \tau)$, where the Greek letter τ states the last Administrative Period under scrutiny in the specific Entity metrics.

t_h A specific Date, in ME either the last day or any day of the Administrative period under analysis, *e.g.* December 31, 20XX. Hence, following the previous notation t_τ represents the last day which is studied.

Eventually, it is quite obvious but just in case, it must be stated that $t_h \neq$ th. The last is used in English language (often as a superscript text), to indicate the n-th object under analysis, *i.e.* the *n-esimo* object under analysis.

E.g., when we write h^{th}, we signify the *h-esimo* Administrative Period, y^{th} is the n-th economic good of the Company, and so on and so forth.

v Volume of a economic good sold or bought and consumed for company's operations. The v symbol is also used by Professor Masini in his metrics. Therefore, we will consider the format $v(x_\alpha)$ or, alternatively, v_{x_α} to indicate the volume of an acquired economic good, consumed to produce one unit of company output, when the company does produce only one economic good, either product or service. If this is not the case, in other words when the company produce more than one economic good, then in order to indicate the volume of an acquired economic good, consumed to produce one unit of each product or service to be sold the following notation has to be adopted, $v(x_{\alpha,\beta})$. Similarly, we will use the format $v(y_\beta)$ or, alternatively, v_{y_β}, to define the quantity of the Enterprise output. Beside, we will use $c_{v(x_\alpha)}$ or the simplify format $c(x_\alpha)$ to indicate the cost of acquisition of a specific product or service, where $c(x_\alpha) = p(x_\alpha)\cdot v(x_\alpha)$.

x_α Company Resource; an acquired either good or service. The x_α Company Resources, where $\alpha = 1 \ldots o$ are transformed in outputs y_β. Hence, the number assumed by α indicates the x^{th} Company Resource consumed in the production process.

y_β Economic good produced by an enterprise, either product of service. The subscript $\beta = 1 \ldots z$. Hence, y_z state among the y^{th} company produced goods the last one under a numerical classification.

π_j Probability related, *e.g.* to a specific state of nature sn_j, where $j = 1 \ldots n$.

Π Inventory (in ME). Hence, Π_{t_h} states the total initial inventory of the h^{th} administrative period, and $\Pi_{t_{h+1}}$ indicates the total ending inventory of the h^{th} administrative period.

μ_β Contribution Margin for each economic good y_β sold, where $\beta = 1 \ldots z$. In the case of budgeted contribution margin, the underlying hypothesis is a first best setting. In fact, in a risky market situation for each β^{th} economic good a vector of contribution margin, μ_β, has to be considered.

c_{x_α} In cost accounting the Acquisition cost of a unit of the x_α^{th} enterprise resource.

${}_eC_h$ Expense of the administrative period, that is "*variazione negativa di esercizio, costo imputato all' esercizio*" in ME. Note, it does not refer to the Expenditures, which are a capitalized cost to be included in the notation Q_{t_h} (please, see below).

e Economic value, normally used as a subscript in ME.

g_{t_h} Reserve, both legal and extraordinary. In ME is one of the three mathematical addenda that define the distribution of periodic net income. The other are the ${}_eP_h$

p_{x_α} p_{y_β} price of acquisition of the x_α^{th} productive resource, and respectively, selling price of the y_β^{th} enterprise economic good exchanged with third entities.

${}_{t_0}P_{t_h}^{(S)}$ In ME, Enterprise Capital at the time t_h; where t_0 states the moment in time of establishment of the Enterprise.

T Total taxation; T is a negative Economic Value, an addend of the ME of Periodic Income, ${}_e\ell_h$.

z Subscript indicating the last of the economic goods produced by the Enterprise.

π_j Probability related, *e.g.* to a specific state of nature sn_j, where $j = 1 \ldots n$.

Π Inventory (in ME). Hence, making use of subscripts: $\Pi_{t_{h-1}}$ states the total initial inventory of the h^{th} administrative period, and Π_{t_h} indicates the total ending inventory of the h^{th} administrative period. The Scholar, for a sake of simplicity, does not differentiate productive conditions inventory, semi-finished products inventories and finished economic goods inventories.

μ_β Contribution Margin for each economic good y_β sold, where $\beta = 1...z$. In the case of budgeted contribution margin, the underlying hypothesis is a first best setting. In fact, in a risky market situation for each β^{th} economic good a vector of contribution margin, μ_β, has to be considered.

$\boldsymbol{\mu}_\beta$ Contribution Margins vector.

$\boldsymbol{M}_{t_h}^{(v)'}$ In ME Net Value of long term financial liabilities and assets.

$\mathbf{M}_\beta$ Contribution Margins (per output) Matrix. Please, note that the total contribution margin is indicated traditionally with CM_i where i indicates the alternative of production, where $i = 1...\ m$. The Contribution Margin Matrix of the Administrative Period h, as well as the *e.g.* the Budget Contribution Margin Matrix of the Administrative Period $h + 1$, is a set of contribution margins per unit of enterprise outputs. Under ideal condition the Contribution Margins Set is a $z \times 1$ matrix, that is a column vector, hence:

$$\mathbf{M}_\beta = \boldsymbol{\mu}_\beta = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_z \end{pmatrix}$$

$\mathbf{CM}_i$ Total (for all enterprise outputs) contribution margin vector (for each state of nature). Instead, as stated before, CM_i is the total (for all enterprise outputs) contribution margin (for each alternative and each state of nature). Therefore, under ideal condition, Contribution Margins Values for all the set of different production alternatives are a vector, in other words, a $z \times 1$ matrix. Given,

$$\mathbf{CM}_\beta = \begin{matrix} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{matrix} \begin{matrix} \begin{matrix} y_1 & y_2 & \cdots & y_z \end{matrix} \\ \begin{pmatrix} y_{11} & y_{21} & \cdots & y_{z1} \\ y_{12} & y_{22} & \cdots & y_{z2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{1m} & y_{2m} & \cdots & y_{zm} \end{pmatrix} \end{matrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_z \end{pmatrix} \tag{5}$$

Hence, the $z \times 1$ matrix of Contribution Margins Values for all the set of different production alternatives is stated below:

$$\mathbf{CM}_{\beta} = \begin{pmatrix} y_{11}\mu_1 + y_{21}\mu_2 + \cdots + y_{z1}\mu_z \\ y_{12}\mu_1 + y_{22}\mu_2 + \cdots + y_{z2}\mu_z \\ \vdots \\ y_{1m}\mu_1 + y_{2m}\mu_2 + \cdots + y_{zm}\mu_z \end{pmatrix} = \begin{pmatrix} \mathrm{CM}_1 \\ \mathrm{CM}_2 \\ \vdots \\ \mathrm{CM}_m \end{pmatrix} \tag{6}$$

The use previous mathematical notations is presented in the following two examples: the first metric presents a $m \times n$ matrix of the forecast periodic income in $h + 1$, *i.e.* matrix of production alternatives, state of natures and payoffs (periodic incomes); the second metric presents the contribution margin optimization equations. Obviously, the optimization could analysed the periodic income or any other economic and "statistic" quantity.

In a probabilistic setting, which is used to define the second best solution of a, *e.g.* management risky decision, m^{th} alternatives a_i and n^{th} states of nature, sn_j will define all the possible values of the periodic income of the next administrative period.

$$\begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccc} sn_1 & sn_2 & \cdots & sn_n \end{array} \\ \begin{pmatrix} \iota_{11} & \iota_{12} & \cdots & \iota_{1n} \\ \iota_{21} & \iota_{22} & \cdots & \iota_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \iota_{m1} & \iota_{m2} & \cdots & \iota_{mn} \end{pmatrix} \end{array} \tag{7}$$

If the states of nature, *e.g.* the prices levels, j are five, that is $j = 1...5$ and the CFO consider only six different alternatives of production, $i = 1...6$, the payoffs (budgeted levels of income at $h + 1$ thirty:

$${}_e\mathscr{I}_{h+1} = \begin{pmatrix} \iota_{1,1} & \iota_{1,2} & \iota_{1,3} & \iota_{1,4} \\ \iota_{2,1} & \iota_{2,2} & \iota_{2,3} & \iota_{2,4} \\ \iota_{3,1} & \iota_{3,2} & \iota_{3,3} & \iota_{3,4} \\ \iota_{4,1} & \iota_{4,2} & \iota_{4,3} & \iota_{4,4} \\ \iota_{5,1} & \iota_{5,2} & \iota_{5,3} & \iota_{5,4} \end{pmatrix} \tag{8}$$

Let's consider a Linear Programming metric of budgeting, in other words the objective of the determination are the flexible (on a single state of nature) budgets. It is like to consider only the j^{th} column of the matrix (7) above. We are missing here the probability approach but we do introduce in the analysis constraints and random generated alternatives, m is extremely large. First let's consider the general metrics:

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & x_j \geq 0 \quad \forall j \in N \end{aligned} \tag{9}$$

Which can be formulated, considering the above mention business decision setting, as:

$$\begin{aligned} \max_{v(y_\beta)} \quad & \mu_\beta^T v(y_\beta) \\ \text{s.t.} \quad & Av(y_\beta) \leq b_\alpha \\ & v(y_\beta) \geq 0 \quad \forall \beta \in N \end{aligned} \tag{10}$$

The constraints $Av(y_\beta) \leq b_\alpha$ are related to the economic goods consumed in the, *e.g.* manufacturing process. The non-negativity constrains $v(y_\beta) \geq 0$ is self explaining. For a sake of simplicity no other constraints are considered. In metric numbered (10):

i $\boldsymbol{\mu}_\beta^T$ is the contribution margins transpose vector of the period $h + 1$, *i.e.* the matrix $(1 \times z)$ of the contribution margin:

$$\mu_{h+1}^T = M_{h+1}(\mu_1 \quad \mu_2 \cdots\cdots \mu_z)$$

ii $\boldsymbol{v}(y_\beta)$ is the vector of the volume produced and sold of company economic goods in the same period of time, $h + 1$:

$$\boldsymbol{v}(y_\beta) = \begin{pmatrix} v(y_1) \\ v(y_2) \\ \vdots \\ v(y_z) \end{pmatrix}$$

iii $\boldsymbol{\mu}_\beta^T \boldsymbol{v}(y_\beta)$ is the equation that total the Contribution Margin value, CM_{h+1}, of the Administrative Period $h + 1$, evaluated by multiplying the transpose vector of the contribution margins of all economic goods y_β by the vector of the volume (produced and sold) of the same goods at time $h + 1$:

$$\boldsymbol{\mu}_\beta^T \boldsymbol{v}(y_\beta) = (\mu_1 \quad \mu_2 \cdots\cdots \mu_z) \begin{pmatrix} v(y_1) \\ v(y_2) \\ \vdots \\ v(y_z) \end{pmatrix}$$

Hence, being $\boldsymbol{\mu}_\beta^T \boldsymbol{v}(y_\beta) = (\mu_1 v(y_1) + \mu_2 v(y_2) + \cdots + \mu_z v(y_z))$:

$$\boldsymbol{\mu}_\beta^T \boldsymbol{v}(y_\beta) = \sum_{\beta=1}^{z} \mu_z v(y_z)$$

Note that quantities of vector $\boldsymbol{v}(y_\beta)$ in the optimization calculations change m^{th} times, in fact by definition m is the number of alternative of production under evaluation. Simulink® integrated with MATLAB®, as well as other software and computational engines like MS Solver, randomly generate m^{th} production alternative (not to mention that this powerful tools are able to evaluate an outstanding number of alternatives); for all the m^{th} randomly generated productions alternatives we will get a set of $\boldsymbol{v}(y_\beta)$-vectors, than:

$$\begin{pmatrix} v(y_1) \\ v(y_2) \\ \vdots \\ v(y_z) \end{pmatrix}_1, \begin{pmatrix} v(y_1) \\ v(y_2) \\ \vdots \\ v(y_z) \end{pmatrix}_2, \dots, \begin{pmatrix} v(y_1) \\ v(y_2) \\ \vdots \\ v(y_z) \end{pmatrix}_i, \dots, \begin{pmatrix} v(y_1) \\ v(y_2) \\ \vdots \\ v(y_z) \end{pmatrix}_m$$

iv $\boldsymbol{A} = \boldsymbol{A}_{o,z}$ is representative of the matrix of the constraints parameters. For a sake of simplicity here we consider only company constraints (*i.e.* constraints related to company inputs). Each single entry of the $(o \times z)$ matrix, state the quantity of input consumed for a unit of output; *e.g.* $v(x_{1,1})$ is the quantity of and acquired economic good (*e.g.*, kilograms of raw material, hours of labour, and so on and so forth) consumed for the production of a single unit of a specific y^{th} output of the Enterprise. Hence, the $\boldsymbol{A}_{o,z}$ is define in the following notation (see equation 11):

$$\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_o \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & \cdots \; y_z \end{array} \\ \begin{pmatrix} v(x_{11}) & v(x_{12}) \cdots v(x_{1z}) \\ v(x_{21}) & v(x_{22}) \cdots v(x_{2z}) \\ \vdots & \vdots \;\; \ddots \;\; \vdots \\ v(x_{o1}) & v(x_{o2}) \cdots v(x_{oz}) \end{pmatrix} \end{array} \tag{11}$$

and $\boldsymbol{A}_{o,z} \cdot \boldsymbol{v}(y_\beta)$ is given by the following product of matrices, *i.e.* the column vector of the cumulative consumption of each single productive condition, x_α, referred to all outputs y_β^{th}:

$$\begin{pmatrix} v(x_{11}) & v(x_{12}) & \cdots & v(x_{1z}) \\ v(x_{21}) & v(x_{22}) & \cdots & v(x_{2z}) \\ \vdots & \vdots & \ddots & \vdots \\ v(x_{o1}) & v(x_{o2}) & \cdots & v(x_{oz}) \end{pmatrix} \begin{pmatrix} v(y_1) \\ v(y_2) \\ \vdots \\ v(y_z) \end{pmatrix} = \begin{pmatrix} \sum_{\beta=1}^{z} v(x_{1\beta}) v(y_\beta) \\ \sum_{\beta=1}^{z} v(x_{2\beta}) v(y_\beta) \\ \vdots \\ \sum_{\beta=1}^{z} v(x_{o\beta}) v(y_\beta) \end{pmatrix} \tag{12}$$

v Eventually, the inequality $\boldsymbol{v}(y_\beta) \geq 0$ implies that all the company outputs y_β^{th} have to be either zero or positive; the volume of each economic good has not to be negative: that is not obvious for the computational engines we do use.

4. Conclusions

Economia Aziendale disciplines are advancing with the systematic observation of the real world in conjunction with the sophisticated application of logical processes. This way it is possible to propose analytical tools for the planning and control of all kinds of businesses. Complex and ultra-complex business systems call for advanced, progressive information systems, and therefore sophisticated Economia Aziendale algorithms, based on ancient and modern logic, on systems methodology, computer science, cybernetics and so on. It seemed that the Economia Aziendale structuring of Prof. Lino Azzine and Prof. Carlo Masini, represented by their excellent summary equations, could make a contribution in this direction. The present paper, after the necessary references to and explorations of the concepts of "uncertainty", "risk" and "probabilistic variables" applied to business decision-making systems, deals with the problems of determination of economic capital, underlining in this context the income criterion (the relationship between collectable future incomes and expected rates), as a central reference point of Economia Aziendale and general economics. Other essential and complementary reference points resulting from analytical evaluation processes, regarding balance sheet capital, will be the subject of subsequent research. With reference to the short term, and therefore not to a final degree of approximation, the overview programme for the h + 1th financial year is presented in a stochastic formalization, also using matrices. In subsequent research it will be integrated into long-term plans and scenarios. The actualization of the optimal value from the many possible expected values of expected incomes, relating to the value matrix, can be extended to all summary values. The formulation obtained this way makes it possible to use advanced business intelligence software for planning and control in businesses. The use of "business simulation" is increasingly widespread. Simulation works by constructing assumed models of complex or ultra-complex systems, in such a way as to be able to calculate the consequences of the movement of given conditions.